Economics 742 Bonus Macro-Labor Lecture 2: Flavors of Search Models

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[1]This lecture synthesizes material from Rogerson, Shimer, and Wright (2005), Pissarides (2000), and excellent lecture notes by Edouard Schaal in addition to the papers indicated. All are gratefully acknowledged.

Search

- Search has become the dominant framework for analyzing labor markets.
  - Many of you have seen a simple search model.
  - Key idea: Trading frictions, which consume time and resources, are important.
  - Today: taxonomy of search models so you have a sense of the toolbox and modeling decisions.
- I will focus on labor search, but search used in many other contexts:
  - Housing
  - Over-the-Counter Financial Markets
  - Money
  - Product Markets
- Big literature; I cannot cover it all.
- Excellent survey: Rogerson, Shimer and Wright (2005).

Key Questions

- How do agents meet?
  - Directed or undirected search.
- How are prices / wages determined?
  - Posting or bargaining.

Outline

1. Partial Equilibrium Search

2. Undirected Search in GE

   2.1 Baseline Pissarides (1985) Model
   2.2 Dynamics
   2.3 Match-Specific Productivity
   2.4 Endogenous Job Destruction (Mortensen and Pissarides, 1994)
   2.5 Efficiency: Hosios (1990) Condition
   2.6 On the Job Search

3. Directed Search in GE

   3.1 Moen (1997) and Shimer (1996)
   3.2 Menzio and Shi (2011)

4. Undirected Search and Posting (Briefly)

Partial Equilibrium

- Search traces its roots to "The Economics of Information" by Stigler (1961) (choose number of stores to search for product).
- Modern search antecedent is McCall (1970) partial equilibrium *sequential* search model.
  - Individual worker repeatedly drawing from wage distribution. Deciding whether to accept or keep searching.
  - I will show in continuous time.
- Unemployed worker maximizes expected discounted utility.
  - Linear utility, discount rate r.
  - Earn w if employed, b if unemployed.
  - When unemployed, at Poisson rate α draw offers iid from distribution $F(w)$. (Undirected search.)
  - When accept, employed forever.

Partial Equilibrium: Bellman Equations

- $W(w)$ is value of accepting wage w.
- U is value of rejecting w, earning b, waiting until next draw.
  - If accept, change in pdv flow utility is $W(w) - U$.

$$\begin{aligned} rW(w) &= w \\ rU &= b + \alpha \int_0^\infty \max\{0, W(w) - U\} \, dF(w) \end{aligned}$$

- $W(w)$ is strictly increasing in w, so optimal strategy is to accept if $w > w_R$, reservation wage, defined by $W(w_R) = U$:

$$\begin{aligned} W(w) - U &= \frac{w - w_R}{r} \\ w_R &= b + \frac{\alpha}{r} \int_{w_R}^\infty (w - w_R) \, dF(w) \\ &= b + \frac{\alpha}{r} \int_{w_R}^\infty (1 - F(w)) \, dw \end{aligned}$$

- This is a contraction in w_R. Endogenously no recall.

Partial Equilibrium: Discussion

$$w_R = b + \frac{\alpha}{r} \int_{w_R}^{\infty} (1 - F(w))\, dw$$

- Can then get hazard rate of job finding $H = \alpha [1 - F(w_R)]$ and average duration of unemployment $D = \frac{1}{H}$.
  - Predictions like $\frac{\partial D}{\partial b} > 0$. Mean preserving spread in $F(\cdot)$ increases D through real option effect.
- Criticism:
  - Partial equilibrium.
  - Diamond (1971) Paradox: Why distribution of wages? In equilibrium, firms should always post reservation wage.
- Responses:
  - General equilibrium search with bargaining instead of posting.
  - Jovanovic (1979): noisily observed match quality and learning.

Undirected Search With Bargaining: Setup

- Classic Pissarides (1985) model. Notation is my hybrid of RSW and Pissarides (2000).
- Mass one of agents.
- Wage w for homogenous workers who produce y output per unit time when matched with firm.
  - Real model, $p = 1$. Profits $\pi = y - w$.
  - y assumed large enough to produce.
  - Linear utility, discount rate r.
- Unemployed get benefit b.
- Firms post vacancy at cost k.
- Separation at Poisson rate λ.
- Worker finds firm at poisson rate α_w, firm finds worker at poisson rate α_e.

