Power laws in firm size and openness to trade: Measurement and implications

di Giovanni, Levchenko and Rancière (JIE 2011)

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Outline

1. Stylized facts on firm-size distribution and exporters
   - Zipf's Law
   - Which firms export
2. Empirical evidence on firm-size distribution, and role of trade
   - Role of trade in estimation and potential bias
   - Detailed estimation using census and customs data (France)

1. Stylized Facts on Firm Size Distribution and Exporters

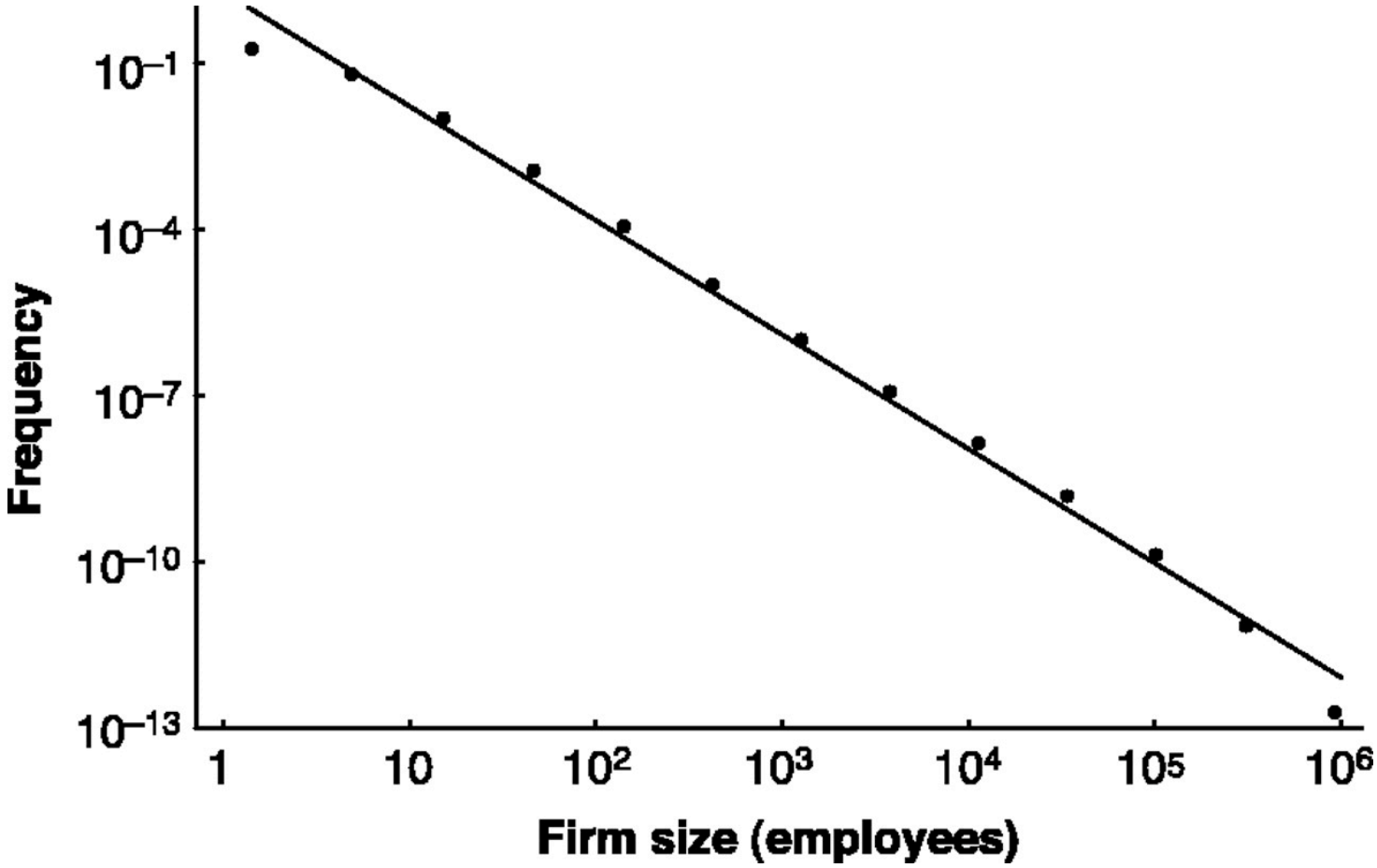
Distribution of Firm Size

- The distribution of firm size is extremely fat-tailed
- x follows a power law if:

