An Elementary Theory of Comparative Advantage

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Overview

Motivation

- Generalize Ricardian and Heckscher-Ohlin models into a unifying and tractable theoretical framework
- Capture *complementarity* of factors, sectors, and institutions.
- Mathematical device "Log-Supermodularity"

Intuition

- "If high X countries are relatively more productive at Y, then they should engage relatively more in Y."
- Simple statement elegantly summarizes gains from trade.
- Leaves open the question: "What is the source of relative productivity differences?"
(technology, factor endowments)

Preview of Results

- Assuming log-supermodularity ("complementarity") of primitive objects generates analogous patterns in output specialization across countries
- All generated in a perfect competition environment without frictions!

Contribution

- This paper can be thought of as a generalized theory of comparative advantage
- Brings (existing) mathematical device of log-supermodularity to trade setting
- Empirical relevance: Krugman (1986), Acemoglu, Antras, and Helpman (2007), among others: "Institutional quality of country, institutional dependence of sector."

Log-Supermodularity: The Basics

- (Watered-Down) Definition: $g : X \to \mathbb{R}^{++}$ is *log-supermodular* if for any $x, x' \in X$,

$$\ln g(\max(x, x')) + \ln g(\min(x, x')) \geq \ln g(x) + \ln g(x')$$

where min and max are component-wise.

- Example: Consider $g : X_1 \times X_2 \to \mathbb{R}^{++}$. Then, $\forall x_1' \geq x_1'', x_2' \geq x_2''$, log-supermodularity of g implies

$$g(x_1', x_2'') \cdot g(x_1'', x_2'') \geq g(x_1', x_2'') \cdot g(x_1'', x_2')$$

or

$$\frac{g(x_1', x_2')}{g(x_1'', x_2')} \geq \frac{g(x_1', x_2'')}{g(x_1'', x_2'')}$$

- In words, the *relative return* to *increasing* x_1 is *increasing* in the level of x_2.

The Only Result You Need To Know!

- If $g : X \to \mathbb{R}^{++}$, $h : X \to \mathbb{R}^{++}$ log-supermodular, then $g \cdot h$ log-supermodular
- If $g : X \to \mathbb{R}^{++}$ is log-supermodular and integrable, then $\int_{X_i} g(x) d\mu_i(x_i)$ is log-supermodular.

Setup

Primatives:

- Factor Productivity $q(\omega, \sigma, \gamma)$
- Factor Supply $f(\omega, \gamma)$ (inelastic but perfectly mobile *within* country),

  where $\omega \in \Omega$ indexes factor types (i.e. higher skilled labor), $\sigma \in \Sigma$ are sector characteristics (i.e. high-skilled labor-intensive), and $\gamma \in \Gamma$ are country characteristics (i.e. strong higher education system).
- NOTE: Γ, Σ, and Ω are totally ordered sets (complete, transitive, and anti-symmetric).

Setup cont'd

We consider $c = 1, \ldots, C$ countries with characteristics $\gamma^c \in \Gamma$, $s = 1, \ldots, S$ goods or sectors with characteristics $\sigma^s \in \Sigma$, and factors $\omega \in \Omega$.

- Aggregate Output in country c, sector s is given as

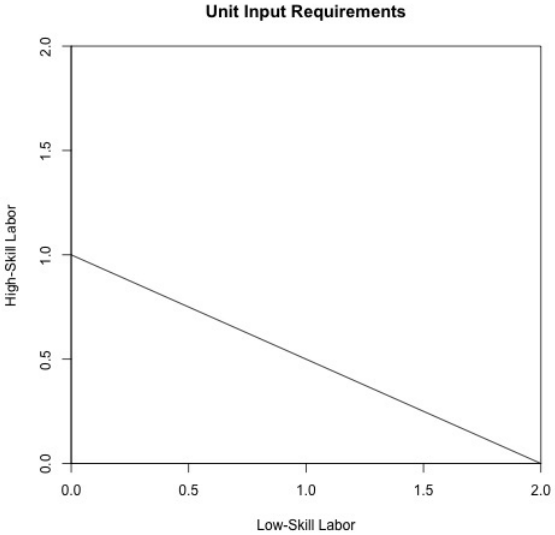
$$Q(\sigma^s, \gamma^c) = \int_\Omega q(\omega, \sigma^s, \gamma^c) l(w, \sigma^s, \gamma^c) d\mu(\omega)$$

where $l(\omega, \sigma^s, \gamma^c) \in f(\omega, \gamma^c)$ is the optimal allocation of factor ω to sector s in country c.

Key Assumptions

- Perfect substitutability of factors
- Unique solution to optimal allocation problem
- Q: What does this buy us? The return becomes output price times quantity, so we solve factor-by-factor
- Implies aggregate sector output is productivity times total factor endowment (i.e. sector gets *all* of the factors it uses)

Simple Two-Factor Example



Comparative Advantage in Technology

- Suppose $q(\omega, \sigma, \gamma) \equiv h(\omega) a(\sigma, \gamma)$
- Productivity of a factor affects all sectors/countries in the same way
- Implies no "real" difference in factors in this economy (i.e., isomorphic to one-factor Ricardian model):

To See This:

- Note, return is
  $r(\omega, \sigma^s, \gamma^c) = p(\sigma^s)q(\omega, \sigma^s, \gamma^c) = p(\sigma^s)h(\omega)a(\sigma^s, \gamma^c)$
- Clear that if there exists a factor which has the highest return in a sector, then *every* factor has its highest return in that sector.
- Relative return does *not* depend on the factor.
- $\Longrightarrow$ Solution to maximization problem allocates *all* factors to the (unique) sector with the highest return (one good produced by each country. Sad!)

