Economics 704a Lecture 1: Business Cycle Facts and Real Business Cycles I

Adam M. Guren[1]

Boston University

Spring 2024

[1] My slides borrow from slides by Simon Gilchrist, Mark Gertler, Emi Nakamura, Jon Steinsson, and Ivan Werning, as well as Jordi Gali's textbook and the cited papers. All are gratefully acknowledged.

Welcome to Economics 704a

- Adam Guren
  - guren@bu.edu
  - 270 Bay State Road, Room 406.
  - Office Hours Until Spring Break: M 9-10:30, W 3:30-5, and by appointment.
    - Will announce when occasionally I need to change
    - Email me if you are coming so I can stagger.
- Research Interests:
  - Macro models and questions with micro data and methods.
  - Housing and the macroeconomy.
- When email about class, **put "Ec 704" in the subject line**.
- Teaching Assistant: Shraddha Mandi (mandis@bu.edu)
  - OH: F 12:30-2:30 in Room 413
  - Section: Tu 3:30-4:45 in CAS 116
- Textbook: Jordi Gali's *Monetary Policy, Inflation, and the Business Cycle*

Course Policies

- I care a lot about teaching.
  - PLEASE let me know how you think the course is going.
  - Particularly with reference to speed and usefulness.
- Slides will be posted online ahead of class.
- There will be (a few) typos. Please point them out and I will repost clean versions of my slides.
  - Corrections will be in blue.
- PLEASE ask questions, challenge my conclusions, etc.
- No electronic devices
  - Contact me if you need an exemption for learning reasons.

Course Requirements For My Half

- I will give 13 lectures from January 18 to February 29.

1. Exam (75%)
   - Tuesday March 5 in class.
   - Review session with Shraddha: Friday 3/1 12:30-1:45 in Room 546 in lieu of 3/5 section.
2. Problem Sets (25%)
   - Due via Blackboard 1/30, 2/6, 2/13, 2/20, 2/27 by 11:59pm.
   - LaTeX policy: Write up your *results* showing key equations but not line-by-line derivations in LaTeX. You may append scanned hand-written derivations.
   - Work in groups, do your own write up (and say who you worked with).
   - Grading: Check+, Check, Check-, Zero.
   - Two brief response email assignments (due 2/27 and 2/29) for the last two classes will together count as one problem set.

Getting the Most Out of the Course

- This section of the course is a bit algebra intensive.
  - Unfortunately, no way around it.
- I will focus on introducing and solving models and intuition in class, with some algebra.
  - Algebra guides with more detailed derivations on course website.
  - More in-depth derivations in section.
  - **Sections are *excellent* and very highly rated**. They will focus on going through some of the more involved derivations in depth and reviewing the intuition.
- Some students in the past have liked pre-reading lecture notes or re-reading and deriving after lecture to fully digest material.

My Views on Macroeconomics

- Macroeconomics has a monopoly on the best questions and worst answers.
  - Great area to do research!
- Macroeconomics is a big tent.
  - Not just what you learn in first year!
  - First year is really "intertemporal economics."
- Took me a while to appreciate macro.
  - Unsettled field in many ways.
    - I will spend substantial time critiquing main models.
  - I will try to add some empirics, interesting papers, etc. to show you why I have come to love it.
    - Especially last few lectures.
  - But will teach you the canon and focus on theory.
- Even if you don't do macro, you will be asked about monetary policy for the rest of your life.

The Big Questions in Macro

- What are the drivers of fluctuations (shocks)? How do fluctuations work?
- Why are responses so big to seemingly small shocks?
- Why are responses so persistent?
- What is the role / optimal conduct of policy, particularly monetary and fiscal?
- What are the roles of non-linearities and how do they change the above questions?

The Big Questions in Monetary Economics

- How is an economy with money different from an economy without money?
- What determines demand for money, the price level, and nominal interest rates?
- What explains business cycles in an economy with money?
- What effects do change in monetary policy have on real activity and inflation?
- How should monetary policy be conducted?
- How do these results change when the nominal interest rate hits zero?

Intellectual History

- Great Depression $\Rightarrow$ Keynes and the Keynesians.
  - IS-LM
  - Ad hoc assumptions (e.g. "consumption function")
- Monetarists.
- Stagflation and Rational Expectations.
- Real Business Cycles and DSGE.
- New Keynesian Model and Synthesis
  - Blanchard (2008): "The state of macro is good.... in the 1970s, the field looked like a battlefield...a largely shared vision both of fluctuations and of methodology has emerged."
- Great Recession upsets consensus model.

Building Towards The "New Keynesian" Model

- Centerpiece of my quarter of the year is NK model and analysis of its implications for policy.
- RBC, DSGE, rational expectations won on methodology.
- RBC model with frictions:
    - Money.
    - Imperfect competition.
    - Nominal rigidities (Calvo).
    - Start with no capital, add it later.
- Need to start with RBC and add each ingredient separately.