$$\Pr(x > s) = cs^{-\zeta}$$

- Power law with exponent $\zeta \approx 1$ in absolute value
  - Also known as "Zipf's Law" (Zipf, 1949)
  - $\zeta < 2 \rightarrow$ infinite variance, finite mean
  - $\zeta < 1 \rightarrow$ infinite variance, infinite mean

Distribution of Firm Size



Source: Axtell (2001)

Firms and Trade

- **Large** firms produce the majority of exports
- Bernard et al. (2007) document that a fraction of U.S. firms are exporters:
  - Of 5.5 million firms operating in the United States in 2000, just 4 percent were exporters
  - Top 10 percent of exporters accounted for 96 percent of total U.S. exports

Firms and Trade

- Melitz (2003) is starting point of "new new" trade theory, which explains distribution of firms and trade. Extends Krugman (1980) to allow for firm-level heterogeneity
- Extensive literature follows (e.g., Chaney, 2008; Eaton et al., 2011), and focus gets even more "granular" (e.g., product dimension)

Importance of Large Firms for Macro and Trade?

- The existence of large firms can potentially have large impact on different macroeconomic phenomenon
- Macroeconomic volatility (Gabaix, 2011; di Giovanni and Levchenko, 2012)
- Welfare implications of policy changes
  - Entry costs
  - Extensive margin of trade

2. Empirical evidence on firm-size distribution, and role of trade

Size Distribution in Economics

- Fat tails have been observed in international trade:
  - Size distribution of exports: Helpman, Melitz, and Yeaple (2004); Hinloopen and van Marrewijk (2008)
  - Fine trade analysis: Eaton, Kortum, and Kramarz (2011), Ghironi and Melitz (2008), Arkolakis and Muendler (2008)
- Zipf's Law has also been observed across:
  - Cities: Zipf (1949), Soo (2007), Gabaix (1999)
  - Firms: Okuyama et al. (1999), Axtell (2001), Luttmer (2007), Rossi-Hansberg and Wright (2007), Gabaix and Landier (2008)

