The Systemic Effects of Benchmarking

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Abstract

We show that the competitive pressure to beat a benchmark may induce institutional trading behavior that exposes retail investors to tail risk. In our model, institutional investors are different from a retail investor because they derive higher utility when their benchmark outperforms. This forces institutional investors to take on leverage to overinvest in the benchmark. Institutional investors execute fire sales when the benchmark experiences shock. This behavior increases market volatility, raising the tail risk exposure of the retail investor. Ex post, tail risk is only short lived. All investors survive in the long run under standard conditions, and the most patient investor dominates. Ex ante, however, benchmarking is welfare reducing for the retail investor, and beneficial only to the impatient institutional investor.

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1 Introduction

The asset management industry has grown exponentially over the past decades. Figure 1 displays the time series of assets under management by hedge funds worldwide, which have grown from $100 billion in 1997 to roughly $2.5 trillion in 2014. Other types of funds, such as mutual funds and index funds, have experienced similar growth. Because of their sheer size, the trading behavior of funds can impact prices and risks in financial markets in non-standard ways. This has sparked a public discussion about the potential risks that asset management may pose to financial stability.\footnote{Several news outlets have reported on the potential threats that the asset management industry may pose for financial stability. See, for example, Condon & Katz (2015) in Bloomberg Business, Marriage (2015) and Oakley (2015) in Financial Times, Eisinger (2014) in The New York Times, Ackerman (2015) and Samuel & Ackerman (2015) in The Wall Street Journal, as well as \url{http://econ.st/WOCeRT} and \url{http://econ.st/1rLdis8} in The Economist.} In this paper, we provide some first theoretical answers to questions about potential threats that the asset management industry may pose to the stability of financial markets.

One key feature of the asset management industry is that managers are compensated for the performance of the managed portfolios in excess of a predetermined benchmark.\footnote{Ma, Tang & Gómez (2015) document that about three-quarters of all mutual fund managers in the U.S. receive performance-linked compensation. Indeed, when an individual investor delegates the management of her portfolio to a manager that can exert effort to become informed about the distribution of returns, then the optimal contract in this principal-agent setting is one that compensates the manager for performance in excess of a benchmark; see, e.g., van Binsbergen, Brandt & Koijen (2008), Cvitanić, Henderson & Lazrak (2014), Cvitanić, Wan & Zhang (2006), Li & Tiwari (2009), and Stoughton (1993).} The so called “benchmarking” incentivizes managers to invest in ways that deviate from the standard theory. For example, Basak & Pavlova (2013), Buffa, Vayanos & Wolley (2015), Cuoco & Kaniel (2011), and others show that benchmarking incites managers to overinvest in stocks that are highly correlated with their benchmarks. This raises the question whether benchmarking may induce trading behavior by asset managers that results in increased risks and instabilities in financial markets. Academics and regulators around the world have recently expressed increased interest in this question.\footnote{For example, the Financial Stability Oversight Council (FSOC) asked in the Notice FSOC-2014-0001 released in December 2014 to what extent benchmarking can “create incentives to alter portfolio allocation in ways that [...] do not take into account risks to the investment vehicle or the broader financial markets?” Similar questions have been asked by Greenwood & Scharfstein (2013), Haldane (2014), IMF (2015), Jones (2015), Morris & Shin (2014), and OFR (2013), among others.}

We answer this question by analyzing a pure-exchange economy consisting of a retail and two institutional investors who can invest in a benchmark and a non-benchmark stock, and who can borrow and lend from each other. Stocks are in positive net supply, and have price processes that may jump. Time runs continuously from 0 to infinity, and investors consume at all periods of time. The retail investor derives standard log-utility from consumption, and can thus be interpreted as an individual mean-variance investor. The institutional investors, on the other hand, derive higher log-utility from consumption.
in states of the world in which the benchmark stock outperforms relative to the non-benchmark stock. These preferences incentivize our institutional investors to gain higher exposures to the benchmark stock than the retail investor, consistent with the behavior of managers whose performances are evaluated relative to a benchmark. Consequently, our institutional investors can be interpreted as asset managers who derive log-utility from the compensation they obtain for managing portfolios.

Investors in our model are heterogenous because they differ from each other through their benchmarking and their time preferences. Despite the complexity of our model, we can solve for all general equilibrium quantities in semi-closed form using a variant of the Fourier inverse methodology of Martin (2013). This allows us to run comparative statistics to analyze the systemic effects of benchmarking by measuring the tail risk exposures of our retail and institutional investors in economies with different benchmarking incentives. It also allows us to carry out a survival and a welfare analysis.

Our main finding can be summarized as follows: Stronger benchmarking incentives lead to higher (lower) tail risk exposure of the retail investor and the aggregate market when the benchmark stock underperforms (outperforms). More precisely, we find that strong benchmarking incentives incite institutional investors to take on leverage to over-expose themselves to the benchmark stock. This results in consumption and portfolio plans of institutional investors that are highly sensitive to the relative performance of the benchmark in states of the world in which the benchmark stock underperforms. Institutional investors react strongly to news in such states of the world. They trade large amounts of the stocks, resulting in large market volatility. If bad news about the benchmark arrives in the form of a jump, then institutional investors initiate fire sales. That is, they sell large amounts of the benchmark stock at a discounted price, and buy large amounts of the non-
benchmark stock at a premium price. This constitutes a flight-to-quality phenomenon. Even though this behavior reduces the tail risk exposure of the institutional investors, higher volatility and fire sales increase the tail risk exposure of the retail investor and the aggregate market in bad states of the benchmark. In contrast, institutional investors carry out buy-and-hold strategies in states of the world in which the benchmark outperforms. This behavior reduces market volatility and therefore also the tail risk exposure of the retail investor and the aggregate market in states of the world in which the benchmark outperforms. However, because institutional investors barely react to news in good states of the benchmark, they end up exposed to higher tail risk. Overall, with large benchmarking incentives, the retail investor and the aggregate market are exposed to high tail risk in states of the world in which institutional investors are exposed to low tail risk, and vice versa. Benchmarking, therefore, introduces a channel through which the trading behavior of institutional investors can impact the tail risk exposure of the retail investor and the aggregate market.

We extend our analysis by considering the costs and benefits of benchmarking ex post and ex ante. We find that benchmarking does not affect the long-term performance of our investors. All investors survive and become infinitely rich in the long run under the mild condition that at least one stock has positive expected dividend rate. We also find that the most patient investor dominates in the long run, regardless of the benchmarking incentives of the institutional investors. As a result, the exposure to tail risk borne by the retail investor is only short lived, and does not affect ex post performance in the long run. Ex ante, however, benchmarking is disadvantageous to the retail investor, and beneficial only to the impatient institutional investor. We establish this fact by measuring the equivalent variation of consumption for economies with and without benchmarking incentives. This analysis reveals that the retail investor always needs to consume less in a world without benchmarking incentives to achieve the same utility as in a world with benchmarking incentives. The opposite result only holds for the impatient institutional investor. Consequently, benchmarking is welfare reducing for the retail investor and potentially also for the patient institutional investor ex ante, even though it does not affect the long-term performance of our investors ex post.

The results of our study have important implications for the regulation of the asset management industry. Our findings indicate that benchmarking can create incentives for asset managers to alter portfolios in ways that do not fully take into account the effects on tail risk exposure of individual investors and the aggregate market. Our results suggest that stronger benchmarking incentives generally make individual investors and potentially also patient fund managers worse off ex ante due to an increased tail risk exposure when compared to a world without benchmarking incentives. Still, tail risk does not materialize in the long run. These results indicate that it is imperative for regulators to formulate precise objectives for a potential regulation of the asset management industry. If the regulator is concerned about investor failure, then our results suggest that there may not be any scope for regulation. On the other hand, if the regulator is concerned about the
tail risk exposure of retail investors, then regulating the compensation packages offered
to fund managers may be one viable option.\textsuperscript{4} However, this option comes at the cost of
making fund managers worse off ex ante. Our results highlight that there is a need for
further research to analyze the costs and the benefits of a potential regulation of the asset
management industry.

In alignment with Jones (2015), our results also suggest that the regulation of the
asset management industry needs to be designed differently than the regulation of banks.
We find that in an economy with benchmarking, the retail investor and the aggregate
market are only exposed to low tail risk in states of the world in which institutional
investors are exposed to high tail risk. Consequently, standard regulatory tools for banks
that target their tail risk exposure, such as value-at-risk measurements and stress testing,
may not be able to identify scenarios in which retail investors and the aggregate market
are at risk of tail events. Additional research is needed to evaluate the effectiveness of
different tools that can be used to regulate the asset management industry.

The rest of this paper is organized in the following way. The remainder of this section
discusses the related literature. Section 2 introduces our model and discusses its main
features. We solve for the general equilibrium of our model in Section 3. Section 4 contains
our main results on the tail risk, survival, and welfare effects of benchmarking. Section 5
concludes. There are two technical appendices.

\subsection{Related literature}

Our results contribute to several strands of literature. First, we contribute to the literature
on institutional investors and their impact on asset prices; see Stracca (2006) for a survey.
As in Brennan (1993), Gómez & Zapatero (2003), Kapur & Timmermann (2005), Roll
(1992), and others, our institutional investors face incentives that incite them to tilt their
portfolios toward their benchmarks. This leads to an asset class effect that raises the price
of the benchmark stock relative to a similar non-benchmark stock (Cuoco & Kaniel (2011),
Basak & Pavlova (2013), and Hodor (2014)). It also induces an increase in the volatility
of our stocks (Basak & Pavlova (2013) and Buffa et al. (2015)). However, we differ from
and extent this literature in several ways. We do not consider the principal-agent problem
underlying the decision of an individual investor to delegate the management of wealth
to a portfolio manager. We also do not derive nor model the optimal contract between an
investor and a portfolio manager in this setting. Both of these problems have been recently
solved in a general equilibrium setting by Buffa et al. (2015). Instead, we follow Basak
& Pavlova (2013) and take a reduced-form approach to modeling the incentives faced
by institutional investors. Theses choices introduce sufficient tractability to be able to
solve the general equilibrium of our model in semi-closed form and to carry out important
comparative statics. In contrast to the existing literature, we also allow for dividend jumps

\textsuperscript{4}We refer to Core & Guay (2010) for a discussion of a potential regulation of compensation packages
in the financial industry.
in our model. We find that even when just one asset has an underlying dividend process that jumps, jump risk may spread to stocks whose dividend processes do not jump. This is due to the fact that all investors update their portfolio holdings when a jump occurs, changing the demand for all stocks and affecting all stock prices. We also find that jumps in asset prices become less severe as institutional investors have stronger benchmarking incentives. This is primarily driven by the fact that institutional investors hold on to their stocks when benchmarking incentives are high.

The paper in this literature that is most closely related to ours is Basak & Pavlova (2013). The key difference, however, is that our focus is to analyze the implications of benchmarking for the tail risk exposure, survival, and welfare of retail and institutional investors. The focus of Basak & Pavlova (2013) is to understand the general equilibrium effects of benchmarking on asset prices. In order to ensure that we have identified the correct benchmarking mechanism, we show that the behavior of asset prices in our general equilibrium is similar to that in Basak & Pavlova (2013). There are also differences in our modeling approaches. In contrast to Basak & Pavlova (2013), our institutional investors consume at all times, and their marginal utility of consumption increases in the dividend ratio attributed to the benchmark rather than in the benchmark dividend level. Our institutional investors are heterogenous, allowing us to also analyze the impact of time discount parameters on prices and risks in our market. We also allow for jumps in the dividend and stock price processes. By incorporating jumps, we can evaluate the impact of unanticipated negative news on portfolio allocations, stock prices, volatilities, and tail risk. This is a key ingredient of our analysis of the systemic effects of benchmarking.

Our results contribute to the literature on the costs and benefits of benchmarking. Admati & Pfleiderer (1997) analyze the use of benchmarks in compensation packages for managers and find that benchmarking may result in suboptimal risk sharing and portfolio choices. However, van Binsbergen et al. (2008) argue that these negative effects may be offset by the benefits of benchmarking in aligning diversification and investment horizon incentives. Das & Sundaram (2002) compare two types of performance-based compensation structures for managers, linear fulcrum fees and option-like incentive fees, and find that investor welfare tends to be higher under option-like fees. Carpenter (2000) finds that option-like fees can push the manager to reduce the managed portfolio's volatility if the manager has HARA utility from terminal wealth. In contrast to this literature, we do not explicitly model the type of performance-based compensation that a manager derives from managing a portfolio. Still, our reduced-form analysis indicates that benchmarking may introduce welfare effects that the literature was unaware of. We show that when managers face strong benchmarking incentives, their trading behavior may affect retail investors with increased tail risk exposure which, in turn, may reduce retail investor welfare.