Undirected Search With Bargaining: Model

- Four value functions: $W(w)$ for worker paying w, U for unemployed, $J(\pi)$ for filled job with profit π, V for vacancy:

$$
\begin{aligned}
rW(w) &= w + \lambda [U - W(w)] \\
rU &= b + \alpha_w [W(w) - U] \\
rJ(\pi) &= \pi + \lambda [V - J(\pi)] \\
rV &= -k + \alpha_e [J(\pi) - V]
\end{aligned}
$$

- Two states, u and e so one law of motion:

$$\dot{u} = \lambda (1 - u) - \alpha_w u$$

- Free entry in job posting:

$$V = 0$$

Undirected Search With Bargaining: Matching Function

- How do firms and workers meet?
    - Reduced form approach through *matching function*.
- With u workers, v vacancies, matches created at Poisson rate $m = m(u, v)$.
    - Assumed continuous, nonnegative, increasing, concave.
- Typically assume CRS and define *market tightness* $\theta = \frac{v}{u}$ so:

$$\begin{aligned} \alpha_e &= \frac{m(u,v)}{v} = m\left(\theta^{-1}, 1\right) \equiv q(\theta) \\ \alpha_w &= \frac{m(u,v)}{u} = \theta q(\theta) \end{aligned}$$

    - $q' < 0$ so that as $\frac{v}{u}$ ratio rises, α_e falls and α_w rises.
    - CRS guarantees unique equilibrium. Multiple with IRS.
- Petrongolo and Pissarides (2001): matching function is roughly CRS and Cobb-Douglas $q(\theta) = \xi\theta^{-\gamma}$, $\gamma \in [.5, .7]$.

Undirected Search With Bargaining: Wages

- Unlike competitive labor markets, when a match occurs there is *bilateral monopoly power*.
    - Worker's outside option is worse than turning away.
    - Firm's outside option is worse than turning away.
- Need a way to split surplus.
    - Typically Nash Bargaining with weight χ for worker:

$$W(w) = U + \chi [J(\pi) - V + W(w) - U]$$

    - Reason we use linear utility is this is easy problem.
    - Implicitly assuming atomistic firms.
    - With IRS or DRS, use Stole and Zwiebel (1996) generalization of Nash bargaining for many workers.

Undirected Search With Bargaining: Steady State Solution

- Manipulate free entry, Nash bargaining, law of motion
- Law of Motion: Inflows equals outflows

$$\begin{aligned} \dot{u} &= \lambda(1-u) - \alpha_w u = 0 \\ u &= \frac{\lambda}{\lambda + \alpha_w} = \frac{\lambda}{\lambda + \theta q(\theta)} \end{aligned}$$

- Free entry and the Bellman for $J(\pi)$ imply:

$$\begin{aligned} k &= q(\theta) J(\pi) \\ \pi &= (r+\lambda) J(\pi) \end{aligned}$$

so

$$\underbrace{\frac{y-w}{r+\lambda}}_{\text{Value of Job to Firm}} = \underbrace{\frac{k}{q(\theta)}}_{\text{Ave Cost of Recruiting Worker}}$$

Undirected Search With Bargaining: Steady State Solution

- Combine Nash bargaining with free entry, $J(\pi) = \frac{y-w}{r+\lambda}$ and $W(w) = \frac{w+\lambda U}{r+\lambda}$:

$$\begin{aligned} W(w) &= U + \chi \left[J(\pi) + W(w) - U \right] \\ w &= rU + \chi (y - rU) \end{aligned}$$

  - Wage is equal to reservation wage rU plus share of surplus.
  - Surplus equal to product minus reservation wage (minus firm reservation wage, zero due to free entry).
- Eliminate rU by combining lots of things (on next slide, which is for your reference only) to get:

$$w = (1-\chi) b + \chi (y + \theta k)$$

  - Worker gets convex combo of y and b.
  - Plus fraction χ of θk, vacancy cost firm saves by hiring worker.
- Alternate solution method: solve for everything in terms of surplus $S = J(\pi) + W(w) - U$. See below.