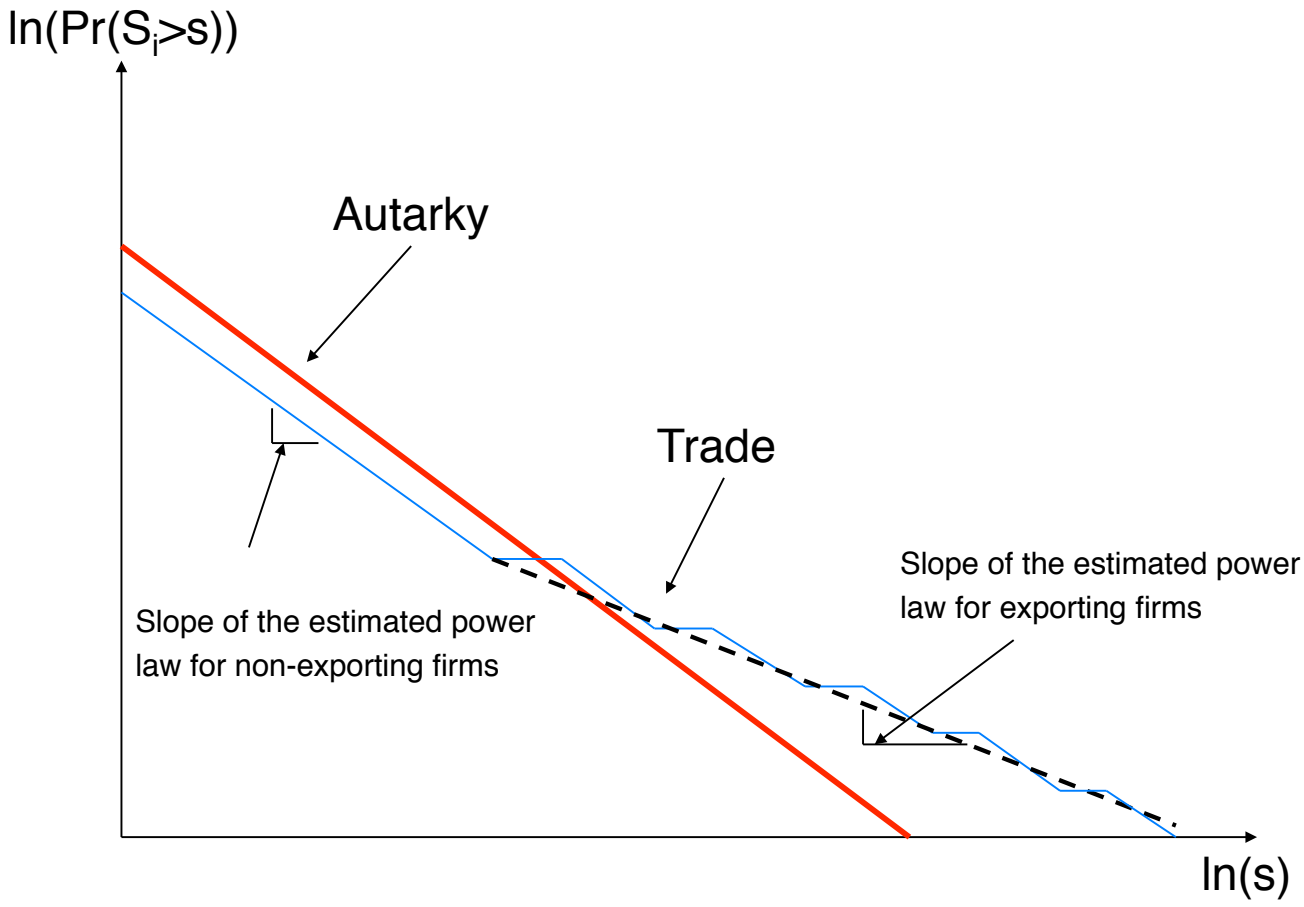
Size Distribution in Economics

- Size distribution theory:
  - Mechanics: Gibrat, Champernowne (1953), Simon (1955), Mandelbrot (1961) , Zanette and Manrubia (1997), Gabaix (1999), Malcai Biham Solomon (1999)
  - Economic models for cities: Gabaix (1999), Cordoba (2008), Rossi-Hansberg and Wright (2007), Duranton (2008)
  - Economic models for firms: Rossi-Hansberg and Wright (2007), Luttmer (2007), Gabaix (2007)

Measurement of Power Laws

- Studies, such as Axtell (2001), have provided empirical evidence of existence of power laws in firm size
- These studies *do not* take into account international trade
- But, opening to international trade will affect the *observed* distribution of firm size given entry/exit of firms
- Canonical heterogeneous firm trade model makes this clear, but we can derive a simple model to show how trade affects estimation of power laws

Measurement of Power Laws: Melitz-Pareto



Power Law in the Melitz-Pareto Framework

- The distribution is firm sales x follows a power law if:

$$\Pr(x > s) = cs^{-\zeta}$$

- Available estimates (Axtell, 2001) put ζ around 1

Model: Domestic Side

- Di Giovanni et al. (2011) consider a canonical monopolistic competition model with CES demand and heterogeneous firms, with Dixit-Stiglitz preferences:

$$\max \left[\int_{J_n} c_{ni}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ s.t. } \int_{J_n} p_{ni} c_{ni} di = Y_n$$

- Firm i in market n has input cost ω_n, marginal cost a_i
- Solution is standard: domestic sales D_i are

$$D_i = M_n \times B_i, \text{ where}$$

$$B_i \equiv a_i^{1-\varepsilon} = \text{baseline firm size}$$

$$M_n \equiv \frac{Y_n}{P_n^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon - 1} \omega_n \right)^{1-\varepsilon} = \text{market size}$$

