Economics 704a Lecture 5: Monopolistic Competition and Markups

Adam M. Guren

Boston University

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Monopolistic Competition and Markups

- Goal: Add nominal rigidity for non-neutrality.
- Problem: How does nominal rigidity work with CRS and perfect competition?
    - Older literature: Rationing with output determined as minimum of supply and demand at given price.
    - Newer Literature: Get rid of CRS and perfect competition and replace with IRS and imperfect competition.
- But how do we have features of oligopoly without modeling the industrial organization, which is a mess in GE?
- Blanchard and Kiyotaki (1987) and subsequent literature: Use *monopolistic competition*.
    - Idea going back to Chamberlain (1933), but popularized by tractable setup of Dixit and Stiglitz (1977).
    - Monopolistic competition is widely used in GE modeling (macro, trade, labor, etc.) and is a tool you should know.
    - Will also allow me to introduce concepts about monopoly.

Monopolistic Competition and Markups

1. Dixit-Stiglitz Preferences and Production
2. Markups and Monopolistic Competition
3. RBC With Monopolistic Competition: The Frictionless Benchmark
4. One Period Nominal Rigidity

Monopolistic Competition

- Continuum of goods ("varieties") $i \in [0, 1]$ with a monopolist for each good.
- Each monopolist faces a downward-sloping demand curve.
    - Substitution between goods imperfect due to "love of variety."
- Each monopolist's optimal choice has an infinitesimal effect on economy-wide aggregates.
    - Industrial organization in GE is simple.
    - Imperfect competition without game theory.
- Start with demand curve from consumer preferences, then firm optimization problem.

Dixit-Stiglitz Preferences

- Idea: CES over a continuum of goods:

$$E_t\left\{\sum_{s=0}^{\infty}\beta^s\left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma}+\zeta\frac{\left(M_{t+s}/P_{t+s}\right)^{1-\nu}}{1-\nu}-\chi\frac{N_{t+s}^{1+\varphi}}{1+\varphi}\right)\right\} \text{ where}$$

$$C_t=\left[\int_0^1 C_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ with } \varepsilon>0$$

- Budget constraint:

$$\begin{aligned}\int_0^1 P_t\left(i\right)C_t\left(i\right)di+B_t+M_t &\leq Q_{t-1}B_{t-1}+M_{t-1}\\ &+W_tN_t+P_t\times\left(TR_t+PR_t\right)\end{aligned}$$

- C_t is sometimes called a “Dixit-Stiglitz aggregate.”

Solving Dixit-Stiglitz: Two-Stage Budgeting

- Two-Stage Budgeting Theorem (Deaton and Muellbauer):
  - If upper stage is separable and lower stage is homothetic, can use two-stage budgeting with nested preferences.
  - Solve the inner nest taking expenditure as given and outer nest by standard utility maximization given inner nest optimization to determine expenditure on bundle purchased in inner nest.
- Example:

$$U = C_t^{\mu} H_t^{1-\mu} \text{ where } C_t = \left[\int_0^1 C_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  - CES is homothetic and C-D is separable (after taking logs).
  - Cost minimize for $C_t\left(i\right)$ as a function of C_t and then use C-D:

$$\mu = C_t P_C / Y_t \text{ and } 1 - \mu = H_t P_H / Y_t$$

- We can use two-stage budgeting here.

Solving Dixit-Stiglitz: Inner Nest Maximization

- Letting X_t be expenditure on Dixit-Stilgitz goods:

$$\max_{\{C_t(i)\}_{i=0}^{1}} \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \lambda\left(\int_0^1 P_t(i)\, C_t(i)\, di - X_t\right)$$

$$C_t(i)^{\frac{-1}{\varepsilon}} C_t^{\frac{1}{\varepsilon}} = \lambda P_t(i)$$

- For any two goods i and j,

$$C_t(i) = C_t(j)\left(\frac{P_t(i)}{P_t(j)}\right)^{-\varepsilon} = \frac{P_t(i)^{-\varepsilon}}{P_t(j)^{1-\varepsilon}} P_t(j)\, C_t(j)$$

- Bring the denominator over and integrate wrt j:

$$C_t(i)\int_0^1 P_t(j)^{1-\varepsilon}\, dj = P_t(i)^{-\varepsilon}\int_0^1 P_t(j)\, C_t(j)\, dj$$

$$C_t(i) = \frac{P_t(i)^{-\varepsilon}}{\int_0^1 P_t(j)^{1-\varepsilon}\, dj} X_t$$

Solving Dixit-Stiglitz: Price Index

- Indirect utility is:

$$
\begin{aligned}
v\left(P_t(i)\big|_{i=0}^{1}, X_t\right) &= \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \frac{X_t}{\left[\int_0^1 P_t(i)^{1-\varepsilon}\, di\right]^{\frac{1}{1-\varepsilon}}}
\end{aligned}
$$

- The cost of buying one unit of utility is:

$$
P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon}\, di\right]^{\frac{1}{1-\varepsilon}}
$$

  - This is an ideal price index.
  - Index is geometric weighted average of individual good prices.

