Risk, Returns, and Multinational Production*

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(PRELIMINARY AND INCOMPLETE)

Abstract

This paper starts by unveiling a new empirical regularity: multinational corporations tend to exhibit systematically higher returns and earnings yields than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings yields and returns than firms selling only in their domestic market. To explain this pattern, we develop a real option value model where firms are heterogeneous in productivity, and have to decide whether and how to sell in a foreign market where demand is risky. Firms can serve the foreign market through trade or foreign direct investment, thus becoming multinationals. Multinational firms are more exposed to risk: following a negative shock, they are reluctant to exit the foreign market because they would forgo the option premium (sunk cost) that they paid to become multinationals. The theory provides a complementary explanation for the cross section of returns by exploiting the production side from an international point of view. We calibrate the model to match U.S. export and FDI dynamics, and use it to explain cross-sectional differences in earnings yields and returns.

Keywords: Multinational firms, option value, cross-sectional returns

JEL Classification : F12, F23, G12

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1 Introduction

Multinational firms tend to have higher returns and earnings yields than non-multinational firms. Among non-multinationals, exporters tend to have higher returns and earnings yields than firms selling only in their domestic market. Many studies in the new trade literature have documented features determining selection into foreign markets: exporters and multinational firms tend to be larger, more productive, to employ more workers, and sell more products than firms selling only domestically.[1] However, none of this literature has studied whether the international status of the firm matters for its investors. Similarly, in the financial literature, explanations of the cross section of returns disregarded the role of the international status of the firm.

In this paper we attempt to fill this gap in the literature. We develop a real option value model where firms' heterogeneity, aggregate uncertainty and sunk costs provide the missing link between firms' international status and their returns on the stock market.

In our framework, firms are heterogeneous in productivity, and choose optimally whether to produce only domestically, export, or set up an offshore affiliate to serve a foreign market where aggregate demand is uncertain. By selling in their domestic market, firms effectively purchase an option that allows them to enter into the foreign market through exports or direct investment (henceforth, FDI). Exercising the option (*i.e.* entering the market) entails a premium, in the form of a sunk cost. Firm-level productivity and prospects of growth of the foreign demand for goods determine the equilibrium choice. The presence of uncertainty generates larger persistence in a status with respect to a deterministic world: when the foreign market is hit by a negative shock, firms are reluctant to exit even if their profits fall below zero, since by exiting they would forgo the option premium they paid to enter. It may be optimal for them to bear losses for a while, and wait for a possible reversal of the shock. Firms' optimal strategies depend on their underlying productivities. As a result, different firms will differ in the covariance of their earnings yields with the aggregate uncertainty. The model endogenously determines cross-sectional differences in earnings-to-price ratios and returns, and provides a complementary explanation for the cross section of returns exploiting the production side from an international point of view.

The selection mechanism is modeled following Helpman, Melitz, and Yeaple (2004). Exports are characterized by low sunk costs and high variable costs, due to the necessity of shipping goods every period, while FDI entails high sunk costs of setting a plant abroad but low variable costs,

[1] See, among others, Bernard, Jensen, and Schott (2009).

since there is no physical separation between production and sales. We adapt this cost structure to a dynamic and stochastic environment, using Dixit (1989) as a benchmark to model entry decision under uncertainty. Since foreign investment is associated to larger sunk costs compared to trade, it generates more persistence in the firm's strategies over time. This implies larger losses if the foreign economy is hit by negative shocks. As a result, multinationals are more exposed to aggregate risk since – in case of a negative shock – they are more reluctant to exit. Pulling back is a more expensive strategy for multinationals than for exporters, who paid a lower option premium to serve the foreign market.

The dynamic model generates an imperfect sorting of firms' productivities into international statuses and is able to explain the observed yields' variation across firms. More productive firms display less persistent dynamics, because they are on average more likely to have positive profits, but have more incentives to become multinationals, and hence self-select into the status that is associated with more persistence. The imperfect sorting of productivities into status, together with the possibility of changing status (the option value) create a wedge between per-period profits and the firm's valuation over time. As a result, earnings yields and returns depend on the choice of whether and how to serve foreign markets, therefore vary across firms with different international status.

The solution of the model delivers a series of predictions relating firms' productivity and the realization of the shocks to the pattern of status changes. First, more productive firms need smaller positive shocks to enter a foreign market, and larger negative shocks to exit. Second, a larger positive shock is needed to induce a domestic firm to become multinational with respect to the one needed to become an exporter. Third, a larger negative shock is needed to induce a multinational to exit the market with respect to the one needed to induce an exporter to exit. These predictions find support in the pattern of status changes in the data. The model is also consistent with other qualitative features of the data on trade and FDI dynamics, like the imperfect sorting of productivity into statuses, and the fact that also relatively large firms exhibit changes of status. We calibrate the theory to match these facts also quantitatively, and with the parameterized model we compute moments of the financial variables from simulated data. We show that the model is able to match features of the financial data which were not targeted in the calibration.

Why are we interested in the cross section of returns and earnings yields? Historically, average returns vary across stocks. Fama and French (1996) is a comprehensive description of the cross-sectional picture of returns. In this paper we address the risk-return trade-off concerning multinational and non-multinational firms. We focus on cash flow dynamics of the firm and how these are determined by endogenous decisions and exogenous risk. Multinational firms are exposed to foreign demand risk for longer due to the higher persistence in their status. This risk must be rewarded by a higher asset returns in equilibrium. Investors will be willing to hold these companies if the returns are high enough to compensate for the risk. We find that this risk is not fully captured by the multifactor model in Fama and French (1993).

The existing financial literature that focuses on cross-sectional differences in earnings-to-price ratios and returns abstracts from the international organization of the firm. Various attempts to explain cross sectional differences in expected returns are based either on different specifications of preferences, or on the presence of persistent shocks to the endowments, or both.[2] The different exposure of firms' cash flows to these two types of shocks determines cross sectional differences in returns observed in the data. We contribute to the financial literature by endogenizing the exposure of cash-flows to these types of shocks. Exposure is directly linked to the decision of when and how to serve the foreign market, which is ultimately driven by the interaction between productivity and cost structure.

To our knowledge, Rob and Vettas (2003) is the only other paper that developed a model of trade and FDI with uncertain demand growth. In their framework FDI is irreversible, so it can generate excess capacity, but has lower marginal cost compared to export. The authors show that uncertainty implies existence of an interior equilibrium where export and FDI coexist. Our work generalizes their model to one with many heterogeneous firms and a more general process for demand growth. Ramondo and Rappoport (2008) introduce idiosyncratic and aggregate shocks in a model where firms can locate plants both domestically and abroad. Multinational production allows firms to match domestic productivity and foreign shocks, and works as a mechanism for risk sharing. We abstract from the risk sharing/diversification motive: the risks in our model are aggregate, hence not insurable. Moreover, we model both trade and multinational production as different modes of dealing with uncertainty in foreign markets.

Our work is related to the literature on trade dynamics with sunk costs. Particularly, Alessandria and Choi (2007) and Irrarazabal and Opromolla (2009) model entry and exit into the export market in a world with idiosyncratic productivity shocks and sunk costs. Our model is closer to the framework in Irrarazabal and Opromolla (2009) for the use of the real option value analogy in

[2]Yogo (2006) and Piazzesi, Schneider, and Tuzel (2007) are examples of non-separable goods in the utility function. Campbell and Cochrane (1999) use internal habits specifications. Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008) use recursive preferences with persistent shocks to the endowment.

solving the firm's optimization problem. While Irrarazabal and Opromolla (2009) concentrate their attention on the impact of idiosyncratic productivity shocks for firm dynamics, we model aggregate demand shocks that affect firms differently only through their endogenous choice of international status. Alessandria and Choi (2007) study the impact of firms' shocks and sunk costs on the business cycle. While the focus of our exercise is different from their paper, we follow their calibration methodology. Both papers analyze the decision to export, but do not consider the possibility of FDI sales. Roberts and Tybout (1997) and Das, Roberts, and Tybout (2007) address empirically the issue of market participation for export. Our model has similar predictions for both exports and FDI sales, and can be estimated by using information from both trade/FDI flows and stock market prices. In general, we contribute to the trade dynamics literature by incorporating in the model the mode of entry (*i.e.*, the decision between export and FDI sales).

While individual elements of our framework are found in other work, this paper is the first to propose a dynamic industry equilibrium model where risk affects firms' international strategies and their financial variables in the stock market. The remainder of the paper is organized as follows. Section 2 presents empirical evidence establishing the ranking in earnings-to-price ratios and returns according to the firms' international status. Sections 3 and 4 develop the model and illustrate its analytical properties. Section 5 brings the model to the data: we report qualitative results on trade dynamics and quantitative results on the earnings yields and returns predicted by a calibrated version of the model. Section 6 is devoted to robustness checks, and Section 7 concludes.

2 Motivating Evidence

In this section we document a new fact distinguishing firms that sell only in their domestic market from exporters and multinational firms. The data display a precise sorting in the financial variables of these firms: multinational firms tend to exhibit significantly higher earnings yields and returns than exporters, and exporters in turn have higher earnings yields and returns than firms selling only in their domestic market.