We also contribute to the literature on heterogenous agents with additively separable utility functions. Gollier & Zeckhauser (2005) analyze models with heterogenous time preferences, and show that the shares of aggregate consumption attributed to each agent
vary dynamically over time. Consistent with Gollier & Zeckhauser (2005), the consumption shares of our retail and institutional investors also change as time evolves. However, in our model this is not only driven by heterogenous time preferences but also by the fact that the consumption shares of our institutional investors depend on the relative performance of the benchmark, which fluctuates stochastically. Tran & Zeckhauser (2014) consider continuous-time, finite-horizon economies driven by Brownian motions in which agents have heterogeneous risk and time preferences, as well as heterogenous beliefs. These authors show that more risk tolerant investors take on more volatile consumption plans.\(^5\) Compared to Tran & Zeckhauser (2014), we allow for jumps in our dividend processes but not for heterogenous beliefs. In our model, the risk tolerance of institutional investors is time-dynamic because it depends on the performance of the benchmark. This yields another channel driving the volatility of consumption plans of institutional investors. Yan (2008) analyzes infinite-horizon Brownian motion economies in which agents have heterogenous time preferences and beliefs, and shows that the investor with the lowest survival index dominates in the long run.\(^6\) We extend the findings of Yan (2008) by showing that the investor with the lowest survival index also dominates in the long run in homogenous belief economies in which dividend processes may jump, and in which institutional investors face benchmarking incentives. Cvitanić, Jouini, Malamud & Napp (2012) solve for the general equilibrium of an infinite-horizon Brownian motion economy in which agents have CRRA utility functions and heterogenous risk and time preferences, as well as heterogeneous beliefs. These authors find that the agents’ optimal portfolios exhibit substantial heterogeneity in equilibrium. Extending these results, we find that benchmarking may undo some of the portfolio heterogeneity across heterogenous institutional investors. This is due to the fact that strong benchmarking incentives force institutional investors to strongly tilt their portfolios toward the benchmark, irrespective of their other preferences.

Finally, our results also contribute to the literature on financial stability. Most of the current debate has focused on the influence of banking on financial stability; see, e.g., the speech by former Federal Reserve Bank Chairman Ben Bernanke at the 2012 Federal Reserve Bank of Atlanta Financial Markets Conference.\(^7\) Like this paper, there are a few other papers that focus on the influence of asset management on financial stability. Bank for International Settlements (2003) discusses how benchmarking may affect financial markets, and concludes that benchmarking can only affect market efficiency and volatility over the short term. We carry out a formal general equilibrium analysis of the effects of benchmarking on financial markets, and find that benchmarking can induce a short-term rise in tail risk exposure, but it cannot affect the long-term performance of financial

\(^5\)Risk tolerance is understood as the marginal propensity to consume out of aggregate wealth.

\(^6\)The survival index of an investor is the sum of her time discount parameter, her optimism bias, and the risk-adjusted expected dividend growth rate. Our investors do not have any optimism bias because they are perfectly rational.

\(^7\)A transcript of this speech is available at http://www.federalreserve.gov/newsevents/speech/bernanke20120409a.pdf.
investors. Garbaravicius & Dierick (2005) analyze the Lipper TASS hedge fund database, and identify three channels through which hedge funds may pose threats to financial stability: (i) through hedge fund failures, (ii) through banks exposures to hedge funds, and (iii) through the impact of hedge funds’ trading behavior on financial markets. Our results show that benchmarking may enable the latter channel. Garbaravicius & Dierick (2005) also document empirically that hedge funds tend to take on leverage, consistent with the behavior of our institutional investors. Danielsson, Taylor & Zigrand (2005) survey the theoretical and empirical literature available at the time, and argue that hedge funds can have systemic effects on financial markets primarily because of the market impact of large hedge fund failures. Extending these results, we show that the trading behavior of hedge fund managers may also have systemic implications because it can raise the tail risk exposure of retail investors.

Danielsson & Shin (2003) coined the term “endogenous risk” as additional financial risk that arises from the interactions and trading behavior of agents in a financial system. Our results show that benchmarking by institutional investors may give rise to endogenous risk. This occurs because benchmarking forces institutional investors to react strongly to news about their benchmarks in states of the world in which the benchmark underperforms. Similarly as in Danielsson, Shin & Zigrand (2013), endogenous risk in our model manifests itself systemically in the form of increased tail risk exposures.

2 Model

We analyze a pure-exchange economy populated by heterogenous agents with CRRA utilities. There are three investors of two different types: a retail investor and two institutional investors. Our economy is comprised of two risky assets and one safe asset. Time is measured continuously and runs from zero to infinity. All dividends and prices are modeled by stochastic processes that are measurable with respect to a probability space \((\Omega, \mathcal{F}, \mathbb{P})\), and adapted to the complete information filtration \((\mathcal{F}_t)_{t\geq 0}\) that represents the flow of information over time.

2.1 Assets

There are two risky stocks with prices \(S_1 = (S_{1,t})_{t\geq 0}\) and \(S_2 = (S_{2,t})_{t\geq 0}\). These assets are in net positive supply. We normalize the supply of each stock to one without loss of generality. Stock \(n \in \{1, 2\}\) produces a stream of dividends \((D_{n,t})_{t\geq 0}\) that satisfies the following stochastic differential equation:

\[
\frac{dD_{n,t}}{D_{n,t-}} = \mu_n dt + \sigma_n dZ_t + J_n dN_t
\]

Here, \(Z = (Z_t)_{t\geq 0}\) is a standard Brownian motion and \(N = (N_t)_{t\geq 0}\) is a Poisson process with constant arrival rate \(\lambda > 0\). For simplicity and to ensure market completeness,
we restrict to the case of two independent sources of risk. However, generalizations to additional sources of risk are possible. We assume that \( \mu_1 \) and \( \mu_2 \) are real-valued scalars, \( \sigma_1, \sigma_2 > 0 \), and \( J_1, J_2 > -1 \).

There is also a safe asset with price \( B = (B_t)_{t \geq 0} \). This asset has net zero supply. The safe asset represents borrowing and lending in the money markets. Borrowing and lending occurs at the interest rate \( r_t \), which is stochastic and satisfies

\[
dB_t = r_t B_t dt.
\]

### 2.2 Investors

There are three investors in our market: a retail investor and two institutional investors. Let the superscript “\( R \)” denote all variables corresponding to the retail investor, and the superscripts “\( A \)” and “\( B \)” denote all variables corresponding to the institutional investors \( A \) and \( B \), respectively.

Agents have non-negative initial endowments \( W_0^R, W_0^A, \) and \( W_0^B \) that correspond to fractions of the total wealth of the economy at the time 0:

\[
W_0 = W_0^R + W_0^A + W_0^B.
\]

Institutional investor \( j \in \{A, B\} \) owns a fraction \( \alpha^j \) of the total assets in positive supply of the economy, while the retail investor \( R \) owns the remainder \( \alpha^R = 1 - \alpha^A - \alpha^B \) of total wealth. At time 0, agents maximize their expected lifetime discounted utility stream subject to their budget constraints by choosing consumption and portfolio plans. We refer to Appendix A for details.

We list some notation. Let \( W_t^j \) denote the wealth of investor \( j \in \{R, A, B\} \) at time \( t > 0 \). Further, let \( c_t^j \) denote the amount of wealth consumed by investor \( j \in \{R, A, B\} \) at time \( t > 0 \). Finally, let \( \pi_t^j = (\pi_{1,t}^j, \pi_{2,t}^j, \pi_{l,t}^j) \) denote the portfolio of investor \( j \in \{R, A, B\} \) at time \( t > 0 \). This portfolio consists of a fraction \( \pi_{1,t}^j \) of wealth invested in asset 1, a fraction \( \pi_{2,t}^j \) of wealth invested in asset 2, and a fraction \( \pi_{l,t}^j \) of wealth offered as lending on the money markets. We assume that any wealth not consumed is invested in either the stock or the money markets. As a result, we have \( \pi_{1,t}^j + \pi_{2,t}^j + \pi_{l,t}^j = 1 \).

We assume that agents have log preferences with respect to consumption as this yields a simple framework in which our results can be easily illustrated. However, generalizations to other CRRA formulations are possible. For simplicity, we illustrate our results in a setting with one retail and two institutional investors. Generalization to settings with more retail and institutional investors are also possible.

### Retail investor

Investor \( R \) derives log-utility from intermediate consumption. She chooses a consumption plan \( (c_t^R)_{t \geq 0} \) and a portfolio plan \( (\pi_t^R)_{t \geq 0} \) that maximizes
subject to the budget constraint that the present value of her consumption plan does not exceed her initial wealth; see Appendix A for details. The retail investor discounts time exponentially with rate $\rho_R > 0$.

Institutional investors

It is well-known that the performance of an institutional investor is measured in relation to a benchmark; see Ma et al. (2015) for empirical evidence. As a result, institutional investors have incentives to post high returns in scenarios in which the underlying benchmark is posting high returns. In order to capture this unique feature of institutional investors’ incentives, we assume that investors $A$ and $B$ benchmark against stock 2, and that their marginal utilities of consumption are increasing in the performance of stock 2. More concretely, we assume that investor $j \in \{A, B\}$ chooses a consumption plan $(c^j_t)_{t \geq 0}$ and a portfolio plan $(\pi^j_t)_{t \geq 0}$ as to maximize

$$
\mathbb{E}\left[ \int_0^\infty e^{-\rho^j_t (1 + I^j_s t)} \log c^j_t \, dt \right]
$$

subject to the budget constraint that the present value of the consumption plan does not exceed the corresponding initial wealth. Here, $I^j_j \geq 0$ is a benchmark importance parameter, and

$$
s_t = \frac{D^j_{2,t}}{D^j_{1,t} + D^j_{2,t}}
$$

is the ratio of dividends attributed to asset 2. Our institutional investors discount time exponentially with rates $\rho_A > 0$ and $\rho_B > 0$, respectively.

2.3 Discussion

Our model of institutional investors can be viewed as a reduced-form model of benchmarking portfolio managers. As we show in Section 3, large values of $I^j_j$ incite institutional investor $j$ to strongly tilt her portfolio toward the benchmark stock. This behavior is consistent with the behavior of portfolio managers who derive compensation from the excess performance of the managed portfolio above a benchmark; see Basak & Pavlova (2013), Brennan (1993), Buffa et al. (2015), Cuoco & Kaniel (2011), Gómez & Zapatero (2003), and Roll (1992), among others. Consequently, we can view our institutional investors as portfolio managers. We also show in Section 3 that the institutional investors’ consumption plans in equilibrium reflect several key features of compensation for managers whose
performances are evaluated relative to a benchmark. Therefore, consumption of our institutional investors can be interpreted as managerial compensation, and $I_A$ and $I_B$ as measures of the strength of the benchmarking incentives that portfolio managers derive from their compensation packages.

Our model is closely related to the model of Basak & Pavlova (2013), who consider a pure-exchange economy with one retail and one benchmarking institutional investor. We ensure that we have identified the correct benchmarking mechanism by showing in Section 3 that the behavior of asset prices in our general equilibrium is similar to that in Basak & Pavlova (2013). However, there are differences in our modeling approaches. Stock prices in Basak & Pavlova (2013) do not jump. The agents in the model of Basak & Pavlova (2013) derive utility from terminal wealth at the finite maturity $T < \infty$; i.e., there is no intermediate consumption. As a result, the agents in Basak & Pavlova (2013) have stronger incentives to consume as time-to-maturity decreases. In contrast, we are interested only in changes in the portfolio-consumption policies of different agents generated by their particular attitudes toward risk. By choosing an infinite time horizon we eliminate incentives to consume stronger as time-to-maturity decreases. The introduction of intermediate consumption allows us to study how benchmarking affects borrowing and lending in the money markets. By allowing for jumps we can analyze how large shocks in one asset spread to other assets, and how agents hedge against jump risk in a dynamic setting with benchmarking. Finally, we allow for two heterogeneous institutional investors. We can therefore study the differences and similarities between the portfolios of our institutional investors, and the impact of heterogeneity on prices and risks.

There are three key benefits of our model specification. First, our model can be seen as a simple but powerful generalization of a Markowitz portfolio selection model with retail and institutional investors that allows us to answer important questions about the systemic effects of benchmarking. Second, our model captures several features of portfolio managers’ incentives that have been established by the extant literature. Third, we can solve for the general equilibrium in our model in semi-closed form up to numerical integration (see Section 3). This allows us to perform important comparative statics. Still, a more realistic objective function for our institutional investors would be

$$\mathbb{E} \left[ \int_0^{\infty} e^{-\rho t} \left( 1 + I_j \frac{S_{2,t}}{S_{1,t} + S_{2,t}} \right) \log c_j t \, dt \right].$$

(3)

Because the prices $S_{1,t}$ and $S_{2,t}$ are determined endogenously in equilibrium, the above formulation of the institutional investors’ incentives is intractable. Given the Markovian structure of our model, we show in Section 3 that there is a one-to-one mapping between prices and dividends of the form $S_n,t = g_n(D_{1,t}, D_{2,t})$ for $n \in \{1, 2\}$. As a result, our

\footnote{In particular, the utilities and consumption plans of our institutional investors satisfy properties (i) and (iii) of Proposition D1 of Appendix D of Basak & Pavlova (2013). Property (ii) is satisfied in some states of the world by the relative consumption of our institutional investors. This provides a possible microfoundation for our model of benchmarking agents.}
formulation of the institutional investors’ objective functions may be seen as a first-order approximation of (3).