Power Law in the Melitz-Pareto Framework

- In the model, $\Pr(1/a < y) = 1 - \left(\frac{b}{y}\right)^{\theta}$, and therefore

$$\Pr(x > s) = \left(b^{\varepsilon-1} C\right)^{\frac{\theta}{\varepsilon-1}} s^{-\frac{\theta}{\varepsilon-1}}.$$

- The distribution of firm sales follows a power law with exponent $\frac{\theta}{\varepsilon-1}$
- Therefore:

$$\frac{\theta}{\varepsilon - 1} \approx 1$$

Model: Export Side

- Assume that there is only *one* export market (consider as composite market of firm sales), m
- Fixed cost of exporting for firm i, from n to m: κ_{mni}
- Iceberg trade cost for shipping from n to m: τ_{mn}
- Define M_m^* such that

$$M_m^* = \frac{Y_m}{P_m^{1-\varepsilon}} \left(\frac{\varepsilon}{\varepsilon - 1} \tau \omega_n \right)^{1-\varepsilon}$$

- Define $\phi = M_m^*/M_n =$ Relative market size, then **export sales** are

$$M_m^* \times B_i = \phi D_i$$

Model: Export Side

- Given fixed costs of exporting, firm i will export to market m only if

$$\frac{M_m^* \times B_i}{\varepsilon} \geq \kappa_{mni}$$

- Define the export probability function as:

$$H(x) = \Pr\left(\kappa_i \leq \frac{cx}{\varepsilon}\right)$$

where $\kappa_i \equiv \kappa_{mni}$

- Probability of exporting is $H(D_i)$

Model: Export Side

- Exports are:

$$X_i = \begin{cases} 0 & \text{if } \frac{\phi D_i}{\varepsilon} < \kappa_i; \quad \text{Probability } 1 - H(D_i) \\ \phi D_i & \text{if } \frac{\phi D_i}{\varepsilon} \geq \kappa_i; \quad \text{Probability } H(D_i) \end{cases}$$

- The total (worldwide) sales of the firms are:

$$\begin{aligned} S_i &= D_i + X_i \\ &= \begin{cases} D_i & \text{if } \frac{\phi D_i}{\varepsilon} < \kappa_i; \quad \text{Probability } 1 - H(D_i) \\ (1+\phi) D_i & \text{if } \frac{\phi D_i}{\varepsilon} \geq \kappa_i; \quad \text{Probability } H(D_i) \end{cases} \end{aligned}$$

Densities of Domestic, Export, and Total Sales

- Postulate that B_i follows a Pareto distribution with exponent ζ: $P(B_i > x) = k_B x^{-\zeta}$, for $x > \underline{B}$
- Some random growth theories (Gabaix, 1999; Luttmer, 2007; Rossi-Hansberg and Wright, 2007) can predict that $\zeta \simeq 1^+$
- Consistent with the common assumption that firm productivity $1/a_i \sim \text{Pareto}(b, \theta)$
  - $\zeta = \frac{\theta}{\varepsilon - 1}$
- Given this distributional assumption, and the structure of the economy, we can derive the densities of domestic, export, and total sales

Densities of Domestic, Export, and Total Sales

Proposition

The densities of domestic sales D_i, exports X_i (when they are nonzero), and worldwide sales S_i are:

$$p_D(x) = kx^{-\zeta-1}1_{x>\underline{D}},$$

$$p_X(x) = Kx^{-\zeta-1}H\left(\frac{x}{\phi}\right)1_{x>\phi\underline{D}},$$

$$p_S(x) = kx^{-\zeta-1}\left[1 - H(x) + H\left(\frac{x}{1+\phi}\right)(1+\phi)^{\zeta}\right] \times 1_{x>(1+\phi)\underline{D}} + kx^{-\zeta-1}1_{\underline{D}<x<(1+\phi)\underline{D}},$$

where $k = \zeta\underline{D}^{\zeta}$, K is a constant ensuring $\int p_X(x)\,dx = 1$, and $1_{\{\cdot\}}$ is the indicator function

