Economics 742 Price Ridigity Bonus Lecture 1:

State-Dependent Pricing and (S,s) Models

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Outline For Price Stickiness Section

1. State-Dependent Pricing and (S,s) Models
2. Evidence on Price Adjustment and Refined (S,s) Models
3. Persistence of Nominal Rigidity and Strategic Complementarity
    - Coordination failures among price setters amplify stickiness.
4. Time Variation in Monetary Policy Effectiveness

- I will largely follow (but go in more depth than) Nakamura and Steinsson's excellent *Annual Review of Economics* article.

Why Price Stickiness?

- Monetary Non-Neutrality.
  - Hard to get non-neutrality *without* price stickiness.
  - But still somewhat open question how important this is, and non-neutrality remains a heated debate.
- Critics of NK:
  - Prices may be sticky at the product level, but this matters little in the aggregate.
  - Price stickiness is overblown. How can small menu costs play such an outsized role business cycles?
- 704: Evidence for price stickiness, NK model
- 742: Digging deeper into price stickiness.
  - Is world Calvo?
  - What does a richer model give us?
  - How much stickiness is reasonable?

(S,s) Models

- Today will be a lot of theory.[1]

1. (S,s) Policies: Sheshinksi and Weiss (1977)
2. Early GE (S,s) Models and Monetary Neutrality
   2.1 Caplin and Spulber (1987)
   2.2 Caplin and Leahy (1991)
3. (S,s) DSGE Model: Golsov and Lucas (2007)
   - Bils and Klenow (2004) in calibration

[1] These notes draw on excellent notes by Virgiliu Midrigan available on the web as well as notes by Emi Nakamura and Jon Steinsson.

Why Menu Costs?

- In the data we see prices change infrequently.
  - Unlikely that optimal prices exhibit this pattern. Makes sense they change smoothly.
  - So large, discrete price changes point to adjustment costs.
- This leads to a menu cost model of price adjustment.
  - Fixed cost to change price.
  - Optimal policy is (S, s)
    - Adjust price when deviates sufficiently from optimal value.
    - (S, s): adjust when real price hits boundary s to target S.
- Similar models used to study lumpy investment, durables consumption, inventories.

Menu Costs: The Questions

- Do firm-level menu costs imply *aggregate price stickiness?*
  - Or does lumpy adjustment "wash out" when we aggregate?
- Distinguish *state-dependent pricing* from *time-dependent*.
  - Time-Dependent Pricing (Calvo, Taylor)
    - Timing of price changes is not affected by state of economy.
    - Change prices optimally taking timing of changes as given.
    - In Calvo, random set of firms change price.
  - State-Dependent Pricing (Menu Costs)
    - Firms choose *both* timing and size of price changes optimally.
    - Timing of price changes depends on state of economy.
    - Firms that change prices have price most "out of line."
- State-dependent pricing generally leads to less aggregate stickiness than time-dependent.
  - How much less is matter of debate.

Menu Costs: Role of Micro Data

- Why do facts about micro-level price-adjustment matter?
    - *Because of the model*.
    - Model provides us with way to aggregate from micro to macro.
- Micro data disciplines model.
    - Tells us what type of model to write.
    - Help choose parameters.
    - Tell us what types of shocks are important and what their features are.
- But model needed to interpret micro facts.
    - Today: Models, new facts, and "second generation" models.
    - Next class: Facts and revising "second generation" models.

Sheshinski and Weiss (1977): Setup

- Sheshinksi and Weiss (1977) show an (S,s) pricing strategy is optimal in a stationary state-dependent framework.
- Consider a firm in partial equilibrium.
  - Aggregate inflation constant at rate g.
  - Profits are a function of the firm's real price.
    - Price index $P(t) = e^{gt}$ with real price $z(t) = p(t)/P(t)$.
    - Iso-Elastic Demand (Dixit-Stiglitz): $q(z) = z^{-\theta}$.
    - Profits: $\Pi(z) = z^{-\theta}(z - c)$ optimized at s^*.
  - Firm pays menu cost χ to change nominal price.
  - Interest rate is r.
- Trade-off: lost profit due to deviation from optimal real price vs. cost of adjusting price.