Solving Dixit-Stiglitz: Demand Function

- $X_t = P_t C_t$, so plugging in price index gives:

$$\begin{aligned} C_t(i) &= \frac{P_t(i)^{-\varepsilon}}{P_t^{1-\varepsilon}} X_t \\ &= \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t \end{aligned}$$

- CES structure delivers *constant elasticity demand function*.
  - Elasticity of demand is elasticity of substitution ε.
  - As $\varepsilon \to \infty$, perfect substitutes and demand perfectly elastic.
  - As $\varepsilon \to 1$, less perfect substitutes and demand more inelastic (but still elastic as $\varepsilon > 1$).
- Each firm has infinitesimal impact on C_t and P_t and treats them as exogenous.

Solving Dixit-Stiglitz: Upper Stage

- With price index P_t, budget constraint can be written as:

$$P_t C_t + B_t + M_t \quad \leq \quad Q_{t-1} B_{t-1} + M_{t-1} + W_t N_t + P_t \left(TR_t + PR_t \right)$$

- Solve upper stage as normal with C_t as Dixit-Stiglitz aggregate:

$$\begin{aligned}
\frac{W_t}{P_t} &= \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \\
1 &= \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} \\
1 &= \beta E_t \left\{ \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} + \zeta \frac{\left(\frac{M_t}{P_t} \right)^{1-\nu}}{C_t^{-\gamma}}
\end{aligned}$$

Uses of Dixit-Stiglitz

- Dixit-Stiglitz is frequently used in GE modeling both in macro and other subfields.
    - Often along with free entry margin that drives profits to zero and endogenously determines number of products.
- Noteworthy Examples:
    - "New" Trade Theory (Krugman, 1980): Love of variety explains high volume of intra-industry trade, e.g. Japan exports Lexus to Germany and Germany exports Mercedes to Japan.
    - New Economic Geography (Krugman, 1990): Urbanization determined by balance between dispersion forces (e.g., housing supply) and agglomeration forces created by increasing returns. As trade costs fall, cities should develop.
    - Endogenous Growth Theory (Romer, 1990): Profits give entrepreneurs incentives to invest in creating new products. Growth through endogenously expanding product variety.

Dixit-Stiglitz Production

- Dixit-Stiglitz is used two ways:
  - Preferences: Households consume each good i, CES preferences over continuum of goods.
  - Production: Households consume final good assembled from intermediates i, CES production fn over continuum of goods.
- These are essentially equivalent.
  - We used utility maximization given to expenditure X_t, but same as cost minimization (duality theory).
  - Cost min s.t. D-S utility level C_t mathematically equivalent to profit max s.t. CES production is C_t (up to sign change).
  - Intuition: Does not matter where continuum is as long as it as CES structure.
- Gali book presents model using Dixit-Stiglitz preferences. I will use Dixit-Stiglitz production.

Dixit-Stiglitz Production

- Final consumption good at time 0, Y_0, is numeraire. Produced from continuum of intermediates $Y_t(i)$, $i \in [0,1]$:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ where } \varepsilon > 1$$

- Choose intermediate input demands by cost min:

$$\min_{\{Y_t(i)\}_{i=0}^1} \int_0^1 P_t(i)\, Y_t(i)\, di \text{ s.t. } Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Dixit-Stiglitz Production

$$\mathcal{L} = \min_{\{Y_t(i)\}_{i=0}^{1},\lambda} \int_0^1 P_t(i)\, Y_t(i)\, di - \lambda_t \left[\left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di \right]^{\frac{\varepsilon}{\varepsilon-1}} - Y_t \right]$$

$$P_t(i) = \lambda_t \left(\frac{Y_t(i)}{Y_t} \right)^{-1/\varepsilon}$$

- Mult both sides by $Y_t(i) / Y_t$ and integrate:

$$\frac{\int_0^1 P_t(i)\, Y_t(i)\, di}{Y_t} = \lambda_t \frac{\int_0^1 Y_t(i)^{1-1/\varepsilon}\, di}{Y_t^{1-1/\varepsilon}} = \lambda_t$$

- From this we see that

$$\lambda_t Y_t = \int_0^1 P_t(i)\, Y_t(i)\, di$$

so by the definition of the ideal price index $\lambda_t = P_t$ is the least cost of producing one unit of Y_t.

Demand and Price Index

- Demand is then:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$$

  - Again, a constant elasticity demand curve.

- Market share is

$$\frac{P_t(i)\,Y_t(i)}{P_t Y_t} = \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon}$$

  - Since $\varepsilon > 1$, market share falls as relative price rises because intermediate inputs are gross substitutes.