3 Equilibrium

An equilibrium at time 0 consists of:

- Consumption plans \((c^R_t)_{t \geq 0}, (c^A_t)_{t \geq 0},\) and \((c^B_t)_{t \geq 0},\) and portfolio plans \((\pi^R_t)_{t \geq 0}, (\pi^A_t)_{t \geq 0},\) and \((\pi^B_t)_{t \geq 0}\) for the retail investor and each institutional investor that maximize (1) and (2) for \(j \in \{A, B\}\) subject to each investor’s budget constraints, and

- Prices \((S^1_{1,t})_{t \geq 0}\) and \((S^2_{2,t})_{t \geq 0},\) as well as an interest rate process \((r_t)_{t \geq 0}\) such that markets are cleared:

\[
0 = \pi^R_{1,t} W^R_t + \pi^A_{1,t} W^A_t + \pi^B_{1,t} W^B_t,
\]
\[
S^1_{1,t} = \pi^R_{1,t} W^R_t + \pi^A_{1,t} W^A_t + \pi^B_{1,t} W^B_t,
\]
\[
S^2_{2,t} = \pi^R_{2,t} W^R_t + \pi^A_{2,t} W^A_t + \pi^B_{2,t} W^B_t.
\]

An important property of our equilibrium is that we can solve for all relevant quantities in semi-closed form up to certain integrals which need to be computed numerically. As a result, our model formulation ensures tractability and allows us to precisely pin down the different mechanisms driving prices and risks in our market. This property also allows us to carry out comparative statics relative to the different model parameters. We refer to Appendix A for a precise characterization of the equilibrium. Proofs of the results are given in Appendix B.

These notations will be used throughout this section. The stock prices \((S^1_{1,t})_{t \geq 0}\) and \((S^2_{2,t})_{t \geq 0}\) are determined endogenously in equilibrium. Let \(\sigma_{n,t}\) denote the instantaneous diffusive volatility of stock \(n \in \{1, 2\}\) at time \(t > 0\) when no jump occurs. That is, if no jump occurs at time \(t > 0,\) then the conditional time-\(t\) variance of the \(n\)-th stock return over the small time period \(\Delta \approx 0\) is

\[
\text{Var}_t \left( \log \frac{S^1_{n,t+\Delta}}{S^1_{n,t}} \right) \approx \sigma_{n,t}^2 \Delta.
\]

In addition, let \(J_{n,t}\) denote the jump size of stock \(n \in \{1, 2\}\) if a jump occurs at time \(t > 0;\) that is,

\[
\frac{S^1_{n,t} - S^1_{n,t-\Delta}}{S^1_{n,t-\Delta}} \approx J_{n,t-\Delta}
\]

for \(\Delta \approx 0\) if a jump occurs at time \(t.\) Define the exposure matrix \(\Sigma_t\) that is composed of the stocks’ exposure to each source of risk as

\[
\Sigma_t = \left[ \begin{array}{cc} \sigma_{1,t} & \sigma_{2,t} \\ J_{1,t} & J_{2,t} \end{array} \right].
\]
The exposure matrix $\Sigma_t$ plays a key role in the consumption-portfolio plans in equilibrium. At last, define the following moments of the benchmark dividend ratio for $j \in \{R, A, B\}$ and $k \in \{1, 2, 3\}$:

$$M_{j,k,t}^j = \mathbb{E}_t \left[ \int_t^\infty e^{-\rho_j(v-t)} s_{v}^{k} dv \right],$$

$$\Delta M_{j,k,t}^j = \mathbb{E}_t \left[ \int_t^\infty e^{-\rho_j(v-t)} \Delta \lambda_{v}^{k} dv \right].$$

$M_{j,k,t}^j$ measures the long-term conditional $k$-th moment of $s_t$ given all information at time $t$ when discounted with the discount rate of investor $j$, while $\Delta M_{j,k,t}^j$ gives the long-term conditional $k$-th moment of the jump magnitude of $s_t$ when a jump occurs given all information at time $t$ after discounting with the discount rate of investor $j$. We derive closed-form expressions for these moments in Appendix A using a variant of the Fourier inverse methodology of Martin (2013). These expressions can be easily computed via numerical integration.

We fix the model parameters as in Table 1 unless specified otherwise. Our parameter choice is inspired by the parameter estimates of Backus, Chernov & Martin (2011) for U.S. equities derived from a Merton model of stock returns. Unlike Backus et al. (2011), though, we assume that jumps are less frequent but more severe in order to illustrate the impact of severe negative shocks on asset prices, risks, and portfolio allocations. We fix our model parameters to match the unconditional means and variances of the growth rates of dividends 1 and 2, while allowing dividend 2, but not dividend 1, to jump. In other words, we set

$$J_1 = 0, \quad \mu_1 - \frac{\sigma_1^2}{2} = \mu_2 - \frac{\sigma_2^2}{2} + \lambda J_2, \quad \text{and} \quad \sigma_1^2 = \sigma_2^2 + \lambda J_2^2.$$

By allowing jumps in $D_{2,t}$ but not in $D_{1,t}$ we can study how negative benchmark shocks impact non-benchmark assets as well as portfolio allocations. By equalizing the means and variances of the dividend growth rates we make a myopic investor indifferent between holding stock 1 and stock 2. As a result, unequal demands for benchmark and non-benchmark assets can be entirely attributed to the benchmarking incentives of institutional investors and their impact on asset prices and risks. Our parameter choice allows us to focus exclusively on the impact of benchmarking on asset prices, risks, and portfolio allocations without having to worry about the influence of idiosyncratic risks. Still, we have experimented with other parameter choices and find that the effects of benchmarking are significantly amplified under more realistic parametrizations.

Given the structure of financial markets with two risky assets and two independent sources of uncertainty, we can show that our model has a unique state price density to price assets in equilibrium, and our market is complete. Let $\xi = (\xi_t)_{t \geq 0}$ denote the unique state price density process. Define $(\theta_t)_{t \geq 0}$ as the process of market price of diffusion risk, and $(\psi_t)_{t \geq 0}$ as the process of market price of jump risk. These processes represent the
compensations that agents request for bearing volatility and jump risk, respectively. The market prices of risks are also determined endogenously in equilibrium.

3.1 State price density

We begin by characterizing the state price density in our market.

**Proposition 3.1.** Define

$$\phi_j = \left( \frac{1}{\rho_j} + I_j M_{j,0} \right)^{-1} > 0.$$  

The unique state price density of the economy is

$$\xi_t = \frac{Q_t}{D_t/D_0},$$  

where

$$D_t = D_{1,t} + D_{2,t},$$

$$Q_t = \alpha^R \rho R e^{-\rho R t} + \alpha^A \phi^A e^{-\rho A t} (1 + I_{A} s_{t}) + \alpha^B \phi^B e^{-\rho B t} (1 + I_{B} s_{t}).$$

The state price density has several interesting features. If there are no benchmarking incentives ($I_A = I_B = 0$) and all agents have the same time discount parameter, we obtain the standard state price density that is inversely proportional to aggregate dividends. When institutional investors have positive benchmarking incentives ($I_A > 0$ or $I_B > 0$), the pricing kernel is sensitive to the performance of the benchmark. In such cases, the benchmark influences the pricing kernel in two ways.

The first influence comes through $Q_t$ in the numerator of (4), which encompasses the pricing contribution of each agent in the market and is an increasing function of benchmark dividend ratio $s_t$. This channel captures the index effect of Basak & Pavlova (2013). In states of the world in which the benchmark is outperforming, institutional investors are exposed to the risk of underperforming relative to the benchmark. These states of the world carry high marginal utility for institutional investors, which is reflected in the pricing kernel. This effect is illustrated in Figure 2(a), which shows that the pricing kernel is an increasing function of $s_t$.

A second channel through which benchmarking affects the pricing kernel is through aggregate dividends in the denominator of (4). This channel reflects that states of the world in which the benchmark outperforms relative to alternative investment opportunities are more risky for institutional investors. In such states it is difficult for institutional investors to match the performance of the benchmark. To understand this effect, consider the opposite scenario: If the benchmark level is high whenever the alternative stock level is also high, then it is easy for institutional investors to post high returns when the benchmark is posting high returns. Thus, states of the world in which both the benchmark and
the alternative investment opportunity are booming are low risk states for our institutional investors. Such states carry low marginal utility for institutional investors, and this is also reflected in the pricing kernel. This is a relative index effect through which benchmarking affects prices, and it complements the absolute index effect of Basak & Pavlova (2013). Figure 2(b) illustrates the relative index effect. It shows that the pricing kernel decreases if the benchmark dividend level increases and the non-benchmark dividend level increases by the same amount.

### 3.2 Prices of Risk

The state price density of Proposition 3.1 yields the prices of risk as well as the interest rate in the market.

**Proposition 3.2.** The drift, volatility, and jump size functions of $D_t$ and $Q_t$ are:

\[ \mu_{d,t} = (1 - s_t)\mu_1 + s_t\mu_2, \quad \sigma_{d,t} = (1 - s_t)\sigma_1 + s_t\sigma_2, \quad J_{d,t} = (1 - s_t)J_1 + s_tJ_2, \]

\[ \mu_{q,t} = -\rho R\alpha e^{-\rho Bt} - \alpha A\phi Ae^{-\rho At}(1 + I_As_t) - \alpha B\phi Be^{-\rho Bt}(1 + I Bs_t) \]

\[ + (I_A\alpha A\phi Ae^{-\rho At} + I_B\alpha B\phi Be^{-\rho Bt})s_t(1 - s_t) [\mu_2 - \mu_1 + (\sigma_1 - \sigma_2)(\sigma_1(1 - s_t) + \sigma_2 s_t)], \]

\[ \sigma_{q,t} = \frac{I_A\alpha A\phi Ae^{-\rho At} + I_B\alpha B\phi Be^{-\rho Bt}}{Q_t}(\sigma_2 - \sigma_1)s_t(1 - s_t), \]

\[ J_{q,t} = \frac{I_A\alpha A\phi Ae^{-\rho At} + I_B\alpha B\phi Be^{-\rho Bt}}{Q_t} s_t(1 - s_t)(J_2 - J_1). \]

The market prices of volatility and jump risk are given by:

\[ \theta_t = \sigma_{d,t} - \sigma_{q,t}, \quad \psi_t = \frac{1 + J_{q,t}}{1 + J_{d,t}}. \]

The interest rate is

\[ r_t = \mu_{d,t} - \mu_{q,t} - \sigma_{d,t}^2 + \sigma_{d,t}\sigma_{q,t} + \lambda(1 - \psi_t). \]

Our prices of risk are not constant despite the fact that our dividend processes have constant coefficients. There are two effects driving the stochasticity of market prices of risk. First, the dividend ratio $s_t$ is a state variable as in Cochrane, Longstaff & Santa-Clara (2008) and Martin (2013). Thus, fluctuations of $s_t$ are mapped onto fluctuations of risk prices. When the dividend ratio changes, agents change their portfolio holdings, and this is reflected in fluctuating market prices of risks. Second, fluctuations of $Q_t$ are also mapped onto fluctuations of market prices of risk when institutional investors have benchmarking incentives ($I_A > 0$ or $I_B > 0$). In such scenarios, changes in the performance of the
benchmark lead to changes in the institutional investors’ portfolios, yielding stronger fluctuations in market prices.

Figure 3 illustrates the behavior of market prices of volatility and jump risk as the benchmark dividend ratio and the benchmark importance parameter vary. From the expressions in Proposition 3.1 we see that the market price of volatility risk is increasing in the benchmark importance parameters $I_A$ and $I_B$ if $\sigma_2 < \sigma_1$ (as in our parametrization in Table 1), and decreasing in $I_A$ and $I_B$ otherwise. Similarly, the market price of jump risk is decreasing in $I_A$ and $I_B$ if $J_2 < J_1$ (as in our parametrization in Table 1), and increasing otherwise. Institutional investors have high demand for the benchmark asset when benchmarking incentives are large, as shown below. As a result, institutional investors are willing to hold the benchmark stock for a small compensation per unit of risk in scenarios in which the benchmark importance parameters are large.

Similar effects also lead to a time-varying interest rate in our model. Figure 4 plots the interest rate $r_t$ as a function of the benchmark dividend ratio $s_t$ and the benchmark importance parameters $I_A$ and $I_B$. When the benchmark dividend ratio is low, the interest rate decreases as institutional investors’ incentives to benchmark increase. On the other hand, the interest rate increases as institutional investors’ incentives to benchmark increase when the benchmark dividend ratio is high.\footnote{We find similar interest rate dynamics for alternative parametrizations of our model.}

3.3 Asset prices

Next, we pin down the stock prices $S_{1,t}$ and $S_{2,t}$, and their volatilities and jump sizes.