Sheshinski and Weiss (1977): Value Functions

- Firm changes its price at dates $\{t_i\}_{i=1}^{\infty}$.
- At times $t \in [t_i, t_{i+1}]$, nominal price is p_i, real price $p_i e^{-gt}$.
- Profit in interval $[t_i, t_{i+1}]$ is V_i:

$$V_i = \int_{t_i}^{t_{i+1}} \Pi\left(p_i e^{-gt}\right) e^{-rt} dt - \chi e^{-rt_{i+1}}$$

- Total value is infinite sum of V_i for all price-constant intervals:

$$\begin{aligned} V &= \sum_{i=0}^{\infty} V_i \\ &= \sum_{i=0}^{\infty} \left[\int_{t_i}^{t_{i+1}} \Pi\left(p_i e^{-gt}\right) e^{-rt} dt - \chi e^{-rt_{i+1}}\right] \end{aligned}$$

Sheshinski and Weiss (1977): First Order Conditions

$$V = \max_{\{t_i, p_i\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \left[\int_{t_i}^{t_{i+1}} \Pi\left(p_i e^{-gt}\right) e^{-rt} dt - \chi e^{-rt_{i+1}} \right]$$

$$\begin{aligned}
\frac{\partial V}{\partial t_i} &= \left[\Pi\left(p_{i-1} e^{-gt_i}\right) - \Pi\left(p_i e^{-gt_i}\right) + \chi r\right] e^{-rt_i} = 0 \\
\frac{\partial V}{\partial p_i} &= \int_{t_i}^{t_{i+1}} \Pi'\left(p_i e^{-gt}\right) e^{-rt} dt = 0
\end{aligned}$$

Sheshinski and Weiss (1977): First Order Conditions

$$\Pi\left(p_i e^{-gt_i}\right) \quad = \quad \Pi\left(p_{i-1} e^{-gt_i}\right) + \chi r$$

- Delay changing price until benefit = cost.
  - LHS: Flow profit if change.
  - RHS: Flow profit if keep old price and wait to pay fixed cost.

$$\int_{t_i}^{t_{i+1}} \Pi'\left(p_i e^{-gt}\right) e^{-rt} dt = 0$$

- Price is right "on average."
  - "Front Loading": Set real price above optimum, let inflation erode and adjust when below optimum.

Sheshinski and Weiss (1977): (S,s) Strategy

- Since problem is stationary, set same nominal price S every time you reset, wait same interval Δ when reset.
  - Equivalent to setting same trigger price $s = Se^{-g\Delta}$.
  - Hence (S, s) strategy.
- Width of bounds expands with inflation and adjustment costs.
- Can also do a two-sided (S, s) problem (Barro, 1972).
  - Lower and upper bounds both trigger jump to same target.

Caplin and Spulber (1987): Overview

- Caplin and Spulber (1987) is the first paper to aggregate individual firms with a menu cost.
  - Present a special case in which money is *completely neutral* even though prices are sticky at micro level.
  - Very surprising!
- Key intuition is *selection effect* (Golosov and Lucas, 2007).
  - Calvo: random cross-section of firms change price. Some change a lot, some change a little.
  - Here, firms that *most need to change their price* (a selected sample) do so.
  - Aggregate price level much more flexible than Calvo.
- I will provide a simplified version.

Caplin and Spulber (1987): Setup

- Demand is $q\left(z\right)=z^{-\theta}$, aggregate price index is Dixit-Stiglitz.
- Money supply:
  - Process for $M\left(t\right)$ is continuous and strictly increasing: $M\left(t'\right)\geq M\left(t\right)\ \forall\ t'\geq t$.
  - Cash in Advance constraint: $M\left(t\right)=\int p\left(t,i\right)c\left(t,i\right)di$.
  - Fixed real cost c.
- Firms follow $\left(S,s\right)$ strategy (never derive even though inflation not constant).
- Distribution of real prices is uniform on $[s,S]$.