Our sample consists of all US-based manufacturing firms in the Compustat Segments database, and tracks about 7400 firms from 1979 to 2006. We define a firm to be a multinational (MN) if it reports the existence of a foreign geographical segment associated with positive sales. Similarly,

Table 1: **Summary Statistics**.

	Domestic	Exporters	Multinationals
total sales (millions $)	162.62	176.19	1547.71
export sales (millions $)	0	26.6	58.89
number of employees (thousands)	1.39	1.65	11.07
book value per share ($)	4.86	5.73	9.58
earnings per share ($)	0.19	0.39	0.88
share price ($)	7.82	8.83	17.09
annual return	0.06	0.1	0.11
number of firms	2454	1883	3091

we define a firm to be an exporter if it reports a positive level of export sales.[3,4] According to this definition, on average, 33.26% of firms sell only in the U.S. market, 24.84% also export to foreign countries, and the remaining 41.9% have positive levels of FDI sales.[5] Table 1 reports descriptive statistics of the sample we use.

In line with the numbers reported by other papers, exporters and multinationals have a size advantage with respect to domestic firms, both in terms of sales and number of employees. Particularly, the size advantage of multinationals is extremely large: on average, multinational firms hire about six times more employees than exporters, and have sales about nine times larger. Consistently with previous evidence, export sales are a small percentage (15%) of exporters' total sales.[6] The novel facts that Table 1 highlights are that the same ordered differences hold for financial variables like book value, earnings, share prices, and returns.

Differences in earnings and market prices do not cancel out: at the contrary, also earnings

[3]Multinational and exporter dummies are constructed based on Compustat geographic and operating segments data. Appendix A contains a summary of the Financial Accounting Standards for data reporting of publicly traded firms present in Compustat, with details on the disclosure requirements. One of the Financial Accounting Standards Board (FASB)'s roles is to *"require significant disclosures about the separate operating segments of an entity's business so that investors can evaluate the differing risks in the diverse operations"*. Appendix A also contains more details about the construction of the sample.

[4]4.5% of firms in the sample report both positive exports and FDI sales. We classify these firms as multinationals, based on the criterium that the existence of FDI sales is sufficient to make them subject to the risk of owning a plan in a foreign market. For robustness, however, we run all the regressions contained in this section also excluding those firms from the sample. The results are unchanged.

[5]Notice that the shares of firms belonging to each group are very different from what reported in other papers that use different data. Particularly, the share of multinational firms is disproportionately large. This is due to the fact that Compustat collects data for publicly listed firms only, which tend to be the largest firms in the population.

[6]Bernard et al. (2003) report an average ratio of export over total sales of 10% for a sample of manufacturing firms in the OECD countries.

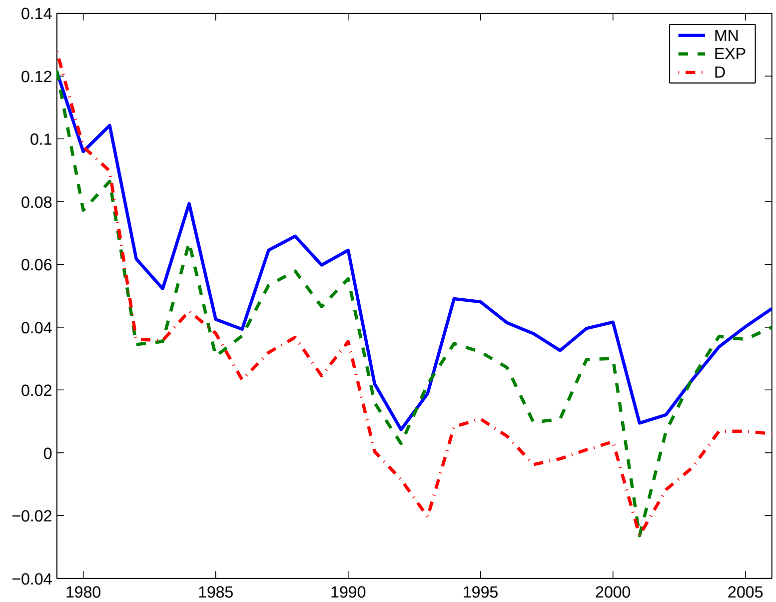

Figure 1: Earnings-to-price ratios, portfolios of firms in each group.

yields (or earnings-to-price ratios) are ordered. Figure 1 shows earnings-to-price ratios over time for three portfolios of firms. Each portfolio is composed by firms with the same international status (only domestic sales, exporters, multinationals).[7] The solid line represents earnings-to-price ratios of multinational firms, the dashed line the ones of exporters, and the dash-dotted line the ones of firms selling only domestically. Multinational firms exhibit higher earnings-to-price ratios than non-multinational firms, consistently over the entire time period. Similarly, exporters exhibit higher earnings-to-price ratios than firms selling only in their domestic market. The average e/p ratios for the three groups are 4.85% for multinational firms, 3.7% for exporters, and 2.12% for firms selling only domestically.

Figure 1 plots the raw data, but the ordering is robust to controlling for the effect of other variables related to size and industry. To separate the effects of the international status from other

[7]The portfolios are constructed as follows. For each firm i, determine its status S ($S = D, EXP, MN$) at the end of year $t-1$, and collect data on earnings (e_t^i) and market capitalization (p_t^i) in year t. Portfolio earnings E_t^S and portfolio value P_t^S are constructed as equally weighted averages of individual values:

$$E_t^S = \frac{1}{N_t^S} \sum_{i \in S} e_t^i \quad , \quad P_t^S = \frac{1}{N_t^S} \sum_{i \in S} p_t^i \ , \ \ \forall \ S$$

where N_t^S denotes the number of firms in status S at time t. Portfolio earnings yields are given by E_t^S / P_t^S.

Table 2: **Earnings-to-Price Regressions.** Firm-level regressions of earnings-to-price ratios on multinational and exporter dummies, market *betas* and other controls, with year and industry fixed effects. (Top and bottom one percent of sample excluded, all dollar values expressed in billions).

	(1)	(2)
MN dummy	0.091	0.092
	(0.004)***	(0.004)***
EXP dummy	0.062	0.064
	(0.004)***	(0.004)***
β^{MKT}		-0.004
		(0.001)***
sales per emp.	4.71e-05	4.71e-05
	(2.67e-05)*	(2.74e-05)*
total revenue	9.49e-07	9.14e-07
	(1.97e-07)***	(2.01e-07)***
constant	-0.098	-0.097
	(0.004)***	(0.005)***
Prob $> F$:		
H_0: MN=EXP	0	0
No. of obs.	47411	47411
R^2	0.072	0.071

firm characteristics, Table 2 displays the results of the following firm-level regression:

$$(e/p)_{it} = \alpha + \gamma_1 D_{it}^{MN} + \gamma_2 D_{it}^{EXP} + \gamma_3 \beta_i^{MKT} + \gamma_4 X_{it} + \delta_{NAICS} + \delta_t + \varepsilon_{it}. \quad (1)$$

The dependent variable $(e/p)_{it}$ is the earnings-to-price ratio of firm i in year t. D_{it}^{MN} and D_{it}^{EXP} are multinational and exporter dummies, respectively, β_i^{MKT} is the market *beta* of the primary security of firm i,[8] X_{it} is a set of controls, including capital/labor ratio, sales per employee (our measure of productivity), total revenues and market capitalization (measures of size). δ_{NAICS} and δ_t are 4-digit industry and year fixed effects, respectively, and ε_{it} is an orthogonal error term.

The coefficients associated to exporters and MN dummies are positive and significant. Moreover, the coefficient associated to multinationality appears to be significantly larger than the one associated to export status, identifying a further difference between these two groups. The Wald test of the null hypothesis that the two dummies' coefficients are the same confirms the differ-

[8]The market *betas* have been computed by running a regression of individual security returns on the market aggregate returns (NYSE, AMEX, and Nasdaq) for the entire sample period. Adding the market *betas* in the regression captures the exposure of the security to aggregate market risk.

Table 3: **Mean Excess Return (% per year).** Portfolios are formed by sorting firms into deciles on the earnings yield (E/P), ratio of cash flow to prices (C/P), dividend yield (D/P), and book-to-market ratio (B/M). Monthly (annualized) data, from 1952 to 2002. From Lettau and Wachter (2007).

	G	Growth to Value							V
Portfolio	1	2	3	4	5	6	7	8	9	10
E/P	4.71	5.02	6.97	7.04	7.00	9.18	9.94	11.18	11.68	12.95
C/P	5.05	6.07	6.49	6.73	8.48	7.72	8.85	9.18	11.47	11.81
D/P	7.35	6.41	7.28	7.41	6.49	7.60	7.73	9.49	8.84	7.45
B/M	5.67	6.55	6.98	6.51	8.00	8.33	8.27	10.08	9.98	10.55

ence in earnings yields of multinational firms *versus* exporters. While Table 2 only contains sales per employee and total revenues as additional controls, we run the regression also adding capital intensity and market capitalization.[9] The purpose of adding the market *betas* is to control for aggregate market risk and to highlight the contribution of the international status to the magnitude of earnings yields once market risk is accounted for.

