

# Economics 742 Bonus Lecture: Aggregation III: Incomplete Information

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# Aggregation and Incomplete Information

- Aggregation of “micro” estimates is a hard problem.
  - Answers likely to come from unexpected places and be creative.
- Today discuss one recent unexpected answer by Angeletos and Lian (2017) that uses *incomplete information*.
  - Will allow me to introduce an exciting literature in macro.
  - But will only scratch the surface of this literature and will focus on aggregation of micro estimates in macro.
- For a complete survey of literature, see recent *Handbook of Macro* chapter by Angeletos and Lian (2017).
  - I will follow a simple treatment by Angeletos and La'O (2010) for first half before turning to aggregation.
  - Jennifer La'O has great notes on her website as well.

# Aggregation and Incomplete Information: Outline

1. Overview of Incomplete Information in Macro
2. Noisy Business Cycles: Angeletos and La'O (2010)<sup>1</sup>
3. Delaying GE: Angeletos and Lian (2017)
4. Expectations in the Data: Coibion and Gorodnichenko (2012)

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<sup>1</sup>These notes borrow from Jennifer La'O's excellent notes.

# Incomplete Information in Macro

- Most macro models assume *common knowledge*.
  - As if all agents gather, reach unanimous consensus on state of economy and in future, and coordinate on commonly-known profile of actions.
  - Example: In RBC, NK, Aiyagari, agg state and model are common knowledge.
- Early literature on incomplete information from Lucas (1972) relaxes this, but limited in tools.
- Recently revived using *games of incomplete information*.
  - Agents have dispersed private info about agg shocks.
    - Leads to lack of consensus among agents about one another's actions, and hence outcomes.
  - Models become *games of incomplete information*.
    - Imperfect Info: State or payoffs not known.
    - Incomplete Info: Distribution of info about state across population not known.

# Two Strands of Literature

## 1. Global Games

- Coordination games with “strong strategic complementarity” gives multiple equilibria with common knowledge.
- Introduction of incomplete info leads to unique equilibrium, even if vanishingly small. But still fragile.
- Applications: Bank runs, currency attacks, etc.
- Not my focus.

## 2. Beauty Contests (a la Keynes)

- “Weak strategic complementarity” gives unique equilibrium with incentives to coordinate.
- Can deliver significant rigidity or inertia at macro level even if not present at micro level.
- Can get “animal spirits” – volatility in equilibrium outcomes without volatility in fundamentals.
  - “Demand driven fluctuations” from correlation in beliefs.
- This will be our focus today.

# Angeletos and La'O: Noisy Business Cycles

- Idea: Relax common knowledge in textbook RBC.
  - Stage 1: Firms and workers meet on “islands,” produce facing uncertainty about other islands.
  - Stage 2: Consumption choices in centralized market with full info and representative consumer.
- Results:
  1. Inertia in response of macro variables, even with small noise.
  2. Business cycles from “noise” shocks that are correlated errors in agents’ expectations of tech shock.
    - 2.1 Formalizes “demand” shocks in RBC setting since works through expectations of agg demand.
    - 2.2 Tech shocks less important, “demand” more *without money*.
    - 2.3 Countercyclical labor wedge, pro-cyclical Solow residual.
- Need both incomplete info and *strategic complementarity*.
  - Best response increasing in action of others.
- Result 1 will be key when we think about aggregation of “micro” effects as GE emerges in long run but not short run.

## Setup: RBC With Islands

- Notes:
  - Start static, extend to dynamic.
  - Relative to the paper, I drop continuum of goods on each island that allows for markups. Simplifies exposition.
- Continuum of islands  $i \in I = [0, 1]$  that produce one good. Household has continuum of  $i$  workers sent to each island.
- Each period has two stages:
  1. Island Stage
    - Households send a worker to each island where labor market equilibrium occurs.
    - Workers and firms have perfect info about local market, but imperfect info about other islands.
  2. Aggregate Stage
    - All info becomes public.
    - Quantities determined by stage 1 decisions, prices clear product market.

# Households

$$\max_{C_t, n_t} \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\gamma}}{1-\gamma} - \int_I \frac{n_{it}^{1+\varepsilon}}{1+\varepsilon} di \right]$$

- Dixit-Stiglitz across islands:

$$C_t = \left[ \int_I (c_{it})^{(\rho-1)/\rho} di \right]^{\rho/(\rho-1)}$$

with  $C_t$  the numeraire.