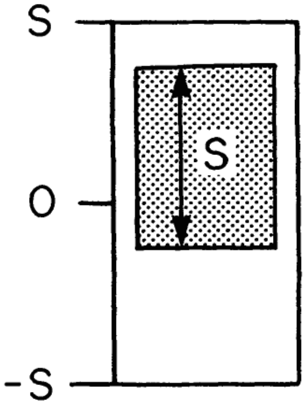
Caplin and Spulber (1987): Monetary Neutrality

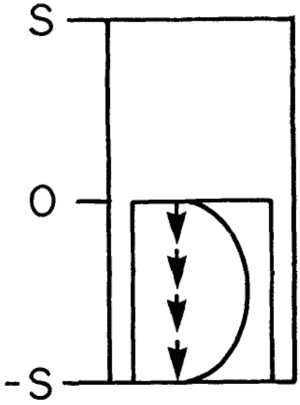
- Money is neutral: P_t grows with M_t, output is fixed.

Caplin and Leahy (1991): "Goldilocks" Economy

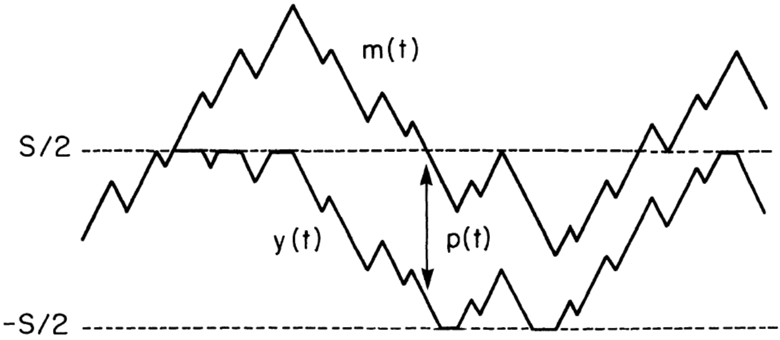
- $M(t)$ can either rise or fall – standard Brownian motion without drift.
  - Simplification where $y = m - p$.
- Firms use symmetric two-sided (S, s) policy: adjust to 0 if hit $-S$ or S.
  - Optimal with quadratic profit function.
- Initial distribution is uniform on $[-S/2, S/2]$.
  - As in Caplin and Spulber, changes in M change location of distribution but not its shape.
  - Monetary neutrality unless lower bound of distribution hits $-S$ or upper bound hits S. Then not neutral.

Caplin and Leahy (1991): "Goldilocks" Economy





Caplin and Leahy (1991): "Goldilocks" Economy



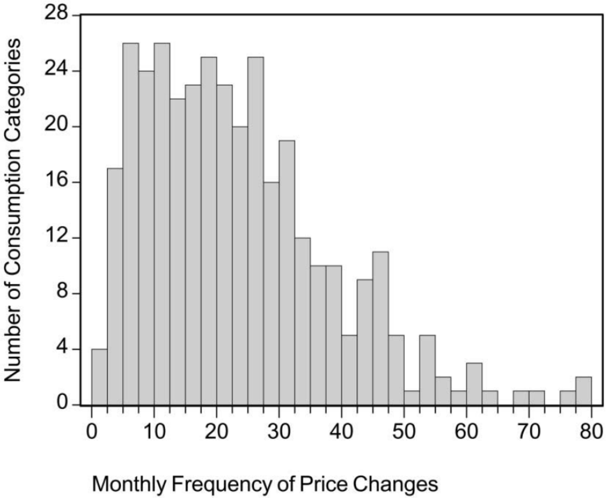
State-Dependent Pricing 1991-2004

- Literature focuses on making these types of models more realistic, which requires computational methods.
- Dotsey, King, and Wolman (1997)
  - Key new feature: Random menu cost.
  - Smooths out kinks in problem created by fixed costs, allowing the model to be solved using perturbation.
- DKW and related papers find that non-neutrality weakened, but not substantially.
  - Mostly because adjust more frequently after monetary shock.
  - Focus on aggregate inflation shock.
  - Calibrated to studies that show prices adjust once a year.