Earnings-to-price ratios carry information about returns on the firms' stocks. Table 3 is taken from Lettau and Wachter (2007), and shows how portfolios delivering higher earnings yields are also associated to higher excess returns. Our data confirm this correlation based on the international status of the firm. Table 4 reports the results of a regression analogous to (1), but with annual firm-level returns as the dependent variable.[10]

The coefficients on the multinational and exporter dummies are positive and significant, confirming that firms selling in foreign markets tend to have higher returns than firms selling only domestically. The coefficient on the multinational dummy is significantly higher than the one on the exporter dummy, indicating even larger excess returns for multinational firms. The ranking holds controlling for market risk, sales per employee, and total revenues.[11] Any cross-sectional differences in returns generated by exposure to aggregate risk is captured by cross sectional differences in their market *betas*. Hence, the significant coefficients on the multinational and exporter dummies identify a separate source of risk.

[9] In the data capital intensity appears to be highly correlated with sales per employee, while market capitalization is highly correlated with total revenues. For this reason, we run the regression with all the combinations of the two controls that are not strongly correlated with each other. The results are basically identical across specifications.

[10] We identify firm-level returns with the returns of the firm's common equity. Data on returns are taken from CRSP.

[11] The discussion about the correlations across controls in footnote 9 still applies.

Table 4: **Returns Regressions.** Firm-level regressions of returns on multinational and exporter dummies, market *betas* and other controls, with year and industry fixed effects. (Top and bottom five percent of sample excluded, all dollar values expressed in billions).

	(1)	(2)
MN dummy	0.044	0.044
	(0.005)***	(0.005)***
EXP dummy	0.031	0.032
	(0.006)***	(0.006)***
β^{MKT}		0.008
		(0.002)***
sales per emp.	4.9e-05	4.64e-05
	(2.89e-05)*	(2.78e-05)*
total revenue	5.56e-07	6.76e-07
	(2.24e-07)***	(2.31e-07)***
constant	0.06	0.045
	(0.005)***	(0.006)***
Prob > F:		
H_0: MN=EXP	0.008	0.015
Obs.	47266	45698
R^2	0.137	0.137

After an exploration of earnings-to-price and returns across the three groups of firms, it seems natural to explore the source of higher returns. Higher returns do not constitute by themselves a puzzle. From a CAPM point of view, higher returns must be explained by higher *betas*, or co-movements with the aggregate risks. Beyond the one-factor CAPM model, Fama and French (1993) introduced a multifactor extension to the original CAPM. Fama and French (1993) argue that a unique source of risk is not able to explain the cross section of returns. Instead, a three-factor model explains a higher fraction of the variation in expected returns. Higher returns must be explained by higher exposures to either of these three factors: market excess returns, high-minus-low book-to-market, or small-minus-big portfolio.[12] Each of the three factors is assumed to mimic a macroeconomic aggregate risk, and the three together span the entire space of assets. Therefore, any asset is a linear combination of the three Fama-French factors.

Table 5 shows the results of running one time-series Fama-French regression for each firm in the sample. We report averages and standard deviations of the sample of estimated coefficients. Strikingly, multinationals feature higher pricing errors on average than exporters and domestic firms, while exporters display higher pricing errors than domestic firms.[13] An explanation for this *anomaly* is that the risk incurred by a firm which decides to serve a foreign market is not fully captured by the three Fama-French factors. The production-based model that we use to explain firms' behavior identifies indeed a "new" source of risk, being *new* defined as a risk factor beyond value or growth.

Firm level regressions are subject to substantial idiosyncratic noise. Next, we compute three portfolios composed by firms according to their international status, and we run the same multifactor regressions at the portfolio level.[14] We present the results in Table 6. It is worth to mention that the risk to which multinationals and exporters are exposed, and the corresponding higher returns that they provide to investors, are not captured by the three existing Fama-French factors. On the contrary, we find that the market *betas* are lower than those of domestic firms. Multinationals and

[12]The small-minus-big (SMB) and high-minus-low (HML) factors are constructed upon 6 portfolios formed on size and book-to-market. The portfolios are the intersection of 2 portfolios formed on size (small and big) and 3 portfolios formed on book equity to market equity (from higher to lower: value, neutral, and growth.) This generates 6 portfolios: small-value, small-neutral, small-growth, big-value, big-neutral, and big-growth. SMB is the average returns on the three small portfolios minus the average returns on the three big portfolios. HML is the average return on the two value portfolios minus the average return on the two growth portfolios. The third factor, the excess return on the market, is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus the one-month Treasury bill rate. For more details see Fama and French (1993).

[13]We refer to the estimated constant coefficients α as the pricing errors, as they reflect returns that are not priced by one of the three factors.

[14]Every year portfolios are formed by equally-weighting firms belonging to each of the three categories. See footnote 7.

Table 5: **Firm-level Regressions**. Summary statistics of the firm-level time series coefficient estimates of Fama-French 3 factor regressions, by international status. Firm level annual excess returns are regressed on the three Fama-French factors: market excess return, SMB (Small Minus Big), and HML (High Minus Low). The α coefficients capture the pricing errors of the three-factor model. There are 1.852 regressions for multinational firms (MN), 1.485 for exporting firms (X), and 2.727 for domestic firms (D).

	Mean	Std. Dev.
α_{MN}	0.11	8.298
α_X	-0.1	1.289
α_D	-0.168	1.971
β_{MN}^{mkt}	-0.115	44.884
β_X^{mkt}	0.747	7.442
β_D^{mkt}	0.935	18.367
β_{MN}^{SMB}	0.526	13.387
β_X^{SMB}	0.572	6.717
β_D^{SMB}	0.519	17.326
β_{MN}^{HML}	-0.422	26.714
β_X^{HML}	0.146	5.669
β_D^{HML}	0.307	14.692

Table 6: **Portfolio Regressions**. Time-series coefficient estimates of Fama-French 3 factor regressions for the three equally-weighted portfolios based on international status. Portfolio annual excess returns are regressed on the three Fama-French factors: market excess return, SMB (Small Minus Big), and HML (High Minus Low). The α coefficients capture the pricing errors of the three-factor model.

	DOM	EXP	MN
R^{mkt}	0.672	0.648	0.666
	(0.07)***	(0.086)***	(0.064)***
R^{SMB}	0.768	0.778	0.538
	(0.081)***	(0.1)***	(0.074)***
R^{HML}	0.234	0.122	0.216
	(0.069)***	(0.085)	(0.063)***
α	-0.074	-0.033	-0.026
	(0.013)***	(0.016)**	(0.012)**
Obs.	28	28	28
R^2	0.903	0.86	0.893

exporters' exposure to the HML factor, related to the value premium, is also significantly lower than the exposure of domestic firms to the same factor. Evidently, if the exposure to the three factors does not explain the higher reward that multinational stocks provide, it must be reflected in the pricing errors of the model. Effectively, the *alpha* of the portfolio of multinational firms is significantly higher than the one of the exporters' portfolio, which in turn is higher than the *alpha* of the portfolio of domestic firms.

Tables 2-6 consistently convey the message that the three groups of firms differ significantly in their financial variables, and that these differences cannot be accounted for either by differences in productivity and size or by traditional risk-factor explanations. In the next section we develop a dynamic structural model in which productivity differences determine the selection of firms into the three statuses, and the presence of sunk costs and aggregate risk gives rise to the observed pattern in their financial variables.

3 Model

3.1 Preferences and Technology

The economy is composed of two countries, a home country and a foreign one. Variables related to consumers and firms from the foreign country are going to be denoted with an asterisk (*). In both countries, agents are infinitely lived, and have time-separable intertemporal preferences. Intra-temporal preferences are defined by:

$$U_t(H(t), Q(t))$$

where H is a homogeneous good, and Q is a CES aggregate of differentiated varieties:

$$Q(t) = \left(\int q_i(t)^{1-1/\eta} di \right)^{\eta/(\eta-1)} .$$

$\eta > 1$ denotes the elasticity of substitution across varieties.

Agents maximize U subject to their budget constraint. While income in the home country is deterministic, we assume that income in the foreign country is hit by aggregate random shocks affecting aggregate demand for the differentiated product Q^*. More precisely, Q^* follows a geometric Brownian motion:

$$\frac{dQ^*}{Q^*} = \mu dt + \sigma dz \tag{2}$$

where $\mu \geq 0$, $\sigma > 0$ and dz is the increment of a standard Wiener process, satisfying: $E(dz) = 0$ and $E(dz^2) = dt$.

Agents are risk averse, and discount future utility with a stochastic discount factor described by the following geometric Brownian motion:

$$\frac{dM}{M} = -rdt + \sigma_M dz \tag{3}$$

where $r > \mu + \sigma\sigma_M$ is the risk-free rate, $\sigma_M > 0$ and dz is the increment of the same Wiener process ruling the evolution of Q^*.[15]

[15] The stochastic discount factor M reflects the fact that consumers' intertemporal rate of substitution is sensitive to aggregate shocks. It can be derived from time-separable constant relative risk aversion preferences and aggregate consumption growth following a Brownian motion. σ^M represents the risk prices, which in this case are constant.

