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us in a similar way to connect the g.t.f. for any region with the g.t.f.'s for the very small sub-regions into which that region may be divided. This connection will contain the quantum analogue of the action principle applied to fields.

The square of the modulus of the transformation function $(q_t|q_T)$ can be interpreted as the probability of an observation of the coordinates at the later time t giving the result q_t for a state for which an observation of the coordinates at the earlier time T is certain to give the result q_T. A corresponding meaning for the square of the modulus of the g.t.f. will exist only when the g.t.f. refers to a region of space-time bounded by two separate (three-dimensional) surfaces, each extending to infinity in the space directions and lying entirely outside any light-cone having its vertex on the surface. The square of the modulus of the g. t. f. then gives the probability of the coordinates having specified values at all points on the later surface for a state for which they are given to have definite values at all points on the earlier surface. The g.t.f. may in this case be considered as a transformation function connecting the values of the coordinates and momenta on one of the surfaces with their values on the other.

We can alternatively consider $|(q_t|q_T)|^2$ as giving the relative a priori probability of any state yielding the results q_T and q_t when observations of the q's are made at time T and at time t (account being taken of the fact that the earlier observation will alter the state and affect the later observation). Correspondingly we can consider the square of the modulus of the g.t.f. for any space-time region as giving the relative a priori probability of specified results being obtained when observations are made of the coordinates at all points on the boundary. This interpretation is more general than the preceding one, since it does not require a restriction on the shape of the space-time region.

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ON THE EXPANDING UNIVERSE.

By M. Bronstein.

(Received on November 14, 1932).

The present state of the problem of the „universe as a whole" is discussed and some cosmological applications of Bohr's ideas concerning the source of stellar radiation are given.

I. General considerations.

This first section will treat the nature of difficulties which are met with in the formulation of the cosmological problem. Part of the views expressed here in this connexion will probably prove to be familiar to the majority of theoretical physicists; they are included for the sake of completeness and continuity of the exposition.

The world, which is not symmetrical in the past and future, i. e. the aspect of which is changed when we survey the events of its history in the reverse order like a cinema film shown from the wrong end, is beyond the reach of explanation by means of any physical theory based on equations which are invariant with respect to the transformation $t \to -t$ (remain unchanged when the sign of the time is reversed). This fact was clearly understood by Boltzmann who, wishing to obtain an explanation of the universe consistent with the laws of classical mechanics and of the statistics founded thereon, was led to believe that the physical world seems not to be symmetrical in $\pm t$ merely because only the small part of its spacetime is accessible to our observations and that, if surveyed in sufficiently great region of space and time, our universe would exhibit perfect symmetry with respect to the interchange of past and future. The universe considered as a whole is, according to Boltzmann, in a state of perfect statistical equilibrium; what we observe by means of our telescopes is the departure from it caused by statistical fluctuations; in any sufficiently great region of spacetime there are as many such fluctuations coming into existence as dying out. The explanation of Boltzmann must be discarded because an analogous departure from statistical equilibrium in the space region of 10^{55} cm^3 (a sphere with radius of the order of distance from the sun to the nearest stars) would be sufficiently great for us to live in and to observe the usual ways of nature; the relative probability of such a fluctuation in the region of space 10^{24} times as great (which is the scope of vision of the great Mt Wilson reflector) equals practically naught, and this is the measure of plausibility of Boltzmann's assumption. We come to the conclusion that the real universe, be it stationary or not, must be highly asymmetrical in $\pm t$, and indeed it can hardly be expected that any rational human being would earnestly believe that any such things as stars absorbing, instead of emitting, energy or as killed soldiers rising up and marching away from the field in perfect order (but backwards) are really possible in nature. A physical theory upon which the solution of the cosmological problem can be based cannot be symmetrical with respect to the interchange of the past and future.