Densities of Domestic, Export, and Total Sales

- The Proposition implies that, conditional on the underlying distribution of productivity, and therefore domestic sales, being Pareto, the presence of exporting behavior implies that the distribution of total sales, as well as export sales, is *systematically different*
- This implies a potential **bias** in estimating power laws using total sales
- This bias can then impact model calibrations if ζ is used to pin down model parameters (e.g., θ given ε)

Densities of Domestic, Export, and Total Sales

- They apply the Proposition to a simple example where fixed cost of exporting is also random:

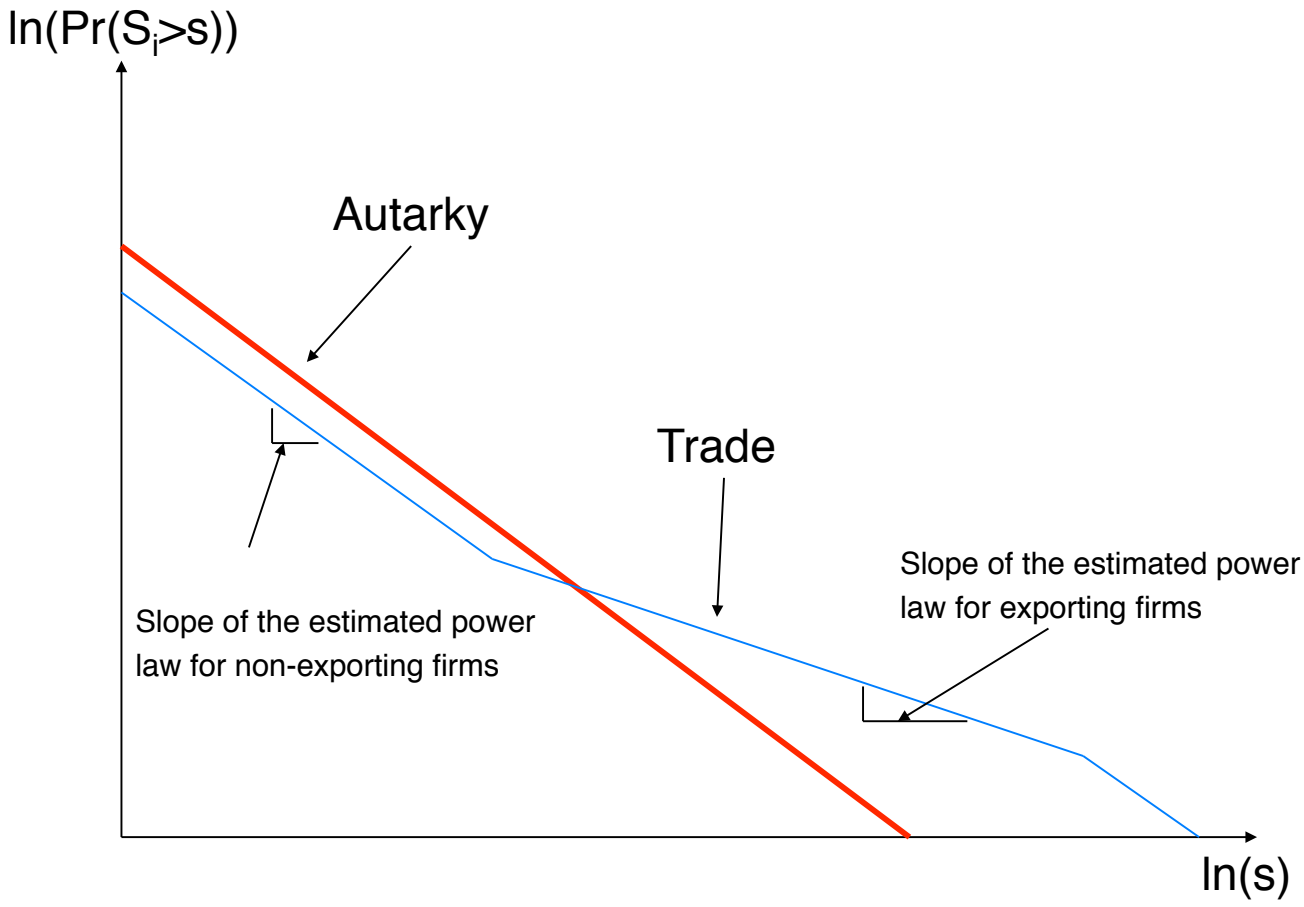
$$H(x/\phi) = \begin{cases} k'' x^{\alpha} & \text{for } x < x^* \\ k''(x^*)^{\alpha} & \text{for } x \geq x^* \end{cases}$$