Course Outline

1. Real Business Cycles
   - Builds off simple RBC model in David's part
2. The New Keynesian Model
   - 2.1 Empirical Motivation for Nominal Rigidity
   - 2.2 Money, Money Demand, and Output
   - 2.3 Monopolistic Competition and Markups
   - 2.4 Full New Keynesian Model
3. Optimal Policy in a New Keynesian Framework
   - 3.1 Discretion
   - 3.2 Commitment
   - 3.3 Monetary Policy in Practice, 2021-23
4. The Liquidity Trap and Policy in a Liquidity Trap
5. New Perspectives on the Monetary Transmission Mechanism
   - 5.1 Heterogenous Agent New Keynesian Models
   - 5.2 Housing and Monetary Policy

A Note On Notation

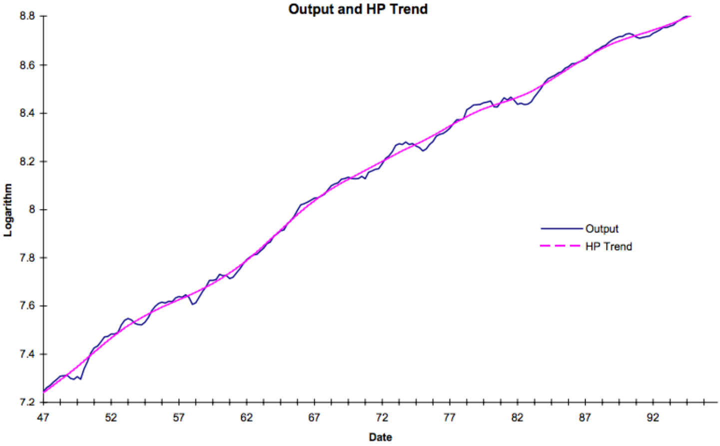
- I will do my best to use consistent notation throughout my portion of the class.
- Largely will follow Gali textbook with a few exceptions:
  - Gali uses σ for CRRA. I use γ for CRRA and $\sigma = 1/\gamma$ for IES.
  - Gali uses D_t for dividends from households. I will split into transfers TR_t and profits PR_t.
  - Gali denotes by r_t the real interest rate between periods t and $t+1$. I denote this by r_{t+1}.
  - Profits are PR_t. Inflation is Π_t.
- Other notational conventions:
  - Upper case will be in levels.
  - Lower case will be in logs.
  - Lower case with hat in log deviations from steady state.

Reading For RBC

- RBC References:
  - Rebelo and King, Handbook Chapter (especially for facts)
  - Note: Gali, Chapter 2 begins with money. Hold off for now.
- Next class, start by critiquing RBC model.
- Read great debate between Summers and Prescott in *Federal Reserve Bank of Minneapolis Quarterly Review* Fall 1986
  - Prescott: "Theory Ahead of Business Cycle Measurement"
  - Summers: "Some Skeptical Observations on Real Business Cycle Theory"
  - Prescott's Rebuttal: "Responses to a Skeptic"
  - https://www.minneapolisfed.org/research/quarterly-review
- Then discuss lots of papers on reading list.

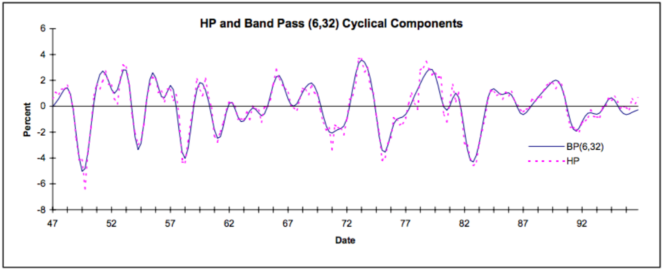
Business Cycle Facts

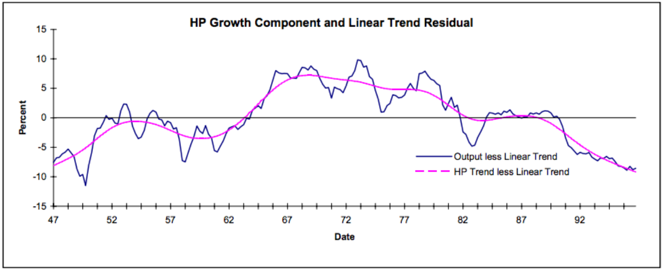
Taking Out Trend: The HP Filter



Source: King and Rebelo (1999)

