Spender Saver Model

Ec 704 · Spring 2024 · Prof. Adam M. Guren

- Households are of two types: λ rule of thumb and $1-\lambda$ optimizing.
- For simplicity ignore money market, but we could always put it in.
- Optimizing households:

$$\max_{C_t^o, N_t^o, B_t^o, M_t^o} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{\left(C_{t+s}^o\right)^{1-\gamma}}{1-\gamma} - \chi \frac{\left(N_{t+s}^o\right)^{1+\varphi}}{1+\varphi} \right) \right\}$$
$$\text{s.t. } C_t^o = \frac{W_t}{P_t} N_t^o - \frac{B_t^o - Q_{t-1} B_{t-1}^o}{P_t} + TR_t^o + PR_t^o - T_t^o$$

  - FOCs:

$$\begin{aligned} \frac{W_t}{P_t} &= \chi \left(N_t^o\right)^{\varphi} \left(C_t^o\right)^{\gamma} \\ 1 &= \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{\left(C_{t+1}^o\right)^{-\gamma}}{\left(C_t^o\right)^{-\gamma}} \right\} = E_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\} \end{aligned}$$

- Non-optimizing households:

$$\max_{N_t} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{\left(C_{t+s}^r\right)^{1-\gamma}}{1-\gamma} - \chi \frac{\left(N_{t+s}^r\right)^{1+\varphi}}{1+\varphi} \right) \right\}$$
$$\text{s.t. } P_t C_t^r = W_t N_t^r - P_t T_t^r$$

  with FOC:

$$\frac{W_t}{P_t} = \chi \left(N_t^o\right)^{\varphi} \left(C_t^r\right)^{\gamma}$$

- Household aggregation:

$$\begin{aligned} C_t &= \lambda C_t^r + (1-\lambda) C_t^o \\ N_t &= \lambda N_t^r + (1-\lambda) N_t^o \end{aligned}$$

- Firm side is exactly as before

$$\begin{aligned} P_t &= \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \\ P_t^* &= (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s}^n P_{t+s}^{\varepsilon} Y_{t+s}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k}^n P_{t+k}^{\varepsilon} Y_{t+k}} \frac{W_{t+s}}{A_{t+s}} \right\} \\ Y_t &= C_t + G_t \\ Y_t &= A_t N_t \left[\int_0^1 \left(\frac{N_t(i)}{N_t} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

- Monetary policy is:

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

  where

$$v_t = \rho_v v_{t-1} + \varepsilon_t$$

- Fiscal policy:

- Government budget constraint is

$$P_tT_t + B_t = Q_{t-1}B_{t-1} + P_tG_t$$

where $T_t = \lambda T_t^r + (1-\lambda)\,T_t^o$.

- Government spending is constant at G.
- Assume that government lump sum taxes or rebates consumers equally when cost of debt changes.

- Market clearing:

$$N_t = \int_0^1 N_t\,(i)\,di$$

- Deriving the NKPC:
  - For firm side,

$$\hat{\pi}_t = \lambda \hat{mc}_t + \beta E_t\left\{\hat{\pi}_{t+1}\right\}$$

  - Marginal costs are

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t$$

where

$$\hat{w}_t - \hat{p}_t = (\gamma + \varphi)\,\hat{y}_t - \varphi \hat{a}_t$$

so

$$\hat{mc}_t = (\gamma + \varphi)\,\hat{y}_t - (1+\varphi)\,\hat{a}_t$$

  - But we do not have technology shocks so

$$\hat{mc}_t = (\gamma + \varphi)\,\hat{y}_t$$

  - Consequently:

$$\begin{aligned}\hat{\pi}_t &= \lambda\,(\gamma+\varphi)\,\hat{y}_t + \beta E_t\left\{\hat{\pi}_{t+1}\right\} \\ &= \kappa \hat{y}_t + \beta E_t\left\{\hat{\pi}_{t+1}\right\}\end{aligned}$$

- Deriving the IS curve:
  - On the aggregate demand side, the Euler equation for the optimizers is:

$$\hat{c}_t^o = -\sigma\left(\hat{i}_t - E_t\left\{\hat{\pi}_{t+1}\right\}\right) + E_t\left\{\hat{c}_{t+1}^o\right\}$$

  - The consumption of the rule of thumb agents is:

$$C_t^r = (W_t/P_t)\,N_t^r - T_t^r$$

log linearized and let

$$t_t = \frac{T_t - T}{Y}$$

we have:

$$\hat{c}_t^r = \frac{WN^r}{PC^r}\left(\hat{w}_t - \hat{p}_t + \hat{n}_t^r\right) - \frac{Y}{C^r}\hat{t}_t^r$$