Labor is the only factor of production. The homogeneous good is produced by perfectly competitive firms in both countries with a one-to-one linear technology, and is perfectly tradeable.[16] We assume that firms from the foreign country only produce the homogeneous good, while firms from the home country produce goods in both sectors and sell both homogeneous and differentiated products in both countries.[17] One could think for example about the foreign country as an economy whose income is subject to natural resource shocks, or whose purchasing power is subject to exchange rate shocks. The homogeneous good could be interpreted as a standard good that both countries produce and consume, and the differentiated good can be seen as a set of more advanced goods that the foreign country does not have the technology to produce.

The differentiated sector is populated by a continuum of firms of total mass n, which operate under a monopolistically competitive market structure. Each firm produces a differentiated variety with a linear technology defined by a unit labor requirement a, which is a random draw from a distribution $G(a)$. a indicates the number of units of labor that a firm needs in order to produce one unit of a differentiated variety.

Let's now turn to the description of the production costs in the differentiated sector. All firms in the differentiated sector produce in the home market. We assume that there are no fixed costs associated to production for the domestic market, so every firm makes positive profits from domestic sales.[18] Besides producing for the domestic market, firms from the home country can produce also for the foreign country one. Production in the foreign market involves fixed operating costs, to be paid every period, and sunk costs of entry. If a firm decides to sell in the foreign market, it can do so either via exports or via foreign direct investment. We call multinationals those firms that decide to serve the foreign market through FDI sales.

We model the choice between trade and FDI along the lines of Helpman, Melitz, and Yeaple (2004): exports entail a relatively small sunk cost of entry F_X, but a per-unit iceberg transportation

[16]The existence of a perfectly tradeable good that is produced and consumed in both countries pins down relative wages as the relative productivity in producing the good. Here we assume that the two countries produce the homogeneous good with identical technologies, so that $w = w^* = 1$.

[17]We introduce this artificial asymmetry between the two countries in order to present in the simplest possible way the problem of entry in a risky foreign market. Solving the model for a symmetric world where both countries produce differentiated varieties and are hit by aggregate shocks is straightforward. We develop the results of the paper in the simpler asymmetric case, and we show that the results easily extend to the symmetric case in Section 6.

[18]We could have introduced positive fixed costs of domestic production, and modeled the initial decision of entry in the domestic market, like in Helpman, Melitz, and Yeaple (2004). This would have introduced additional complications in solving for firms' optimal dynamics, without any gains for our empirical analysis. Compustat includes only publicly listed firms, so when a firm enters or exits Compustat we do not have any information about whether the firm is infact entering or exiting the market, or whether it just started or stopped being listed.

cost τ to be paid every period,[19] while FDI is associated to a larger sunk cost F_I ($F_I > F_X$), but there are no transportation costs to be covered every period, as both production and sales happen in the foreign market. Both options also entail fixed operating costs to be paid every period, that we denote with f_I and f_X for FDI and exports, respectively. Once entered the foreign market, a firm can exit at no cost. However, if it decides to re-enter, it will have to pay the sunk cost again.[20] Sunk costs and stochastic demand imply that firms decide to enter when their expected profits are well above zero, and – once entered – are reluctant to exit even in case of losses due to negative shocks. We show that this dynamic behavior, labeled "hysteresis" by the literature (see Dixit and Pindyck (1994)), is more severe for multinational firms than for exporters, due to the larger sunk costs of FDI. Notice also that the cost structure and the nature of uncertainty imply that if a firm decides to enter the foreign market, it will do so either as an exporter or as a multinational firm, but it will never adopt the two strategies at the same time.[21]

Hence for a given realization of Q^*, a firm with productivity $1/a$ must choose its optimal status S ($S \in \{D, X, I\}$, *i.e.* domestic, exporter, or multinational), the current selling price $p_S(a)$ and an updating rule (how to change the optimal price and status following changes in aggregate foreign demand). The state of the economy is described by the realization of foreign aggregate demand Q^*.

The CES aggregation over individual varieties implies that individual pricing rules are independent on Q^*. However, marginal costs of production and optimal pricing rules vary with the status of the firm. Let w, w^* denote the wages in the home and foreign countries, respectively. The marginal cost of domestic production is given by the labor requirement times the domestic wage, $MC_D = aw$. The marginal cost of exporting is augmented by the iceberg transportation cost: $MC_X = \tau aw$.

[19] $\tau > 1$ units of good need to be shipped for one unit of good to arrive to the destination country.

[20] Roberts and Tybout (1997) report evidence on the fact that previous exporting experience matters as long as firms do not exit the foreign market. They find that sunk costs of entry for first-time exporters are not statistically different from sunk-costs for second-time exporters, *i.e.* firms that were once selling in the foreign market, exited, and decided to re-enter. Our assumptions on the structure of sunk costs reflect these findings.

[21] This feature of the model is the same as in Helpman, Melitz, and Yeaple (2004). Rob and Vettas (2003) obtain the existence of an equilibrium where firms can optimally choose to adopt simultaneously the two strategies because in their model firms choose the amount of the foreign investment, and given the structure of demand there may be the possibility of overinvestment. In their framework, FDI can be adopted to cover certain demand, while exports are used to serve the additional random excess demand without incurring the cost of a larger investment that could be underutilized. In the data we do observe firms that both exports and have FDI sales (about 5% of the total). This fact could be rationalized within our framework by having multiproduct firms that choose different strategies for different product lines, or in a multi-country model where firms choose different strategies to enter different countries. Unfortunately, there is not enough information in the Compustat Segments data to check whether any of these is the case. Explaining the choice of firms to adopt both entry strategies would probably need a differently tailored framework, and is beyond the scope of this paper.

When the firm serves the foreign market through FDI, firm-specific productivity is transferred to the foreign country and the firm employs foreign labor: $MC_I = aw^*$. CES preferences across varieties of the differentiated good imply that the optimal prices are $p_S(a) = \dfrac{\eta}{\eta - 1} MC_S(a)$.

Let $\pi_D(a;Q)$, $\pi_X(a;Q^*)$ and $\pi_I(a;Q^*)$ denote the per-period profits from domestic sales, from exports and from FDI sales abroad, respectively, for a firm with productivity $1/a$, given a realization of the aggregate quantity demanded in the domestic (foreign) market equal to Q (Q^*):

$$\pi_D(a;Q) = B(aw)^{1-\eta}P^{\eta}Q \quad (4)$$

$$\pi_X(a;Q^*) = B(\tau aw)^{1-\eta}(P^*)^{\eta}Q^* - f_X \quad (5)$$

$$\pi_I(a;Q^*) = B(aw^*)^{1-\eta}(P^*)^{\eta}Q^* - f_I \quad (6)$$

where $B \equiv \frac{1}{\eta-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta}$, and P (P^*) is the aggregate price of the differentiated good in the domestic (foreign) market, that firms take as given while solving their maximization problem. In order to assure the existence of exporters in equilibrium, we assume that the cost parameters satisfy the following inequality:[22]

Assumption 1.

$$\left(\frac{w^*}{w}\right)^{\eta-1}(f_I + rF_I) > \tau^{\eta-1}(f_X + rF_X). \quad (7)$$

3.2 Value Functions

We solve the model along the lines of Dixit (1989). Let $V_S(a;Q^*)$ denote the expected net present value for a firm with productivity $1/a$ in status S ($S = D, X, I$) starting with aggregate demand Q^* in the foreign market and following optimal policy. As we assume no uncertainty in the domestic market, firms in all statuses S have positive profits $\pi_D(a;Q)$ from domestic sales. We abstract from those profits and interpret the value functions as net of domestic profits.

Over a generic time interval Δt, the Bellman equation for a firm that is currently selling only in the domestic market is:

$$V_D(a,Q^*) = \max\left\{E[M\Delta t \cdot V_D(a,(Q^*)')|Q^*]\ ;\ V_X(a,Q^*) - F_X\ ;\ V_I(a,Q^*) - F_I\right\}. \quad (8)$$

[22] Condition (7) is the "present discounted value equivalent" of the analogous assumption in Helpman, Melitz, and Yeaple (2004). It is derived by imposing that the profit functions of an exporter and of a multinational firm – expressed as functions of the productivity level $1/a$ – cross at a point associated with positive profits.

The right-hand side of the Bellman equation expresses the firm's possibilities. If it sells only domestically, it gets the continuation value from not changing status, equal to the expected discounted value of the firm conditional on the current realization of foreign demand Q^*. If it decides to switch to export (FDI) it gets the value of being an exporter, V_X (multinational, V_I) minus the sunk cost of entry F_X (F_I). The Bellman equation for an exporter is:

$$V_X(a, Q^*) = \max\left\{\pi_X(a, Q^*)\Delta t + E[M\Delta t \cdot V_X(a, (Q^*)')|Q^*]\ ;\ V_D(a, Q^*)\ ;\ V_I(a, Q^*) - F_I\right\} \quad (9)$$

and for a multinational:

$$V_I(a, Q^*) = \max\left\{\pi_I(a, Q^*)\Delta t + E[M\Delta t \cdot V_I(a, (Q^*)')|Q^*]\ ;\ V_D(a, Q^*)\ ;\ V_X(a, Q^*) - F_X\right\}. \quad (10)$$

Notice that the continuation value of an exporter (a multinational) also includes the flow profits from sales in the foreign market $\pi_X(a, Q^*)\Delta t$ ($\pi_I(a, Q^*)\Delta t$). There are no costs of exiting the foreign market: if a firm decides to exit, its value is simply the one of a domestic firm: $V_D(a, Q^*)$.