Taking Out Trend: The HP Filter



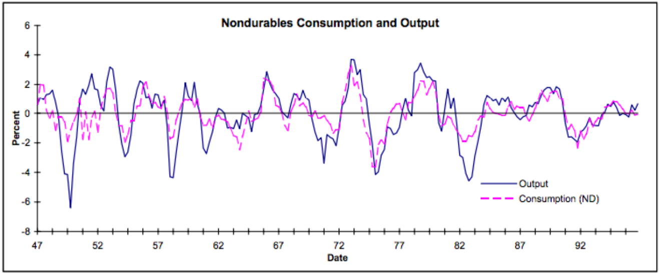


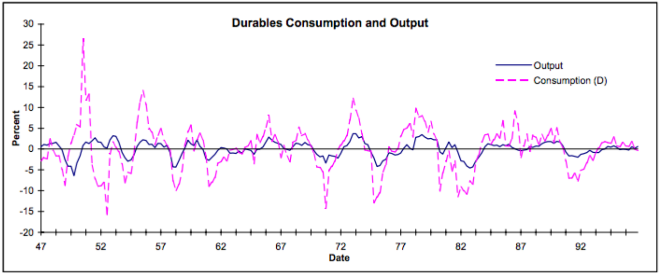
Business Cycle Facts

Business Cycle Statistics for the U.S. Economy

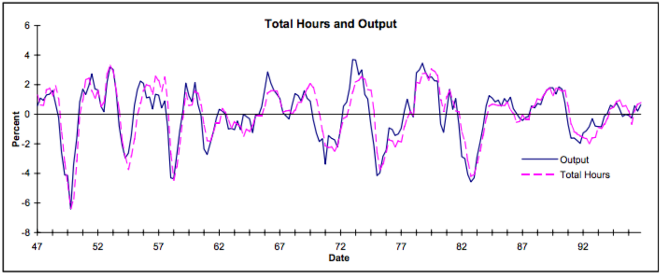
	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

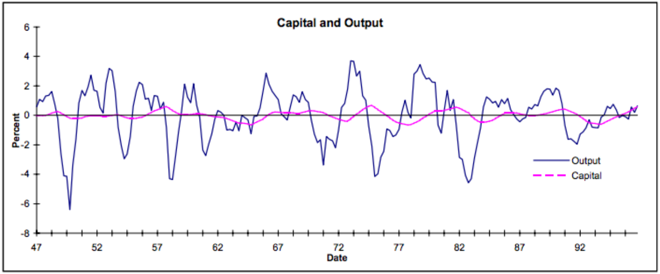
Business Cycle Facts: Consumption



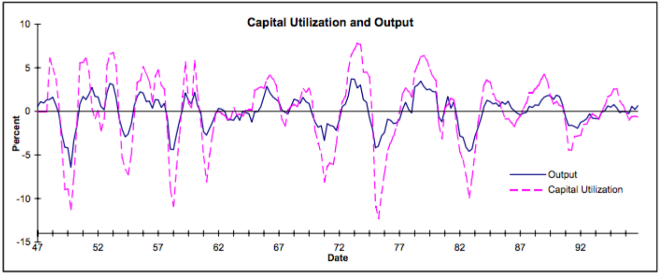


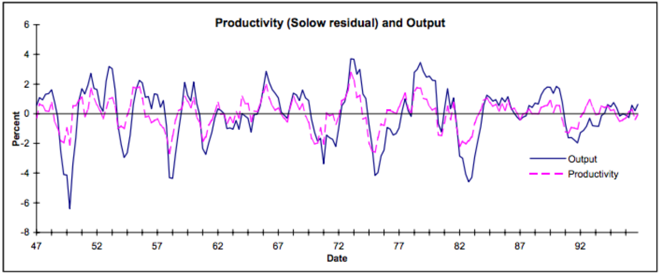
Business Cycle Facts: Total Hours and Capital



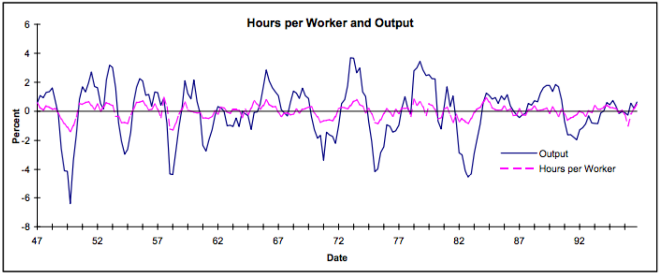


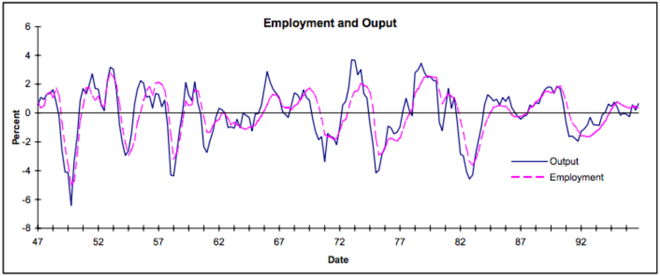
Business Cycle Facts: Utilization and Productivity