Bils and Klenow (2004)

- First to use BLS CPI micro data to assess pricing behavior.
- Find prices change every 4.3 months, on average.
  - Even if one considers sales, prices change every 5.5 months.
  - Substantial heterogeneity across goods.
  - Frequency of price resets does not vary substantially over cycle.
- Cross-sectional patterns do not match predictions of time-dependent models.
  - Inflation at good level is volatile and transient.
  - Many price decreases.
  - Persistence and volatility not related to freq of price change.
- Klenow and Kryvtsov (2005, 2008): Average absolute size of price changes is large: about 10% even though inflation is 2%.

Bils and Klenow (2004)



Bils and Klenow (2004)

Year	Median Frequency (%)	Median Duration (Months)
1995	21.3	4.2
1996	20.8	4.3
1997	19.9	4.5
1998	21.2	4.2
1999	21.4	4.1
2000	21.7	4.1
2001–2	22.0	4.0

Golosov and Lucas (2007)

- "Second Generation" State-Dependent Model.
- Key New Feature: Idiosyncratic cost shock.
  - Rationalizes large size of price changes when inflation is low, acyclical frequency of price resets.
  - Mean reverting so variance of relative prices does not grow over time as in data.
- Find very strong selection effect:
  - Effect of price stickiness is six times weaker than Calvo.
  - Because firms that need to change the most change, so prices quite flexible in practice (like Caplin-Spulber).
  - Unlike prior literature, freq of price changes not very cyclical because idiosyncratic shock dominates.
- Will go through modified model in detail.

Golosov and Lucas (2007): Modifications

- Discrete time rather than continuous (must discretize to solve).
  - Value function is over price not time to adjust and price.
- log utility in consumption rather than CRRA.
- Cash-in-Advance / Central Bank Determining nominal GDP rather than money in utility function.
- Model in my notes similar to subsequent literature, e.g. Nakamura-Steinsson (2010) and Midrigan (2011).

Modified GL (2007): Households

- Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \omega L_t\right]$$

- C_t is a Dixit-Stiglitz index over $z \in [0,1]$ with price index P_t:

$$C_t = \left[\int_0^1 c_t\left(z\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

- Optimize subject to transversality and budget constraint:

$$P_t C_t + E_t\left[Q_{t,t+1} B_{t+1}\right] \leq B_t + W_t L_t + \int_0^1 \Pi_t\left(z\right) dz$$

  - Complete markets. A-D security payoffs B_{t+1}, prices $Q_{t,t+1}$.

Modified GL (2007): Households

- Given total consumption C_t, allocate to goods $c_t(z)$ by cost minimization:
$$c_t(z) = C_t \left(\frac{p_t(z)}{P_t} \right)^{-\theta}$$
- Choose C_t, L_t, and B_{t+1} to maximize:
$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \omega L_t - \Lambda_t \begin{pmatrix} P_t C_t + Q_{t,t+1} B_{t+1} \\ -B_t - W_t L_t - \int_0^1 \Pi_t(z)\, dz \end{pmatrix} \right\} \right]$$

Modified GL (2007): Household FOCs

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left\{\log C_t-\omega L_t-\Lambda_t\left(\begin{array}{c}P_tC_t+Q_{t,t+1}B_{t+1}\\ -B_t-W_tL_t-\int_0^1\Pi_t(z)\,dz\end{array}\right)\right\}\right]$$

- FOCs:

$$\begin{aligned}1/C_t &= P_t\Lambda_t\\ W_t\Lambda_t &= \omega\\ \beta\Lambda_{t+1} &= \Lambda_t Q_{t,t+1}\end{aligned}$$

- Combining,

$$\begin{aligned}Q_{t,t+1} &= \beta\left(\frac{P_{t+1}C_{t+1}}{P_tC_t}\right)^{-1}\\ W_t &= \omega P_tC_t\end{aligned}$$

  - Functional forms mean nominal wages proportional to nominal GDP, saving us a state variable.