Now, it is well known, that not only classical mechanics but also its two generalizations, the theory of relativity (both special and general) and the wave mechanics, are perfectly symmetrical in $\pm t$. From the above stated fact it follows that every attempt to obtain the solution of the cosmological problem within the scope of these theories must be in principle erroneous and hopeless. This remark refers also to the theory of Lemaître. It is stated usually that Lemaître's theory of the expanding universe has succeeded in explaining the recession of spiral nebulae, but this statement is far from being exact. It is true that this theory was able to deduce the equality of sign of velocities of very far distant objects and the proportionality of the velocity to the distance, but it cannot explain why the direction of the motion is that of recession and not that of approach; and indeed the collapsing universe would satisfy the equations of Lemaître's theory as well as the expanding one. The recent attempt made by Lemaître[1] to prove the contrary cannot be considered as satisfactory. Now, it has been pointed out by Bohr[2] that there exist in nature very important regions which are beyond the reach of the usual wave mechanics and the usual theory of relativity: these regions are the internal layers of the stars. The existence of stellar nuclei governed by the laws of the relativistic quantum theory has been confirmed by Landau.[3] A considerable part of the mass of the whole universe is likely to be contained in these nuclei, to which the consequences of the general theory of relativity (and therefore the law of the conservation of energy) cannot be applied a priori, and the flow of energy coming from them must play a most important part in the household of nature. The universe should be considered therefore as consisting of two interacting parts; the one of them (part A) obeys the laws of the general theory of relativity, and the other (part B) does not. This is the reason why this theory cannot be applied to the universe as a whole.

It is highly probable that the relativistic quantum theory, whatever modifications the concept of time has to undergo in it, must prove to be asymmetrical with respect to the interchange of past and future. It is impossible to obtain a complete and consistent solution of the cosmological problem before the relativistic quantum theory has been built up. Nevertheless it would be perhaps not impossible to try to obtain some information about the general aspect of part A, to which the theory of relativity can be applied, if the nature of its interaction with part B be taken into consideration as a matter of observation. The observation shows that part B can be considered as a set of sources of gravitational field and of radiation scattered pretty uniformly through part A. As to the nature of this interaction two different

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[1] G. Lemaître, Revue des Questions Scientif., novembre 1932, p. 391.

[2] N. Bohr, Convegno di fisica nucleare (Roma, Ottobre 1931), p. 119.

[3] L. Landau, Sow. Phys. 1, 285, 1932.

points of view can be now looked upon as consistent with known observational and theoretical facts:

1. It is well known that there exist several (at least two) one-parameter-sequences of stars; in each of these sequences the luminosity of the star, its radius etc. can be considered as definite functions of its mass (this statement is not absolutely exact because of the comparatively small deviations caused perhaps by the different chemical constitution of the stars). It is possible that these sequences are usual courses of stellar evolution, which is interrupted only by sudden jumps of the star from one sequence to another (outbursts of Novae). If it be so, the mass of the star diminishes very considerably in the process of its evolution, which can be accounted for by the „annihilation of matter“ going on in the pathological (as Landau has named it) stellar nucleus. Every star must be considered as being in equilibrium corresponding to the instantaneous mass, radius and radiative outflow of the nucleus. The conditions of this equilibrium are changing very slowly owing to the decay of the nucleus. The duration of the evolution cannot be calculated, because it is not known whether the process of the generation of energy in the nucleus is governed by the law of conservation of mass or not.

2. The alternative hypothesis is as follows: the one-parameter-sequences of stars are not evolutionary sequences at all. Their physical significance consists only in the fact that each given stellar mass possesses several definite (if the chemical differences can be neglected) more or less stable configurations of equilibrium. The only source of stellar radiation is the violation of the law of energy going on in the nucleus. The star is a perpetuum mobile of the first kind and does not evolve at all, with the exception of spontaneous jumps from one configuration of equilibrium to the other more stable configuration. Such jumps have but little influence on part A as a whole, because the masses and luminosities of the post-outburst stars differ, as far as we know, very little from those of the pre-outburst stars.[1]

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[1] Cf. E. A. Milne, The Observatory 54, 126, 1931 (Nr. 684).

It seems at first sight that owing to the existence of very dense stellar nuclei it would be possible to propose a third hypothesis, namely the Kelvin-Helmholtz contraction theory in a somewhat modernized form. The amount of energy $\frac{GM^2}{R}$, which is too small when M and R are the mass and the radius of a star, can be made perhaps sufficiently great if M and R are supposed to be the mass and radius of the enormously dense nucleus. The decrease of the stellar mass during its evolution would come out to be a consequence of the ordinary packing effect without any „annihilation of matter“. It seems nevertheless that even in this modified form the contraction hypothesis cannot be made to agree with the facts. The extreme slowness of the evolution can be explained only by means of the assumption that the star possesses the configurations of equilibrium in which it always remains and that the parameters defining this equilibrium are changing at a rate which is quite independent of the rate of transitions into the equilibrium state and is very slow compared to it. The contraction hypothesis does not fulfil this condition.