Technology cont'd

Now, assume that $a(\sigma, \gamma)$ is log-supermodular. Then by definition,

$$\forall \quad (\sigma^{s_1}, \gamma^{c_2}), (\sigma^{s_2}, \gamma^{c_1})$$

with

$$\gamma^{c_1} \geq \gamma^{c_2}, \sigma^{s_1} \geq \sigma^{s_2},$$

we have

$$a(\sigma^{s_1}, \gamma^{c_2}) \cdot a(\sigma^{s_2}, \gamma^{c_1}) \leq a(\sigma^{s_1}, \gamma^{c_1}) \cdot a(\sigma^{s_2}, \gamma^{c_2})$$

$$\iff \frac{a(\sigma^{s_1}, \gamma^{c_1})}{a(\sigma^{s_1}, \gamma^{c_2})} \geq \frac{a(\sigma^{s_2}, \gamma^{c_1})}{a(\sigma^{s_2}, \gamma^{c_2})}$$

provided that $a(\sigma^{s_1}, \gamma^{c_2}), a(\sigma^{s_2}, \gamma^{c_2}) \neq 0$

What does this mean?

- In words, the relative productivity of a country's output is increasing in sectors that it is "better suited for" technologically.
- Strong assumption generates a prediction that holds for any pair of sector/country characteristics
- Recall math result: log-supermodularity of $a(\sigma, \gamma) \Longrightarrow$ log-supermodularity of $Q(\sigma, \gamma)$.
- Thus in equilibrium, we have a "ladder" of production such that high γ countries produce high σ goods, medium γ countries produce medium σ goods $\ldots$ etc.

A Concrete Example: Krugman ('86)

- Let σ^s be the technological intensity of sector s, and γ^c be a measure of "closeness" of a country to the world technological frontier
- Take $a(\sigma^s, \gamma^c) = e^{(\sigma^s \times \gamma^c)}$. Satisfies

$$\frac{\partial^2 \ln a()}{\partial\sigma\partial\gamma} > 0 \ \forall(\sigma, \gamma)$$

(an alternative characterization of log-supermodularity)

Comparative Advantage in Factor Endowments

- Now suppose we let factor productivity vary only in a Hicks neutral fashion: $q(\omega, \sigma, \gamma) = a(\gamma)h(\omega, \sigma)$ with $a(\gamma) > 0, h(\omega, \sigma) \geq 0$
- Each factor assigned to sector with highest return (depends only on $h(\omega, \sigma)$ and not absolute advantage $a(\gamma)$):
- $r(\omega, \sigma^s, \gamma^c) > r(\omega, \sigma^{s'}, \gamma^c)$
  $\iff p(\sigma^s)a(\gamma^c)h(\omega, \sigma^s) > p(\sigma^{s'})a(\gamma^c)h(\omega, \sigma^{s'})$
  $\iff p(\sigma^s)h(\omega, \sigma^s) > p(\sigma^{s'})h(\omega, \sigma^{s'})$
- Same assignment for all countries (no γ dependence above), implies specialization driven by factor endowments

Factor Endowments cont'd

- Suppose now that $h(\omega, \sigma)$ is log-supermodular (high ω factors are more productive in high σ sectors regardless of location). Further, assume that $f(\omega, \gamma)$ (High γ countries are relatively more abundant in high ω factors).
- Note, these are analogous to the ordinal assumptions on factor intensity and abundance in the $2 \times 2 \times 2$ Heckscher-Ohlin model.

Implications For Output

- Math results on log-supermodularity imply $Q(\sigma, \gamma)$ is log-supermodular
- Countries c_1, c_2, goods s_1, s_2 with $\gamma^{c_1} \geq \gamma^{c_2}$, and $\sigma^{s_1} \geq \sigma^{s_2}$

$$\frac{Q(\sigma^{s_1}, \gamma^{c_1})}{Q(\sigma^{s_1}, \gamma^{c_2})} \geq \frac{Q(\sigma^{s_2}, \gamma^{c_1})}{Q(\sigma^{s_2}, \gamma^{c_2})},$$

  i.e. the relative output increasing in "better matched" sector/country pairs.
- In fact, the more general result, comparing 2 countries and J sectors, high γ countries specialize in high σ sectors:

$$\frac{Q^{s_1 c_1}}{Q^{s_1 c_2}} \geq \cdots \geq \frac{Q^{s_J c_1}}{Q^{s_J c_2}}$$

Both Sources of Comparative Advantage?

- A natural question is "Can we combine these two sources of comparative advantage and generate clean predictions on output specialization patterns?"
- Short answer is: No. (At least, in the absence of stronger functional form restrictions)
- However, log-supermodularity of $q(\omega, \sigma, \gamma)$ does pin down specialization in the "best" and "worst" sectors:
- With log-supermodularity of $f()$ and $q()$,
  $Q(\underline{\sigma}, \gamma^{c_1})Q(\bar{\sigma}, \gamma^{c_2}) \geq Q(\underline{\sigma}, \gamma^{c_2})Q(\bar{\sigma}, \gamma^{c_1})$
  for any $\gamma^{c_1} \leq \gamma^{c_2}$