- Budget constraint:

$$\int p_{it} c_{it} di \leq \int \pi_{it} di + \int w_{it} n_{it} di$$

where  $\pi_{it}$  is profit,  $w_{it}$  is wage,  $p_{it}$  is price of  $i$ 's good.



# Firms and Fundamentals

- Representative firm on island  $i$  produces DRS with labor:

$$q_{it} = A_{it} n_{it}^{\theta}$$

- Firm maximizes profit:

$$\pi_{it} = p_{it} q_{it} - w_{it} n_{it}$$

discount at rep consumer's valuation of profit  $U'(C_t) \pi_{it}$ .

- Productivity is log-normally distributed across islands:

$$a_{it} \equiv \log A_{it} = \bar{a}_t + \xi_{it}$$

where  $\xi_{it} \sim N(0, 1/k_{\xi})$  and  $\bar{a}_t \sim N(\mu, 1/\kappa_0)$ .

## Information Structure

- Let  $x_{it}$  be a sufficient statistic of all private information of aggregate productivity:

$$x_{it} = \bar{a}_t + u_{it}, \quad u_{it} \sim N(0, 1/\kappa_x)$$

- This combines observed local productivity and private signal (see paper).
- Let  $y_t$  be a sufficient statistic of all public information of aggregate productivity

$$y_t = \bar{a}_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, 1/\kappa_y)$$

- By Kalman filter,

$$E_i[\bar{a}_t] = \frac{\kappa_0}{\kappa_0 + \kappa_x + \kappa_y} \mu + \frac{\kappa_x}{\kappa_0 + \kappa_x + \kappa_y} x_t + \frac{\kappa_y}{\kappa_0 + \kappa_x + \kappa_y} y_t$$

## Equilibrium Concept: Game of Incomplete Info

- Let  $\omega_{it}$  be the “type” of the island  $\omega_{it} = (a_{it}, x_{it}, y_t)$ .
  - The aggregate state of the economy is the entire cross-sectional distribution of  $\omega_{it}$ , denoted  $\Omega_t$ .
  - We can think of nature as drawing  $\Omega_t$  and then drawing a distribution of  $\omega_{it}$ s from  $\Omega_t$  according to  $F(\omega|\Omega)$ .
- Equilibrium will be a combination of Walrasian equilibrium in stage 2 and Bayesian Nash (or equivalently Perfect Bayesian) equilibrium in period 2.
  - Recasting equilibrium as a game of imperfect information.
  - Strategy: Solve for best response function and aggregate.
- Since the problem is static, drop  $t$  for now.

# Equilibrium Definition

## Definition

An equilibrium is  $\{n(\omega), q(\omega), w(\omega), Q(\Omega), N(\Omega), p(\omega, \Omega), c(p, Q)\}$  s.t.:

1. The aggregate price is the numeraire:

$$P(\Omega) \equiv \left[ \int p(\omega, \Omega)^{1-\rho} dF(\omega|\Omega) \right]^{\frac{1}{1-\rho}} = 1 \quad \forall \Omega_t.$$

2.  $c(p, Q)$  is rep consumer's optimal demand for a commodity whose price is  $p$  when agg output (income) is  $Q$ .
3.  $n(\omega)$  and  $q(\omega)$  are optimal for rep firm on island  $\omega$ , where  $q(\omega) = A(\omega) n(\omega)^\theta$ .
4.  $w(\omega)$  is such that  $n(\omega)$  is the optimal labor supply on  $\omega$ .
5. Price that clears market on  $\omega$  is  $p(\omega, \Omega)$  and the aggregate output and employment are:

$$Q(\Omega) = \left[ \int q(\omega)^{\frac{\rho-1}{\rho}} dF(\omega, \Omega) \right]^{\frac{\rho}{\rho-1}} \quad \text{and} \quad N(\Omega) = \int n(\omega) dF(\omega|\Omega)$$

## Equilibrium Characterization: Work Backwards

- In stage 2, optimal demand gives price:

$$p(\omega, \Omega) = \left( \frac{q(\omega)}{Q(\Omega)} \right)^{-\frac{1}{\rho}}$$