  for some $k'' > 0$ and $\alpha > 0$

- Then the distribution of export sales is given by:

$$p_X(x) \propto \begin{cases} x^{-\zeta-1+\alpha} & \text{for } x < x^* \\ x^{-\zeta-1} & \text{for } x \geq x^* \end{cases}$$

Measurement of Power Laws: Stochastic Fixed Costs



Data

- Source: Income Statement of French firms from Tax Filings (BRN), Year 2006
- 2,182,571 firms, of which 194,444 (roughly 9%) are exporters
- Full Universe of Firms with domestic sales larger than 750K Euros (threshold used; drops 7.7% of sample). Results robust to 100K
- Variables: Domestic Sales D, Total sales S, Exports X
- Tradeable Sector
  - Drop industries for which total exports are less than 5% of total sales

Empirical Methodology

- They apply three different estimation methodologies to sales (s_i), as well as employees
- Consider different sub-samples:
  - Exporters vs. non-exporters
  - Domestic sales only
- Consider impact of trade openness on deviations in power law estimates by sectors

Empirical Methodology

- Regression 1 (Axtell, 2011):

$$\ln\left(\Pr(S_i > s)\right) = \ln\left(C\right) - \zeta \ln\left(s\right)$$

- Regression 2 (PDF):

$$f(s) = C\zeta s^{-(\zeta+1)}$$

  Assign firms to bins, and calculate PDF by frequency

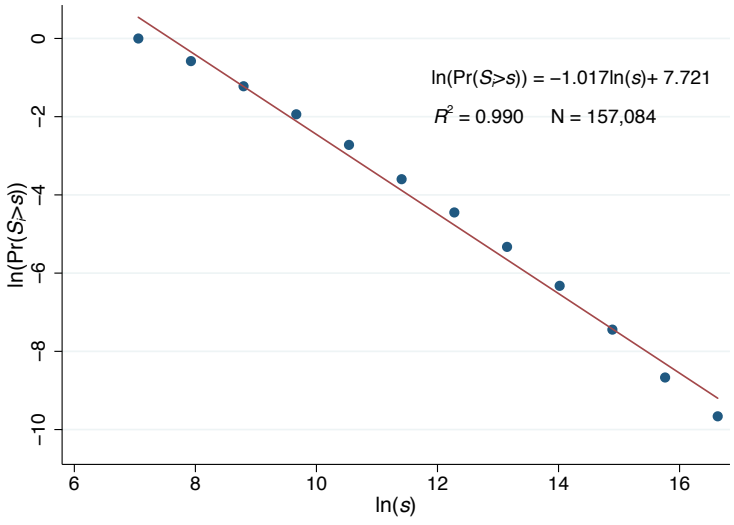
- Regression 3 (Gabaix and Ibragimov, 2011):

$$\ln\left(\text{Rank}_i - \frac{1}{2}\right) = \text{Constant} + \widehat{\zeta}_{LR} \ln S_i + \epsilon_i$$

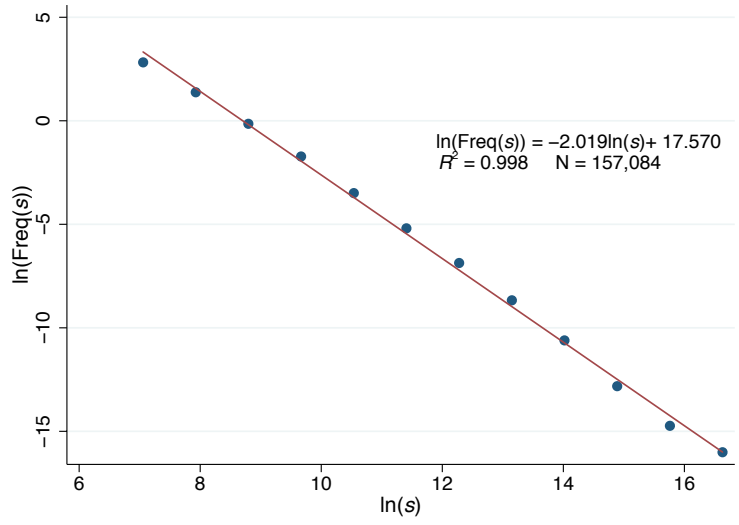
Power Law in Firm Size, All Firms: Sales and Employees

