Economics 704a Lecture 4: Money and Output

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Introducing Money into RBC

- RBC is an entirely real model. Normalize $P = 1$.
- To think about price level determination and inflation, need to introduce money.
  - Today add money to RBC
- Also money is non-neutral in the data.
  - What model features do we need to obtain non-neutrality?

What is Money?

1. Store of value
    - Zero nominal interest rate.
    - Loses value with inflation.
2. Unit of account
    - Facilitates pricing.
3. Medium of Exchange
    - Eliminates need for “double coincidence of wants” in barter economy.
    - Facilitates specialization
    - See New Monetary Economics literature that micro founds money with search.

- What gives value?
    - Used to be precious metal.
    - Now, fiat money.

Household Problem: Setup and Notation

- P_t is price level at time t, $p_t = \log(P_t)$.
  - $\pi_{t+1} = p_{t+1} - p_t$ is gross inflation between t and $t+1$.
  - P_t/P_{t+1} is real return on money between t and $t+1$.
- M_t is quantity of money households hold at end of period t.
  - $1/P_t$ is the value of one unit of money.
  - "Real Money Balances" are M_t/P_t.
  - Note: Different timing from Gali.
- Bonds will now be nominal, with nominal gross return of Q_t.
  - Nominal interest rate is $i_t = \log Q_t$.

A Note on Timing

- Instead of giving a claim to a return on capital determined at time $t+1$, nominal bonds are *coupon bonds*.
  - Buy at face price of 1 at t, know that will pay Q at $t+1$.
  - Bond return between t and $t+1$ now determined *at time t*.
  - Consequently, gross t to $t+1$ return is denoted as Q_t.
  - And bond holdings of bonds bought at t and maturing at $t+1$ are B_t not B_{t+1} with the real bonds in RBC.
- Money follows the same timing.
- Real interest rate between t and $t+1$ is still determined at $t+1$, so we denote it as R_{t+1}.
- This timing helps clarify what an "expectation at time t" means and is consistent with literature.

Fisher Equation

$$E_t\left\{R_{t+1}\right\} = E_t\left\{\frac{P_t}{P_{t+1}}\right\} Q_t$$

- This is an no-arbitrage identity.
- Nominal bond costs 1 dollar today at price level P_t.
- Returns Q_t dollars tomorrow at price level $E_t\left\{P_{t+1}\right\}$.
    - Inflation is uncertain.
    - Consequently, real interest rate is determined at $t+1$ but nominal interest rate is determined at t.
- Often written as linear approximation in logs:

$$E_t\left\{r_{t+1}\right\} = i_t - E_t\left\{\pi_{t+1}\right\}$$

Money Demand: Ideas

- Money has no nominal return. If $i_t > 0$, why hold money?
  - Money provides "liquidity services."
    - Costly and time consuming to buy and sell bonds every time you want to buy something.
    - You also don't want to seek out someone who wants to trade for exactly what you have.
- But i_t is the opportunity cost of holding money.
  - As it goes up, hold less $\Rightarrow$ money demand.
- To focus on interesting questions about how money changes economy, punt on why people hold it and just put real money balances M_t/P_t in the utility function.
  - Choose convenient isoelastic form.
  - Alternative: Cash in advance constraint.

Household Problem With Money

$$\max_{C_t, N_t, B_t, M_t} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{t+s}^{1-\gamma}}{1-\gamma} + \zeta \frac{(M_{t+s}/P_{t+s})^{1-\nu}}{1-\nu} - \chi \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right) \right\}$$

- Budget constraint:

$$P_t C_t + B_t + M_t \leq Q_{t-1} B_{t-1} + M_{t-1} + W_t N_t + P_t \times (TR_t + PR_t)$$

- Real Budget Constraint:

$$C_t = \frac{W_t}{P_t} N_t - \frac{B_t - Q_{t-1} B_{t-1}}{P_t} - \frac{M_t - M_{t-1}}{P_t} + TR_t + PR_t$$

  - TR_t are real transfers and PR_t are real rebated profits.
    - When govt prints money revenue rebated to households.
    - Households own firms and get any profits as transfers.
- Note: Money in utility function is money you have at *end of period* (e.g. taking into period $t+1$).