In the next section we shall give some cosmological applications of the second point of view discussed above.

II. The expansion of the universe.

If the information afforded by the observations made with great modern astronomical instruments is applicable to the universe in general, we can suppose that the whole space is filled fairly uniformly with galaxies placed at distances of about 10^{24} cm apart. The mass of an average galaxy is about 10^{11} times greater than the Sun's mass, which gives a mean density of matter in the universe of the order of magnitude of 10^{-28} gr/cm^3. The line-element of such a homogeneous universe can be presented in the form

$$ds^2 = -R^2\left[d\chi^2 + \sin^2\chi\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right] + c^2dt^2. \qquad (1)$$

The general form of the gravitational equations is

$$G_{ik} - \frac{1}{2}\, g_{ik}\, G + \lambda g_{ik} + \varkappa T_{ik} = 0 \qquad (2)$$

with

$$\varkappa = 1{,}87 \cdot 10^{-27}\,\mathrm{gr}^{-1}\,\mathrm{cm} \text{ and}$$

$$T_{ik} = \sum_{j,l} g_{ij}\, g_{kl} \left(\rho + \frac{p}{c^2}\right) \frac{dx_j}{ds} \frac{dx_l}{ds} - g_{ik} \frac{p}{c^2} \tag{3}$$

(tensor of energy and momentum). In this last equation ρ is interpreted as the full density of material and radiative mass and p as the pressure of radiation. The pressure arising from the transfer of momentum by moving galaxies is neglected. The density of the matter ρ_0 is connected with the full density ρ by means of $\rho_0 + \frac{3p}{c^2} = \rho$.

In the following we shall suppose that R and λ are functions of time, not because we wish to use any a priori hypothesis, but merely for the sake of generality. In fact it is one of the most general assumptions that can be made about a world which is homogeneous with respect to space. The physical meaning of the special assumption $\lambda = \text{const.}$ will become clear further on. The formulae (1) and (2) with $x_1 = \chi$, $x_2 = \vartheta$, $x_3 = \varphi$, $x_4 = t$ give

$$T_{11} = \frac{\lambda R^2 - 1}{\varkappa} - \frac{1}{\varkappa c^2}\left[\left(\frac{dR}{dt}\right)^2 + 2R\frac{d^2R}{dt^2}\right],$$

$$T_{22} = T_{11}\,\sin^2\chi, \quad T_{33} = T_{22}\,\sin^2\vartheta,$$

$$T_{44} = \frac{c^2}{\varkappa}\left(\frac{3}{R^2} - \lambda\right) + \frac{3}{\varkappa R^2}\left(\frac{dR}{dt}\right)^2,$$

$$T_{ik} = 0 \quad (i \neq k).$$

Comparison with (3) gives

$$\frac{d\chi}{ds} = \frac{d\vartheta}{ds} = \frac{d\varphi}{ds} = 0, \quad \left(\frac{dt}{ds}\right)^2 = \frac{1}{c^2}$$

and

$$\left.\begin{aligned} p &= \frac{c^2}{\varkappa}\left(\lambda - \frac{1}{R^2}\right) - \frac{1}{\varkappa}\left[\frac{1}{R^2}\left(\frac{dR}{dt}\right) + \frac{2}{R}\frac{d^2R}{dt^2}\right], \\ \rho &= \frac{1}{\varkappa}\left[\frac{3}{R^2} - \lambda + \frac{3}{c^2R^2}\left(\frac{dR}{dt}\right)^2\right]. \end{aligned}\right\} \tag{4}$$

We have also

$$\rho_0 = \frac{6}{\varkappa R^2}\left[\frac{1}{c^2}R\frac{d^2R}{dt^2} + \frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 + 1\right] - \frac{4\lambda}{\varkappa}. \tag{5}$$