In the continuation region, taking the limit for $\Delta t \to 0$, the Bellman equation of a domestic firm reduces to:

$$E[d(M \cdot V_D(a, Q^*))] = 0. \quad (11)$$

Using (3), applying Ito's Lemma, and rearranging, we obtain:[23]

$$rV_D(a, Q^*) = V_D'(a, Q^*)\tilde{\mu}Q^* + \frac{1}{2}V_D''(a, Q^*)\sigma^2(Q^*)^2 \quad (12)$$

where $\tilde{\mu} = \mu + \sigma\sigma_M$.[24] The solution takes the form $V_D(Q^*) = (Q^*)^\xi$, where:

$$\xi = \frac{(1-m) \pm \sqrt{(1-m)^2 + 4\bar{r}}}{2}$$

and $m = 2\tilde{\mu}/\sigma^2$, $\bar{r} = 2r/\sigma^2$. Hence the value function of a domestic firm takes the form:

$$V_D(a, Q^*) = A_D(a)(Q^*)^\alpha + B_D(a)(Q^*)^\beta \quad (13)$$

where α and β are the negative and positive values of ξ, respectively, and $A_D(a)$ and $B_D(a)$ are parameters to be determined.

[23] Details on the derivation of equations (12), (14), (15) are contained in Appendix B.

[24] $\tilde{\mu}$ represents the risk-neutral drift, which is a result of taking expectations of the value function under the risk-neutral measure.

To compute the value functions of exporters and multinationals, we need to take into account also the current profits from sales in the foreign market. In the continuation region, the value of a firm starting as an exporter with foreign aggregate demand Q^* satisfies:

$$rV_X(a, Q^*) = B(\tau a w)^{1-\eta}(P^*)^\eta Q^* - f_X + V_X'(a, Q^*)\tilde{\mu}Q^* + \frac{1}{2}V_X''(a, Q^*)\sigma^2(Q^*)^2 \tag{14}$$

and the value of a firm starting as a multinational with foreign aggregate demand Q^* satisfies:

$$rV_I(a, Q^*) = B(aw^*)^{1-\eta}(P^*)^\eta Q^* - f_I + V_I'(a, Q^*)\tilde{\mu}Q^* + \frac{1}{2}V_I''(a, Q^*)\sigma^2(Q^*)^2. \tag{15}$$

For $S = X, I$, the value function takes the affine form $V_S(Q^*) = (Q^*)^\xi + c_{s0} + c_{s1}Q^*$. By substituting it in the expressions above, the value functions of an exporter and a multinational firm can be written as:

$$V_X(a, Q^*) = A_X(a)(Q^*)^\alpha + B_X(a)(Q^*)^\beta + \frac{B(\tau a w)^{1-\eta}(P^*)^\eta Q^*}{r - \tilde{\mu}} - \frac{f_X}{r} \tag{16}$$

$$V_I(a, Q^*) = A_I(a)(Q^*)^\alpha + B_I(a)(Q^*)^\beta + \frac{B(aw^*)^{1-\eta}(P^*)^\eta Q^*}{r - \tilde{\mu}} - \frac{f_I}{r} \tag{17}$$

where $A_X(a)$, $B_X(a)$, $A_I(a)$ and $B_I(a)$ are parameters to be determined.

Equations (13), (16), and (17) describe the value functions in their continuation regions. We still need to solve for the updating rule, which in this case consists of thresholds in the realizations of the state Q^* that induce firms to switch status. Let $Q^*_{SR}(a)$ denote the quantity threshold at which a firm with productivity $1/a$ switches from status S to status R, for $S, R \in \{D, X, I\}$. In order to find the six quantity thresholds $Q^*_{SR}(a)$ and the six value function parameters $A_S(a)$, $B_S(a)$, for $S \in \{D, X, I\}$, we impose the following value-matching and smooth-pasting conditions:

$$
\begin{aligned}
V_D(a, Q^*_{DX}(a)) &= V_X(a, Q^*_{DX(a)}) - F_X && (18)\\
V_D(a, Q^*_{DI}(a)) &= V_I(a, Q^*_{DI}(a)) - F_I && (19)\\
V_X(a, Q^*_{XD}(a)) &= V_D(a, Q^*_{XD}(a)) && (20)\\
V_X(a, Q^*_{XI}(a)) &= V_I(a, Q^*_{XI}(a)) - F_I && (21)\\
V_I(a, Q^*_{ID}(a)) &= V_D(a, Q^*_{ID}(a)) && (22)\\
V_I(a, Q^*_{IX}(a)) &= V_X(a, Q^*_{IX}(a)) - F_X && (23)\\
V'_R(a, Q^*_{RS}(a)) &= V'_S(a, Q^*_{RS}(a)) \ , \ for \ S, R \in \{D, X, I\}. && (24)
\end{aligned}
$$

For each a, equations (18)-(24) are a system of twelve equations in the twelve unknowns given by the six quantity thresholds $Q^*_{SR}(a)$ and by the value functions parameters $A_S(a)$, $B_S(a)$, for $S, R \in \{D, X, I\}$. The system is highly nonlinear, and as such is associated to multiple solutions. To get an economically sensible solution, we follow Dixit (1989) and impose $A_D(a) = 0$, $\forall a$: the option of entering a foreign market is nearly worthless for a domestic firm experiencing a very low Q^*. Consistently, it must be that $B_D(a) \geq 0$ to insure non-negativity of $V_D(a, Q^*)$. The option of quitting FDI for another strategy is nearly worthless for a multinational firm experiencing an extremely high Q^*, hence $B_I(a) = 0$. Similarly, a multinational firm has expected value $\frac{B(aw^*)^{1-\eta}(P^*)^\eta Q^*}{r-\tilde{\mu}} - \frac{f_I}{r}$ from the strategy of never changing status, hence the optimal strategy must yield a no lesser value: $A_I(a) \geq 0$. Finally, an exporter has expected value $\frac{B(\tau aw)^{1-\eta}(P^*)^\eta Q^*}{r-\tilde{\mu}} - \frac{f_X}{r}$ from the strategy of never changing status, hence its optimal strategy must yield a no lesser value for any realization of Q^*: $A_X(a), B_X(a) \geq 0$. Consequently, the value function of a domestic firm V_D is increasing on the entire domain, indicating the fact that, as the realized aggregate demand in the foreign market Q^* increases, the value of the option of entering the foreign market (either through trade or FDI) increases. The value functions of an exporter and of a multinational (V_X and V_I respectively) are U-shaped: for low levels of Q^*, the term with the negative exponent α dominates, and the value is high thanks to the option of leaving the market. Conversely, for high levels of Q^* the value is high due to the profit stream that the firm derives from staying in the market and – for exporters – due to the additional option value of becoming a multinational firm (the term with the positive exponent β).

3.3 Equilibrium

Under our assumption, whereby firms from the foreign country only produce the homogeneous good, the aggregate price of the differentiated good in the two countries is given by:

$$P^{1-\eta} = n \int \left(\frac{\eta a w}{\eta - 1} \right)^{1-\eta} dG(a) \tag{25}$$

$$(P^*)^{1-\eta} = n \left[\int_{\Omega_X(Q^*)} \left(\frac{\eta \tau a w}{\eta - 1} \right)^{1-\eta} dG(a) + \int_{\Omega_I(Q^*)} \left(\frac{\eta a w^*}{\eta - 1} \right)^{1-\eta} dG(a) \right] \tag{26}$$

where n is the mass of firms from the home country in the differentiated goods sector, and $\Omega_X(Q^*)$ ($\Omega_I(Q^*)$) is the subset of these firms that export (have multinational sales) when the realization of the state is Q^*.

Since we abstract from the problem of entry in the domestic market, we take the mass of firms n as given[25]. The aggregate quantity demanded in the domestic market Q and the initial value of Q^*, $Q^*(0)$, are also taken as given. The existence of a perfectly tradeable homogeneous good produced in both countries implies: $w = w^* = 1$.

3.4 Earnings-to-Price Ratios and Returns

In Section 2 we showed data on earnings-to-price ratios and returns, displaying a ranking across firms with different international status. We need to compute earnings yields and returns in the model to be able to compare the predictions of the theory with the data.

Our earnings yields measure in the model is given by the ratio π_t/V_t, where π_t represents per-period profits and V_t is the market value of the firm. In a static model, or in a dynamic but deterministic model, π_t/V_t is constant and independent on the firm's status, since per-period profits and value of the firm coincide. Dynamics and uncertainty introduce a wedge between these two magnitudes, which reflects the option value.