- In stage one, worker FOC is:

$$w(\omega) = \frac{n(\omega)^\varepsilon}{E[U'(Q(\Omega)) | \omega]}$$

- Firms solve:

$$\max_q E \left[ U'(Q(\Omega)) (p(\omega, \Omega)) q(\omega) - w(\omega) \left( \frac{q(\omega)}{A(\omega)} \right)^{1/\theta} \mid \omega \right]$$

with FOC:

$$E \left[ U'(Q(\Omega)) \left( p(\omega, \Omega) - \frac{1}{\theta} w(\omega) \frac{n(\omega)}{q(\omega)} \right) \mid \omega \right] = 0$$

## Equilibrium Characterization (Continued)

- Substitute in worker FOC, price, and production function:

$$E \left[ U' (Q (\Omega)) \left( \frac{q (\omega)}{Q (\Omega)} \right)^{-\frac{1}{\rho}} - \frac{1}{\theta} \left( \frac{q (\omega)}{A (\omega)} \right)^{\frac{\varepsilon+1}{\theta}} \frac{1}{q (\omega)} | \omega \right] = 0$$

- Solve for  $q (\omega)$  so strategy  $q : \Omega \rightarrow R$  is fixed point to:

$$q (\omega)^{\frac{\varepsilon+1}{\theta} + \frac{1}{\rho} - 1} = \theta A (\omega)^{\frac{\varepsilon+1}{\theta}} E \left[ Q (\Omega)^{\frac{1}{\rho} - \gamma} | \omega \right] \quad \forall \omega$$

with

$$Q (\Omega) = \left[ \int q (\omega)^{\frac{\rho-1}{\rho}} d\Omega (\omega) \right]^{\frac{\rho}{\rho-1}} \quad \forall \Omega$$

- Interpretation: Can rewrite as marginal cost of labor equals marginal utility of commodity from island times MPL:

$$n (\omega)^{\varepsilon} = E \left[ U' (Q (\Omega)) \left( \frac{q (\omega)}{Q (\Omega)} \right)^{-\frac{1}{\rho}} \theta \frac{q (\omega)}{n (\omega)} | \omega \right]$$

## Deriving the Best Response Function

- Given info structure, conditional on  $\omega$  posterior of  $Q(\omega)$  is lognormal with variance independent of  $\omega$ .
- Take logs of fixed point equation and use log normality to get:

$$\left(\frac{\varepsilon+1}{\theta} + \frac{1}{\rho} - 1\right) \log q(\omega) = \log \theta + \frac{\varepsilon+1}{\theta} \log A(\omega) + \left(\frac{1}{\rho} - \gamma\right) E[\log Q(\Omega) | \omega] \\ + \frac{1}{2} \left(\frac{1}{\rho} - \gamma\right)^2 \text{Var}[\log Q(\Omega) | \omega]$$

- Proposition: Equilibrium strategy is unique fixed point to:

$$\log q(\omega) = \zeta + (1 - \alpha) \chi a(\omega) + \alpha E[\log Q(\Omega) | \omega] \quad \forall \omega$$

$$\text{where } \chi \equiv \frac{\frac{\varepsilon+1}{\theta}}{\frac{\varepsilon+1}{\theta} + \gamma - 1} > 0, \quad \alpha \equiv \frac{\frac{1}{\rho} - \gamma}{\frac{\varepsilon+1}{\theta} + \frac{1}{\rho} - 1} < 1$$

$$\zeta = \frac{1}{\frac{\varepsilon+1}{\theta} + \frac{1}{\rho} - 1} \left\{ \log \theta + \frac{1}{2} \left(\frac{1}{\rho} - \gamma\right)^2 \text{Var}[\log Q(\Omega)] \right\}$$

# Best Response Function and Strategic Complementarity

$$\log q(\omega) = \zeta + (1 - \alpha) \chi a(\omega) + \alpha E[\log Q(\Omega) | \omega]$$

- Best response function gives interpretation of economy as Bayesian-Nash equilibrium of incomplete info game.
  - Players = islands, action = production, types = local info.
- $\chi > 0$  determines elasticity of local output to local prod.
- $\alpha$  is elasticity of local output to expected aggregate output.
  - This is a measure of strategic interaction.
  - $\alpha > 0 \Rightarrow$  strategic complements,  $\alpha < 0 \Rightarrow$  strategic substitutes.
- Strategic complementarity summarized by  $\alpha$ .
  - Comes from optimal quantity increasing in “aggregate demand,” strength increasing in  $1/\rho$ .
  - Muted by income effect on labor supply controlled by  $\gamma$ .
  - Also increasing in  $\theta$  (less DRS, produce more if agg output rises) and decreasing in  $\varepsilon$  (more disutility of labor, less incentive to produce more when agg output rises).