Undirected Search With Bargaining: Math For Reference

- Combine $\frac{k}{q(\theta)} = J(\pi)$ and $rU = b + \theta q(\theta)[W(w) - U]$:

$$rU = b + \theta \frac{k}{J(\pi)}[W(w) - U]$$

- Write Nash bargaining as $J(\pi) = \frac{1-\chi}{\chi}[W(w) - U]$, plug in:

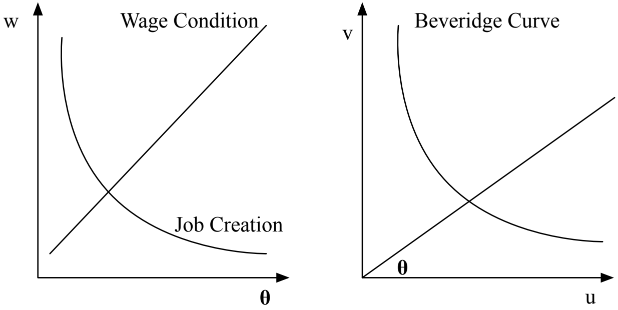
$$rU = b + \frac{\chi}{1-\chi}\theta k$$

- Plug in to $w = rU + \chi(y - rU)$:

$$\begin{aligned} w &= b + \frac{\chi}{1-\chi}\theta k + \beta\left(y - b - \frac{\chi}{1-\chi}\theta k\right) \\ &= (1-\chi)b + \chi(y + \theta k) \end{aligned}$$

Undirected Search With Bargaining: Steady State Solution

$$
\begin{aligned}
\text{Wage Equation:} \quad & w &= (1-\chi)\, b + \chi\,(y + \theta k) \\
\text{Job Creation:} \quad & y - w &= (r+\lambda)\,\frac{k}{q(\theta)} \\
\text{Beveridge Curve:} \quad & u &= \frac{\lambda}{\lambda + \alpha_w} = \frac{\lambda}{\lambda + \theta q(\theta)}
\end{aligned}
$$



Undirected Search With Bargaining: Steady State Solution

- Combine wage equation and job creation to get single easily solved condition in θ:

$$(1-\chi)\frac{y-b}{k} = \frac{r+\lambda}{q(\theta)} + \chi\theta$$

- Interpretation:

$$\underbrace{(1-\chi)\frac{y-b}{r+\lambda+\chi\theta q(\theta)}}_{\text{Value of Job to Firm}} = \underbrace{\frac{k}{q(\theta)}}_{\text{Ave Cost of Recruiting Worker}}$$

- Can do comparative statics.
    - Intuition in terms of firm incentives to post.
    - Example: Increase in b. Workers' outside option improves, they get higher wages, reduces incentive to post, less vacancy posting, θ falls and u rises.

Undirected Search With Bargaining: Dynamics

- w and v are jump variables. u is not.
- Value functions:

$$\begin{aligned} rW(w) &= w + \dot{W} + \lambda \left[U - W(w)\right] \\ rU &= b + \dot{U} + \theta q(\theta)\left[W(w) - U\right] \\ rJ(\pi) &= \pi + \dot{J} + \lambda\left[V - J(\pi)\right] \\ rV &= -k + \dot{V} + q(\theta)\left[J(\pi) - V\right] \end{aligned}$$

- All profits realized immediately: $V = \dot{V} = 0$.
- Wage equation exactly as before.
  - Previous derivation shows it holds in and out of steady state.

Undirected Search With Bargaining: Dynamics

- Following previous steps gives single equation in θ:

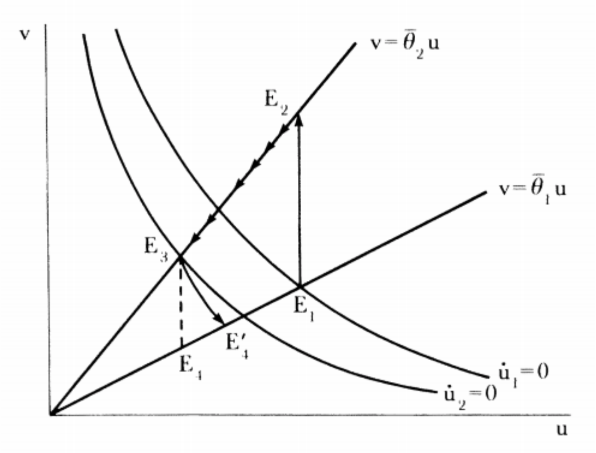
$$\dot{\theta} = \frac{(r+\lambda)\frac{k}{q(\theta)} - (1-\chi)(y-b) + \chi k\theta}{\frac{-k}{[q(\theta)]^2} q'(\theta)}$$