Let $eps_S(a, Q^*)$ denote the earnings yield of a firm with productivity $1/a$ in status S when the

[25] Since we do not impose free-entry, we can normalize the mass of firms to $n = 1$, and present the results in terms of shares of total firms.

realization of aggregate foreign demand is Q^*. Earnings yields in the model are given by:

$$ep_D(a,Q^*) = \frac{\pi_D(a)}{\frac{\pi_D(a)}{r} + V_D(a,Q^*)} \tag{27}$$

$$ep_X(a,Q^*) = \frac{\pi_D(a) + \pi_X(a,Q^*)}{\frac{\pi_D(a)}{r} + V_X(a,Q^*)} \tag{28}$$

$$ep_I(a,Q^*) = \frac{\pi_D(a) + \pi_I(a,Q^*)}{\frac{\pi_D(a)}{r} + V_I(a,Q^*)}. \tag{29}$$

Similarly, our model-based measure of returns $r_S(a,Q^*)$ is given by:

$$ret_S(a,Q^*) = ep_S(a,Q^*) + \frac{E[dV_S(a,Q^*)]}{V_S(a,Q^*)}\ ,\ for\ S \in \{D,X,I\}. \tag{30}$$

Returns are given by the earnings yield plus the expected change in the valuation of the firm.

The empirical evidence presented in Section 2 suggests the following ordering in aggregate earnings yields and returns across groups:

$$\int_{\Omega_D(Q^*)} ep_D(a,Q^*)dG(a) < \int_{\Omega_X(Q^*)} ep_X(a,Q^*)dG(a) < \int_{\Omega_I(Q^*)} ep_I(a,Q^*)dG(a) \tag{31}$$

$$\int_{\Omega_D(Q^*)} ret_D(a,Q^*)dG(a) < \int_{\Omega_X(Q^*)} ret_X(a,Q^*)dG(a) < \int_{\Omega_I(Q^*)} ret_I(a,Q^*)dG(a) \tag{32}$$

where $\Omega_D(Q^*)$, $\Omega_X(Q^*)$, and $\Omega_I(Q^*)$ are the sets of firms that have domestic sales only, exporters, and multinationals, respectively.

In Section 5 we will parameterize the model to match quantitatively features of trade and multinational production dynamics that we observe in the data. With this parameterization, we will show that the model is also able to generate the observed ranking in average earnings-to-price ratios and returns. Before these results, in the next section we show a series of qualitative properties of the model that illustrate its amenability to reproduce features of the trade dynamics data.

4 Qualitative Properties of the Solution

In this section we illustrate the workings of the model with a series of analytical properties of the solution. These properties illustrate the potential of the model to account for the export and FDI dynamics of heterogeneous firms, and the departure from the deterministic model which allows to match the ranking in the financial variables.

4.1 Ordering of the Quantity Thresholds

The relationship between the sunk costs of exporting and FDI, $F_I > F_X$, implies a precise ordering of the quantity thresholds that are solution of (18)-(24). Theorem 1 contains this result.

Theorem 1. *If $F_I > F_X$, the quantity thresholds $Q^*_{RS}(a)$, for $R, S \in \{D, X, I\}$, solution of system (18)-(24), satisfy the following ordering:*

$$Q^*_{IX}(a) < Q^*_{ID}(a) < Q^*_{XD}(a) < Q^*_{DX}(a) < Q^*_{DI}(a) < Q^*_{XI}(a)$$

for each given productivity level $1/a$.

Proof: See Appendix B.

Like in Dixit (1989), the pure presence of sunk costs implies that "entry" thresholds are higher than "exit" thresholds: $Q_{DX} > Q_{XD}$, $Q_{DI} > Q_{ID}$, and $Q_{XI} > Q_{IX}$.[26] A higher quantity demanded Q^* is needed to induce a firm to enter a foreign market through FDI with respect to the quantity necessary to induce the firm to export: $Q_{DI} > Q_{DX}$. An even larger positive shock is needed to induce and exporter to become a multinational, since he is already serving the foreign market with exports: $Q_{XI} > Q_{DI}$. Similarly, a larger negative shock is needed to induce a multinational to exit the foreign market with respect to the shock needed to induce an exporter to exit: $Q_{ID} < Q_{XD}$. Finally, an even larger negative shock is needed to induce a multinational to divest but serve the foreign market as an exporter: $Q_{IX} < Q_{ID}$.

The result of Theorem 1 applies for any given productivity level $1/a$. The following subsection illustrates the behavior of the value functions and quantity thresholds while letting vary firm-level productivity.

4.2 Comparative Statics: Value and Productivity

System (18)-(24) makes clear that both the quantity thresholds and the parameters of the value functions depend on the productivity level $1/a$. Figure 2 shows the value function of a domestic

[26] Results about hysteresis, together with the analytical proof of the ordering of the thresholds Q_{RS}, for $R, S, \in \{D, X, I\}$, are contained in Appendix B.

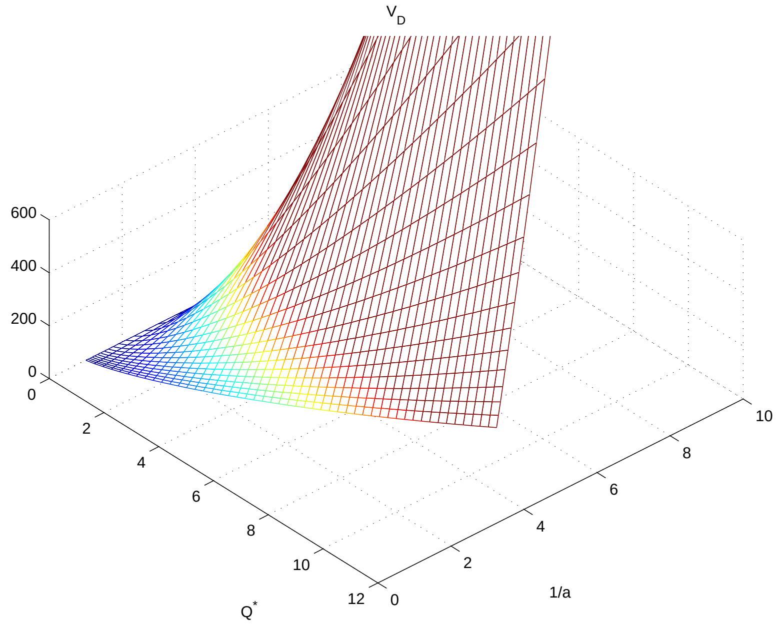

Figure 2: Value function of a domestic firm.

firm as a function of the aggregate quantity demanded in the foreign market Q^* and of productivity $1/a$. As observed in the previous section, V_D is increasing in Q^*, as the option value of entering the foreign market is increasing in the quantity demanded. V_D is also increasing in firm's productivity, as more productive firms can get higher profits from entering the foreign market.

Figure 3 shows the value functions of an exporter and of a multinational firm as functions of Q^* and $1/a$. As previously observed, V_X and V_I are U-shaped functions of Q^*, indicating the high option value of exiting for low realizations of Q^* and the high option value of not changing status for high realizations of Q^*.[27] The behavior of the value functions for $Q^* \to 0$ does not vary much across the productivity dimension: when Q^* is low, the value is high as firms of all productivity levels associate a high value to the option of exiting. The behavior of the value functions when Q^* is "large", conversely, varies with individual productivity: the value function is steeper for higher productivity firms, indicating that more productive firms obtain higher returns from staying in the foreign market when the realized aggregate demand is high.

From Figure 3, the qualitative behavior of V_X and V_I appears very similar. Figure 4 plots the difference between the value functions of firms serving the foreign market and firms selling only

[27] Notice that for $Q^* \to \infty$, the value funcion of an exporter is steeper than the one of a multinational, because the exporter gets high value both from staying in the market as an exporter and from the option value of becoming a multinational.

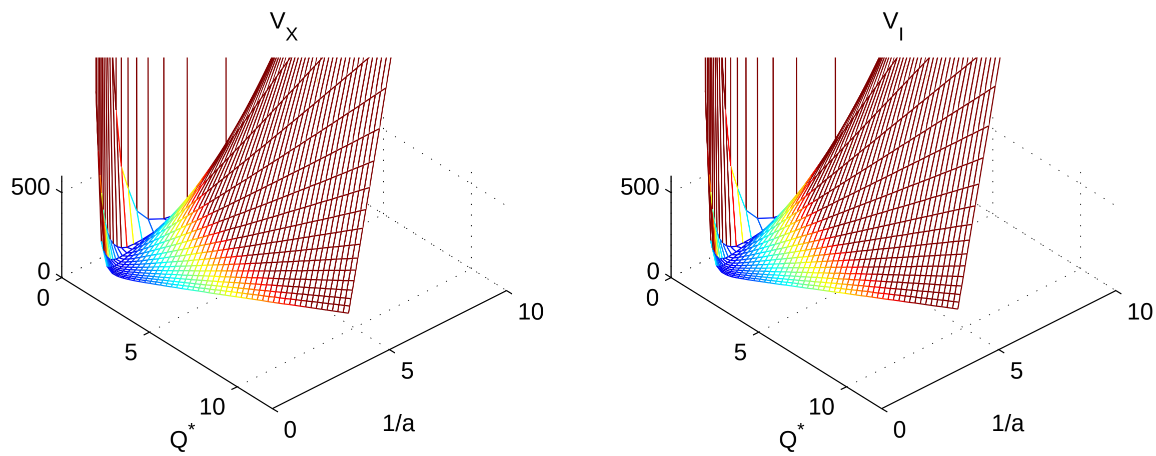

Figure 3: Value functions of an exporter and of a multinational firm.

domestically, $V_X - V_D$ and $V_I - V_D$. For each productivity level $1/a$, each plot has two stationary points, a local maximum and a local minimum. The value matching and smooth pasting conditions imply that the local maxima correspond to the "entry" thresholds (Q^*_{DX} and Q^*_{DI} in the left and right plot respectively), while the local minima correspond to the "exit" thresholds (Q^*_{XD} and Q^*_{ID}). The picture shows that both entry and exit thresholds are decreasing in $1/a$, indicating that more productive firms enter the foreign market for lower realizations of aggregate demand Q^* with respect to less productive firms. Similarly, more productive firms need larger negative shocks to demand to be induced to exit the foreign market with respect to less productive firms.