## Closed Form Log-Linear Solution

Guess and check that equilibrium output on each island is given by:

$$\log q(\omega) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y$$

where

$$\varphi_a = (1 - \alpha) \chi$$

$$\varphi_x = \left\{ \frac{(1 - \alpha) \kappa_x}{(1 - \alpha) \kappa_x + \kappa_y + \kappa_0} \right\} \alpha \chi$$

$$\varphi_y = \left\{ \frac{\kappa_y}{(1 - \alpha) \kappa_x + \kappa_y + \kappa_0} \right\} \alpha \chi$$

and aggregate output is given by:

$$\log Q(\Omega) = \varphi'_0 + \varphi_{\bar{a}} \bar{a} + \varphi_{\varepsilon} \varepsilon$$

where

$$\varphi_{\bar{a}} = \varphi_a + \varphi_x + \varphi_y \text{ and } \varphi_{\varepsilon} = \varphi_y$$

# Implication 1: Need Strategic Complementarity For Lack of Common Knowledge to Matter

- When  $\alpha = 0$ ,

$$\log q(\omega) = \varphi_0 + \chi a(\omega)$$

$$\log Q(\Omega) = \varphi'_0 + \chi \bar{a}(\Omega)$$

- Equivalent to common knowledge economy.
- When  $\alpha \neq 0$ , expectations of what others will do are critical.
  - Action that depends on what you expect others will do.
  - Get cross-sectional dispersion in employment, output, and relative prices that cannot be justified by the heterogeneity of local productivity.
  - Common noise in the public information contributes to fluctuations that are orthogonal to aggregate productivity.

## Implication 2: Dampening of Aggregates

$$\log q(\omega) = \varphi_0 + \varphi_a a + \varphi_x x + \varphi_y y$$

$$\frac{\partial \varphi_x}{\partial \alpha} < 0, \frac{\partial \varphi_y}{\partial \alpha} > 0 \text{ for given } \chi$$

- Stronger strategic complementarity  $\alpha \Rightarrow$  equilibrium output on  $i$  depends more on local forecast of agg output, less on current fundamentals.
  - $\alpha$  induces output to be anchored to prior, less sensitive to private info, more sensitive to public info.
  - Intuition: Stronger complementarity  $\Rightarrow$  common info is better predictor of others' activity.
- Aggregate responses are *endogenously sluggish*.

## Implication 3: Noisy Cycles With “Demand Shocks”

- Stronger  $\alpha \Rightarrow$  greater noise-driven aggregate fluctuations and smaller technology-driven fluctuations.
  - Anchoring to prior explains why technology matters less.
  - Heightened sensitivity to noisy public info explains why aggregate output responds more to noise.
- Agg employment:

$$\begin{aligned}\log N(\Omega) &= \text{const} + \frac{1}{\theta} (\log Q(\Omega) - \bar{a}) \\ &= \text{const} + \frac{1}{\theta} [(\varphi_{\bar{a}} - 1) \bar{a} + \varphi_{\varepsilon} \varepsilon]\end{aligned}$$

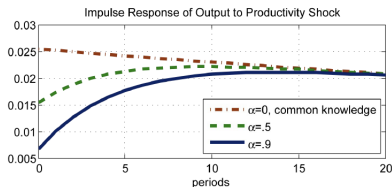
- Noise-driven fluctuations (from  $\varepsilon$ , noise in common signal  $y$ ) feature co-movement in  $N$ ,  $Q$ , and  $C$  orthogonal to  $A$ .
- Driven by *correlated errors about aggregate demand*.
- Microfoundation for “demand shocks.”