It is easy to verify that the divergence of the tensor of momentum and energy can be presented in the form of a covariant vector, whose first three components are identically zero and whose fourth component is

$$\frac{d\rho}{dt} + \left(\frac{p}{c^2} + \rho\right)\frac{3}{R}\frac{dR}{dt}.$$

A direct **calculation** by means of (4) makes it equal to $-\frac{1}{\varkappa}\frac{d\lambda}{dt}$. **It makes clear** the significance of the hypothesis $\lambda = \text{const.}$ in the theory of Lemaître. This hypothesis makes the divergence of the tensor of energy and momentum equal to naught, which is equivalent to the law of conservation of momentum and energy. Without this special hypothesis we obtain the equation

$$\frac{d}{dt}(R^3\rho) + \frac{p}{c^2}\frac{d}{dt}(R^3) = -\frac{1}{\varkappa}\frac{d\lambda}{dt}R^3$$

or, after introducing the volume of the universe $V = \pi^2 R^3$ and its energy $E = \rho c^2 V$,

$$\frac{dE}{dt} + p\frac{dV}{dt} = -\frac{\pi^2 c^2 R^3}{\varkappa}\frac{d\lambda}{dt}. \tag{6}$$

The assumption $\lambda = \text{const.}$ by Lemaître meant that the change of the radius R is going on adiabatically, i. e. that the universe receives no energy from without. In our theory the equations of general relativity are applied to part A of the universe, not to the universe as a whole. The part A is no closed system; it does receive energy from without, or, to speak more exactly, from within, i. e. from the nuclei of B scattered uniformly through A. If we make the second hypothesis about the nature of interaction of A and B (hypothesis based on Bohr's idea), we must assume that the material mass of the world M_0 is constant and that the energy generation pro unit mass and unit time is also constant. We obtain

$$\left.\begin{aligned} \frac{dE}{dt} + p\frac{dV}{dt} &= \alpha M_0, \\ \rho_0 R^3 &= \frac{M_0}{\pi^2}. \end{aligned}\right\} \tag{7}$$

The universal constant α cannot be calculated theoretically without the aid of the relativistic quantum theory; on comparing the luminosities of spiral nebulae with their supposed masses de Sitter finds[1] its observational value to be about $2{,}7 \cdot 10^{-3}$ cm² sec⁻³. The other constant M_0 is at present unknown.

On comparing with (5) and (6) we find that (7) is equivalent to the system of differential equations

$$\left.\begin{aligned} \frac{d^2R}{dt^2} &= \frac{\varkappa c^2 M_0}{6\pi^2 R^2} + \frac{2\lambda c^2}{3} R - \frac{c^2}{R} - \frac{1}{R}\left(\frac{dR}{dt}\right)^2, \\ \frac{d\lambda}{dt} &= -\frac{\varkappa \alpha M_0}{\pi^2 c^2 R^3}. \end{aligned}\right\} \qquad (8)$$

This system is of the third order. Its solution would involve five constants (including α and M_0). Only three equations for these five constants can be furnished at present by the observational data (these three equations are $\alpha = 2{,}7 \cdot 10^{-3}\,\mathrm{cm}^2\,\mathrm{sec}^{-1}$, $\frac{M_0}{\pi^2 R^3} = 10^{-28}\,\mathrm{gr.\,cm}^{-3}$ and $\frac{1}{R}\frac{dR}{dt} = 1{,}5 \cdot 10^{-17}$ as deduced by de Sitter[1] from the observations of the recession of spiral nebulae). It follows from this that no computation of the actual value of the radius of the universe is possible at present.

The equations (8) become identical with those of Lemaître[2] when $\alpha \rightarrow 0$. The most striking feature of the equations (8) is that they are not invariant with respect to the change of sign of t and that they are not satisfied by $R = \mathrm{const.}$ When we compare this result with Lemaître's theory we become aware of the true physical cause of the expansion of the universe. While working out the theory of Lemaître Eddington[3] has tried to show that Einstein's universe is unstable with respect to small changes of R; in this he saw the cause of the expansion of the universe. This argument is based on the assumption of the possibility of small changes of R and therefore it does not show why R is variable at all; all that it shows is that if the radius can change it will change, but we do not see why the universe with absolutely constant radius cannot exist. Now we know that the universe with constant radius would not be consistent with physical processes going on in it, first of all with the process of generation of energy in the „pathological“ nuclei of stars.