**Proposition 3.3.** The price of the benchmark stock is

$$S_{2,t} = \frac{D_t}{Q_t} \left( \alpha^R e^{-\rho_R t} M_{1,t}^R + \alpha^A \phi_A e^{-\rho_A t} (M_{1,t}^A + I_A M_{2,t}^A) + \alpha^B \phi_B e^{-\rho_B t} (M_{1,t}^B + I_B M_{2,t}^B) \right),$$

and the price of the non-benchmark stock is

$$S_{1,t} = \frac{D_t}{Q_t} \left( \alpha^R e^{-\rho_R t} + \alpha^A \phi_A e^{-\rho_A t} (\rho_A^{-1} + I_A M_{1,t}^A) + \alpha^B \phi_B e^{-\rho_B t} (\rho_B^{-1} + I_B M_{1,t}^B) \right) - S_{2,t}.$$

The stock volatility functions are

$$\sigma_{2,t} = \theta_t + \frac{D_t}{Q_t S_{2,t}} \left[ \alpha^R \rho_R e^{-\rho_R t} (M_{1,t}^R - M_{2,t}^R) + \alpha^A \phi_A e^{-\rho_A t} (M_{1,t}^A - M_{2,t}^A) \right.$$

$$\left. + \alpha^B \phi_B e^{-\rho_B t} (M_{1,t}^B - M_{2,t}^B) + 2I_A \alpha^A \phi_A e^{-\rho_A t} (M_{2,t}^A - M_{3,t}^A) + 2I_B \alpha^B \phi_B e^{-\rho_B t} (M_{2,t}^B - M_{3,t}^B) \right],$$
\[ \sigma_{1,t} = \theta_t \left(1 + \frac{S_{2,t}}{S_{1,t}}\right) + \frac{D_t(\sigma_2 - \sigma_1)}{Q_t S_{1,t}} \left[I_A \alpha^A \phi_A e^{-\rho_A t}(M^A_{1,t} - M^A_{2,t}) + I_B \alpha^B \phi_B e^{-\rho_B t}(M^B_{1,t} - M^B_{2,t})\right] - \sigma_{2,t} \frac{S_{2,t}}{S_{1,t}}. \]

The jump sizes of the stocks are

\[ J_{2,t} = \psi_t^{-1} - 1 + \frac{D_t}{\psi_t Q_t S_{2,t}} \left(\alpha^R \rho_R e^{-\rho_R t} \Delta M^R_{1,t} + \alpha^A \phi_A e^{-\rho_A t} \Delta M^A_{1,t} + \alpha^B \phi_B e^{-\rho_B t} \Delta M^B_{1,t} + I_A \alpha^A \phi_A e^{-\rho_A t} \Delta M^A_{2,t} + I_B \alpha^B \phi_B e^{-\rho_B t} \Delta M^B_{2,t}\right), \]

\[ J_{1,t} = \psi_t^{-1} \left(1 + \frac{S_{2,t}}{S_{1,t}} \frac{D_t}{Q_t} (I_A \alpha^A \phi_A e^{-\rho_A t} \Delta M^A_{1,t} + I_B \alpha^B \phi_B e^{-\rho_B t} \Delta M^B_{1,t})\right) - 1 - \frac{S_{2,t}}{S_{1,t}}(1 + J_{2,t}). \]

Figure 5 displays the stock prices as functions of the benchmark dividend ratio and the benchmark importance parameters. Naturally, the benchmark stock is more expensive than the non-benchmark stock whenever the benchmark outperforms; i.e., whenever \( s_t \) is large. On the other hand, the benchmark stock is cheaper than the non-benchmark stock when \( s_t \) is low. We also see that the price \( S_{1,t} \) of the non-benchmark stock is highly sensitive to the benchmark importance parameters \( I_A \) and \( I_B \), while the price \( S_{2,t} \) of the benchmark stock is not very sensitive to the benchmark importance parameters. These price effects are driven by the institutional investors’ needs to hedge against underperforming relative to the benchmark as we will demonstrate in the next sections.

Figure 6 displays stock volatilities and jumps as functions of the benchmark dividend ratio and the benchmark importance parameters. The volatility of both stocks is increasing functions of the benchmark importance parameter. Jumps sizes are also increasing functions of the benchmark importance parameters so that jumps become less severe as institutional investors have stronger incentives to benchmark. In addition, we see that stock volatilities are highest when the benchmark dividend ratio is low to intermediate, while jumps are most severe when the benchmark dividend ratio is extremely high or extremely low. Again, these effects are driven by the institutional investors’ needs to hedge against underperformance relative to the benchmark, as we will show below. Figure 6 also illustrates the feedback effect from benchmark shocks to the non-benchmark stock: even though the non-benchmark dividend process does not jump, the stock price associated with the non-benchmark dividend stream does jump. This is due to the fact that after a jump occurs, all investors update their portfolios and alter the demand for all stocks, which affects all stock prices.
3.4 Optimal portfolio plans

We are now in a position to describe the optimal portfolio plans of our investors. An important assumption is that the exposure matrix $\Sigma_t$ is of full rank. This assumption is closely related to market completeness as indicated by Bardhan & Chao (1996) and Hugonnier, Malamud & Trubowitz (2012). Sufficient conditions ensuring this assumption are ad-hoc and technical. However, our simulations show that this assumption is satisfied in most scenarios.

Proposition 3.4. Suppose that $\Sigma_t$ has full rank for all $t \geq 0$. For the retail investor, the optimal portfolio plan is given by

$$
\begin{pmatrix}
\pi_{1,t}^R \\
\pi_{2,t}^R
\end{pmatrix} = \Sigma_t^{-1} \begin{pmatrix}
\theta_t \\
\psi_t^{-1} - 1
\end{pmatrix}.
$$

For institutional investor $j \in \{A,B\}$, the optimal portfolio plan is given by

$$
\begin{pmatrix}
\pi_{1,t}^j \\
\pi_{2,t}^j
\end{pmatrix} = \Sigma_t^{-1} \begin{pmatrix}
\theta_t \\
\psi_t^{-1} - 1
\end{pmatrix} + \Sigma_t^{-1} \begin{pmatrix}
(\sigma_2 - \sigma_1)(M_{1,t}^j - M_{2,t}^j) \\
\Delta M_{1,t}^j \psi_t^{-1}
\end{pmatrix} \frac{I_j}{\rho_j^{-1} + I_j M_{1,t}^j}.
$$

Finally, we have

$$
\pi_{1,t}^j = 1 - \pi_{1,t}^j - \pi_{2,t}^j,
$$

for $j \in \{R,A,B\}$.

At any point in time, the portfolio held by the retail investor coincides with the standard mean-variance portfolio given the log-utility formulation. The portfolio of an institutional investor at time $t$ is decomposed into two components: a component that accounts for the standard mean-variance portfolio (the first summand) and arises due to the institutional investors’ needs to hedge against volatility and jump risk, and a component that hedges against fluctuations of the benchmark dividend ratio $s_t$ (the second summand). Our results indicate that institutional investors have hedging motives beyond the mean-variance motives of Duffie & Richardson (1991). The additional hedging affects the demand for all assets. When an institutional investor has small incentives to benchmark ($I_j \approx 0$), her optimal portfolio is close to the optimal portfolio of the retail investor. Large incentives to benchmark shift the institutional investors’ portfolios away from the retail investor’s portfolio, and closer toward the benchmark stock. Figure 7 illustrates the portfolio plans of our retail and institutional investors, and showcases that institutional investors tilt their portfolios toward the benchmark stock as their incentives to benchmark grow large. It is noticeable that institutional investors take on leverage to invest in the benchmark stock as the benchmarking incentives grow large.

It is well understood that benchmarking incentives lead to deviations from the standard mean-variance portfolio allocation; among many others, see Basak, Pavlova & Shapiro.
(2007), Brennan (1993), Gómez & Zapatero (2003), Jorion (2003), and Roll (1992). Our findings on the institutional investors’ portfolios complement the existing literature by allowing asset prices to jump, by allowing agents to consume at intermediate times, and by allowing for heterogenous institutional investors. Basak & Pavlova (2013) find that institutional investors who benchmark against the absolute level of a benchmark and consume only at terminal time have additional demand for stocks that are highly correlated with the benchmark. Similarly, our institutional investors have additional demand for the benchmark stock. Nevertheless, the additional demand for the benchmark stock depends on the risk preferences of each institutional investor, as well as on the risk profiles of each stock. Consistent with risk aversion, Figure 8 shows that the institutional investors’ demand for the benchmark stock decreases and their demand for the non-benchmark stock increases as benchmark dividend jumps become more severe. Figure 8 also indicates that, against the standard mean-variance intuition, increases in the volatility of a dividend stream raise the institutional investors’ demand for the corresponding risky security. Both of these effects are driven by the need of institutional investors to hedge against fluctuations of the benchmark.

How do the portfolios of our institutional investors compare to each other? Our institutional investors are heterogeneous and differ from each other through their benchmark importance parameters and their time preferences. Thus, different institutional investors hold different portfolios. Note that the portfolio of institutional investor \( j \) converges to

\[
\left( \frac{\pi_{1,t}^j}{\pi_{2,t}^j} \right) = \Sigma_t^{-1} \left( \frac{\theta_t}{\psi_t^{-1} - 1} + \Sigma_t^{-1} \left( \frac{(\sigma_2 - \sigma_1)(1 - M_{1,t}^j/M_{1,t}^A)}{\psi_t^{-1} \Delta M_{1,t}^j/M_{1,t}^j} \right) \right)
\]

as \( I_j \to \infty \) given that \( I_j/(\rho_A^{-1} + I_j M_{1,t}^A) \to 1/M_{1,t}^A \). The formulation of the moments of \( s_t \) in Appendix A implies that \( M_{2,t}^j/M_{1,t}^j \approx s_t \) and \( \Delta M_{1,t}^j/M_{1,t}^j \approx \Delta s_t \) conditional on a jump at time \( t \) in the realistic setting \( \rho_J \approx 0 \). Therefore, Proposition 3.4 tells us that the differences between the portfolios of our institutional investors are only very small whenever the benchmark importance parameters are large.

### 3.5 Consumption plans

The following proposition characterizes the optimal consumption policies of our agents.

**Proposition 3.5.** The optimal consumption plan for institutional investor \( j \in \{A, B\} \) is

\[
c^j_t = \rho_j \frac{1 + I_j s_t}{1 + I_j \rho_j M_{1,t}^j} W_t^j = \frac{\alpha^j W_0 \phi_j e^{-\rho_j t} (1 + I_j s_t)}{\xi_t}.
\]

For the retail investor, the optimal consumption plan is

\[
c^R_t = \rho_R W_t^R = \frac{\alpha^R W_0 \rho_RE^{-\rho_R t}}{\xi_t}.
\]
The consumptions plans of our investors are increasing functions of wealth. As Figure 9(a) indicates, consumption is also nonlinearly increasing in the benchmark dividend ratio $s_t$, and linearly increasing in the benchmark dividend level $D_{2,t}$ when $s_t$ is kept fixed. Consistent with our log-utility formulation, the retail investor consumes a constant fraction of wealth at any point of time. Institutional investors, on the other hand, consume a time-varying fraction of wealth. The wealth-consumption ratios of institutional investors are nonlinear functions of the benchmark dividend ratio $s_t$ and the benchmark importance parameters $I_A$ and $I_B$. Figure 9(b) displays the wealth-consumption ratios of institutional investors as functions of $s_t$, $I_A$, and $I_B$. We see that the wealth-consumption ratio of an institutional investor is increasing in her benchmark importance parameter. An institutional investor consumes the largest fraction of wealth when the benchmark slightly outperforms and her benchmark importance parameter is large.

Consumption by our institutional investors can be interpreted as compensation for benchmarking portfolio managers. The literature has extensively studied optimal contracts for portfolio managers in principal-agent settings in which an investor (principal) delegates the management of her portfolio to a manager (agent) who must exert effort to become informed about the distributions of returns (see Stracca (2006) for a literature survey). There is consensus that optimal contracts specify compensation packages that are nonlinear and increasing in the performance of the managed portfolio over and above a predetermined benchmark (see, e.g., Li & Tiwari (2009) and Stoughton (1993)). The compensation plans of our institutional investors reflect this feature. In Figure 10 we plot the consumption of institutional investor $A$ in excess of what is consumed by the retail investor. In order to make the consumption of both investors comparable, we temporarily neglect institutional investor $B$ and set $\alpha_B = 0$, $\alpha_A = \alpha_R = 0.5$, and $\rho_A = \rho_R = 0.02$ for this figure. We see that the institutional investor consumes more than the retail investor if $I_A > 0$. In particular, the institutional excess consumption is increasing in the benchmark importance parameter. We also see that the institutional excess consumption is nonlinearly increasing in the benchmark dividend ratio $s_t$, and linearly increasing in the benchmark dividend level $D_{2,t}$ when $s_t$ is kept fixed. We know from Figure 7 that institutional investors allocate large fractions of their wealth on the benchmark when the benchmark importance parameters are large. In such cases, the performance of an institutional investor is highly linked to the performance of the benchmark stock. As a result, institutional investors perform well when the benchmark performs well, and in such scenarios they also consume more than a comparable retail investor. We conclude that institutional investors in our model enjoy a consumption bonus when their portfolios perform well, which is consistent with the optimal contacts for benchmarking portfolio managers established by the literature.
4 Systemic Effects

We study the systemic implications of benchmarking incentives based on the general equilibrium of Section 3. We proceed as follows. First, we measure the impact of benchmarking on tail risk for investors in our market. To this extent we measure value-at-risk of the portfolio returns of each institutional and retail investor, as well as of the aggregate market. Value-at-risk is the percentage loss we can forecast for a future period of time with a 1% probability. It gives a measure of the risk of large systemic losses faced by our investors and the aggregate market. In a second step, we analyze how the trading behavior of institutional investors may expose retail investors to large systemic losses. In particular, we study how benchmarking incentives may lead to large trading volumes and fire sales by institutional investors, resulting in large volatilities. We then study how benchmarking affects the long-term survival of our investors. In a final step, we measure the impact of benchmarking on welfare.