Household Problem: Three FOCs

- Static FOC WRT labor:

$$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

- Dynamic FOC WRT B_t:

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} = E_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\}$$

- Dynamic FOC WRT M_t:

$$1 = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\} + \zeta \frac{\left(\frac{M_t}{P_t} \right)^{-\nu}}{C_t^{-\gamma}}$$

Bonds Vs. Money

- The bonds and money FOCs can also be written as:

$$
\begin{aligned}
U_{C_t} &= \beta Q_t E_t \left\{ \frac{P_t}{P_{t+1}} U_{C_{t+1}} \right\} \\
U_{C_t} &= \beta E_t \left\{ \frac{P_t}{P_{t+1}} U_{C_{t+1}} \right\} + U_{M_t/P_t}
\end{aligned}
$$

- Euler: MU cost of buying ε more bonds today = discounted price-level and return adjusted benefit of having ε more bonds tomorrow.
- Money: MU cost of holding ε more money today = discounted price-level adjusted benefit of having ε more money tomorrow plus liquidity benefits of holding ε more money overnight.
- Money Demand Trade-Off: Can buy bond and get return or money and get utility benefit

Money Demand

- Combining bonds and money FOCs gives money demand:

$$\frac{M_t}{P_t} = \zeta^{1/\nu} \left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu}$$

  - Increasing in C_t: Consume more, demand more money.
    - Intuition: Strength related to γ tells us this is a wealth effect!
    - When consume more, marginal utility of consumption is lower but liquidity benefit of money is same so you hold more money.
  - Decreasing in Q_t: Decreasing in opportunity cost of holding money, the nominal interest rate. Strength determined by ν, curvature of utility of holding money.
  - Often summarize as reduced-form function:

$$\frac{M_t}{P_t} = \Phi\left(C_t, Q_t\right)$$

- Log-linearize:

$$\hat{m}_t - \hat{p}_t = \frac{\gamma}{\nu}\hat{c}_t - \eta \hat{i}_t \text{ where } \eta \approx \frac{1}{\nu\left(i+1\right)}.$$

Introducing Money into RBC

- Add money to full RBC model.
  - To highlight role of money, we will get rid of capital for now.
  - Means there is no inter-temporal tradeoff and economy adjusts immediately to its steady state.
- Firms produce DRS with labor according to:

$$Y_t = A_t N_t^{1-\alpha}$$

- Each period firms solve:

$$\max_{N_t} P_t A_t N_t^{1-\alpha} - W_t N_t$$

- FOC provides a period-by-period labor demand schedule:

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

Completing the Model

- Aggregate resource constraint:

$$C_t = Y_t$$

- Government rebates seigniorage revenue:

$$TR_t = \frac{M_t - M_{t-1}}{P_t}$$

- Bonds are in zero net supply.
- Finally, there are exogenous processes for A_t and M_t.
  - The money supply M_t^S process is the monetary policy.
  - Assume mean reverting so there is a steady state.
  - $M^S = M^D \equiv M$ so money market clearing is in background not in equilibrium conditions.

Equilibrium

Definition

An equilibrium is an allocation $\{C_{t+s}, N_{t+s}, Y_{t+s},\}_{s=0}^{\infty}$ and set of prices $\{P_{t+s}, Q_{t+s}, W_{t+s}\}_{s=0}^{\infty}$ along with exogenous processes $\{A_{t+s}, M_{t+s}\}_{s=0}^{\infty}$ such that:

$$
\begin{aligned}
Y_t &= A_t N_t^{1-\alpha} \\
\frac{W_t}{P_t} &= (1-\alpha) A_t N_t^{-\alpha} \\
\frac{W_t}{P_t} &= \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \\
Y_t &= C_t \\
\frac{M_t}{P_t} &= \zeta^{1/\nu} (1 - 1/Q_t)^{-1/\nu} C_t^{\gamma/\nu} \\
1 &= \beta E_t \left\{ Q_t P_t C_{t+1}^{-\gamma} / \left(P_{t+1} C_t^{-\gamma} \right) \right\}
\end{aligned}
$$

Monetary Neutrality: The Real Block

- Fist four equations in the "real block" pin down output, employment, the real wage, and consumption.

$$\text{Labor Supply: } \frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$$

$$\text{Labor Demand: } \frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

where $C_t = Y_t = A_t N_t^{1-\alpha}$.