  - Denom ≥ 0, num increasing in $\theta \Rightarrow$ unstable equation in θ.

- Thus only rational expectations equilibrium is $\dot{\theta} = \dot{J} = 0$.
  - Wage equation and job creation jump to steady state values instantaneously.
  - Beveridge Curve shifts instantaneously in response to shock.
  - θ and w jump to steady state values.
  - u moves gradually on Beveridge diagram along θ constant line.

Undirected Search With Bargaining: Dynamics

- Counter-clockwise loops around Beveridge as in data.



Source: Pissarides (1985)

Relation to Shimer (2005)

- Next class we will discuss Shimer (2005), so I want to set up a point of departure.
- Shimer adds shocks to y or λ that occur at poisson rate ζ.
  - Previously had one equilibrium condition:

$$(1-\chi)\frac{y-b}{k}=\frac{r+\lambda}{q(\theta)}+\chi\theta$$

  - This becomes:

$$(1-\chi)\frac{y-b}{k}+\zeta E_{y,\lambda}\frac{1}{q(\theta_{y',\lambda'})}=\frac{r+\lambda+\zeta}{q(\theta_{\theta,\lambda})}+\chi\theta_{y,\lambda}$$

- Trivial to solve this on a grid for (y,λ).
  - Shimer uses approximation to Oronstein-Uhlenbeck process.
  - When shock hits, new value y' moves up or down one grid point with asymmetric probability.

Match-Specific Productivity

- When firms and workers meet, draw match-specific productivity y iid from $F(\cdot)$.
    - Match if $y \geq y_R$, defined by zero surplus from match.
    - Index value functions and wage by y: $W_y(w)$, $J_y(\pi)$, w_y.

$$
\begin{aligned}
rW_y(w) &= w_y + \lambda\left[U - W_y(w)\right] \\
rU &= b + \theta q(\theta) \int_{y_R}^{\infty} \left(W_y(w) - U\right) dF(y) \\
rJ_y(\pi) &= y - w_y + \lambda\left[V - J_y(\pi)\right] \\
rV &= -k + q(\theta) \int_{y_R}^{\infty} \left(J(y - w_y) - v\right) dF(y)
\end{aligned}
$$

- As before, $V = 0$, $\dot{u} = \lambda(1-u) - \theta q(\theta)(1 - F(y_R))u$, $W_y(w) = U + \chi\left[J_y(\pi) + W_y(w) - U\right]$.
    - Shock is multiplicative to y so distribution $F(y)$ is fixed.

Match-Specific Productivity: Math For Reference

- Work with surplus $S_y = [J_y(\pi) - V - W_y(w) - U] = \frac{y - rU}{r+\lambda}$.
- Nash bargaining and free entry give:

$$\begin{aligned} rU &= b + \theta q(\theta) \chi \int_{y_R}^{\infty} S_y dF(y) \\ k &= (1-\chi) q(\theta) \int_{y_R}^{\infty} S_y dF(y) \end{aligned}$$

- Combine to get $rU = b + \frac{\chi}{1-\chi}\theta k$ and plug in to get:

$$S_y = \frac{y - b - \frac{\chi}{1-\chi}\theta k}{r+\lambda}$$

- $S_{y_R} = 0 \Rightarrow y_R = b + \frac{\chi}{1-\chi}\theta k$
- Then $S_y = \frac{y - y_R}{r+\lambda}$ so with free entry have:

$$(r+\lambda) k = q(\theta)(1-\chi) \int_{y_R}^{\infty} (y - y_R) dF(y)$$

Match-Specific Productivity: Summary

- System (back wage out) from surplus split:

$$
\begin{aligned}
y_r &= b + \frac{\chi}{1-\chi}\theta k \\
(r+\lambda)\,k &= q(\theta)(1-\chi)\int_{y_R}^{\infty}(y-y_R)\,dF(y) \\
u &= \frac{\lambda}{\lambda+\theta q(\theta)(1-F(y_R))}
\end{aligned}
$$