4.1 Value-at-risk

We compute values-at-risk for our institutional investors, our retail investor, and the aggregate market via exact Monte Carlo simulation. Figure 11 plots the 1-year values-at-risk against the benchmark importance parameter $I_j$ and the dividend ratio $s_0$. Across the board, we see that values-at-risk are highest in periods of extremely positive or extremely negative benchmark performance; i.e., in periods in which the dividend ratio $s_0$ is very large or very small. However, we see that a shifting of tail risk occurs as the benchmark dividend ratio falls from 1 to 0. Institutional investors are most exposed to value-at-risk when $s_0$ is large and the benchmark outperforms. This is primarily due to institutional investors investing large fractions of their wealth in the benchmark stock whenever $s_0$ is large. The benchmark stock has large negative jumps when $s_0$ is large (Figure 6). Figure 7 suggests that the institutional investors’ portfolio weights for stocks are least sensitive to fluctuations in the dividend ratio $s_0$ when the benchmark importance parameters $I_A$ and $I_B$ are large and $s_0$ is large. As a result, institutional investors carry out buy-and-hold strategies when $s_0$, $I_A$, and $I_B$ are large, with large exposures to tail risk arising from the large negative jumps of the benchmark.

In contrast, the retail investor is most exposed to value-at-risk when $s_0$ is small and the benchmark importance parameters are large. We see that value-at-risk for the retail investor strongly increases when the benchmark importance parameters $I_A$ and $I_B$ grow large in scenarios in which the dividend ratio $s_0$ is low. The retail investor reduces her investment in the safe asset when $s_0$ falls beyond a certain threshold (Figure 7). Consequently, the retail investor is more exposed to the stock market in scenarios in which $s_0$ is low. When $s_0$ is low and $I_A$ and $I_B$ are large, the volatility of both stocks is also large and their jumps are severe; see Figure 6. As a result, the retail investor is more exposed to tail risk whenever $s_0$ is small.
Aggregate market capitalization is equal to the sum of the wealth of all investors due to market clearing. Because of the strong investment of institutional investors in the benchmark when $s_0$ is high, value-at-risk for the aggregate market is highest when the benchmark dividend ratio is high. Still, we see that aggregate market value-at-risk is increasing in the benchmark importance parameters $I_A$ and $I_B$ whenever $s_0$ is low. The large tail risk assumed by the retail investor when the benchmark dividend ratio is low dominates the tail risk exposures of the institutional investors. Consequently, the aggregate market is more exposed to tail risk when benchmark incentives grow stronger in scenarios in which the benchmark underperforms.

4.2 Volatility and fire sales

Figure 7 indicates that the sensitivities of the institutional investors’ portfolio weights for stocks are highest whenever the benchmark importance parameters $I_A$ and $I_B$ are large and the dividend ratio $s_0$ is small. As a result, small fluctuations of the benchmark dividend ratio can result in large changes in the portfolios of institutional investors. Institutional investors sell (buy) large amounts of the benchmark and buy (sell) large amounts of the non-benchmark asset as a result of a small drop (rise) in the dividend ratio when $s_0$ is low. This behavior generates large trading volumes, which induces large volatilities for both stocks and severe jumps for the benchmark stock in scenarios in which $s_0$ is small and $I_A$ and $I_B$ are large (Figure 6). As a result, the portfolio volatilities of institutional investors decrease and the portfolio volatility of the retail investor increases with increasing benchmarking importance parameters when the benchmark dividend ratio is small (Figure 12). This naturally increases the retail investor’s exposure to tail risk.

Large portfolio sensitivities open up the possibility of fire sales by institutional investors after a jump occurs. Investors experience a cash flow shock when the benchmark dividend process jumps, prompting them to adjust their portfolio holdings. The degree to which investors’ portfolios are adjusted depends on the benchmark importance parameters and the dividend ratio. Consider a market in which there are no institutional investors; i.e., $I_A = I_B = 0$. Because the benchmark dividend falls drastically when a jump occurs, the cash flow of the benchmark stock become less profitable than the cash flow of the non-benchmark stock. This results in a substitution effect in which investors reduce their exposure to the benchmark stock and increase their exposure to the non-benchmark stock. The outlook of smaller cash flow also induces an income effect. Because investors expect less cash flow from their investments, they become less risk tolerant and reduce their exposure to both risky stocks. Figure 13 indicates that the substitution effect dominates when there are no institutional investors and when the dividend ratio is low. All investors reduce their exposure to the benchmark. Figure 13 indicates that the income effect for institutional investors is exacerbated when benchmarking incentives are large and the dividend ratio is low. When the benchmark importance parameters $I_A$ and $I_B$ are large, institutional investors are under pressure...
to beat the benchmark. After a jump, the outlook of lower cash flow incites institutional investors to strongly adjust their portfolios. They will further reduce their exposure to the benchmark asset, and increase their exposure to the non-benchmark asset. They also slightly increase their exposure to the safe asset; i.e., institutional investors reduce their leverage after a jump. These are flight-to-quality phenomena.

Because of the large portfolio sensitivities, institutional investors sell off large fractions of the benchmark stock and buy large fractions of the non-benchmark stock after a jump when their benchmarking incentives are large and the dividend ratio is low. They can only sell the benchmark stock at a discount and buy the non-benchmark stock at a premium after a jump; see Figure 6. This phenomenon constitutes fire sales in our model. Retail investors perceive fire sales as an opportunity to acquire the benchmark stock and sell the non-benchmark stock. The substitution effect for the retail investor is weak when benchmarking incentives are strong. The income effect invites the retail investor to buy the benchmark stock. When combined, these effects reduce the degree to which the retail investor cuts down her exposure to the benchmark stock after a jump when benchmarking incentives are large. Consequently, the retail investor is more exposed to the benchmark stock after a jump when the dividend ratio is low and benchmarking incentives are large. This results in a higher exposure to tail risk.

4.3 Survival

How does the exposure to tail risk impact the long-term survival of our investors? We can answer this question by looking at the long-term consumption plans and shares. We can rewrite the consumption $c^j_t$ of institutional investor $j \in \{A, B\}$ as

$$(1 + I_j s_t) \frac{Q_0 \alpha^j W_0 \phi_j}{\bar{c}(t)} \left[ (1 - s_0) e^{(\mu_1 - \frac{\sigma^2_1}{2} + \lambda J_1 - \rho_j + \bar{\rho}) t + \sigma_1 W_t + J_1 \tilde{N}_t} + s_0 e^{(\mu_2 - \frac{\sigma^2_2}{2} + \lambda J_2 - \rho_j + \bar{\rho}) t + \sigma_2 W_t + J_2 \tilde{N}_t} \right]$$

where $\bar{I} = \max\{I_A, I_B\}$, $\bar{\rho} = \max\{\rho_R, \rho_A, \rho_B\}$, $\tilde{N}_t = N_t - \lambda t$ is the compensated Poisson jump martingale, and

$$\bar{c}(t) = \alpha^R \rho_R e^{(\bar{\rho} - \rho_R) t} + \alpha^A \phi_A (1 + I_{A,s_t}) e^{(\bar{\rho} - \rho_A) t} + \alpha^B \phi_B (1 + I_{B,s_t}) e^{(\bar{\rho} - \rho_B) t}.$$ 

By construction, we have $0 < \lim_{t \to \infty} \bar{c}(t) < \infty$ and $0 < s_t < 1$ almost surely for all $t > 0$. As a result, the asymptotic behavior of $c^j_t$ as $t \to \infty$ is driven only by the asymptotic behavior of discounted stock prices. The expressions above show that $c^j_t \to \infty$ as $t \to \infty$ almost surely if

$$\bar{\rho} + \mu_1 - \frac{\sigma^2_1}{2} + \lambda J_1 > \rho_j \quad \text{or} \quad \bar{\rho} + \mu_2 - \frac{\sigma^2_2}{2} + \lambda J_2 > \rho_j.$$ 

In this case we also have $W^j_t \to \infty$ as $t \to \infty$ almost surely and institutional investor $j$ becomes infinitely rich in the long run through her investments. On the other hand, we

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10 Given our log-utility formulation, consumption at any point of time is a fraction of total wealth.
have \( c_t^j \rightarrow 0 \) as \( t \rightarrow \infty \) almost surely if

\[
\bar{\rho} + \mu_1 - \frac{\sigma_1^2}{2} + \lambda J_1 < \rho_j \quad \text{and} \quad \bar{\rho} + \mu_2 - \frac{\sigma_2^2}{2} + \lambda J_2 < \rho_j.
\]

(9)

In this case, institutional investor \( j \) becomes extinct in the long run. Given that \( \bar{\rho} \geq \rho_j \), an inspection of (9) reveals that extinction can occur only if both dividend processes have negative expected growth rates.

Similarly, we can rewrite the consumption \( c_t^R \) of the retail investor as

\[
\frac{Q_0 \alpha^R W_0 \rho_R}{c(t)} \left[ (1 - s_0)e^{(\mu_1 - \frac{\sigma_1^2}{2} + \lambda J_1 - \rho_R + \bar{\rho})t + \sigma_1 W_t + J_1 N_t} + s_0 e^{(\mu_2 - \frac{\sigma_2^2}{2} + \lambda J_2 - \rho_R + \bar{\rho})t + \sigma_2 W_t + J_2 N_t} \right].
\]

We see that the retail investor becomes infinitely rich in the long run under condition (8), and she goes extinct almost surely if condition (9) holds. As for institutional investors, the retail investor can only go extinct in the long run if both dividend processes have negative expected growth rate.

The analysis of consumption plans indicates that, in the long run, either all investors survive and become infinitely rich, or all investors fail. By studying consumption shares we can determine which investor dominates in the long run. The consumption share of institutional investor \( A \) is

\[
\frac{c_t^A}{c_t^R + c_t^A + c_t^B} = \frac{1}{1 + \frac{\alpha^B \phi_B(1 + I_B s_t)}{\alpha^A \phi_A(1 + I_A s_t)} e^{(\rho_A - \rho_R)t} + \frac{\alpha^B \phi_B(1 + I_B s_t)}{\alpha^A \phi_A(1 + I_A s_t)} e^{(\rho_A - \rho_B)t}}.
\]

An analogous representation can be derived from the consumption share of institutional investor \( B \). The consumption share of the retail investor is

\[
\frac{c_t^R}{c_t^R + c_t^A + c_t^B} = \frac{1}{1 + \frac{\alpha^A \phi_A(1 + I_A s_t)}{\alpha^R \phi_R} e^{(\rho_R - \rho_A)t} + \frac{\alpha^B \phi_B(1 + I_B s_t)}{\alpha^R \phi_R} e^{(\rho_R - \rho_B)t}}.
\]

Given that \( 1 \leq 1 + I_j s_t \leq 1 + \bar{I} < \infty \) for \( j \in \{A, B\} \), consumption shares are only driven by the ranking of the time discount coefficients of our investors. The investor who dominates in terms of relative consumption is the one with the smallest time discount coefficient; i.e., the investor with \( \rho_j = \min \{\rho_R, \rho_A, \rho_B\} \). All other investors will have consumption ratios that converge to zero. The most patient investor will be the richest in the long run.

The survival conditions we derive do not depend on the benchmarking importance parameters \( I_A \) and \( I_B \). As a result, survival occurs independently of the benchmarking behavior of our institutional investors. In the same vein, the most patient investor dominates in the long run independently of the behavior of institutional investors. We conclude that albeit benchmarking exposes the retail investor to tail risk in the short term, tail risk does not materialize in the long run. The long-term performance of our investors is unaffected by the benchmarking incentives of our institutional investors. These results extend the findings of Yan (2008), who shows that in a general class of infinite-horizon models driven
by Brownian motions, the investor with the lowest survival index dominates in the long run.\footnote{The ranking of survival indices in our model is equal to the ranking of time discount parameters of our investors given that our investors are perfectly rational and do not have optimism biases.} We show that this result also holds when dividends are allowed to jump, and when institutional investors have benchmarking incentives.

Condition (8) is satisfied in our numerical case study parametrized in Table 1. All investors in our example, both institutional and retail, become infinitely rich in the long run. However, the retail investor has the smallest time discount coefficient. Therefore, the retail investor will be the richest investor in the long run. Although the retail investor is exposed to large tail risk when the benchmark stock underperforms due to the trading behavior of institutional investors, these tail risks are only short-lived. Our institutional investors take on large bets when the benchmark underperforms (fire sales as in Figure 13), and they consume large fractions of their wealth (Figure 9). This behavior reduces the wealth of institutional investors relative to the retail investor.

### 4.4 Welfare

For fixed benchmark importance parameters $I_A$ and $I_B$, the preferences formulated in (1) and (2) are locally nonsatiated, the utility possibility set is convex, and the equilibrium described in Section 3 is Pareto efficient. However, the utility possibility set may no longer be convex as $I_A$ and $I_B$ change. As a result, we cannot analyze the impact of benchmarking on welfare in our model by measuring changes in social welfare as responses to changes in $I_A$ and $I_B$. In order to circumvent this issue, we adopt the notion of equivalent variation and evaluate the impact of benchmarking on welfare by computing how much additional consumption our investors would have to consume at any given point of time in a world in which there are no benchmarking incentives ($I_A = I_B = 0$) in order to achieve the same level of utility as in a world with positive benchmarking incentives ($I_A > 0$ or $I_B > 0$). Because for $I_A = I_B = 0$ all investors have log preferences, positive equivalent variations of consumption for investor $j$ imply welfare gains for investor $j$, while negative equivalent variations imply welfare losses for investor $j$.