# Comments on Angeletos and La'O

- Models rely a lot on special assumptions.
  - Timing.
  - Information structure (normality, etc.).
  - What is public and what is not.
- Example: If prices and wages on each island are observed, then return to RBC equilibrium and noisy signals are irrelevant.
- Broader concern in these types of frameworks of “anything goes” with the right signals and market structure.
  - Angeletos and La'O do a good job getting away from this.
  - But as you read literature, ask yourself how authors impose this discipline and what model assumptions matter.

# Dynamic Version of Angeletos and La'O

- Angeletos and La'O extend model to allow for dynamics.
  - Productivity is AR(1).
  - Agg state is not common knowledge in second half of period.
- Key assumption: Aggregate prices and quantities are not observed. Only history of signals.
  - Avoids “forecasting the forecasts of others” problem from Townsend (1983) that creates an  $\infty$  dimensional state space.
  - Some methods to solve. See La'O notes, Huo and Takayama (2015a, 2015b), Huo and Pedroni (2017).



## Aggregation and Common Knowledge: Delaying GE

- Angeletos and Lian (2017) apply incomplete info framework to think about aggregation of identified micro evidence.
- Idea:
  - “GE adjustment takes time.”
  - GE effects hinge on strong assumptions about rational expectations equilibrium. When relax, slow down GE effects.
  - Two relaxations of REE: tatonnement and common knowledge.
- Main result is formal equivalence between:
  - Tatonnement process where aggregate price gradually converges to Walrasian equilibrium.
  - Imperfect common knowledge.
- Imperfect common knowledge is thus equivalent to a case where prices adjust slowly.
  - Since prices transmit GE, GE kicks in gradually.
  - In “short run” get pure partial equilibrium effect.

## Angeletos and Lian: Static Setup

- $i \in [0, 1]$  islands with rep household and firm.
  - Numeraire  $q_i^z$ , non-tradable  $q_i$ , and tradable  $q^*$
  - Prices 1,  $p_i$ ,  $p^*$ .
- FOCs of firm and household problem define local supply and demand functions:

$$q_i = S(p_i, p^*, a_i), \quad q_i^* = S^*(p_i, p^*, a_i), \quad q_i^z = S^z(p_i, p^*, a_i)$$

$$c_i = D(p_i, p^*, \xi_i, y_i), \quad c_i^* = D^*(p_i, p^*, \xi_i, y_i), \quad c_i^z = D^z(p_i, p^*, \xi_i, y_i)$$

- For any  $p^*$ , impose local clearing and let  $\theta = (\xi_i, a_i)$  to get:

$$q_i = c_i = Q(p^*, \theta_i) \text{ and } p_i = P(p^*, \theta_i)$$

$$q_i^* = Q^*(p^*, \theta_i) \text{ and } c_i^* = C^*(p^*, \theta_i) \text{ (and similar for } z)$$

- Net excess demand is:

$$n_i^* = c_i^* - q_i^* = N^*(p^*, \theta_i)$$

- Global equilibrium:  $\int N^*(p^*, \theta_i) di = 0$



# Angeletos and Lian: PE vs. GE

- Work with log-linear model.
  - Abuse notation and redefine every variable in log-deviations.
  - Let  $\bar{\theta} = \int \theta_i di$  denote the aggregate fundamental, which changes by  $\Delta\bar{\theta} = \bar{\theta}_{new} - \bar{\theta}_{old}$ . Can write  $p^* = P^*(\bar{\theta})$
  - Work with non-tradable expenditure:  $x_i \equiv p_i + q_i$ .
    - Agg is  $\bar{x} = \int x_i di$
    - Does not matter, but simplifies exposition

- Proposition: There exist  $\varepsilon^{micro}$  and  $\varepsilon^{macro}$  such that:

$$\Delta x_i = \Delta \bar{x} + \varepsilon^{micro} (\Delta \bar{\theta}_i - \Delta \bar{\theta}) \text{ and } \Delta \bar{x} = \varepsilon^{macro} \Delta \bar{\theta}$$

- Let  $PE$  be response if one island alone is hit by shock,  $GE$  be additional response if all are hit by a shock.
  - $PE$  is for given  $p^*$ ,  $GE$  additional response with endog  $p^*$ .
    - $PE$  here includes "local GE effects."
  - Then  $PE = \varepsilon^{micro}$  and  $PE + GE = \varepsilon^{micro} + \varepsilon^{macro}$ .