  - Assuming $C^r = C^o = C$ which implies $N^r = N^o = N$, we have:

$$\begin{aligned}\hat{c}_t &= \lambda \hat{c}_t^r + (1-\lambda)\,\hat{c}_t^o \\ \hat{n}_t &= \lambda \hat{n}_t^r + (1-\lambda)\,\hat{n}_t^o \\ \hat{w}_t - \hat{p}_t &= \gamma \hat{c}_t + \varphi \hat{n}_t\end{aligned}$$

and defining $\gamma_c = C/Y$ we can rewrite the equation for $\hat{c}_t^r$ as:

$$\gamma_c \hat{c}_t^r = \frac{WN}{PY}\left(\hat{w}_t - \hat{p}_t + \hat{n}_t^r\right) - \hat{t}_t^r$$

– But we know that

$$\hat{n}_t^r = \frac{1}{\varphi}\left(\hat{w}_t - \hat{p}_t - \gamma \hat{c}_t^r\right)$$

so

$$\left(\varphi\gamma_c + \frac{WN}{PY}\gamma\right)\hat{c}_t^r = \frac{WN}{PY}\left(1+\varphi\right)\left(\hat{w}_t - \hat{p}_t\right) - \varphi\hat{t}_t^r$$

– But

$$\hat{w}_t - \hat{p}_t = \gamma\hat{c}_t + \varphi\hat{n}_t$$

so

$$\left(\varphi\gamma_c + \frac{WN}{PY}\gamma\right)\hat{c}_t^r = \frac{WN}{PY}\left(1+\varphi\right)\left(\gamma\hat{c}_t + \varphi\hat{n}_t\right) - \varphi\hat{t}_t^r$$

– Finally,

$$\frac{WN}{PY} = \frac{1}{\mu}$$

so

$$\left(\mu\varphi\gamma_c + \gamma\right)\hat{c}_t^r = \left(1+\varphi\right)\left(\gamma\hat{c}_t + \varphi\hat{n}_t\right) - \mu\varphi\hat{t}_t^r$$

or

$$\hat{c}_t^r = \frac{\gamma\left(1+\varphi\right)}{\mu\varphi\gamma_c + \gamma}\hat{c}_t + \frac{\varphi\left(1+\varphi\right)}{\mu\varphi\gamma_c + \gamma}\hat{n}_t - \frac{\mu\varphi}{\mu\varphi\gamma_c + \gamma}\hat{t}_t^r$$

– We have that

$$\hat{c}_t = \lambda\hat{c}_t^r + \left(1-\lambda\right)\hat{c}_t^o$$

so

$$\begin{aligned} c_t - E_t\left\{c_{t+1}\right\} &= \lambda\left[c_t^r - E_t\left\{c_{t+1}^r\right\}\right] + \left(1-\lambda\right)\left[c_t^o - E_t\left\{c_{t+1}^o\right\}\right] \\ &= \lambda\left[c_t^r - E_t\left\{c_{t+1}^r\right\}\right] - \sigma\left(1-\lambda\right)\left(\hat{i}_t - E_t\left\{\hat{\pi}_{t+1}\right\}\right) \end{aligned}$$

– Plugging in the previous result gives:

$$\begin{aligned} c_t - E_t\left\{c_{t+1}\right\} &= \lambda\left[c_t^r - E_t\left\{c_{t+1}^r\right\}\right] - \sigma\left(1-\lambda\right)\left(\hat{i}_t - E_t\left\{\hat{\pi}_{t+1}\right\}\right) \\ &= \lambda\left[\begin{array}{c} \frac{\gamma(1+\varphi)}{\mu\varphi\gamma_c+\gamma}\hat{c}_t + \frac{\varphi(1+\varphi)}{\mu\varphi\gamma_c+\gamma}\hat{n}_t - \frac{\mu\varphi}{\mu\varphi\gamma_c+\gamma}\hat{t}_t^r \\ -E_t\left\{\frac{\gamma(1+\varphi)}{\mu\varphi\gamma_c+\gamma}\hat{c}_{t+1} + \frac{\varphi(1+\varphi)}{\mu\varphi\gamma_c+\gamma}\hat{n}_{t+1} - \frac{\mu\varphi}{\mu\varphi\gamma_c+\gamma}\hat{t}_{t+1}^r\right\} \end{array}\right] \\ &\quad -\sigma\left(1-\lambda\right)\left(\hat{i}_t - E_t\left\{\hat{\pi}_{t+1}\right\}\right) \\ \left(\frac{\mu\varphi\gamma_c + \gamma - \lambda\gamma\left(1+\varphi\right)}{\mu\varphi\gamma_c+\gamma}\right)\left(\hat{c}_t - E_t\left\{\hat{c}_{t+1}\right\}\right) &= -\lambda\frac{\varphi\left(1+\varphi\right)}{\mu\varphi\gamma_c+\gamma}E_t\left\{\Delta\hat{n}_{t+1}\right\} + \frac{\lambda\mu\varphi}{\mu\varphi\gamma_c+\gamma}E_t\left\{\Delta\hat{t}_{t+1}^r\right\} \\ &\quad -\sigma\left(1-\lambda\right)\left(\hat{i}_t - E_t\left\{\hat{\pi}_{t+1}\right\}\right) \end{aligned}$$