Figure 14 displays the equivalent variations of consumption for our different investors at inception. The equivalent variation of consumption of the retail investor is negative for every value of the benchmark dividend ratio $s_0$. The same result holds for all other times $t > 0$. As a result, the retail investor is worse off in a world in which institutional investors have benchmarking incentives. The analysis is subtle for our institutional investors. We see that institutional investor $A$ with the highest discount rate has positive equivalent variation of consumption in every state of the world. This implies that the most impatient institutional investor is better off in a world with benchmarking incentives. However, institutional investor $B$ with a low discount rate has negative equivalent variation in states of the world in which the benchmark underperforms. Unlike for the retail investor, for whom benchmarking is always welfare reducing, benchmarking may be welfare reducing...
or increasing for institutional investors depending on how patient they are.

We obtain similar results for alternative choices of discount parameters. The retail investor is always worse off and the impatient institutional investor is always better off when institutional investors have benchmarking incentives.

5 Conclusion

We show that benchmarking may induce trading behavior by institutional investors which exposes a retail investor and the aggregate market to tail risk. To show this we solve in semi-closed form the general equilibrium of a pure-exchange economy with one retail investor and two institutional investors who can invest in a benchmark stock, a non-benchmark stock, and a safe asset. Institutional investors have marginal utility of consumption that increases in the relative performance of their benchmark. This incites them to tilt their portfolios toward the benchmark stock. It also increases their portfolio sensitivities with respect to the relative performance of the benchmark. As a result, institutional investors trade large volumes of the stocks in states of the world in which the benchmark underperforms, raising market volatility. This naturally results in high tail risk exposure of the retail investor and the aggregate market.

In spite of the higher tail risk exposure, we find that benchmarking does not affect the long-term performance of our investors. All investors survive in the long run if at least one stock has positive expected dividend rate, and the most patient investor dominates in terms of relative wealth. Still, benchmarking is welfare reducing for the retail investor ex ante, and is only welfare increasing for the impatient institutional investor.

Our results have important implications for the regulation of the asset management industry. We find that benchmarking may incentivize portfolio managers to invest in ways that induce additional risks in financial markets. We also find that retail investors may be worse off in economies with benchmarking than in economies without benchmarking due to a high tail risk exposure, even though their long-term performance is unaffected by the trading behavior of institutional investors. These results indicate that it is imperative for regulators to formulate precise objectives for a potential regulation of the asset management industry. Do regulators wish to control for the tail risk exposure of the retail investor or the aggregate market? Do regulators wish to prevent investor failures? These questions need to be answered first before any effective regulatory policy can be formulated.

Our results indicate that the regulation of the asset management industry will require different regulatory tools than the regulation of banks. We find that the retail investor and the aggregate market are exposed to high tail risk whenever benchmarking institutional investors are exposed to low tail risk, and vice versa. As a result, standard regulatory tools that measure tail risk exposures of financial institutions, such as value-at-risk and stress tests, may be unable to detect situations in which retail investors and the aggregate market are at risk of tail events.
This paper is the first to analyze the systemic implications of benchmarking from a theoretical point of view by considering its general equilibrium effects. Still, our model is of reduced form. We do not address the principal-agent problem underlying the decision of individual investors to allocate their wealth among portfolio managers. We also do not address the optimal contracting problem between individual investors and portfolio managers that gives rise to benchmarking. We leave it to future research to analyze the systemic effects of benchmarking in a principal-agent setting in which these problems can be addressed. Furthermore, we do not analyze potential regulatory tools for controlling for the systemic effects of benchmarking. Our results indicate that more research is needed to be able to carefully evaluate the effectiveness of a potential regulation of benchmarking. A first step in this direction is taken by Jones (2015).

A  Equilibrium formulation and solution

An equilibrium in our model consists of consumption plans \((c_{R t})_{t\geq 0}, (c_{A t})_{t\geq 0}, (c_{B t})_{t\geq 0}\), portfolio plans \((\pi_{R t})_{t\geq 0}, (\pi_{A t})_{t\geq 0}, (\pi_{B t})_{t\geq 0}\), stock prices \((S_{1,t})_{t\geq 0}\) and \((S_{2,t})_{t\geq 0}\), prices of diffusion \((\theta_{t})_{t\geq 0}\) and jump \((\psi_{t})_{t\geq 0}\) risk, and an interest rate process \((r_{t})_{t\geq 0}\) such that the following conditions are satisfied:

- \((c_{R t})_{t\geq 0}\) and \((\pi_{R t})_{t\geq 0}\) solve
  \[
  \sup \mathbb{E} \left[ \int_{0}^{\infty} e^{-\rho R t} \log c_{R t} d t \right], \quad \text{subject to} \quad \mathbb{E} \left[ \int_{0}^{\infty} \xi_t c_{R t} d t \right] \leq \alpha^R W_0.
  \]

- For \(j \in \{A, B\}\), \((c_{j t})_{t\geq 0}\) and \((\pi_{j t})_{t\geq 0}\) solve
  \[
  \sup \mathbb{E} \left[ \int_{0}^{\infty} e^{-\rho j t} (1 + I_j s_t) \log c_{j t} d t \right], \quad \text{subject to} \quad \mathbb{E} \left[ \int_{0}^{\infty} \xi_t c_{j t} d t \right] \leq \alpha^B W_0
  \]

- Markets are cleared:
  \[
  W_{t}^{R} \pi_{t}^{R} + W_{t}^{A} \pi_{t}^{A} + W_{t}^{B} \pi_{t}^{B} = 0, \\
  W_{t}^{R} \pi_{1,t}^{R} + W_{t}^{A} \pi_{1,t}^{A} + W_{t}^{B} \pi_{1,t}^{B} = S_{1,t}, \\
  W_{t}^{R} \pi_{2,t}^{R} + W_{t}^{A} \pi_{2,t}^{A} + W_{t}^{B} \pi_{2,t}^{B} = S_{2,t}.
  \]

Here, \(\xi = (\xi_t)_{t\geq 0}\) is the state price density in the market so that the process

\[
  t \mapsto \xi_t W^j_t + \int_{0}^{t} c^j_s \xi_s d s
\]

is a martingale relative to the complete information filtration \((\mathcal{F}_t)_{t\geq 0}\) for each \(j \in \{R, A, B\}\).

We follow the approach of Martin (2013), and derive semi-analytical expressions for the dividend ratio moments of Section (3). These expressions can be computed exactly.
Fourier inversion implies that except for some integrals which need to be computed numerically. Write \( \tilde{y}_{2,v,t} = y_{2,v} - y_{2,t} \) for \( y_{2,t} = \log D_{2,t} \). We can express the dividend share as

\[
s_v = \frac{e^{\tilde{y}_{2,v,t} + y_{2,t}}}{e^{\tilde{y}_{2,v,t} + y_{2,t}} + e^{\tilde{y}_{1,v,t} + y_{1,t}}}
\]

for \( v \geq t \). Using this transformation, it follows that

\[
\mathbb{E}_t \left[ \int_t^\infty e^{-\rho t (v-t)} s_v dv \right] = \mathbb{E}_t \left[ \int_t^\infty e^{-\rho t (v-t)} \frac{e^{\tilde{y}_{2,v,t} + y_{2,t}}}{e^{\tilde{y}_{2,v,t} + y_{2,t}} + e^{\tilde{y}_{1,v,t} + y_{1,t}}} dv \right]. \tag{10}
\]

By multiplying and dividing by \( e^{-\left( \tilde{y}_{2,v,t} + y_{2,t} + \tilde{y}_{1,v,t} + y_{1,t} \right)/2} \) and denoting \( u_v,t = \tilde{y}_{2,v,t} + y_{2,t} - \tilde{y}_{1,v,t} - y_{1,t} \), we can rewrite the integrand as

\[
\frac{e^{u_v,t/2}}{2\cosh(u_v,t/2)}. \tag{11}
\]

Fourier inversion implies that

\[
\frac{1}{2\cosh(u_v,t/2)} = \int \mathcal{G}_1(z)e^{izu_v,t} dz,
\]

where \( \mathcal{G}_1(z) = \frac{1}{2} \frac{\sech(\pi z)}{\sigma z} \). Because all integrands are bounded, the first moment can be expressed as

\[
M_{1,t}^j = \mathbb{E}_t \left[ \int_t^\infty e^{-\rho_j (v-t)} \int \mathcal{G}_1(z)e^{(iz+1/2)u_v,t} dz dv \right] = \int \frac{\mathcal{G}_1(z)e^{(iz+1/2)u_t,t}}{\rho_j - c(-iz - 1/2, iz + 1/2)} dz
\]

with the function \( c(\cdot, \cdot) \) defined as

\[
c(\theta_1, \theta_2) = \left( \mu_1 - \frac{1}{2} \sigma_1^2 \right) \theta_1 + \left( \mu_2 - \frac{1}{2} \sigma_2^2 \right) \theta_2 + \frac{1}{2} \sigma_1^2 \theta_1^2 + \frac{1}{2} \sigma_2^2 \theta_2^2 + \sigma_1 \sigma_2 \theta_1 \theta_2 + \lambda \left( (1 + J_1)^{\theta_1} + (1 + J_2)^{\theta_2} - 2 \right),
\]

and \( j \in \{A, B, R\} \).

The second and third moments can be calculated in an analogous way, leading to the expressions

\[
M_{2,t}^j = \int \frac{\mathcal{G}_2(z)e^{(iz+1)u_t,t}}{\rho_j - c(-iz - 1, iz + 1)} dz, \quad M_{3,t}^j = \int \frac{\mathcal{G}_3(z)e^{(iz+3/2)u_t,t}}{\rho_j - c(-iz - 3/2, iz + 3/2)} dz,
\]

where

\[
\mathcal{G}_\gamma(z) = \frac{1}{\Gamma(\gamma)} \frac{\Gamma(\gamma + iz)}{\Gamma(\gamma + \gamma/2)} \frac{\Gamma(\gamma/2 - iz)}{\Gamma(\gamma/2)}, \quad \gamma \in \{2, 3\},
\]

and \( \Gamma(\cdot) \) represents the standard Euler gamma function. For the jump magnitude moments we can use a similar approach and obtain

\[
\Delta M_{\gamma,t}^j = \int \frac{\mathcal{G}_\gamma(z)e^{\left( iz + \gamma/2 \right)u_t}}{\rho_j - c(-iz - \gamma/2, iz + \gamma/2)} \left( \frac{1 + J_2}{1 + J_1} \right)^{iz + \gamma/2} dz, \quad \gamma \in \{1, 2\}.
\]
B Proofs

Proof of Proposition 3.1 and 3.2. Since the state price density depends on aggregate dividends and the dividend share \( s_t \), we derive its dynamics in terms of these state variables. An application of Ito’s Lemma gives

\[
dD_t = D_t \left( (1-s_t) \mu_1 + s_t \mu_2 \right) dt + D_t \left( (1-s_t) \sigma_1 + s_t \sigma_2 \right) dZ_t + D_t \left( (1-s_t) J_1 + s_t J_2 \right) dN_t.
\]

Market clearing implies that, at any time \( t \), aggregate consumption is equal to aggregate dividend:

\[
D_t = c^A_t + c^B_t + c^R_t.
\]

As we show in Proposition 3.5,

\[
c^j_t = e^{-\rho^A t} (1 + I_A s_t) \alpha^A W_0 \phi_A,
\]

for \( j \in \{A, B\} \) and

\[
c^R_t = \rho_R W^R_t = \frac{\alpha^R W_0 \rho_R e^{-\rho^R t}}{\xi_t}.
\]

By plugging these in, we have

\[
D_t = \frac{W_0 \xi_t^{-1} (\gamma_{1t} + \gamma_{2t} + \gamma_{3t} s_t)}{\xi_t},
\]

where

\[
\gamma_{1t} = \alpha^R \rho_R e^{-\rho^R t},
\]

\[
\gamma_{2t} = \alpha^A \phi_A e^{-\rho^A t} + \alpha^B \phi_B e^{-\rho^B t},
\]

\[
\gamma_{3t} = \alpha^A I_A \phi_A e^{-\rho^A t} + \alpha^B I_B \phi_B e^{-\rho^B t}.
\]

This gives the characterization of the state price density as

\[
\xi_t = W_0 \frac{Q_t}{D_t},
\]

where we defined \( Q_t = \gamma_{1t} + \gamma_{2t} + \gamma_{3t} s_t \). At \( t = 0 \) we have \( \xi_0 = 1 \), which implies that initial wealth can be expressed as \( W_0 = \frac{D_0}{Q_0} \).