Modified GL (2007): Firms

- Linear production function:

$$y_t(z) = A_t(z) L_t(z)$$

- Maximize discounted profits:

$$E_t \sum_{\tau=0}^{\infty} Q_{t,t+\tau} \Pi_{t+\tau}(z)$$

where nominal profits are:

$$\Pi_t(z) = p_t(z) y_t(z) - W_t L_t(z) - \chi W_t I_t(z)$$

  - χ is menu cost, $I_t(z)$ indicator for price change in period t.

Modified GL (2007): Shocks and Equilibrium

- Aggregate shocks from money supply adjusted so that nominal GDP $S_t = P_t C_t$ follows:

$$\log S_t = \mu + \log S_{t-1} + \eta_t,\ \ \eta \sim N\left(0, \sigma_S^2\right)$$

- Idiosyncratic productivity follows mean-reverting process:

$$\log A_t\left(z\right) = \rho \log A_{t-1}\left(z\right) + \varepsilon_t\left(z\right),\ \ \varepsilon_t\left(z\right) \sim N\left(0, \sigma_A^2\right)$$

- Can complete with labor market clearing or goods clearing:

$$C_t = \left[\int_0^1 \left(\frac{\omega p_t\left(z\right)}{W_t}\right)^{1-\theta} dz\right]^{\frac{1}{\theta-1}}$$

  - Integrating over joint distribution of $p_t\left(z\right)$ and $A_t\left(z\right)$, $\phi_t\left(p/P, A\right)$.

Modified GL (2007): State Variables

- Normalize problem by price, so in real terms.
  - Let $\Pi_t^R = \Pi_t / P_t$ and $Q_{t,t+1}^R = Q_{t,t+1} P_{t+1} / P_t$
- State variables for firm problem are:
  1. Firm's aggregate productivity $A_t(z)$
  2. Real wage $\frac{W_t}{P_t}$ or equivalently C_t as $\frac{W_t}{P_t} = \omega C_t$
  3. To forecast $Q_t^R = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1}$, need C_{t+1} which requires *joint distribution* $\phi_t(p/P, A)$.
     - This is what makes things computationally tricky.

Modified GL (2007): Firm's Bellman Equation

$$V\left(A_t, C_t, \phi_t\right) = \max_{p_t} \left\{ \Pi_t^R\left(z\right) + E_t\left[Q_{t,t+1}^R V\left(A_{t+1}, C_{t+1}, \phi_{t+1}\right)\right]\right\}$$

where

$$\begin{aligned}
\Pi_t^R\left(z\right) &= C_t \left(\frac{p_t\left(z\right)}{P_t}\right)^{-\theta} \left(\frac{p_t\left(z\right)}{P_t} - \frac{1}{A_t\left(z\right)} \frac{W_t}{P_t}\right) - \chi \frac{W_t}{P_t} I_t\left(z\right) \\
Q_{t,t+1}^R &= \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \\
S_t &= P_t C_t = W_t / \omega
\end{aligned}$$

Need to solve jointly with market clearing.

Modified GL (2007): GL Solution For Stationary Equilibrium

- Consider case where $\sigma_S^2 = 0$ so no agg uncertainty.
- Exploit that distribution ϕ_t only enters Bellman through C_{t+1}.
  - Conjecture equilibrium with invariant distribution $\phi_t = \tilde{\phi}\ \forall t$ and $C_t = \bar{C}\ \forall t$.
  - ϕ is no longer a state variable.
- Solve numerically for fixed point between Bellman and $C_t = \bar{C}$ using discrete state space approximation.
  1. Guess $\bar{C}$ and solve Bellman.
  2. From optimal policy, determine invariant distribution $\tilde{\phi}$.
  3. Use optimal policy, $\tilde{\phi}$, and market clearing to solve for C_t , which becomes the new $\bar{C}$.
  4. Iterate until convergence.