## RE Relaxation 1: Tattonement

- Walrasian auctioneer adjusts  $p^*$  in response to  $N^*$ .
  - Let  $T$  determine the number of “updates” the auctioneer makes.
  - Formally Tattonement price  $\hat{p}^* = \hat{P}^*(T)$ , the solution to ODE:

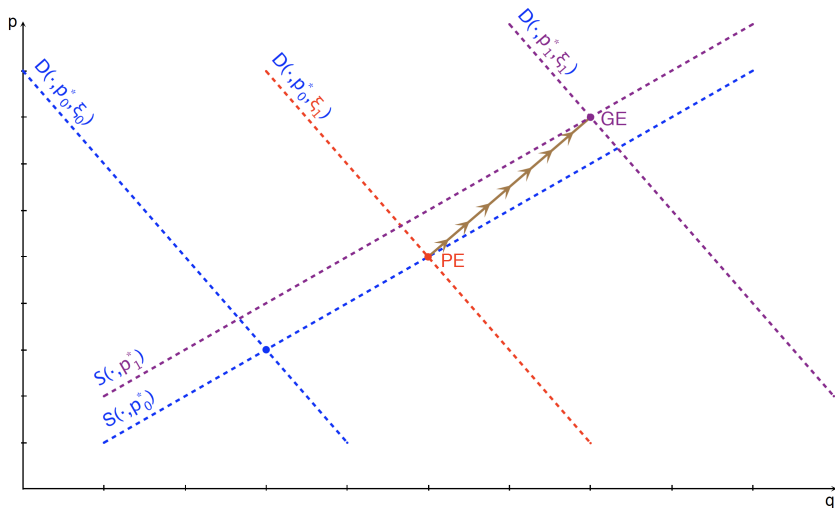
$$\frac{d\hat{P}^*(t)}{dt} = N^*(\bar{\theta}_{new}, \hat{P}^*(t))$$

with initial condition  $\hat{p}^*(0) = p^*(\bar{\theta}_{old})$ .

- Lemma 1:  $\exists w(t)$  with  $w' > 0$ ,  $w(0) = 0$ ,  
 $\lim_{T \rightarrow \infty} w(T) = 1$  s.t.  $\hat{p}^* = p_{old}^* + w(T)(p_{new}^* - p_{old}^*)$ .
- Lemma 2:  $q_i = Q(\hat{p}^*, \theta_i)$ ,  $p_i = (\hat{p}^*, \theta_i)$  are Tattonement  $p, q$ .
- Proposition: Macro elasticity in Tattonement economy is:

$$\varepsilon_{Tat}(T) = \varepsilon^{micro} + \underbrace{w(T)(\varepsilon^{macro} - \varepsilon^{micro})}_{\text{GE after } T \text{ updates}}$$

# RE Relaxation 1: Tattonement



# Angeletos and Lian: Incomplete Info

- Information Structure
  - Location-specific signals  $s_i = \Delta \bar{\theta} + v_i$ ,  $v_i \sim N(0, \sigma_v^2)$  and  $\Delta \bar{\theta} \sim N(0, \sigma_\theta^2)$ .
  - Let  $\lambda \equiv \frac{\sigma_v^{-2}}{\sigma_v^{-2} + \sigma_\theta^{-2}} \in [0, 1]$  be the degree of common knowledge.
  - Then the  $h$ th order belief is  $\bar{E}^h[\bar{\theta}] = \bar{\theta}_{old} + \lambda^h \Delta \bar{\theta}$
- Equilibrium with Incomplete Info
  - Game as in Angeletos and La'O
  - Define  $q_i = Q(E_i[p^*], \theta_i)$ ,  $p_i = P(E_i[p^*], \theta_i)$ ,  $x_i = q_i + p_i$ .
  - Then  $\bar{q} = Q(\bar{E}[p^*], \bar{\theta})$ .
  - In equilibrium it must be that  $p^*$  clears the aggregate market.
- Rewrite as game with linear best responses:

$$q_i = BR(\theta_i, E_i[\bar{\theta}], E_i[\bar{q}])$$

with slope of BR w.r.t  $E_i[\bar{q}]$  of  $\alpha \in (-1, 1)$ .