Next, an application of Ito’s Lemma on \( s_t \) leads to the following stochastic differential equation,

\[
ds_t = s_t(1 - s_t) \left[ \mu_2 - \mu_1 + (\sigma_1 - \sigma_2) (\sigma_1 (1-s_t) + \sigma_2 s_t) \right] dt + s_t(1 - s_t) (\sigma_2 - \sigma_1) dZ_t
\]
\[
+ s_t (1 - s_t) (J_2 - J_1) \\
\frac{1}{1 + (1 - s_t) J_1 + s_t - J_2} dN_t.
\]

In order to use the method of undetermined coefficients to pin down market prices of risk and interest rate, we also need the dynamics of \( D_t^{-1} \). Ito’s Lemma yields:

\[
dD_t^{-1} = (\sigma^2_t - \mu_d,t) D_t^{-1} dt - \sigma_d,t D_t^{-1} dZ_t - \frac{J_d,t^{-1}}{1 + J_d,t^{-1}} D_t^{-1} dN_t.
\]

Using the dynamics derived for \( s_t \), we can express the dynamics of \( Q_t \) as

\[
dQ_t = Q_t d\xi_t + \gamma_3 t \sigma_s,t dZ_t + \gamma_3 t J_{s,t}^{-1} dN_t,
\]

where

\[
\mu_{q,t} = -\rho_R \gamma_3 t - \alpha^A \phi_A \rho_A e^{-\rho_A t} (1 + I_A s_t) - \alpha^B \phi_B \rho_B e^{-\rho_B t} (1 + I_B s_t) + \gamma_3 t \mu_{s,t}.
\]

An additional application of Ito’s Lemma on the right side of (12) gives

\[
d\xi_t = -\xi_t \left( \mu_{q,t} - \sigma^2_{d,t} + \sigma_{d,t} \sigma_{q,t} + \left[ \frac{1 + J_{q,t}}{1 + J_{d,t}} \right] \lambda \right) dt
\]

\[-\xi_t (\sigma_{d,t} - \sigma_{q,t}) dZ_t + \xi_t \left[ \frac{1 + J_{q,t}}{1 + J_{d,t}} - 1 \right] (dN_t - \lambda dt).
\]

By matching the coefficients we obtain the following expressions for risk prices:

\[
\theta_t = \sigma_{d,t} - \sigma_{q,t} ; \quad \psi_t = \frac{1 + J_{q,t}}{1 + J_{d,t}} ;
\]

\[
\rho_t = \mu_{d,t} - \mu_{q,t} - \sigma^2_{d,t} + \sigma_{d,t} \sigma_{q,t} + (1 - \psi_t) \lambda.
\]

\[\square\]

**Proof of Proposition 3.3.** Market completeness implies that \( \xi_t S_{2,t} = \mathbb{E}_t \left[ \int_t^\infty \xi_v dZ_v dv \right] \). Therefore,

\[
\frac{Q_t}{D_t} S_{2,t} = \mathbb{E}_t \left[ \int_t^\infty \left( \alpha^R \rho_R e^{-\rho_R t} S_v + (\alpha^A \phi_A e^{-\rho_A t} + \alpha^B \phi_B e^{-\rho_B t}) S_v + (I_A \alpha^A \phi_A e^{-\rho_A t} + I_B \alpha^B \phi_B e^{-\rho_B t}) S_v \right) dv \right]
\]

\[
= \alpha^R \rho_R e^{-\rho_R t} M_{1,t}^R + \alpha^A \phi_A e^{-\rho_A t} M_{1,t}^A + \alpha^B \phi_B e^{-\rho_B t} M_{1,t}^B
\]

\[+ I_A \alpha^A \phi_A e^{-\rho_A t} M_{2,t}^A + I_B \alpha^B \phi_B e^{-\rho_B t} M_{2,t}^B. \quad \text{(13)}\]
We apply the same approach for asset $S_d$ and obtain:

$$S_{2,t} = \frac{D_t}{Q_t} (\alpha^R \rho e^{-\rho t} M_{1,t}^R + \alpha^A \phi_A e^{-\rho t} (M_{1,t}^A + I_A M_{2,t}^A) + \alpha^B \phi_B e^{-\rho t} (M_{1,t}^B + I_B M_{2,t}^B)).$$

We apply the same approach for asset $S_{1,t}$ and obtain:

$$S_{1,t} = \frac{D_t}{Q_t} (\alpha^R e^{-\rho t} + \alpha^A \phi_A e^{-\rho t} (\rho_A^{-1} + I_A M_{1,t}^A) + \alpha^B \phi_B e^{-\rho t} (\rho_B^{-1} + I_B M_{1,t}^B)) - S_{2,t}.$$

Next, we compute the volatility and jump size functions for both asset prices by the method of coefficient matching. Define the function $f$ as

$$f(s_t; \gamma, \sigma) \equiv E^{(i \gamma + \frac{\gamma}{2})\log \left( \frac{s_t}{1 - s_t} \right)} \left( \frac{s_t}{1 - s_t} \right)^{i \gamma + \frac{\gamma}{2}},$$

such that

$$M_{\gamma,t}^2 = \int \frac{G_\gamma(z) e^{(i \gamma + \frac{\gamma}{2})\log (\frac{s_t}{1 - s_t})}}{\rho_j - c(-i \gamma - \frac{\gamma}{2}, i \gamma + \frac{\gamma}{2})} dz = \int \frac{G_\gamma(z) f(s_t; \gamma, \sigma)}{\rho_j - c(-i \gamma - \frac{\gamma}{2}, i \gamma + \frac{\gamma}{2})} dz.$$

Ito’s formula implies that

$$dM_{\gamma,t}^2 = (\cdots) dt + \int \frac{G_\gamma(z) f'(s_t; \gamma, \sigma) + \sigma_{s,t} - \sigma_1}{\rho_j - c(-i \gamma - \frac{\gamma}{2}, i \gamma + \frac{\gamma}{2})} dz dZ_t + \int \frac{G_\gamma(z)(f(s_t; \sigma_2 - \sigma_1))}{\rho_j - c(-i \gamma - \frac{\gamma}{2}, i \gamma + \frac{\gamma}{2})} dz dN_t$$

$$= (\cdots) dt + \int \frac{G_\gamma(z)(\gamma + \frac{\gamma}{2})}{\rho_j - c(-i \gamma - \frac{\gamma}{2}, i \gamma + \frac{\gamma}{2})} dz dZ_t$$

$$+ \int \frac{G_\gamma(z) e^{(i \gamma + \frac{\gamma}{2})\log (\frac{s_t}{1 - s_t})}}{\rho_j - c(-i \gamma - \frac{\gamma}{2}, i \gamma + \frac{\gamma}{2})} \left[ \left( \frac{1 + J_2}{1 + J_1} \right)^{i \gamma + \frac{\gamma}{2}} - 1 \right] dz dN_t.$$

Because we are only matching the uncertainty coefficients, the drift is irrelevant to our calculations. We have for $\gamma \in \{1, 2\}$:

$$\int \frac{G_\gamma(z) e^{(i \gamma + \frac{\gamma}{2})\log (\frac{s_t}{1 - s_t})}}{\rho_A - c(-i \gamma - \frac{\gamma}{2}, i \gamma + \frac{\gamma}{2})} dz = \gamma \left( M_{\gamma,t}^2 - M_{\gamma+1,t}^2 \right),$$

$$\int \frac{G_\gamma(z) e^{(i \gamma + \frac{\gamma}{2})\log (\frac{s_t}{1 - s_t})}}{\rho_j - c(-i \gamma - \frac{\gamma}{2}, i \gamma + \frac{\gamma}{2})} \left[ \left( \frac{1 + J_2}{1 + J_1} \right)^{i \gamma + \frac{\gamma}{2}} - 1 \right] dz = \Delta M_{\gamma,t}^2.$$

Therefore,

$$dM_{\gamma,t}^2 = (\cdots) dt + \gamma (\sigma_2 - \sigma_1) (M_{\gamma,t}^2 - M_{\gamma+1,t}^2) dZ_t + \Delta M_{\gamma,t}^2 dN_t.$$

On the other hand, the product rule implies that

$$d\xi_t S_{2,t} = (\cdots) dt + \xi_t S_{2,t} (\sigma_{2,t} - \theta_t) dZ_t + \xi_t \xi_t S_{2,t} - (1 + J_{2,t-\psi t} - 1)dN_t.$$
A match of the coefficients on both sides of (13) and using the relations above results in:

\[
\sigma_{2,t} = \theta_t + \frac{D_t(\sigma_2 - \sigma_1)}{Q_t S_{2,t}} \left[ \alpha^R \rho R e^{-\rho R t} (M^{R}_{1,t} - M^{R}_{2,t}) + \alpha^A \phi_A e^{-\rho A t} (M^{A}_{1,t} - M^{A}_{2,t}) 
+ \alpha^B \phi_B e^{-\rho B t} (M^{B}_{1,t} - M^{B}_{2,t}) + 2I_A \alpha^A \phi_A e^{-\rho A t} (M^{A}_{2,t} - M^{A}_{3,t}) + 2I_B \alpha^B \phi_B e^{-\rho B t} (M^{B}_{2,t} - M^{B}_{3,t}) \right],
\]

\[
J_{2,t} = \psi_t^{-1} - 1 + \frac{D_t}{\psi_t Q_t S_{2,t}} \left( \alpha^R \rho R e^{-\rho R t} \Delta M^{R}_{1,t} + \alpha^A \phi_A e^{-\rho A t} \Delta M^{A}_{1,t} + \alpha^B \phi_B e^{-\rho B t} \Delta M^{B}_{1,t} 
+ I_A \alpha^A \phi_A e^{-\rho A t} \Delta M^{A}_{2,t} + I_B \alpha^B \phi_B e^{-\rho B t} \Delta M^{B}_{2,t} \right).
\]

An analogous approach for the price of asset 1 yields:

\[
\sigma_{1,t} = \theta_t \left( 1 + \frac{S_{2,t}}{S_{1,t}} \right) + \frac{D_t(\sigma_2 - \sigma_1)}{Q_t S_{1,t}} \left[ I_A \alpha^A \phi_A e^{-\rho A t} (M^{A}_{1,t} - M^{A}_{2,t}) + I_B \alpha^B \phi_B e^{-\rho B t} (M^{B}_{2,t} - M^{B}_{1,t}) \right] - \sigma_{2,t} \frac{S_{2,t}}{S_{1,t}}
\]

\[
J_{1,t} = \psi_t^{-1} \left( 1 + \frac{S_{2,t}}{S_{1,t}} + \frac{D_t}{S_{1,t}} Q_t \right) \left( I_A \alpha^A \phi_A e^{-\rho A t} \Delta M^{A}_{1,t} + I_B \alpha^B \phi_B e^{-\rho B t} \Delta M^{B}_{1,t} \right) - 1 - \frac{S_{2,t}}{S_{1,t}} (1 + J_{2,t})
\]

\[\square\]

**Proof of Proposition 3.4.** With the optimal consumption policies and prices of risk determined, we turn to the characterization of the optimal allocation rules. To simplify the exposition of the result, we introduce the matrix notation:

\[
\pi^A_t = [\pi^A_{1,t} \pi^A_{2,t}], \quad \pi^B_t = [\pi^B_{1,t} \pi^B_{2,t}], \quad \pi^R_t = [\pi^R_{1,t} \pi^R_{2,t}],
\]

\[
\sigma_t = [\sigma_{1,t} \sigma_{2,t}], \quad J_t = [J_{1,t} J_{2,t}],
\]

\[
\Sigma_t = [\sigma_t J_t], \quad \mu_t = [\mu_{1,t} \mu_{2,t}], \quad r_t = [r_t r_t].
\]

Then, the wealth process for agent \(j\), with \(j \in \{R, A, B\}\), satisfies

\[
dW^j_t = (W^j_t (r_t + [\mu_t - r_t] \pi^j_t) - c^j_t) dt + W^j_t [dZ_t \cdot dN_t] \cdot (\Sigma_t \pi^j_t).
\]

Given that the process \(\xi_t W^j_t + \int_0^t c^j_s \xi_s ds\) is a martingale with dynamics

\[
d(\xi_t W^j_t) + c^j_t \xi_t dt = \left( \sigma^j_t \pi^j_t - \theta_t \right) dZ_t + \left( (1 + J^j_t \pi^j_t) \psi_t - 1 \right) (dN_t - \lambda dt), \quad (14)
\]

we use the equality \(\xi_t W^j_t + \int_0^t \xi_s c^j_s ds = \mathbb{E}_t \left[ \int_0^\infty \xi_s c^j_s ds \right] \) and the optimal policies \(c^j_t\) characterized above, to match uncertainty loadings and solve for the optimal allocation. Starting with institutional investor \(A\), we have the following equality

\[
\xi_t W^A_t = \mathbb{E}_t \left[ \int_t^\infty \xi_s c^A_s ds \right] = \mathbb{E}_t \left[ \int_t^\infty e^{-\rho A (v-t)} (1 + I_A s_v) dv \right] \alpha^A W_0 \phi_A e^{-\rho A t}
\]

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\[ \xi_t W_t^A = \alpha^A W_0 \phi_A e^{-\rho_A t} (\rho_A^{-1} + I_A M_{1,t}^A), \]  

where the last equality follows from \( M_{1,t}^A = \mathbb{E}_t \left[ \int_t^\infty e^{-\rho_A (v-t)} s_v dv \right] \).