Modified GL (2007): GL Solution For Impulse Response

- Consider stationary equilibrium: $\log S_t$ growing by μ.
- Hit with unanticipated jump from $\log S_t$ to $\log (1+h) S_t$, solve for perfect-foresight rational expectations equilibrium.
  1. Choose $\{C_t\}_{t=0}^{n}$ converging to new stationary $\bar{C}$ in n periods.
  2. Define sequence of value functions $\{V_t\}_{t=0}^{n}$, solve by backwards induction from $V_n = \bar{V}$, new stationary value.
  3. Use pricing strategies to construct sequence of distributions $\{\phi_t (p/P, A)\}_{t=0}^{n}$.
  4. Construct$\{C_t\}_{t=0}^{n}$ from pricing strategies, $\{\phi_t (p/P, A)\}$, and market clearing.
  5. Mapping from $\{C_t\}_{t=0}^{n}$ to new $\{C_t\}_{t=0}^{n}$ is a contraction. Iterate until convergence.
- Approx. $\sigma_S^2 \neq 0$ assuming $C_t = \bar{C}$ for stationary equilibrium.

Modified GL (2007): Krussel-Smith Solution Method

- Alternate solution method pursued is Krussel-Smith approach.
  - $C_{t+1} = S_{t+1}/P_{t+1}$, so to forecast $Q_{t,t+1}$, forecast P_{t+1}.

1. Nakamura and Steinsson (2010) assume forecast rule:

$$\frac{P_t}{P_{t-1}} = \Gamma\left(\frac{S_t}{P_{t-1}}\right)$$

   where problem is discretized and Γ is a matrix.

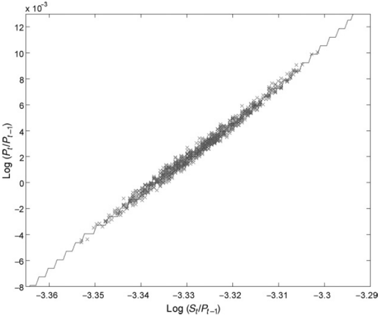
2. Midrigan (2011) assumes forecast rule:

$$\log\left(\frac{P_t}{S_t}\right) = \alpha_0 + \alpha_1 \log\left(\frac{S_t}{S_{t-1}}\right) + \alpha_2 \log\left(\frac{P_{t-1}}{S_{t-1}}\right)$$

   where problem is continuous and solved by collocation.

Modified GL (2007): Krussel-Smith Solution Algorithm

1. Assume forecast rule Γ.
2. Solve model given Γ.
   - This requires making sure the C_t clears the market, which is subtle.
3. Simulate data and estimate Γ.
4. Update Γ to estimated Γ and iterate until convergence.



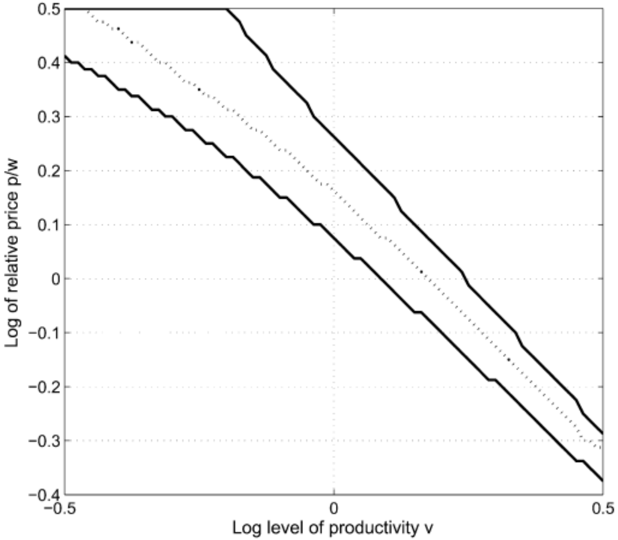
Source: Nakamura and Steinsson (2010)