## Angeletos and Lian: Incomplete Info and Equivalence

- Lemma: Equilibrium  $p^*$  is such that for some  $\delta \neq 0$ ,  $a \in (0, 1)$ ,

$$\bar{E}[\bar{\theta}] = \delta \sum_{h=1}^{\infty} a^{h-1} \bar{E}^h[\bar{\theta}] = \bar{\theta}_{old} + \delta \sum_{h=1}^{\infty} a^{h-1} \lambda^h \Delta \bar{\theta}$$

- Given belief hierarchy, can show:

$$\bar{E}[p^*] = p_{old}^* + \pi(\lambda)(p_{new}^* - p_{old}^*)$$

where  $\pi' > 0$ ,  $\pi(0) = 0$ ,  $\pi(1) = 1$ .

- Proposition: Letting  $\varepsilon_{Inc}$  be the info macro elasticity:

$$\varepsilon_{Inc}(\lambda) = \varepsilon^{micro} + \underbrace{\pi(\lambda)(\varepsilon^{macro} - \varepsilon^{micro})}_{\text{GE Parameterized by } \lambda}$$

- $\pi(\lambda)$  is decreasing in  $\lambda$ , increasing in  $\alpha$ .
- Intuition: higher  $\alpha$ , higher weight on actions of others and higher order beliefs.

# Angeletos and Lian: Observational Equivalence

- Theorem: For any Tatonnement economy with  $T$  updates, there exists an incomplete info economy with a degree of common knowledge  $\lambda \in (0, 1)$  such that:
  1. For any  $\Delta\bar{\theta}$ , the rational expectation  $\bar{E}[p^*]$  coincides with  $\hat{p}_T^*$ .
  2. The two economies feature the same macro elasticity.
    - And vice-versa.
- In other words, these are *observationally equivalent*.
- Shows how simple relaxations of standard rational expectations equilibrium delay GE effects in similar ways.
  - A-L are comparing to other relaxations in upcoming draft.

## Angeletos and Lian: Dynamic Extension

- Angeletos and Lian then extend to a dynamic economy where Tattonement takes time or gradual learning occurs.
  - To make sure islands do not learn fundamental from tradable price, they face global price with noise.
  - One-time shock, so endogenous public signal not an issue.
  - All results from above hold in dynamic versions!
    - $T$  in Tattonement is a time.
    - Learning: Degree of common knowledge  $\lambda$  increases over time.
- Furthermore, in incomplete info case, *bigger GE effects take longer to emerge*.
  - Bigger GE effect  $\Rightarrow$  bigger weight on actions of others.
  - With strategic complementarity, higher order beliefs matter more, take longer to emerge.
    - Recall  $\bar{E}^h[\bar{\theta}] = \bar{\theta}_{old} + \lambda^h \Delta \bar{\theta}$

## Angeletos and Lian: Comments

- This is a very interesting paper!
- Take away: If care about  $\%Explain = \frac{PE+GE}{Agg\ Change}$  and approximate with  $\%\hat{Explain} = \frac{PE}{Agg\ Change}$ , as long as measuring with “short enough” lag you are okay if relax REE.
  - Big GE  $\Rightarrow$  good for longer with incomplete info.
  - Small GE  $\Rightarrow$  good regardless of speed.
- The issue is mapping it to time. What is “short enough”?
  - Mian and Sufi (2014) use  $\Delta Emp_{2007-2009}$ . Is two years “short enough” to interpret the fraction of the employment decline they explain as immune to GE critiques?
  - Chodorow-Reich (2014) uses a one year change. Is this short enough to interpret the fraction of the employment decline he explains as immune to GE critiques?
- Open question I am thinking about.



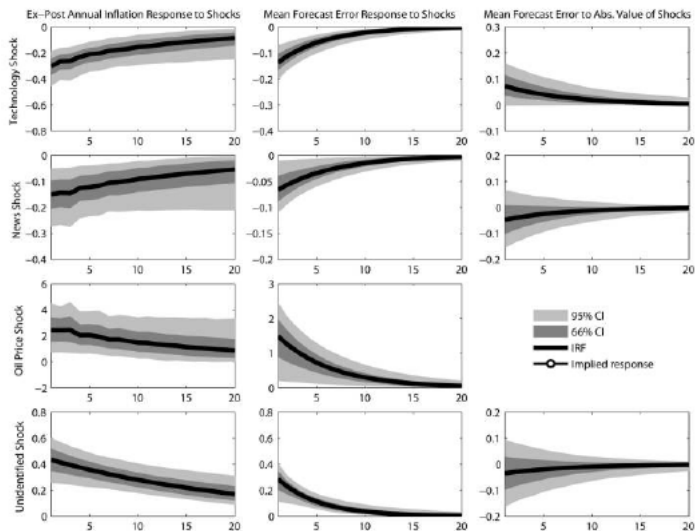
## Coibion and Gorodnichenko: Survey Expectations