Notice that for $Q^* \to 0$, $V_X - V_D$ and $V_I - V_D$ tend to infinity, because the option value of exiting the foreign market is extremely high for very low realizations of Q^* (and irrespective of firm's productivity). Conversely, for $Q^* \to \infty$, $V_X - V_D$ and $V_I - V_D$ tend to negative infinity, because the domestic firms' option value of entering the foreign market is exremely high, compared to the flow profits of staying for firms that are already serving that market.

Figure 5 plots the difference between the value functions of a multinational firm and of an exporter, $V_I - V_X$, as a function of Q^* and $1/a$. In this picture, for each value of $1/a$, the peak of the surface represents the quantity threshold where the firm switches from being an exporter to being a multinational, Q_{XI}. The figure shows that also the threshold Q_{XI} is decreasing in $1/a$, consistent with the prediction of Helpman, Melitz, and Yeaple (2004), according to which the most productive firms are more likely to become multinationals than exporters. Notice also that, for constant Q^*, the excess value $V_I - V_X$ decreases as productivity $1/a$ increases: for the same level of

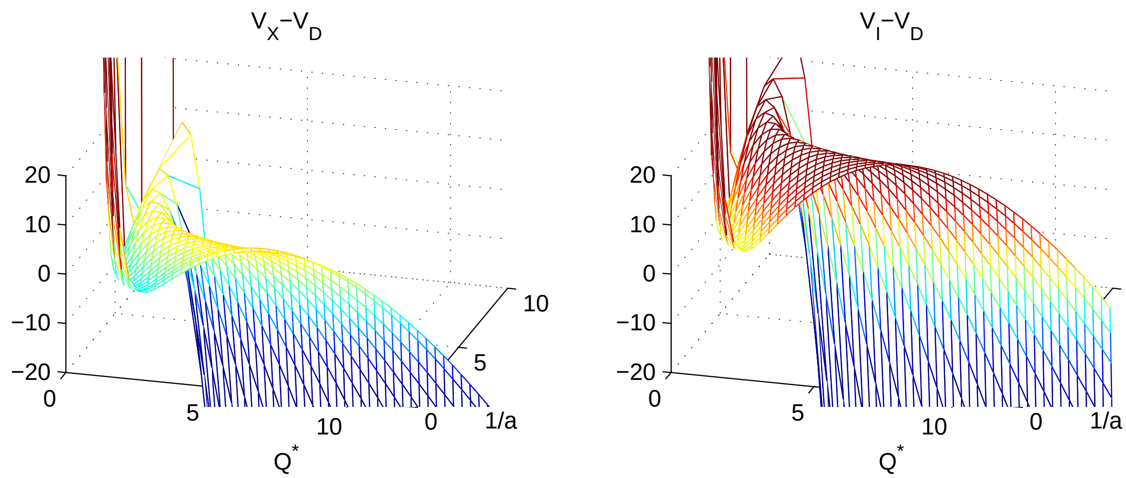

Figure 4: Difference between the value functions of exporters and multinationals and the value function of domestic firms.

Q^*, a more productive exporter has a higher option value of switching to FDI compared to a less productive one.

Figures 4 and 5 suggest a systematic relationship between the quantity thresholds Q^*_{RS} and the firm productivity level $1/a$. Theorem 2 establishes this result.

Theorem 2.

$$\frac{\partial Q^*_{RS}(a)}{\partial a} \geq 0, \quad for \quad R, S \in \{D, X, I\}, \ \forall a. \tag{33}$$

Proof: See Appendix B.

Theorem 2 establishes that the six thresholds $Q^*_{RS}(a)$ are decreasing in productivity $1/a$, indicating that more productive firms need smaller positive shocks to demand to enter the foreign market, and larger negative shocks to exit. The one-to-one correspondence between productivities and quantity thresholds established by Theorem 2 implies that the functions $Q^*_{RS}(a)$ are invertible, hence for each realization of aggregate foreign demand Q^* we can compute six productivity thresholds $a_{RS}(Q^*)$, for $R, S \in \{D, X, I\}$, that determine the selection of heterogeneous firms into the three statuses and their likelihood of switching across statuses. This redefinition of the thresholds in terms of productivity is extremely helpful to compute the model numerically. The sets of firms belonging to each status can be written as functions of the productivity thresholds a_{RS}. At time

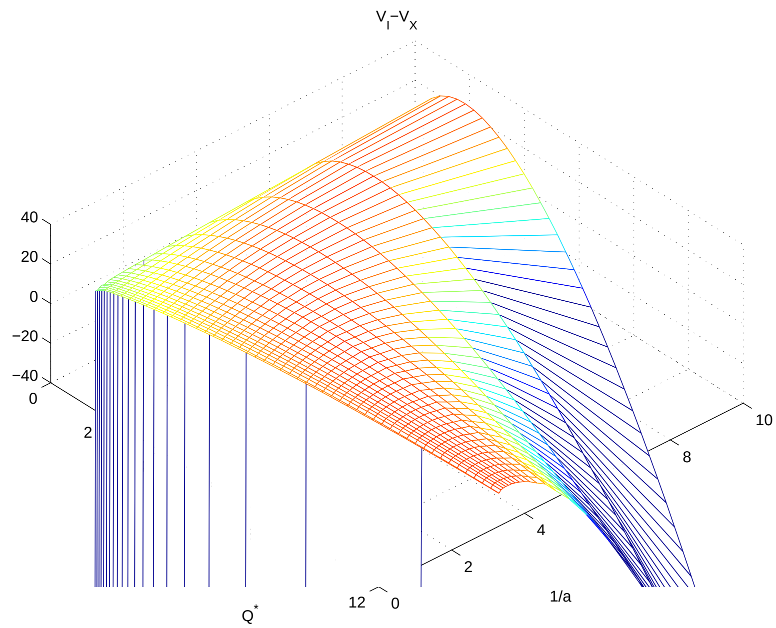

Figure 5: Difference between the value functions of an exporter and of a multinational firm.

$t = 0$:

$$\Omega_{D0} = [a_{DX}, \infty) \quad (34)$$

$$\Omega_{X0} = [a_{XI}, a_{XD}] \quad (35)$$

$$\Omega_{I0} = (0, a_{ID}] \quad (36)$$

and for the following periods:

$$\Omega_{Dt+1} = \left\{\left\{\Omega_{Dt} \bigcap [a_{DX}, \infty)\right\} \bigcup \left\{\Omega_{Xt} \bigcap [a_{XD}, \infty)\right\} \bigcup \left\{\Omega_{It} \bigcap [a_{ID}, a_{IX}]\right\}\right\} \quad (37)$$

$$\Omega_{Xt+1} = \left\{\left\{\Omega_{Dt} \bigcap [a_{DI}, a_{DX}]\right\} \bigcup \left\{\Omega_{Xt} \bigcap [a_{XI}, a_{XD}]\right\} \bigcup \left\{\Omega_{It} \bigcap [a_{IX}, \infty)\right\}\right\} \quad (38)$$

$$\Omega_{It+1} = \left\{\left\{\Omega_{Dt} \bigcap (0, a_{DI}]\right\} \bigcup \left\{\Omega_{Xt} \bigcap (0, a_{XI}]\right\} \bigcup \left\{\Omega_{It} \bigcap (0, a_{ID}]\right\}\right\} \quad (39)$$

where we omitted the dependence of Ω_{St} and a_{RS} on Q^* to ease the notation. Notice that the sets Ω_S vary with the realization of Q^*, as firms may switch status, but only depend on the firms' status in the previous period, due to the Markov property of Brownian motions.