- Integrate over market share to get price index as before:

$$\frac{\int_0^1 P_t(i)^{1-\varepsilon}\, di}{P_t^{1-\varepsilon}} = 1 \Rightarrow P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon}\, di\right]^{\frac{1}{1-\varepsilon}}$$

Monopolists and the Markups Formula

- General monopolist problem:

$$\max_{Q} P(Q)\,Q - C(Q)$$

- FOC is:

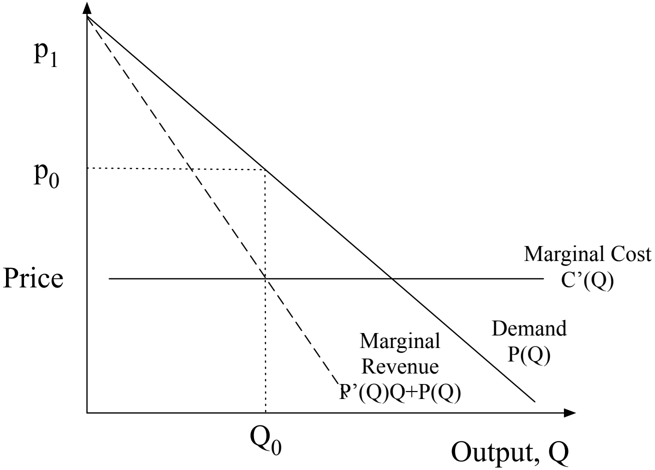
$$P'(Q)\,Q + P(Q) = C'(Q)$$

- Recalling $\varepsilon_{demand} = -\frac{P}{Q}\frac{\partial Q}{\partial P}$,

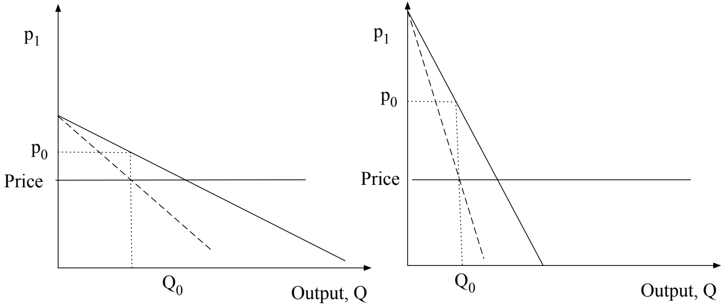
$$\begin{aligned}
\frac{1}{\varepsilon_{demand}} &= -\frac{Q}{P}P'(Q) = 1 - \frac{C'(Q)}{P(Q)} \\
\frac{P(Q)}{C'(Q)} &= \frac{1}{1-\frac{1}{\varepsilon_{demand}}} = \frac{\varepsilon_{demand}}{\varepsilon_{demand}-1}
\end{aligned}$$

- Price is a multiplicative markup over marginal cost.
    - Markup is inversely related to elasticity of demand.
    - Monopolist always on elastic portion of demand curve.

Monopoly Diagram



Monopoly Diagram: Markups and Elasticity



- More inelastic ⇒ bigger markup

Dixit-Stiglitz: Fixed Markup

- Each producer is a monopolist in its own variety and faces a demand curve with elasticity ε.
- Consequently,

$$\begin{aligned} \frac{P_t(i)}{P_t} &= \frac{\varepsilon}{\varepsilon - 1} MC_t \\ \text{Real Price} &= (1+\mu) \text{ Real Marginal Cost} \end{aligned}$$

- In RBC framework, we typically assume:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

- Producing an additional unit of $Y_t(i)$ requires $(1-\alpha) Y_t(i) / N_t(i)$ units of labor at real cost W_t/P_t so:

$$\frac{P_t(i)}{P_t} = (1+\mu) \frac{W_t/P_t}{(1-\alpha) Y_t(i) / N_t(i)}.$$

Alternatives to Dixit-Stiglitz

- Dixit-Stiglitz is convenient, but fixed markup is stark.
    - Variable markup important in practice and gives different economics.
    - Example: When expose to international competition, markups fall. D-S does not allow.
- As a result, trade economists have come up with tractable alternatives that give variable markups.
    - Quasilinear quadratic (e.g., Melitz and Ottaviano, 2008)
    - Translog (e.g., Feenstra)
    - Atkeson and Burstein (2008): CES with Bertrand within sectors
    - Kimball (1995) generalization of CES to variable markup
    - No time to cover: Want you to be aware of issue.
- Also, in models with endogenous entry margin and determination of number of varieties, need fixed cost.