	I. Sales		
	(1) CDF	(2) PDF	(3) ln(Rank-0.5)
ζ	1.017	1.019	0.825
	(0.032)	(0.031)	(0.004)
R^2	0.990	0.998	0.991
No. of firms	157,084	157,084	157,084
	II. Employees		
	(1) CDF	(2) PDF	(3) ln(Rank-0.5)
ζ	1.078	1.093	0.790
	(0.072)	(0.083)	(0.003)
R^2	0.958	0.985	0.906
No. of firms	152,429	152,429	152,429

Power Law in Firm Size, All Firms: Sales



(a) CDF

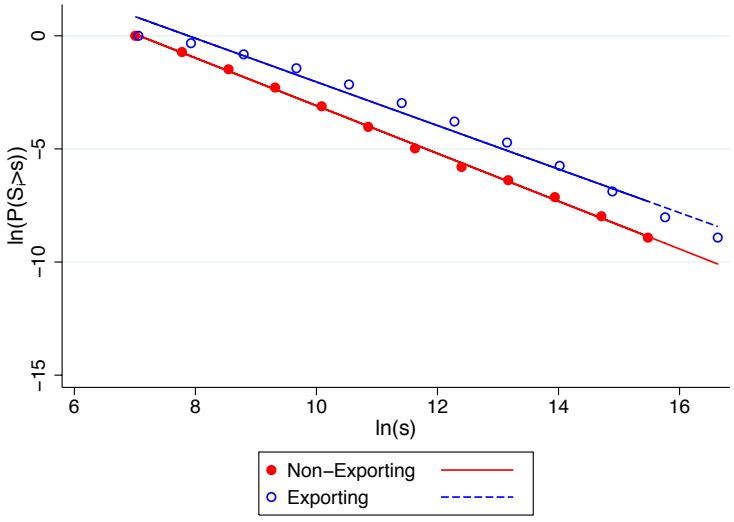


(b) PDF

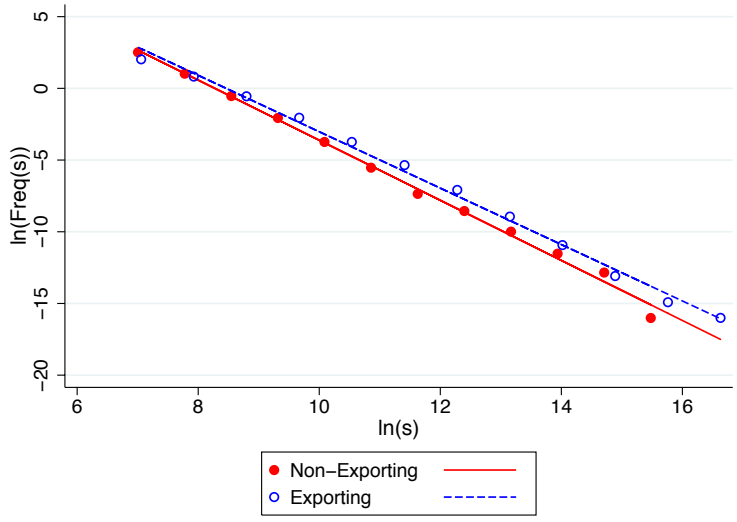
Power Laws in Firm Size, Non-Exporting and Exporting Firms: Sales and Employees