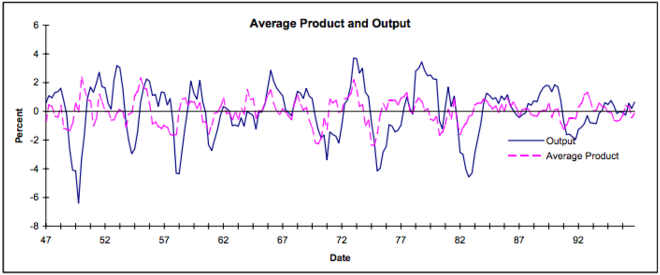


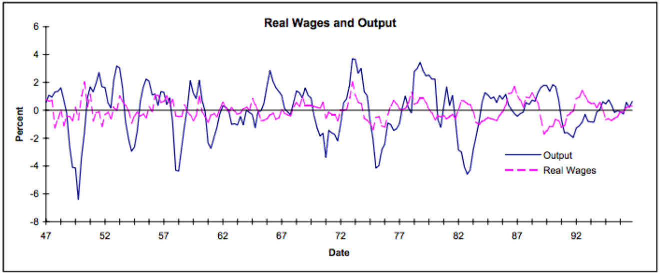
Business Cycle Facts: Intensive and Extensive Margins





Business Cycle Facts: Labor Productivity and Wages





Real Business Cycle Model

Review From David's Part

- David showed you an RBC model with:
    1. No labor supply.
    2. IID productivity (initially), persistent productivity with 2-state Markov process.
- Solution method:
    1. Solve for optimal policy rules.
    2. Simulate Solow residuals, compute optimal $\{c_t, y_t, i_t\}$ in response to Solow shocks.
    3. Repeat many times, ask whether properties of $\{c_t, y_t, i_t\}$ look like Business Cycle Facts.
- Lessons:
    - Propagation proportional to Capital Share α.
      With realistic α, modest propagation.
    - Model succeeds at investment, not as great for consumption and output (but can improve).

Agenda For Today: Full RBC Model

- Relax some assumptions of David's part:
  1. Add in labor supply.
  2. Generalized AR(1) technology (Solow residual) process.
- Solve for optimal policy rules that characterize equilibrium of $\{C_t, N_t, K_{t+1}, Y_t\}$ two ways:
  1. Planner's problem and decentralized equilibrium.
  2. Show they are equivalent (second welfare theorem).
- New solution method: Log-Linearization.
- Calibration and quantitative analysis.

RBC Section Outline

1. RBC Model: Setup and Solutions (assume away growth)
   1.1 Planner's Problem
   1.2 Decentralized Equilibrium
   1.3 Log Linearization
   1.4 Model Performance
2. Calibrated RBC: Successes and Failures
   2.1 Fit to Business Cycle Facts and Intuitions
   2.2 Internal Propagation
   2.3 Labor Supply Elasticity and Extensive Margin
   2.4 Solow Residual, Technology Shocks, and Capital Utilization
   2.5 Rotemberg-Woodford Critique
3. Where Next? Business Cycle Accounting

Setup: Households

- Preferences:

$$U\left(C_t, N_t\right) = E_t\left\{\sum_{s=0}^{\infty}\beta^s\left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} - \chi\frac{N_{t+s}^{1+\varphi}}{1+\varphi}\right)\right\}$$

  - Discount factor $\beta \in (0,1)$, $\rho = -\log\beta$ is the discount rate.
  - $\gamma > 0$ is CRRA, $\sigma = 1/\gamma$ is IES.
  - $\varphi > 0$ where $1/\varphi$ is the Frisch elasticity of labor supply (more on this next class).
- Notes:
  - All that matters is U being twice continuously differentiable with $U_c > 0$, $U_{cc} < 0$, $U_n < 0$, $U_{nn} < 0$.
  - Eventually will add discount rate shock as in Gali.

$$U\left(C_t, N_t\right) = E_t\left\{\sum_{s=0}^{\infty}Z_t\beta^s\left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} - \chi\frac{N_{t+s}^{1+\varphi}}{1+\varphi}\right)\right\}$$

so increase in Z_t raises MU of consumption.

Setup: Households

$$U\left(C_t, N_t\right) = E_t\left\{\sum_{s=0}^{\infty}\beta^s\left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} - \chi\frac{N_{t+s}^{1+\varphi}}{1+\varphi}\right)\right\}$$

- Budget Constraint:

$$P_t C_t + B_{t+1} \leq R_t B_t + W_t N_t$$

  - P_t is price of output C_t, for now normalized to one.
  - B_{t+1} is holdings of real bond bought at price 1 at time t and yielding R_{t+1} at time $t+1$.
  - R_t is gross real interest rate between periods $t-1$ and t.
  - W_t is real wage.