Since we are only interested in the loadings on uncertainty sources, we focus on the last term on the right side in the expression above and match the coefficients with the one in (14). Proceeding as we did in the previous section, we apply Ito’s Lemma and match the uncertainty loadings. It follows that

\[ \xi_t W_t^A (\sigma_t' \pi_t^A - \theta_t) = I_A \phi_A \alpha^A W_0 e^{-\rho_A t} \mathbb{E}_t \left[ \int_t^\infty e^{-\rho_A (v-t)} (\sigma_2 - \sigma_1) s_v (1 - s_v) dv \right] = I_A \phi_A \alpha^A W_0 e^{-\rho_A t} (\sigma_2 - \sigma_1) (M_{1,t}^A - M_{2,t}^A), \]

We can further simplify by substituting (15) in the expression above to obtain

\[ \sigma_t' \pi_t^A = \theta_t + \frac{I_A (\sigma_2 - \sigma_1) (M_{1,t}^A - M_{2,t}^A)}{\rho_A^{-1} + I_A M_{1,t}^A}. \]

Similarly, we have

\[ \xi_t W_t^A ((1 + J_t' \pi_t^A) \psi_t - 1) = I_A \phi_A \alpha^A W_0 e^{-\rho_A t} \Delta M_{1,t}^A, \]

which can be simplified to

\[ J_t' \pi_t^A = \psi_t^{-1} - 1 + \frac{I_A \Delta M_{1,t}^A}{\psi_t (\rho_A^{-1} + I_A M_{1,t}^A)}. \]

The optimal portfolio is therefore characterized by

\[ \pi_t^A = \sum_{t} \left[ \frac{\theta_t}{\psi_t^{-1} - 1} \right] + \sum_{t} \left[ \frac{(\sigma_2 - \sigma_1) (M_{1,t}^A - M_{2,t}^A)}{\Delta M_{1,t}^A \psi_t^{-1}} \right] \frac{I_A}{\rho_A^{-1} + I_A M_{1,t}^A}. \]

To conclude the proof, we solve the retail investor’s problem. From the martingale relation, we have

\[ \xi_t W_t^R = \mathbb{E}_t \left[ \int_t^\infty \xi_s e_s^R dv \right] = \mathbb{E}_t \left[ \int_t^\infty \alpha^R W_0 \rho_R e^{-\rho_R v} dv \right] = \xi_t W_t^R = \alpha^R W_0 e^{-\rho_R t}. \]  

An application of Ito’s Lemma on the last term on (16) and matching the uncertainties coefficients as done before leads to the following characterization of the optimal weights:

\[ \pi_t^R = \sum_{t} \left[ \frac{\theta_t}{\psi_t^{-1} - 1} \right]. \]
Proof of Proposition 3.5. We start solving the institutional investors’ problems. We focus on investor A; the derivation for investor B is analogous. From the first order condition we have

$$e^{-\rho_A t}(1 + I_A s_t)(c^A_t)^{-1} = y\xi_t \Rightarrow c^A_t = e^{-\rho_A t}(1 + I_A s_t)\xi_t^{-1}y^{-1},$$

(17)

where $y$ is the Lagrange multiplier obtained by substituting (17) in the budget constraint:

$$\mathbb{E}\left[\int_0^\infty \xi_t c^A_t dt\right] = \mathbb{E}\left[\int_0^\infty e^{-\rho_A t}(1 + I_A s_t)y^{-1}dt\right] = \alpha^A W_0 \rightarrow y^{-1} = \frac{\alpha^A W_0}{\mathbb{E}\left[\int_0^\infty e^{-\rho_A t}(1 + I_A s_t)dt\right]}.$$

By denoting

$$\phi_A = \left(\frac{1}{\rho_A} + I_A\mathbb{E}\left[\int_0^\infty e^{-\rho_A t}s_t dt\right]\right)^{-1},$$

and substituting $y^{-1}$ in (17), we have the following expression for the optimal consumption level for institutional investors:

$$c^A_t = \frac{e^{-\rho_A t}(1 + I_A s_t)\alpha^A W_0 \phi_A}{\xi_t}.$$

Now, we analyze the optimization problem for the retail investor. This agent’s first order condition is

$$e^{-\rho_R t}(c^R_t)^{-1} = y_R \xi_t \Rightarrow c^R_t = e^{-\rho_R t}\xi_t^{-1}y^{-1}.\quad(18)$$

Plugging (18) in the investor’s budget constraint gives that

$$\mathbb{E}\left[\int_0^\infty \xi_t c^R_t dt\right] = \mathbb{E}\left[\int_0^\infty e^{-\rho_R t}y^{-1}dt\right] = \alpha^R W_0 \rightarrow y^{-1} = \frac{\alpha^R W_0}{\mathbb{E}\left[\int_0^\infty e^{-\rho_R t}dt\right]} = \alpha^R W_0 \rho.$$ 

Substituting $y^{-1}$ back into (18) yields:

$$c^R_t = \frac{e^{-\rho_R t}\alpha^R W_0 \rho}{\xi_t}.$$

References


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Table 1: Model parameters. These parameters are chosen to roughly match the estimates of Backus et al. (2011) for U.S. equities and options. We assume that jumps are larger but less frequent than Backus et al. (2011). Similar parameters for jump sizes and jump frequencies of U.S. equities are estimated by Barro (2006) and Wachter (2013).
(a) State price density $\xi_t$ versus benchmark dividend ratio $s_t$ at $t = 1$ for $I_A = I_B = 2$.

(b) State price density $\xi_t$ versus benchmark dividend level $D_{2,t}$ at $t = 1$. Here, we take $I_A = I_B = 2$ and $D_{1,t} = D_{2,t}$ so that $s_t = 0.5$ is kept fixed.

Figure 2: *State price density.* These figures plot the state price density $\xi_t$ against the benchmark dividend ratio $s_t$ and the benchmark dividend level $D_{2,t}$.
Figure 3: Market prices of risk. These figures give comparative statics of the market prices of volatility and jump risk, $\theta_t$ and $\psi_t$, relative to changes in the benchmark dividend ratio $s_t$ and the benchmark importance parameters $I_A$ and $I_B$. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$. 

(a) Market price of volatility risk $\theta_t$ at time $t = 0$.

(b) Market price of jump risk $\psi_t$ at time $t = 0$. 

Figure 4: Interest rate. This figure gives comparative statics of the interest rate $r_t$ relative to changes in the benchmark dividend ratio $s_t$ and the benchmark importance parameters $I_A$ and $I_B$. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$. 
(a) Price of the non-benchmark stock, $S_{1,t}$, for different values of $I_A$ and $I_B$.

(b) Price of the benchmark stock, $S_{2,t}$, for different values of $I_A$ and $I_B$. The price is close to insensitive to the benchmark importance parameters so that the lines for different values of $I_A$ and $I_B$ lie on top of each other.

Figure 5: *Stock prices.* These figures give comparative statics of the prices of the benchmark and the non-benchmark stocks, $S_{1,t}$ and $S_{2,t}$, relative to changes in the benchmark dividend ratio $s_t$ and the benchmark importance parameters $I_A$ and $I_B$. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$. 

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Figure 6: Stock volatilities and jump sizes. These figures give comparative statics of the volatilities ($\sigma_{1,t}$ and $\sigma_{2,t}$) and jump sizes ($J_{1,t}$ and $J_{2,t}$) relative to changes in the benchmark dividend ratio $s_t$ and the benchmark importance parameters $I_A$ and $I_B$. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$. 
(a) Portfolio weights for the non-benchmark stock for different values of $I_A$ and $I_B$.

(b) Portfolio weights for the benchmark stock for different values of $I_A$ and $I_B$.

(c) Portfolio weights for the safe asset for different values of $I_A$ and $I_B$.

Figure 7: Portfolios. These figures show the portfolio plans of different investors for different values of the benchmark dividend ratio $s_t$ and the benchmark importance parameters $I_A$ and $I_B$. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$. 

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(a) Institutional investors’ portfolio weights for the non-benchmark stock, $\pi_{1,t}^A$ (solid blue) and $\pi_{1,t}^B$ (dashed blue), and the benchmark stock, $\pi_{2,t}^A$ (solid red) and $\pi_{2,t}^B$ (dashed red), as functions of the benchmark dividend jump size $J_2$.

(b) Institutional investors’ portfolio weights for the non-benchmark stock, $\pi_{1,t}^A$ (solid blue) and $\pi_{1,t}^B$ (dashed blue), and the benchmark stock, $\pi_{2,t}^A$ (solid red) and $\pi_{2,t}^B$ (dashed red), as functions of the non-benchmark and benchmark dividend volatilities, $\sigma_1$ and $\sigma_2$.

Figure 8: Comparative statics of portfolios of institutional investors. These figures are generated under the assumption that $t = 0$, $I_A = I_B = 2$, and $s_0 = 0.5$. They reflect ceteris paribus changes relative to the parametrization of Table 1.
(a) Consumption $c^j_t$ for $j \in \{R, A, B\}$ as a function of the benchmark dividend ratio $s_t$ and level $D_{2,t}$. We take $t = 0$ and $I_A = I_B = 2$. For the right plot, we assume $D_{1,t} = D_{2,t}$.

(b) Wealth to consumption ratio $c^j_t / W^j_t$ for $j \in \{R, A, B\}$ as a function of the benchmark dividend ratio $s_t$ and the benchmark importance parameters $I_A$ and $I_B$. We take $t = 0$ and $I_A = I_B$.

Figure 9: Consumption plans. These charts plot consumption as functions of the dividend ratio $s_t$, the dividend level $D_{2,t}$, and the benchmark importance parameters $I_A$ and $I_B$. 
Figure 10: Institutional excess consumption. These figures plot the excess consumption of institutional investor, $c^A_t - c^R_t$, as a function of the benchmark dividend ratio $s_t$, the dividend level $D_{2,t}$, and benchmark importance parameter $I_A$. We take $t = 0$. To make the retail and institutional investors comparable, we assume here that $\alpha_A = \alpha_R = 0.5$, $\alpha_B = 0$, and $\rho_A = \rho_R = 0.02$. For the right plot, we also assume $D_{1,t} = D_{2,t}$.
Figure 11: Value-at-Risk. These figures plot the 1-year value-at-risk of the institutional and retail investors, and of the aggregate market against the benchmark dividend ratio \( s_t \) and the benchmark importance parameters \( I_A \) and \( I_B \). The analysis is carried out at time \( t = 0 \) under the assumption that \( I_A = I_B \). We simulate \( 10^5 \) exact samples of \( D_{1,t} \) and \( D_{2,t} \) for \( t = 1 \), and use these to construct samples of the benchmark dividend ratio \( s_t \), as well as wealth \( W^j_t \) and realized portfolio returns \( (W^j_t - W^j_0)/W^j_0 \) for \( j \in \{R, A, B\} \) at \( t = 1 \). Value-at-risk is the negative 1% quantile of the simulated realized return distribution.
Figure 12: *Portfolio volatilities.* This figure plots portfolio volatilities for investors A, B, and R at time $t = 0$ as a function of the benchmark dividend ratio $s_0$ and the benchmark importance parameters $I_A$ and $I_B$. The portfolio volatility of investor $j \in \{R, A, B\}$ can be computed as the square root of $(\sigma_{1,t} \pi_{1,t}^j + \sigma_{2,t} \pi_{2,t}^j)^2 + \lambda (J_{1,t} \pi_{1,t}^j + J_{2,t} \pi_{2,t}^j)^2$. 
Figure 13: Portfolios changes after a jump when the dividend ratio is low. These figures plot portfolio changes as functions of the benchmark importance parameter when a jump occurs. We assume $s_0 = 0.2$ and $I_A = I_B$. If a jump occurs at time $t$, then the portfolio change of investor $j$ is measured as $\pi^t_jW^t_j - \pi^{t-}_jW^{t-}_j$, where $\pi^{t-}_j$ and $W^{t-}_j$ are the portfolio weights and wealth of investor $j$ right before the jump occurs. Portfolio changes are decomposed into substitution effects $((\pi^t_j - \pi^{t-}_j)W^{t-}_j)$ and income effects $\pi^t_j(W^t_j - W^{t-}_j)$.
Figure 14: Equivalent variation of consumption. These figures show the equivalent variation of consumption; i.e., the amount of additional consumption that investor \( j \in \{ R, A, B \} \) has to consume at any point of time in a world in which \( I_A = I_B = 0 \) in order to achieve the same level of utility as in a world with \( I_A > 0 \) or \( I_B > 0 \). Formally, letting \( c_{0,t}^j \) denote the consumption at time \( t \) of investor \( j \) when \( I_A = I_B = 0 \), the equivalent variation of consumption \((EV_R^j, EV_A^j, EV_B^j)\) satisfy \( \mathbb{E}[\int_0^\infty e^{-\rho t} \log(c_{0,t}^R + EV_R^R)dt] \), \( \mathbb{E}[\int_0^\infty e^{-\rho t}(1 + I_A s_t) \log(c_{0,t}^A + EV_A^A)dt] \), and \( \mathbb{E}[\int_0^\infty e^{-\rho t}(1 + I_B s_t) \log(c_{0,t}^B + EV_B^B)dt] \). We only plot \( EV_I^j \) for \( t = 0 \); analogous plots hold for all \( t > 0 \).