where $\Delta\hat{n}_{t+1} = E_t\left\{\hat{n}_{t+1}\right\} - \hat{n}_t$ and $\Delta\hat{t}_{t+1}^r = E_t\left\{\hat{t}_{t+1}^r\right\} - \hat{t}_t^r$.

– Letting $\Gamma = \left(\mu\varphi\gamma_c + \gamma - \lambda\gamma\left(1+\varphi\right)\right)^{-1}$,

$$\begin{aligned} \hat{c}_t - E_t\left\{\hat{c}_{t+1}\right\} &= -\lambda\Gamma\varphi\left(1+\varphi\right)E_t\left\{\Delta\hat{n}_{t+1}\right\} + \lambda\mu\varphi\Gamma E_t\left\{\Delta\hat{t}_{t+1}^r\right\} \\ &\quad -\sigma\left(1-\lambda\right)\Gamma\left(\mu\varphi\gamma_c+\gamma\right)\left(\hat{i}_t - E_t\left\{\hat{\pi}_{t+1}\right\}\right) \end{aligned}$$

or equivalently

$$\hat{c}_t = E_t\left\{\hat{c}_{t+1}\right\} - \tilde{\sigma}\left(\hat{i}_t - E_t\left\{\hat{\pi}_{t+1}\right\}\right) - \Theta_n E_t\left\{\Delta\hat{n}_{t+1}\right\} + \Theta_\tau E_t\left\{\Delta\hat{t}_{t+1}^r\right\}$$

where

$$
\begin{aligned}
\tilde{\sigma} &= \sigma(1-\lambda)\Gamma(\mu\varphi\gamma_c+\gamma) \\
\Theta_n &= \lambda\Gamma\varphi(1+\varphi) \\
\Theta_\tau &= \lambda\mu\varphi\Gamma
\end{aligned}
$$

which is a dynamic IS curve!

- Government budget constraint:
  - Linearize
  $$P_tT_t + B_t = Q_{t-1}B_{t-1} + P_tG_t$$
  where $T_t = \lambda T_t^r + (1-\lambda) T_t^o$.
  - Rewrite as:
  $$P_tT_t + \frac{\bar{B}_t}{Q_t} = \bar{B}_{t-1} + P_tG_t$$
  - Let $t_t = \frac{T_t - T}{Y}$ and linearize around a steady state with debt B and a balanced budget. Let $\gamma_b = \frac{B}{PG+B}$. Then with no government spending shock:
  $$(1-\gamma_b)\hat{t}_t + = \gamma_b\left(\hat{i}_t - \hat{b}_t\right) + \gamma_b\hat{b}_{t+1}$$
  - However, we always have zero debt and lump sum rebate so:
  $$\hat{t}_t = \frac{\gamma_b}{1-\gamma_b}\hat{i}_t$$
  - Also given our assumption that it is rebated proportionally,
  $$\hat{t}_t^r = \hat{t}_t$$
  and so expansionary monetary policy increases transfers.
- Market clearing
$$\hat{y} = \gamma_c\hat{c}_t$$
- The full equilibrium system:

$$
\begin{aligned}
\hat{c}_t &= E_t\{\hat{c}_{t+1}\} - \tilde{\sigma}\left(\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}\right) - \Theta_n E_t\{\Delta\hat{n}_{t+1}\} + \Theta_\tau E_t\left\{\Delta\hat{t}_{t+1}^r\right\} \\
\hat{\pi}_t &= \kappa\hat{y}_t + \beta E_t\{\hat{\pi}_{t+1}\} \\
\hat{y}_t &= \hat{n}_t \\
\hat{y}_t &= \gamma_c\hat{c}_t \\
\hat{t}_t &= \frac{\gamma_b}{1-\gamma_b}\hat{i}_t \\
\hat{i}_t &= \phi_\pi\hat{\pi}_t + \hat{v}_t \\
\hat{v}_t &= \rho_v\hat{v}_{t-1} + \varepsilon_t
\end{aligned}
$$

- This can be simplified, by combining everything into an AS and AD equation, but I will just load it into Dynare as is.