Setup: Firms

- Firms produce output Y_t CRS with labor N_t and capital K_t.
  - Notation: K_t determined in previous period, produced one to one with output.
  - Technology:
    $$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$
  - A_t is TFP, $a_t = \log A_t$ follows an AR(1) process:
    $$a_t = \rho_a a_{t-1} + \varepsilon_t^a \text{ with } \rho_a \in [0,1] \text{ and } \varepsilon_t^a \sim N\left(0, \sigma_\varepsilon^2\right)$$
- Firms maximize discounted profits PR:
  $$PR_t = P_t Y_t - W_t N_t - P_t I_t + B_{t+1} - R_t B_t$$
  - B_t is bond issuance by firms to finance capital purchases (can also think of as dividend with B being equity holdings).
  - Capital depreciates at rate $\delta \in (0,1]$ so:
    $$K_{t+1} = I_t + (1-\delta) K_t$$
- Discount according to household (shareholder) preferences.

Setup: Markets

- Three markets clear:
  - Labor: $N_t^{firms} = N_t^{households}$
  - Bond: $B_t^{firms} = B_t^{households}$
  - Output: $Y_t = I_t + C_t$, or equivalently $I_t = S_t$
- Plugging aggregate resource constraint $Y_t = I_t + C_t$ into law of motion for capital gives **aggregate law of motion**:

$$K_{t+1} = Y_t - C_t + (1 - \delta) K_t$$

Equilibrium Definition

Definition

An equilibrium is an allocation $\{C_{t+s}, N_{t+s}, K_{t+s+1}, Y_{t+s}, B_{t+s+1}\}_{s=0}^{\infty}$, a set of prices $\{W_{t+s}, R_{t+s+1}\}_{s=0}^{\infty}$, an exogenous technology process $\{A_{t+s}\}_{s=0}^{\infty}$, and initial conditions for bonds and capital such that:

1. Households maximize utility subject to budget constraints.
2. Firms maximize discounted profits given their technology.
3. Markets clear:
   3.1 Labor demanded equals labor supplied.
   3.2 Bond issuance by firms equals bond holding by households
   3.3 Output equals consumption plus investment.

- Good discipline to provide these sorts of equilibrium definitions; we will do so repeatedly.
  - Always check N equations with N unknowns.

Planner's Problem

- Until Lecture 4, normalize $P_t = 1$ (real model).
- Welfare theorems: Frictionless markets and no externalities implies planning problem $\iff$ decentralized equilibrium.
- The planner's problem is:

$$\max_{\{C_{t+s}, N_{t+s}, K_{t+s+1}, Y_{t+s}\}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} - \chi \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right) \right\} \text{ s.t.}$$

$$Y_t = C_t + K_{t+1} - (1-\delta) K_t$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

- Convex so optimality is defined by FOCs and transversality:

$$\lim_{t\to\infty} \beta^t E_t \{U_{c,t} F_{k,t} K_t\} = 0$$

Planner's Problem: Intratemporal FOC WRT Labor

$$(1-\alpha)\frac{Y_t}{N_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

- LHS is the marginal product of labor. With Cobb-Douglas, MPL is $(1-\alpha)$ times the average product.
- The RHS is the marginal rate of substitution between labor and consumption.
- In equilibrium, $MPL_t = MRS_t$.

Planner's Problem: Intertemporal FOC WRT Capital

$$E_t \left\{ \frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) \right] \right\} = 1$$

- Because $U_{c,t} = C_t^{-\gamma}$, this is an Euler equation:

$$U_{c,t} = \beta E_t \left\{ (1 + r_{t+1})\, U_{c,t+1} \right\}$$

- The implicit interest rate r_{t+1} is:

$$r_{t+1} = \alpha \frac{Y_{t+1}}{K_{t+1}} - \delta = MPK_{t+1} - \delta$$

  - Household savings channeled to capital investment.
  - Return on savings is return on capital net of depreciation.

The Stochastic Discount Factor

- Define:

$$\Lambda_{t,t+1} = \frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}}$$

  as the household's ***stochastic discount factor***.
  - It is a stochastic process that is the *effective discount rate* of agents in the economy.
- The SDF pins down economy's interest rate as Euler is:

$$1 = E_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\}$$

- The SDF is the implicit discount rate of firms if households own firms as this is how shareholders discount cashflows.

The Stochastic Discount Factor in Asset Pricing

- Rewrite Euler as function of payoff X_{t+1} of the bond where return is $R_{t+1} = X_{t+1}/Q_t$ and Q_t is the price of the bond:

$$Q_t = E_t \left[\Lambda_{t,t+1} X_{t+1}\right]$$

- In a world of many assets, *this equation holds for each asset*.
  - **Asset prices are determined by covariance between payoffs and the SDF**.
  - Intuition: If an asset pays off in states where consumption is low, asset is a hedge against consumption risk and has lower return (e.g. gold); opposite true for stocks.
- **Equity premium puzzle** is that with reasonable parameterization, return on equities is too low in data.
- Field of asset pricing is built off of SDF! In fact Cochrane's seminal textbook uses SDF as unifying lens for entire field.
  - Leave further exploration to EC 745, Asset Pricing.