- Compare to before:

$$
\begin{aligned}
w &= (1-\chi)\,b + \chi(y+\theta k) \\
(r+\lambda)\,k &= q(\theta)(y-w) \\
u &= \frac{\lambda}{\lambda+\theta q(\theta)}
\end{aligned}
$$

- Similar-looking diagram in (θ, y_R) space. Wage distribution.

Endogenous Job Destruction: Setup

- Mortensen and Pissarides (1994) endogenize job destruction as well as job creation.
  - At poisson rate λ new draw from $F(y'|y)$ with $F(y'|y_2)$ first order stochastically dominating $F(y'|y_1)$ if $y_2 > y_1$.
  - All jobs begin at y_0 sufficiently high.
- Similar to match-specific productivity but now y_R is a *job destruction threshold*.

$$
\begin{aligned}
rW_y(w) &= w_y + \lambda\chi \int_{y_R}^{\infty} \left(\max\left\{S_{y'}, 0\right\} - S_y\right) dF\left(y'|y\right) \\
rU &= b + \theta q(\theta)\left(W_{y_0}(w) - U\right) \\
rJ_y(\pi) &= y - w_y + \lambda(1-\chi) \int_{y_R}^{\infty} \left(\max\left\{S_{y'}, 0\right\} - S_y\right) dF\left(y'|y\right. \\
rV &= -k + q(\theta)\left(J\left(y_0 - w_y\right) - v\right)
\end{aligned}
$$

- $V = 0$, $\dot{u} = \lambda F(y_R)(1-u) - \theta q(\theta) u$

Endogenous Job Destruction: Math Reference

- The surplus is now:

$$S_y = \frac{y - rU + \lambda \int_{y_R}^{\infty} S_{y'} dF\left(y'|y\right)}{r + \lambda}$$

- Writing U in terms of S_y, $rU = b + \theta q\left(\theta\right) \chi S_{y_0}$, plugging in, using $S_y = \frac{y - y_R}{r+\lambda}$ and noting $S_{y_r} = 0$ gives:

$$y_R = b + \theta q\left(\theta\right) \chi \frac{y_0 - y_R}{r + \lambda} - \frac{\lambda}{r + \lambda} \int_{y_R}^{\infty} \left(y' - y_R\right) dF\left(y'|y_R\right)$$

- This *job destruction condition* is increasing in (θ, y_R) space.
- Job creation as in match-specific, decreasing in (θ, y_R) space:

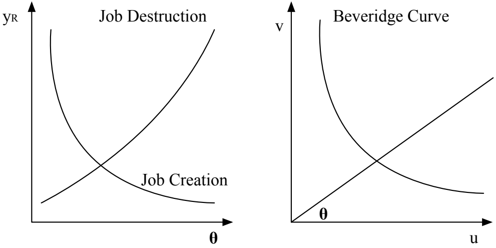
$$\frac{k}{q\left(\theta\right)} = \frac{\left(1 - \chi\right)\left(y_0 - y_R\right)}{r + \lambda}$$

Endogenous Job Destruction: Summary

$$y_R = b + \theta q(\theta)\chi\frac{y_0 - y_R}{r+\lambda} - \frac{\lambda}{r+\lambda}\int_{y_R}^{\infty}(y' - y_R)\, dF(y'|y_R)$$

$$\frac{k}{q(\theta)} = \frac{(1-\chi)(y_0 - y_R)}{r+\lambda}$$

$$u = \frac{\lambda F(y_R)}{\lambda F(y_R) + \theta q(\theta)}$$



Endogenous Job Destruction: Implications

- θ and y_R are jump variables, and the reservation productivity and job creation jump to steady state on impact of shock.
- Creates *asymmetric unemployment dynamics*.
    - If y_R falls, job creation increases (less picky) and destruction decreases as unemployment adjusts gradually.
    - If y_R rises in equilibrium to y_R', all matches with productivity in $[y_R, y_R']$ are suddenly laid off in a burst of job destruction.
    - Unemployment quickly in recessions, falls slowly in recoveries.
- Large and brief spike in separations early in recession consistent with Davis and Haltiwanger (1990, 1991) facts.
- Mortensen and Pissarides (1994) simulate with 3 state Markov process (so don't track endogenously deforming y distribution).
- Note: All separations are mutual.