I. Sales							
	CDF		*PDF*		*ln(Rank-0.5)*		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Exporters	Non-Exporters	Exporters	Non-Exporters	Exporters	Non-Exporters	t-stat
ζ	0.964	1.055	0.967	1.095	0.738	1.029	37.03**
	(0.042)	(0.011)	(0.041)	(0.044)	(0.006)	(0.005)	
R^2	0.981	0.999	0.996	0.996	0.972	0.998	
No. of firms	67,078	90,006	67,078	90,006	67,078	90,006	
II. Employees							
	CDF		*PDF*		*ln(Rank-0.5)*		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Exporters	Non-Exporters	Exporters	Non-Exporters	Exporters	Non-Exporters	t-stat
ζ	0.967	1.251	0.949	1.111	0.724	0.891	28.54**
	(0.078)	(0.074)	(0.100)	(0.041)	(0.004)	(0.004)	
R^2	0.939	0.967	0.974	0.991	0.906	0.870	
No. of firms	66,040	86,389	66,040	86,389	66,040	86,389	

Power Laws in Firm Size, Non-Exporting and Exporting Firms: Sales



(a) CDF

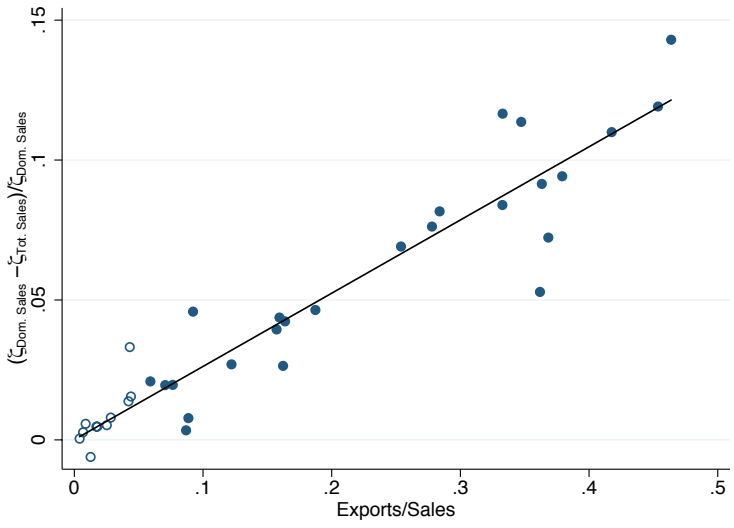


(b) PDF

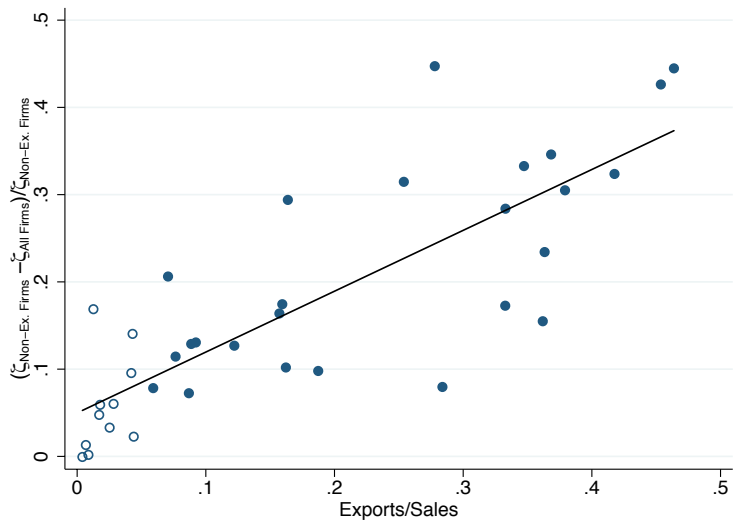
Power Law in Firm Size, All Firms, Domestic Sales Only

	(1) CDF	(2) PDF	(3) ln(Rank-0.5)
ζ	1.048	1.055	0.869
	(0.030)	(0.027)	(0.004)
R^2	0.992	0.998	0.992
No. of firms	157,084	157,084	157,084

Deviations in Power Law Estimates and Openness at Sector Level



(a) Domestic – Total



(b) Non-exporters – Total

NB: The non-tradeable sectors are denoted by hollow dots, and the tradeable sectors by solid dots