Planner's Problem: Solution

- Given the current state of the economy (A_t, K_t), the allocation $\{C_t, N_t, K_{t+1}, Y_t\}$ is determined by:

$$\begin{aligned}
(1-\alpha)\frac{Y_t}{N_t} &= \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \\
1 &= E_t\left\{\frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)\right]\right\} \\
Y_t &= C_t + K_{t+1} - (1-\delta)K_t \\
Y_t &= A_t K_t^{\alpha} N_t^{1-\alpha}
\end{aligned}$$

- Prices and bond holdings can be backed out by FOCs for one side of the market.

Decentralized Equilibrium: Households

$$\max_{C_t, N_t, B_{t+1}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} - \chi \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right) \right\}$$
$$\text{s.t. } C_t + B_{t+1} = R_t B_t + W_t N_t$$

- Static FOC WRT labor:

$$W_t = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

- Dynamic FOC WRT B_t:

$$1 = E_t \left\{ \frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}} R_{t+1} \right\} = E_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\}$$

Decentralized Equilibrium: Firms

$$\max_{N_t, I_t, B_{t+1}} E_t \left\{ \sum_{s=0}^{\infty} \Lambda_{t,t+s} PR_{t+s} \right\} \quad \text{s.t.} \quad K_{t+1} = I_t + (1-\delta) K_t$$

$$PR_t = A_t K_t^{\alpha} N_t^{1-\alpha} - W_t N_t$$
$$-I_t + B_{t+1} - R_t B_t$$

- Static FOC WRT labor:

$$W_t = (1-\alpha) \frac{Y_t}{N_t}$$

- Dynamic FOC WRT capital and bonds together imply:

$$E_t \{\Lambda_{t,t+1} R_{t+1}\} = E_t \left\{ \Lambda_{t,t+1} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right) \right\}.$$

Decentralized Equilibrium: Solution

- Combining the firm and household static FOCs gives:

$$(1-\alpha)\frac{Y_t}{N_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

- Combining the firm and household dynamic FOCs gives:

$$1 = E_t\left\{\frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)\right]\right\}$$

- Same as planner's optimality conditions $\Rightarrow$ same solution.
    - Bond market clears by Walras' Law.

Bellman Equation Approach

- Write planner's problem as a Bellman equation:

$$V\left(A_t, K_t\right) = \max_{C_t, N_t, K_{t+1}} U\left(C_t, N_t\right) + \beta E_t\left\{V\left(A_{t+1}, K_{t+1}\right)\right\}$$
$$\text{s.t. } A_t K_t^{\alpha} N_t^{1-\alpha} = C_t + K_{t+1} - \left(1-\delta\right) K_t$$

  - Contraction by Blackwell's theorem $\Rightarrow$ unique solution.
  - Solve numerically by standard techniques (*e.g.*, value fn or policy fn iteration) on computer to get policy functions:

$$C\left(A_t, K_t\right),\ N\left(A_t, K_t\right), K_{t+1}\left(A_t, K_t\right)$$

- Key insight: Recursive solution with optimum depending on state $s^t = \left(A_t, K_t\right)$.
  - Solves the model globally.
  - But slow, untransparent.

Log Linearization

- Log-linear approximation of $Y_t = f(X_t)$ around a point X.
  - First-order Taylor approx to $\log(f(x))$
- Define $x_t = \log(X_t)$ and $\hat{x}_t = x_t - x$. Then:

$$
\begin{aligned}
y_t &= \log(f(\exp(x_t))) \\
y_t &\approx \log(f(\exp(x))) + \frac{f'(\exp(x))\exp(x)}{f(\exp(x))}(x_t - x) \\
\hat{y}_t &\approx \frac{f'(X)X}{f(X)}\hat{x}_t
\end{aligned}
$$

- Can also derive using $\hat{x}_t \approx dX_t/X_t$:

$$
\begin{aligned}
Y + dY_t &\approx f(X) + f'(X)\,dX_t \\
\frac{dY_t}{Y}Y &\approx f'(X)X\frac{dX_t}{X} \\
\hat{y}_t &\approx \frac{f'(X)X}{f(X)}\hat{x}_t
\end{aligned}
$$

Log Linearization Rules

- The rules of differentiation have analogues in log linearization that can be useful:
  1. **Multiplicatives**: $\frac{X_t Y_t}{Z_t} = \gamma$ log linearizes to: $\hat{x}_t + \hat{y}_t - \hat{z}_t = 0$.
     - This is exact and not an approximation. Divide by steady state values:
       $$\frac{\frac{X_t}{X}\frac{Y_t}{Y}}{\frac{Z_t}{Z}} = \frac{\gamma}{\gamma} = 1$$
       and take logs: $\log\left(\frac{X_t}{X}\right) + \log\left(\frac{Y_t}{Y}\right) - \log\left(\frac{Z_t}{Z}\right) = \log(1) = 0$.
  2. **Exponentials**: $X_t^{\alpha} = 1$ log linearizes to: $\alpha\hat{x}_t = 0$.
     - $f(x) = x^{\alpha}$, $f'(x) = \alpha x^{\alpha-1}$. Log linearization is $\frac{f'(X)X}{f(X)}\hat{x}_t = \alpha\hat{x}_t$.
  3. **Resource Constraints**: $Y_t = C_t + I_t$ log linearizes to $\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{i}_t$
- The first TA session will be devoted to log linearization.