On-The-Job Search

- Lots of permutations. See RSW (2005) and Pissarides (2000).
- Briefly discuss extending match-specific productivity model by adding on-the-job search to account for E-E flows (2.9%, as opposed to 2.6 for E-U).
- $m = m(u + e, v)$ where e is on-the-job searchers.
    - Costs ψ to search.
    - Two reservation productivities, y_R and y_s.
- Key finding: Sorting. Reject $[0, y_R)$, accept and on-the-job search $[y_R, y_s)$, accept and no on-the-job search $[y_s, \infty)$.
    - On-the-job search taken into account in Nash bargaining.
    - Lower wage to compensate firm for possibility may leave.
    - For $[y_R, y_s)$, willing to accept lower wage and pay ψ. But for high enough productivity, wage is high enough and odds do better are low enough that not willing.
- Gives "job ladder."

Efficiency: The Hosios Condition

- Write planner's problem in basic Pissarides (1985) model:

$$\max_{u,\theta} \int_0^\infty e^{-rt} \left[y\left(1-u\right) + bu - k\theta u \right]$$
$$\text{s.t. } \dot{u} = \lambda\left(1-u\right) - \theta q\left(\theta\right) u$$

  - Linear utility $\Rightarrow$ no distributional concerns.
- Set up Hamiltonian (for convenience costate variable is $-$) :

$$H = e^{-rt}\left[y\left(1-u\right) + bu - k\theta u \right] - \mu\left[\lambda\left(1-u\right) - \theta q\left(\theta\right) u \right]$$

- FOC $H_\theta = 0$ and $H_u = \dot{\mu}$ are:

$$\begin{aligned} -e^{-rt} ku + \mu u q\left(\theta\right)\left(1-\eta\left(\theta\right)\right) &= 0 \\ -e^{-rt}\left(y - b + k\theta\right) + \mu\left[\lambda + \theta q\left(\theta\right)\right] &= \dot{\mu} \end{aligned}$$

  - $\eta\left(\theta\right) = -\frac{\theta q'(\theta)}{q(\theta)}$ is the elasticity of the matching function.

Efficiency: The Hosios Condition

- Solve for μ from first equation to get $\mu = \frac{e^{-rt}k}{q(\theta)(1-\eta(\theta))}$ so in steady state $\dot{\mu} = -r\frac{e^{-rt}k}{q(\theta)(1-\eta(\theta))}$.
- Plug both into H_u FOC and manipulate to get:

$$(1-\eta(\theta))\frac{y-b}{k} = \frac{r+\lambda}{q(\theta)} + \eta(\theta)\theta q(\theta)$$

- Recall that:

$$(1-\chi)\frac{y-b}{k} = \frac{r+\lambda}{q(\theta)} + \chi\theta q(\theta)$$

- Efficiency thus occurs when the *Hosios (1990) condition* holds:

$$\chi = \eta(\theta)$$

Efficiency: Hosios Condition Intuition

- Think about *search (congestion) externalities*:
  - Additional *u* imposes externality on other *u*s by $\downarrow \alpha_w$.
  - Additional *v* imposes externality on other *v*s by $\downarrow \alpha_e$.
  - Social planner wants to balance two externalities.