- End with a neat paper that uses survey expectations to show:
  1. Incomplete Information Models Are Consistent With Data
  2. How Long Learning May Take in These Models.
- Background: In 2000s “information” models about inflation have a resurgence.
  - Woodford (2003) (will discuss in lecture 9).
  - Mankiw and Reis (2002) “Sticky Info”
  - Sims (2003) Rational Inattention
- Coibion and Gorodnichenko show all these NK-style models have a common prediction:  $E_t[\pi_t]$  responds more sluggishly than  $\pi_t$  to innovations in fundamentals.
  - Test: Is forecast error serially correlated with same sign?
  - Also test ancillary predictions of each model.
    - Favorable to incomplete and noisy info, but I won't cover.

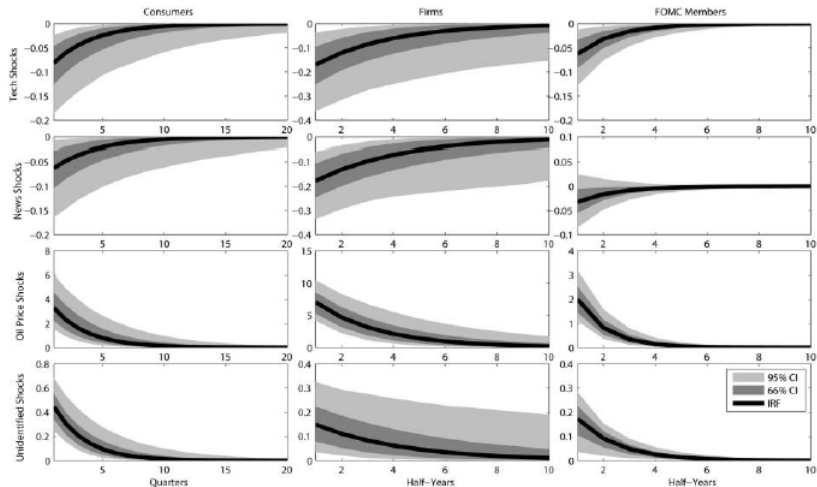
## Coibion and Gorodnichenko: Data

- Two-Step Approach: Identify shocks, look at IRFs of agent expectations to shocks.
- Shocks:
  1. Tech shocks as in Gali (1999)
  2. Oil shocks as in Hamilton (1996)
  3. News shocks as in Barsky and Sims (2011)
- Inflation Expectations:
  1. Survey of Professional Forecasters
  2. Michigan Survey of Consumers
  3. Firms in Livingston Survey
  4. FOMC Members

# Professional Forecasters: Serial Corr



# Consumers, Firms, Central Bankers Too



# Magnitudes

CONVERGENCE RATES OF FORECASTS

	Professional Forecasters	Consumers	Firms	FOMC Members	<i>p</i> -Value for Equality across Agents
Technology shocks	.86 (.05)	.80 (.10)	.89 (.09)	.86 (.08)	.91
News shocks	.89 (.05)	.81 (.10)	.89 (.08)	.88 (.09)	.89
Oil price shocks	.88 (.05)	.74 (.07)	.86 (.07)	.59 (.13)	.04
Unidentified shocks	.88 (.05)	.74 (.09)	.89 (.06)	.87 (.08)	.44
<i>p</i> -value for equality across shocks	.98	.90	.98	.19	.60

- Implication: Weight of  $< 20\%$  on new info.
  - This is quarterly, so forecast error has half life of  $> 3$  quarters.
  - Concern with using: Assumes that inflation is exogenous to forecast, while models have it endogenous.
- Lots of interesting research being done and to be done on using expectations surveys to discipline learning models.