Log Linearized RBC Model

- Nonlinear equilibrium conditions: $(1-\alpha)\frac{Y_t}{N_t} = \chi \frac{N_t^{\varphi}}{C_t^{-\gamma}}$, $Y_t = C_t + K_{t+1} - (1-\delta) K_t$, $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$, and

$$1 = E_t \left\{ \frac{\beta C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) \right] \right\}$$

- These become:

$$\begin{aligned}
\hat{y}_t - \hat{n}_t &= \varphi \hat{n}_t + \gamma \hat{c}_t \\
\hat{c}_t &= -\sigma \beta \alpha \frac{Y}{K} E_t \left\{ \left(\hat{y}_{t+1} - \hat{k}_{t+1} \right) \right\} + E_t \left\{ \hat{c}_{t+1} \right\} \\
\hat{k}_{t+1} - \hat{k}_t &= \frac{Y}{K} \hat{y}_t - \frac{C}{K} \hat{c}_t - \delta \hat{k}_t \\
\hat{y}_t &= \hat{a}_t + \alpha \hat{k}_t + (1-\alpha) \hat{n}_t
\end{aligned}$$

  - Plus $\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_t^a$

Log Linearized RBC Model: Intuition

- We are going to be working a lot with log-linearized models. Before solving, pause to inspect for intuition.
- Labor-Leisure: $\widehat{mpl}_t = \hat{w}_t = \widehat{mrs}_t \Rightarrow \hat{y}_t - \hat{n}_t = \varphi \hat{n}_t + \gamma \hat{c}_t$
  - LHS: With C-D, MPL moves with relative log deviation of y and n from steady state.
    - If y is higher than n, means more capital rel to labor and since complements MPL is higher.
  - RHS: MRS log deviation from steady state.
    - Rises with $\hat{n}_t$ due to higher disutility of labor, stronger with high φ (next class: $1/\varphi$ is Frisch elasticity of labor supply).
    - Rises with $\hat{c}_t$ because when richer want to work less due to wealth effect. Strength determined by γ because with CRRA, $\gamma = 1/IES$. If IES low, less benefit to working more to raise C.
    - Alternate intuition: When C high, MU of C falls (strength related to IES), so return to working in MU of C terms falls.

Log Linearized RBC Model: Intuition

- Solve labor-leisure for $\hat{y}_t$, plug into production function and solve for $\hat{n}_t$:

$$\hat{n}_t = \frac{1}{\alpha+\varphi}\left(\hat{a}_t + \alpha \hat{k}_t\right) - \frac{\gamma}{\alpha+\varphi}\hat{c}_t$$

- Labor is increasing in $\hat{a}_t + \alpha \hat{k}_t$
    - Tech shock and increase in capital both increase MPL.
    - Higher MPL has bigger effect on hiring when labor supply is more elastic (lower φ) and when labor share is higher (capital share α lower).
- Labor is decreasing in $\hat{c}_t$ due to *wealth effect*.
    - Again strength mediated by γ due to IES logic.
    - $\alpha + \varphi$ also determines strength of wealth effect because acts through labor-leisure tradeoff.

Log Linearized RBC Model: Intuition

- Plug back into production function to obtain:

$$\hat{y}_t = \left(1 + \frac{1-\alpha}{\alpha+\varphi}\right)\left(\hat{a}_t + \alpha \hat{k}_t\right) - \frac{\gamma\left(1-\alpha\right)}{\alpha+\varphi}\hat{c}_t$$

- $\hat{a}_t + \alpha \hat{k}_t$ has two effects:
    - A direct effect in $\hat{y}_t$ through production function (the 1).
    - And an indirect effect on $\hat{y}_t$ through labor demand ($\frac{1-\alpha}{\alpha+\varphi}$), which is stronger with higher labor share and more elastic labor supply.
- $\frac{-\gamma(1-\alpha)}{\alpha+\varphi}\hat{c}_t$ reflects the negative wealth effect on labor supply.
- Call solution for $\hat{y}_t = \hat{y}\left(\hat{a}_t, \hat{k}_t, \hat{c}_t\right)$.