- Yields one equation in N_t:

$$\frac{\chi N_t^{\varphi}}{\left(A_t N_t^{1-\alpha}\right)^{-\gamma}} = (1-\alpha) A_t N_t^{-\alpha} \Rightarrow N_t = \left(\frac{1-\alpha}{\chi} A_t^{1-\gamma}\right)^{\frac{1}{\varphi+\gamma+\alpha(1-\gamma)}}$$

Monetary Neutrality: The Real Block

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- **Monetary Neutrality**: *Real outcomes are independent of the price level and unaffected by nominal variables.*
    - Super Neutrality: Not only does the level of money not matter, but its long-run growth rate also does not matter.

Determining Prices: The Nominal Block

- Given these real outcomes, the nominal block together with the process for money supply determines the price level and the nominal interest rate:

$$\begin{aligned} \frac{M_t}{P_t} &= \zeta^{1/\nu} \left(1 - 1/Q_t\right)^{-1/\nu} C_t^{\gamma/\nu} \\ 1 &= Q_t E_t \left\{\Lambda_{t,t+1}/\Pi_{t+1}\right\} \end{aligned}$$

- Expected inflation determines nominal interest rate with real interest rate pinned down by SDF in real block.
- Money market equilibrium determines price level.
    - Money demand shifted by nominal interest rate or consumption.

How General Is The Neutrality Result?

- Look at real block imposing $Y_t = C_t$:

$$\begin{aligned} C_t &= F(N_t; A_t) \\ \frac{W_t}{P_t} &= F_N(N_t; A_t) \\ \frac{W_t}{P_t} &= \frac{U_{N_t}}{U_{C_t}} \end{aligned}$$

- Money cannot show up in aggregate resource constraint or production function.
- From labor supply, see key condition is *separability between money and consumption / labor in utility function.*
  - M_t/P_t does not affect U_{N_t} or U_{C_t} and thus does not affect MRS or labor supply curve.

Non-Separable Money In Utility

- We can generate non-neutrality from creating a cross-partial between M_t/P_t and N_t or C_t in the utility function.
  - See Gali book.
- I find this to be a tremendously unsatisfying way to obtain non-neutrality.
  - Money in the utility function is a short cut.
  - What does it mean for liquidity services to increase when labor or consumption changes?
  - How is this effect not weak?
- Gali also discusses empirical issues.
  - Relationship between money supply and price level violates empirical evidence.

What About Capital?

- Assuming away capital is not driving neutrality.
- Non-capital equations and resource constraint:

$$\begin{aligned} Y_t &= F\left(K_t, N_t; A_t\right) \\ \frac{U_{N_t}}{U_{C_t}} &= F_N\left(K_t, N_t; A_t\right) \\ Y_t &= C_t + K_{t+1} - (1-\delta) K_t \end{aligned}$$

- Household Euler is in terms of real rate, as is firm capital FOC:

$$\begin{aligned} 1 &= \beta E_t \left\{ \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} R_{t+1} \right\} \\ R_{t+1} &= F_n\left(K_{t+1}, N_{t+1}; A_{t+1}\right) + (1-\delta) \end{aligned}$$

- Real side pinned down independent of nominal side.

Price Level Determinacy

- Is price level $\{P_t\}$ *uniquely* determined given central bank's actions? Or are there multiple potential price levels?
- First, log-linearize nominal block
  - Pull Euler equation into real block (it determines R_{t+1}).
  - Nominal block is then money demand and Fisher equation, which in log-linear form are:

$$\begin{aligned} \hat{m}_t - \hat{p}_t &= \hat{y}_t - \eta \hat{i}_t \\ \hat{i}_t &= E_t \{\hat{r}_{t+1}\} + E_t \{\hat{\pi}_{t+1}\} \end{aligned}$$

    - $\hat{r}_t$ and $\hat{y}_t$ can be thought of as exogenous to nominal block.
    - Simplification for algebra:$\gamma = \nu$ so no coefficient on $\hat{y}_t$.
- Then solve forward for p_t, see if there is unique solution (e.g., does infinite sum converge).
- Note: This is a bit esoteric and will not be on the test. But I want to use it to introduce Taylor Rules and why Central Banks target interest rates in stead of the money supply.