Golosov and Lucas (2007): Calibration Moments

1. Frequency of price change.
2. Average log price increase.
3. Standard dev of new prices for price increases.

Moment	Data (1)	Model (2)
Quarterly inflation rate	.0064	.0064
Standard deviation of inflation	.0062	0
Frequency of change	.219	.239
Mean price increase	.095	.097
Standard deviation of new prices	.087	.090

Golosov and Lucas (2007): Stationary (S,s) Policy



GL (2007): Importance of Idiosyncratic Shock For Data

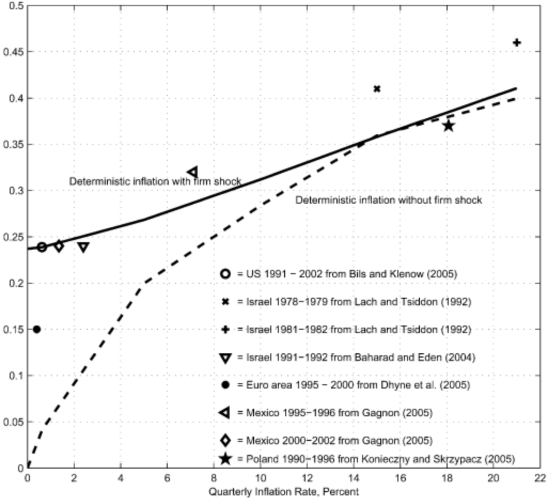


FIG. 3.—Fraction of prices changed each month

GL (2007): Six Times Less Non-Neutrality than Calvo

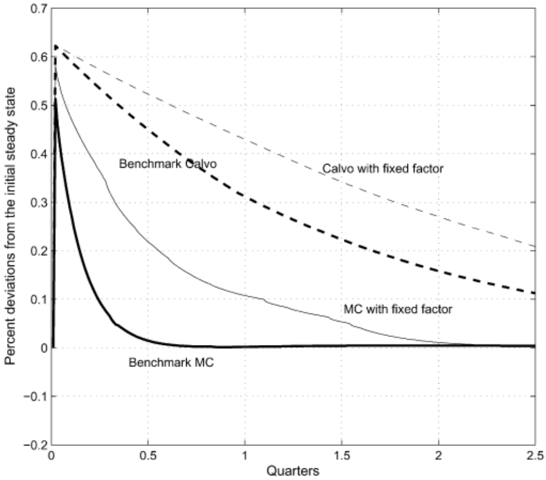


FIG. 5.—Output responses in menu cost and Calvo models

GL (2007): Selection Effect Intuition

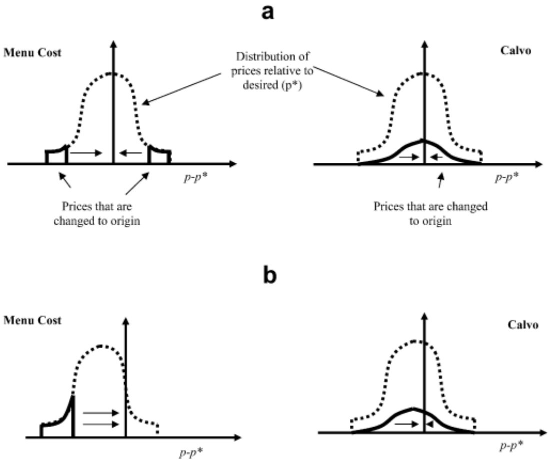


FIG. 6.—Price adjustment in menu cost and Calvo models. *a*, Price adjustment before aggregate shock. *b*, Price adjustment after aggregate shock.