Log Linearized RBC Model: Solution

- Plug $\hat{y}_t = \hat{y}\left(\hat{a}_t, \hat{k}_t, \hat{c}_t\right)$ into Euler and capital accumulation:

$$\hat{c}_t = -\sigma\beta E_t \left\{ \alpha \frac{Y}{K} \left(\hat{y}\left(\hat{a}_{t+1}, \hat{k}_{t+1}, \hat{c}_{t+1}\right) - \hat{k}_{t+1} \right) \right\} + E_t \left\{ \hat{c}_{t+1} \right\}$$

$$\hat{k}_{t+1} - \hat{k}_t = \frac{Y}{K} \hat{y}\left(\hat{a}_t, \hat{k}_t, \hat{c}_t\right) - \frac{C}{K}\hat{c}_t - \delta \hat{k}_t$$

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_t^a$$

- This is a system of difference equations for $\hat{a}_t$, $\hat{c}_t$, and $\hat{k}_{t+1}$ where $\hat{a}_{t-1}$ and $\hat{k}_t$ are predetermined.
  - Second order difference equation with two characteristic roots.
  - Unstable: Associated with forward-looking consumption.
  - Stable: Associated with capital.

Log Linearized RBC: Solution

- Assume reduced-form policy functions for $\hat{c}_t$ and $\hat{k}_{t+1}$ as a function of state $\left(\hat{a}_t, \hat{k}_t\right)$:

$$\begin{aligned} \hat{c}_t &= \psi_{ca}\hat{a}_t + \psi_{ck}\hat{k}_t \\ \hat{k}_{t+1} &= \psi_{ka}\hat{a}_t + \psi_{kk}\hat{k}_t \end{aligned}$$

  - Solve using method of undetermined coefficients.
  - Plug in to get $\hat{n}_t$, $\hat{y}_t$.
- Can get intuition by noting that $\hat{k}_t \approx 0$ over the cycle:

$$\begin{aligned} \hat{c}_t &= \psi_{ca}\hat{a}_t \\ \hat{n}_t &= \frac{1-\gamma\psi_{ca}}{\alpha+\varphi}\hat{a}_t \\ \hat{y}_t &= \left(1 + \frac{1-\alpha}{\alpha+\varphi}(1-\gamma\psi_{ca})\right)\hat{a}_t \end{aligned}$$

  - As $\psi_{ca} > 0$, when a_t increases, c_t, n_t, and y_t also increase.

Log Linearized RBC Model: Intuition

- Get intuition from $\hat{k}_t \approx 0$ over the cycle.

$$
\begin{aligned}
\hat{c}_t &\approx \psi_{ca}\hat{a}_t \\
\hat{n}_t &\approx \frac{1-\gamma\psi_{ca}}{\alpha+\varphi}\hat{a}_t \\
\hat{y}_t &\approx \left(1+\frac{1-\alpha}{\alpha+\varphi}\left(1-\gamma\psi_{ca}\right)\right)\hat{a}_t
\end{aligned}
$$

- Business Cycle: When a_t increases, c_t, n_t, and y_t also increase.
  - Consumption is direct effect.
  - Employment has direct effect offset by wealth effect. Stronger with higher labor share and more elastic labor supply.
  - Output has direct and indirect effect amplified by labor demand partially offset by wealth effect.

Calibration

- Choose "reasonable" parameters, solve on a computer.
  - Target steady-state moments like long-run interest rates, fraction of time spent working, labor share etc.
  - Choose others: CRRA of 1 or 2, labor supply elasticity of 1, depreciation of 10% per year.
  - Technology shocks fit to time series properties of Solow resid.
- See how well matches business cycle facts.
  - Standard deviations, autocorrelations, correlations with output.
  - With policy functions solved for from theory, feed in actual Solow residual to get simulated series.

Reminder: Business Cycle Facts

Business Cycle Statistics for the U.S. Economy

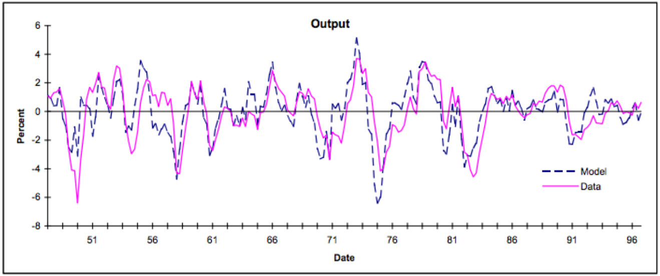
	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

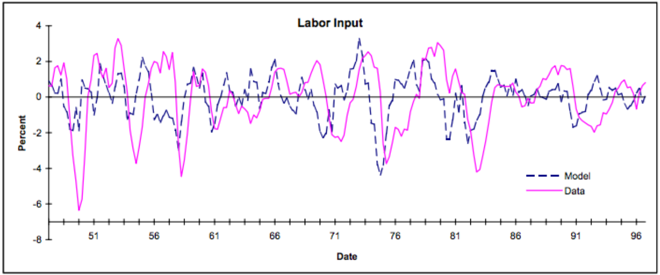
Simulated Model Moments

Business Cycle Statistics for Basic RBC Model[35]

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

Model Fit: Output and Labor





Model Fit: Consumption and Investment