- Elasticity of duration of unemployment wrt *u* is $1 - \eta(\theta)$, elasticity of duration of vacancy wrt *v* is $\eta(\theta)$.
  - If $\eta(\theta)$ is high, marginal firm imposing bigger search externality on other firms than marginal worker imposing on other workers.
  - So social planner wants to tax firms by giving more of wage to workers by increasing χ.
  - Things balance when:

$$\frac{1-\eta(\theta)}{\eta(\theta)} = \frac{1-\chi}{\chi}$$

Efficiency: Hosios Condition Intuition[2]

- Another way of seeing it is that with a fixed prob of match $\bar{q}$, the optimality condition would be:

$$\underbrace{k}_{\text{Per-Period Cost}} = \underbrace{\mu}_{\text{Social Value of Job}} \times \underbrace{\bar{q}}_{\text{Prob of Match}}$$

- However, q is not fixed so:

$$\begin{aligned} k &= \mu\left(q(\theta) + \theta q'(\theta)\right) \\ &= \mu \times q(\theta)\left(1 + \underbrace{\eta(\theta)}_{\text{Neg Congestion Externality}}\right) \end{aligned}$$

- To internalize externality, reduce reward for job creation by fraction $\eta(\theta)$ of value of match.

[2]Thanks to Edouard Schaal for this intuition.

Directed Search and Wage Posting: Idea

- Undirected search with bargaining is inefficient because agents are "too passive."
  - No competitive forces.
- Directed search provides an alternative.
  - Workers can choose where to search (e.g. firms with different wages).
  - Also alternative to Nash bargaining: wages are posted by profit-maximizing firms.
  - Issue of commitment to posted wage, which I will ignore.
- Directed search is more tractable in some respects.
  - Easier to introduce risk aversion (Acemoglu and Shimer, 1999)
  - Block recursive equilibrium (Menzio and Shi, 2011) allows for substantial heterogeneity without ∞-dimensional state space.

Directed Search and Wage Posting: Idea

- An alternative to DMP framework is directed search with wage posting as in Moen (1997) and Shimer (1996).
- Unemployed direct search to "submarkets" with different ws.
  - Once in submarket, random with frictions, so each submarket has matching function $q(\theta)$.
  - Higher $w \Rightarrow$ more $u \Rightarrow$ lower $\theta \Rightarrow \alpha_e = q(\theta) \uparrow$, $\alpha_w \downarrow$.
  - Workers sort across submarkets so in equilibrium indifferent.
- Firms post jobs *anticipating worker behavior*.
  - Optimize given worker indifference constraint, which determines $\theta(w)$.
- Competitive equilibrium given search frictions.
  - Efficiency (Hosios) endogenously holds.
  - Sometimes called "competitive search."

Directed Search and Wage Posting: Setup

- Value functions for worker:

$$
\begin{aligned}
rU &= b + \max_{w} \{\theta(w)\, q(\theta(w))\, [W(w) - U]\} \\
rW(w) &= w + \lambda\, [U - W(w)]
\end{aligned}
$$

- In equilibrium, workers are indifferent, so $\theta(w)$ implicitly defined by:

$$
rU = b + \frac{\theta(w)\, q(\theta(w))\, (w - rU)}{r + \lambda}
$$

- Value functions for firms with free entry:

$$
\begin{aligned}
rJ(y - w) &= y - w - \lambda J(y - w) \\
k &= \frac{q(\theta)(y - w)}{r + \lambda}
\end{aligned}
$$

Directed Search and Wage Posting: Solution

- Firms max value of vacancy taking worker search as given:

$$\max_{w} q(\theta(w))(y-w) \text{ s.t. } rU = b + \frac{\theta(w)\, q(\theta(w))(w-rU)}{r+\lambda}$$

- Focus on symmetric equilibrium. Convex problem $\Rightarrow$ all firms choose same w; one active submarket. FOC:

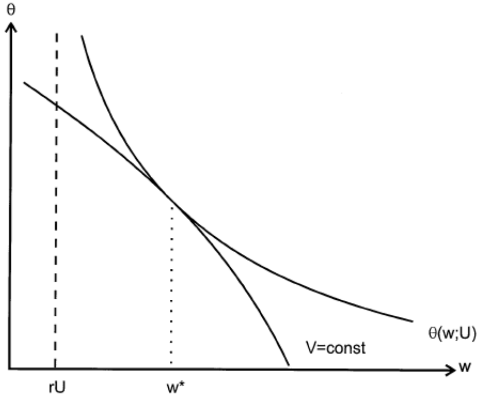
$$q'(\theta)\frac{d\theta}{dw}(y-w) = q(\theta)$$

- Differentiate constraint to get:

$$\begin{aligned}(1-\eta(\theta))\frac{d\theta}{dw} &= -\frac{\theta}{w-rU}\\ \frac{1-\eta(\theta)}{\eta(\theta)} &= \frac{y-w}{w-rU} = \frac{J(y-w)}{W(w)-U}\end{aligned}$$

- Equivalent to Nash bargaining with weights $\eta(\theta)$
  $\Rightarrow$ Hosios endogenously satisfied.
  - Can show constrained efficiency.