Introducing Monopolistic Competition into RBC

- Add labor market clearing, bond market clearing, and aggregate resource constraint:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

$$Y_t = C_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} = A_t N_t^{1-\alpha}\left[\int_0^1 \left(\frac{N_t(i)}{N_t}\right)^{\frac{\varepsilon-1}{\varepsilon}(1-\alpha)} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Assume symmetric equilibrium (turns out to be unique):

$$P_t(i) = P_t,\ N_t(i) = N_t,\ Y_t(i) = Y_t$$

  - Since $P_t(i)/P_t = 1$, optimal pricing implies:

$$\frac{W_t}{P_t} = \frac{1-\alpha}{1+\mu} Y_t / N_t$$

Equilibrium With Monopolistic Competition

Definition

A symmetric equilibrium is an allocation $\{C_{t+s}, N_{t+s}, Y_{t+s}, B_{t+s}\}_{s=0}^{\infty}$ and set of prices $\{P_{t+s}, Q_{t+s}, W_{t+s}\}_{s=0}^{\infty}$ along with exogenous processes $\{A_{t+s}, M_{t+s}\}_{s=0}^{\infty}$ such that:

$$
\begin{aligned}
Y_t &= A_t N_t^{1-\alpha} \\
\frac{W_t}{P_t} &= \frac{1-\alpha}{1+\mu} A_t N_t^{-\alpha} \\
\frac{W_t}{P_t} &= \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \\
Y_t &= C_t \\
\frac{M_t}{P_t} &= \zeta^{1/\nu} \left(1 - 1/Q_t\right)^{-1/\nu} C_t^{\gamma/\nu} \\
1 &= \beta E_t \left\{ Q_t P_t C_{t+1}^{-\gamma} / \left(P_{t+1} C_t^{-\gamma} \right) \right\}
\end{aligned}
$$

What Does Monopolistic Competition Change?

- *Exact same* equilibrium definition as from last class except:

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

is now:

$$\frac{W_t}{P_t} = \frac{1-\alpha}{1+\mu} A_t N_t^{-\alpha}$$

  - Labor *demand* curve from optimal price setting replaces profit maximization.
  - But optimal price setting is profit maximization (that's how we derived the markup formula).

- Markup is wedge between real wage and marginal product.
  - In fact, *the markup is the labor wedge in this model*.
  - In markup form, $1+\mu_t^L = \frac{MPL_t}{MRS_t} = \left(1+\mu_t^P\right)\left(1+\mu_t^W\right)$.
  - No labor market distortion, so labor wedge is product markup.

Markups and Dynamics

- Again have monetary neutrality.
- In real block, equilibrium gives:

$$N_t = \left(\frac{1-\alpha}{\chi\left(1+\mu\right)} A_t^{1-\gamma} \right)^{\frac{1}{\varphi+\gamma+\alpha(1-\gamma)}}$$

  - Due to markup, firms produce too little and hire too little relative to perfect competition.

- Log-linearize:

$$\hat{n}_t = \frac{1-\gamma}{\varphi+\gamma+\alpha\left(1-\gamma\right)} \hat{a}_t$$

- Note that markup does not affect dynamics, only steady state.
  - Because markups are constant.

Dynamics With Variable Markups

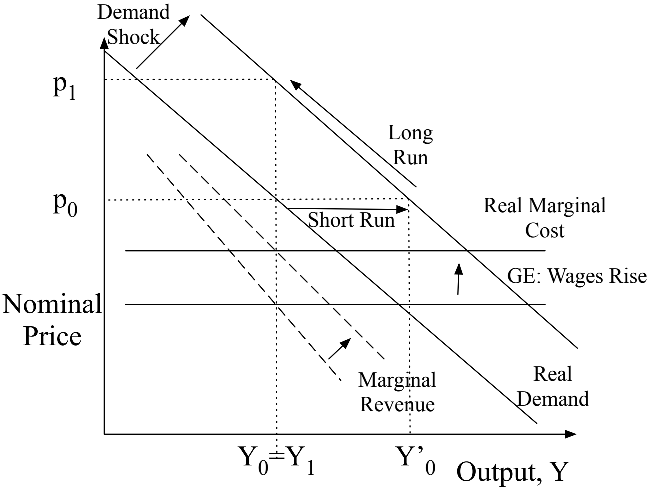
- Way forward: What would happen if markups were time-varying?

$$\hat{n}_t = \frac{1-\gamma}{\varphi+\gamma+\alpha(1-\gamma)}\hat{a}_t - \frac{1}{\varphi+\gamma+\alpha(1-\gamma)}\hat{\mu}_t$$

- Countercyclical markups can be a source of business cycle fluctuations.
  - Way to generate a counter-cyclical labor wedge as in the data.
- How to get countercyclical markups? Sticky prices!
- For next class, read Gali Ch. 3.

Monopoly Diagram: Demand Shock With Sticky Prices

- Prices fixed in short run, flexible in long run.
- I have rigged the following diagram so money is neutral.



Monopoly Diagram: Tech Shock With Sticky Prices

- Prices fixed in short run, flexible in long run.