Example: Determinacy With Money Process

- Combining Fisher and money demand,

$$p_t = \frac{\eta}{1+\eta} E_t\left\{p_{t+1}\right\} + \frac{1}{1+\eta} m_t + u_t \text{ where } u_t = \frac{\eta E_t\left\{r_{t+1}\right\} - y_t}{1+\eta}$$

- Solving forward,

$$p_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\left\{m_{t+k}\right\} + \bar{u}_t$$

where $\bar{u}_t$ is independent of monetary policy.

- This is a convergent sum and the price level is determinate if m_t mean reverts.

Money or Interest Rates?

- What matters for inflation is really nominal interest rate:

$$1 = Q_t E_t \left\{ \Lambda_{t,t+1} / \Pi_{t+1} \right\}$$

- Central bank can choose i_t rather than $\{M_t\}$:

$$\frac{M_t}{P_t} = \Phi\left(Q_t, Y_t\right)$$

- Targeting a nominal interest rate gives central bank more direct control over inflation.
  - If target M, shifts in money demand move nominal interest rate and affect inflation.
  - If target i, will not happen.
- Consequently, modern central banks set i_t not M_t.
  - We will drop money market from our models and instead impose a process or rule for i_t.

Example: Exogenous Interest Rate Process

- Assume we have an exogenous process $\{i_t\}$.
- Now only restriction is Fisher equation:

$$\hat{i}_t = E_t\left\{\hat{r}_{t+1}\right\} + E_t\left\{\hat{\pi}_{t+1}\right\}$$

- This pins down expected inflation, but *actual inflation is not determinate and there are multiple equilibria*.
    - Any solution with:

$$\hat{\pi}_t = \hat{i}_{t-1} - \hat{r}_t + \xi_t$$

    where ξ_t is mean zero in expectation is an equilibrium.

Interest Rate ("Taylor") Rule

- An alternative to an exogenous interest rate process is an interest rate rule:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + v_t,\ \phi_\pi \geq 0.$$

  - The monetary policy shock v_t is not a forecast error, and so it is serially correlated and often an AR(1): $v_t = \rho_v v_{t-1} + \varepsilon_t^v$.
- Sometimes called a "Taylor rule" after Taylor's (1993) famous analysis of Volcker-Greenspan monetary policy.
- Combine Taylor rule with Fisher equation to get

$$\phi_\pi \hat{\pi}_t = E_t\left\{\hat{\pi}_{t+1}\right\} + E_t\left\{\hat{r}_{t+1}\right\} - v_t$$

- If $\phi_\pi > 1$, this has a unique non-explosive solution:

$$\hat{\pi}_t = \sum_{s=0}^{\infty} \phi_\pi^{-(s+1)} E_t\left\{\hat{r}_{t+s+1} - v_{t+s}\right\}$$

  if $\phi_\pi < 1$ we have indeterminacy.

The Taylor Principle

- $\phi_\pi > 1$, which guarantees determinacy, is known as the **Taylor principle**, and we typically assume it holds.
  - In words, *Central Banks should adjust nominal interest rate more than one-for-one in response to a change in inflation*.
- Intuition from Fisher:

$$\hat{i}_t = E_t \left\{ \hat{r}_{t+1} \right\} + E_t \left\{ \hat{\pi}_{t+1} \right\}$$

  - If π_t high, raise i_t and hence $E_t \left\{ \hat{\pi}_{t+1} \right\}$ more than one for one.
  - If π_t low, lower i_t and hence $E_t \left\{ \hat{\pi}_{t+1} \right\}$ more than one for one.
  - Leads to unique π_t that does not explode.
    - Determinacy from interest rate rule holding both on and off equilibrium path.
    - Central bank is threatening to explode economy off equilibrium path!
    - Is this satisfying?

Cochrane (2011) Critique of Taylor Principle

- Generally we like determinacy.
  - Many argue that by following Taylor principle, Volcker and Greenspan brought inflation under control.
- But Cochrane (2011) argues bad economics:

  "Many proposals to trim equilibria sound superficially like sensible descriptions of what governments do to stop extreme inflation or deflation...however, stopping an inflation or deflation is completely different from disallowing an equilibrium....

  Transversality conditions can rule out real explosions, but not nominal explosions. Hyperinflations are historic realities. This condition did not come from any economics of the model. I conclude that there is nothing wrong with [the other equilibria], and this model does not determine inflation."