Directed Search and Wage Posting: Diagram (Moen, 1997)



- Constrained efficient because off-equilibrium-path option of posting different wage leads to $\theta(w)$ tightness schedule.
  - Firms internalize effect wage choice has on tightness.
  - Internalizes search externalities.

Block Recursivity: Menzio and Shi (2010)

- Menzio and Shi (2010) show directed search *with free entry* is convenient because one can handle heterogeneity easily.
- This is because the equilibrium is *block recursive.*
  - Value and policy functions do not depend on distribution of heterogenous workers across employment states.
  - Paper is about on-the-job search.
    - Employment states are unemployed or employed of match with various idiosyncratic productivities.
    - Aggregate state is aggregate component of productivity.
  - BRE: Can solve on a grid of the aggregate productivity.
- Schaal (2015) extends to multi-worker firms with DRS and both worker and firm heterogeneity.

Random Search Not Block Recursive

- By contrast, random search is *not* block recursive.
    - Wages depend on workers' and firms' outside options, which depend on distribution of workers across firms.
    - Hard to solve out of steady state.
- Need to make restrictive assumptions with random search. Examples:
    - No free entry: exogenous contact rates.
    - All bargaining occurs relative to unemployment with no permanent worker heterogeneity.

Menzio and Shi (2010): Block Recursivity Intuition

- Different types of workers (in productivity of existing job with on-the-job search) search for different types of vacancies independently of how many are in each state.
- If firm opens vacancy in given submarket, knows exactly what type of worker it will meet, so expected value of meeting worker independent of distribution.
- Free entry $\Rightarrow$ probability of matching independent of dist $\Rightarrow$ value and policy functions independent of dist.

Schaal (2015): Alternate Block Recursivity Intuition

- Wages are choice variables, so no need to forecast.
    - Only place where distribution may matter is market tightness.
- Free entry equalizes cost of opening vacancy to value of job, which depends on probability job is created (and thus market tightness) and surplus of match but not on distribution.
    - Cost of opening vacancy is constant, so can invert free entry to pin down market tightness as function of value of new job.
- Value of new job is not directly affected by distribution, only indirectly through future market tightness.
    - But free entry pins down future market tightness as a function of aggregate productivity not distribution.
- So equilibrium exists that does not depend on distribution.

Menzio and Shi (2010): Note on Proof

- The proof is technical but fairly simple:

1. Write down single value function for unemployed worker and employed workers of each type.
2. Write down free entry condition, invert and plug into value function. Note free entry condition does not depend on distributions.
3. Value and free entry together are contraction. Invoke Blackwell.
4. Value and free entry do not depend on distributions, only aggregate productivity. Contraction maps to new value function with same property. So unique solution by Blackwell must have same property.

Undirected Search and Posting

- Random matching with posting is another paradigm.
  - Diamond (1971) paradox applies $\Rightarrow$ all firms post w_R.
  - Get around this with heterogenous leisure or on-the-job search.
- Burdett and Mortensen (1998) is famous paper here.
  - On-the-job search.
  - Get endogenous distribution of wages. Firms pay different wages to increase inflow and reduce outflow of workers. With free entry, indifferent between posting different wages.
  - This sort of mixed-strategy equilibrium is typical with undirected search and posting.
- See Rogerson, Shimer, and Wright (2005) for details.

Where to Go From Here

- See Pissarides (2000) and Rogerson, Shimer, and Wright (2005) for more permutations.
- Other tools I have not covered:
  - Models of frictional wage dispersion.
  - Structurally estimated search models and bargaining protocols that make SMM feasible (people like Postel-Vinay and Robin).
  - Will touch on these in future lectures.
- Next class: How well do search models explain the world?