The history of the real physical world is asymmetrical with respect to time: the function λ can only decrease, but it cannot increase. The universe never returns into the state in which it was before (if the state be defined by the values of R, $\frac{dR}{dt}$ and $\frac{d^2R}{dt^2}$ simultaneously). It cannot be shown that all solutions of (8) coming into consideration correspond to the radius always increasing; we cannot say at present whether the increase of the radius can be followed by its decrease, because our knowledge about the constants of integration of (8) is insufficient. We do not know for instance the value of the pressure of radiation in interstellar space, partly because of the uncertainty as to the nature of penetrating radiation and as to the laws of its absorption. The phenomenon of penetrating radiation may prove to be very important for the solution of the cosmological problem. It would be of interest to study the general properties of the solutions of (8).

Note added on January 13th 1933. In the meantime L. Landau has drawn my attention to the fact that the fulfilment of the gravitational equations of Einstein's theory in the empty space surrounding a material body is incompatible with the nonconservation of the body's mass. This circumstance can be rigorously verified in the case of Schwarzschild's solution (spherical symmetry); physically it is connected with the fact that Einstein's gravitational equations allow only transverse gravitational waves and no longitudinal (this can be shown when the gravitational waves are defined by means of the Riemann tensor G_{iklm} thus excluding the fictitious waves connected with the special

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[1] W. de Sitter, Proc. Nat. Acad. U. S. A. **16**, 474, 1930; Proc. Acad. Amsterdam **39**, 82, 1930. It is doubtful whether these numerical data are really reliable, but we need them only for the sake of illustration.

[2] G. Lemaître, Ann. Soc. Scientif. Bruxelles 47 (A) 29, 1927.

[3] A. S. Eddington, Mont. Not. Roy. Astr. Soc. **90**, 668, 1930.

choice of coordinate systems). The difficulty becomes apparent when we try for instance to formulate the gravitational equations in the space surrounding a box that contains one single atom of RaE.

The avoidance of this difficulty in my paper is based on the use of macroscopical equations instead of microscopical; the treatment of the generation of radiant energy by stars' nuclei is in my paper formally equivalent to the introduction of a new form of energy connected with the λ-field which compensates Bohr's nonconservation. This way out of the difficulty seems to be a very unpleasant one; no other means of getting out of it can be seen at present. The paradox is indeed very puzzling and very characteristic of the difficulties arising in connection to the cosmological problem.

It is a pleasure to thank P. Ehrenfest and L. Landau for the interesting discussion of my paper.

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ON THE ANNEALING OF PLASTICALLY DEFORMED ROCK-SALT CRYSTALS.

By N. A. Brilliantow and I. V. Obreimow.

(Received on November 27, 1932).

The present work is the continuation of a previous paper[1]. In the latter it was shown that in a plastically deformed specimen of rock-salt residual strain remains. The magnitude of this strain can be estimated by means of the double refraction (piezo-optical effect) that it produces.

The problem was much simplified by the fact that the residual strain is distributed in a very simple manner. The planes of optical symmetry of the strained portions lie parallel to the planes (110); i. e. between crossed nicols the strained crystal appears dark if the direction (110) is parallel to the principal plane of the nicol. If the planes of the nicol are turned to make an angle of 45° with the direction (110), the crystal will appear bright in the strained places and dark in the unstrained. The magnitude of strain can easily be found if the piezo-optical constant of rock-salt is known.

The distribution of strain is also very simple. The strained parts penetrate the entire crystal in thin layers parallel to (110). The (110) plane in rock-salt has long been known to mineralogists as the gliding plane. These planes can be seen on Fig. 1, which shows a photograph of a parallelipipedon of rock-salt, which has undergone a pressure of 2000 gramms per sq. mm. in the direction shown by the arrows, as on Fig. 2 seen in a polarisation microscope. The magnification is about 60 times. The crystal is viewed through the face (110). The principal planes of the nicols are parallel to (010) and (001). The gliding planes are parallel to (011).

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[1] Obreimow and Schubnikoff. ZS. f. Phys. 41, 907, 1927.