Figure 6 illustrates Theorem 2. For an arbitrary parameterization, we plot the six quantity thresholds as functions of firm-level productivity. The picture also shows an additional property of the thresholds: hysteresis, defined as the horizontal distance between "entry" and "exit" thresholds,

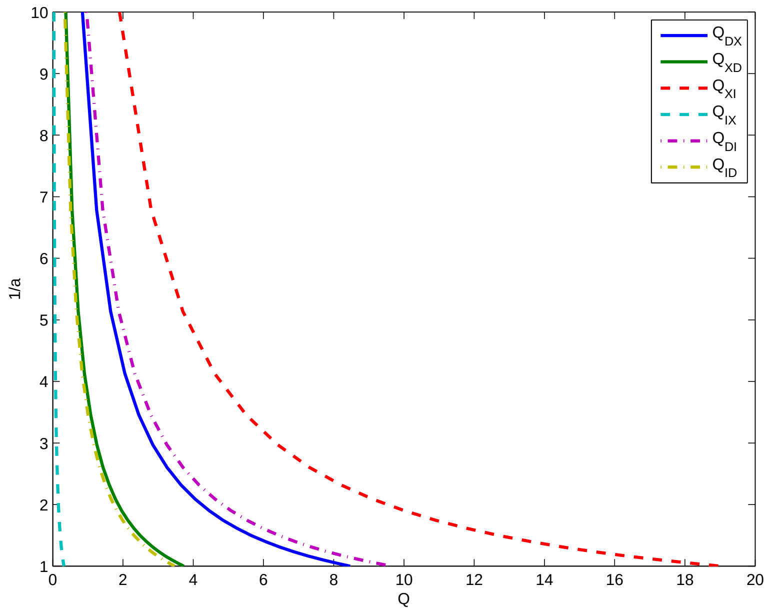

Figure 6: Quantity thresholds as functions of firm's productivity.

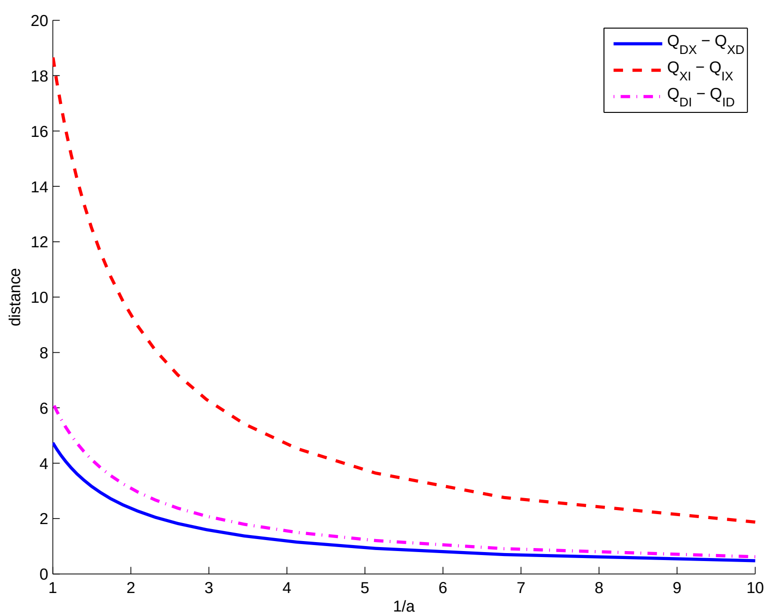

Figure 7: Distances between entry and exit thresholds as functions of firm's productivity.

is also decreasing in productivity. Figure 7 plots the three distances $Q_{DX} - Q_{XD}$, $Q_{DI} - Q_{ID}$, and $Q_{XI} - Q_{IX}$ as functions of firm productivity: the distance between entry and exit thresholds is decreasing in productivity, indicating that more productive firms suffer less from being locked into a market by the presence of sunk entry costs. This result is established by Theorem 3:

Theorem 3.

$$\frac{\partial[|Q^*_{RS}(a) - Q^*_{SR}(a)|]}{\partial a} \geq 0, \quad for \quad R, S \in \{D, X, I\}, \ \forall a. \tag{40}$$

Proof: See Appendix B.

Theorem 3 is useful to investigate how hysteresis combines with productivity. On the one hand, for the same choice of status, the theorem implies that more productive firms exhibit less hysteresis. On the other hand, more productive firms self-select into the status (I) that is associated with more hysteresis.[28] This generates an imperfect sorting of productivities into status, which is a well-documented feature of the data.[29]

5 Empirical Results

Objective of this section is an evaluation of the performance of the model in matching qualitatively and quantitatively features of the data on trade and FDI dynamics, and of the pattern of earnings yields and returns across firms. In Section 5.1 we show that the ordering of thresholds established by Theorem 1 is consistent with the pattern of firms switching status in the data. In Section 5.2 we calibrate the parameters of the model to match quantitatively moments related to the switching pattern, and to relative size and presence of the three types of firms in the data. In Section 5.3 we use the calibrated version of the model to compute earnings yields and returns, which are not targeted in the calibration. We show that the quantitative theory is able to replicate the observed ranking in financial variables that we documented in Section 2.

5.1 Qualitative Results: Switching Pattern in the Data

The ordering in the quantity thresholds that is the result of Theorem 1, associated with the Markov properties of Brownian motions, has implications for the expected switching pattern of firms across

[28] Dixit and Pindyck (1994) show that hysteresis is increasing in the sunk cost of investment.
[29] See Armenter and Koren (2009).

Table 7: **Switching Behavior.** Average percentages of firms switching status each year (Source: Compustat).

t \ $t+1$	D		X		I
D			1.063	$>$	1.061
					$\wedge$
X	0.986				1.675
	$\vee$				
I	0.679	$>$	0.572		

statuses. Since $Q^*_{DX} < Q^*_{DI} < Q^*_{XI}$, on average, it should be more likely for a domestic firm to start exporting than to do FDI, and it should be more likely for an exporter to start FDI than for a domestic firm. Similarly, $Q^*_{IX} < Q^*_{ID} < Q^*_{XD}$ implies that it should be more likely that multinationals exit completely the foreign market instead than switching to exports following a negative shock, and it should be more likely for an exporter to exit the foreign market than for a multinational firm.

We find that the average shares of firms switching across statuses in the data are consistent with these preditions. In Compustat, each year on average 6.04% of firms switch status. Of those, 1.063% switch from domestic to exporters, more than the 1.061% that switch from domestic to multinational. 1.675% of firms switch from exporters to multinational, more than the 1.061% that switch from domestic to multinational. 0.679% of firms switch back from being multinational to domestic only, more than the 0.572% that switch back from being multinational to being an exporter. Finally, 0.986% of firms switch back from exports to domestic sales only, more than the 0.679% that switch back from being a multinational to selling only domestically. Table 7 summarizes the average percentages of firms switching status in the data.

This qualitative evidence raises our confidence on the fact that the model can be successfully calibrated to match quantitatively the switching pattern and other firm characteristics. This will be the object of the next section.

5.2 Calibration

The calibration exercise that we present in this section is designed to match a series of facts on exports and FDI sales dynamics. We do not make any use of financial variables in this exercise.

In the next section we show that the model calibrated by targeting trade facts only performs well also in matching non-targeted moments like earnings yields and returns.

To calibrate the model, we need to choose a functional form for the cost distribution $G(a)$, and assign values to its parameters. We need to parameterize the Brownian motions, and to choose values for preference parameters and parameters describing trade and FDI costs. We refer to the literature to assign parameters to preferences, Brownian motions, and to the firms' productivity distribution. We choose the remaining parameters such that the model matches a series of moments from the data. We start describing the calibration with the parameters we adopt from the literature.

Several studies document that the tail of the empirical firm size distribution is well approximated by a Pareto distribution (see for example Luttmer (2007)). As in the model firm's size (sales) is linked to the productivity distribution, we assume that firms' productivities $1/a$ are distributed according to a Pareto law with location parameter b and shape parameter k.[30] We normalize $b = 1$, and choose k to match the coefficient of the empirical sales distribution: if productivity is Pareto-distributed with shape parameter k, sales in the model are also Pareto-distributed with shape parameter $k/(\eta-1)$. By regressing firm rank on firm size, Luttmer (2007) finds that $k/(\eta-1) = 1.06$. We then choose k accordingly, given a value for η. There is little agreement in the literature on the value to attribute to the elasticity of substitution across differentiated varieties, η. Many papers focusing on long-run macroeconomic predictions use a standard value of 2. Other papers, focusing on matching data at business cycle frequencies, choose much higher values. Alessandria and Choi (2007), for example, set $\eta \approx 10$ (to match markups of about 11%). We set $\eta = 2.54$, equal to the median value in Broda and Weinstein (2006) 3-digit estimates.[31] This choice implies $k = 1.63$. The existence of a perfectly tradeable homogeneous good that is produced in both countries ensures that $w = w^* = 1$.[32]

We impose that the drift of the Brownian motion ruling the evolution of Q^* has value $\mu = 0$. The need to impose zero expected demand growth arises from the fact that we abstract from firms' productivity growth.[33] We compute σ as the standard deviation of consumption growth for a set

[30] The Pareto distribution is also a convenient choice for computational reasons, since it allows to solve explicitly for the aggregate prices P, P^* as functions of the productivity thresholds a_{RS} and of the other parameters of the model.

[31] Broda and Weinstein (2006) estimate sectoral elasticities of substitution from price and volume data on U.S. consumption of imported goods. By using data at the 10-digit Harmonized System, they estimate how much demand shifts between 10-digit varieties when relative prices vary, within each 3-digit SITC sector.

[32] In general equilibrium, in the absence of a homogeneous sector, wages would depend on Q, Q^*.

[33] If $\mu > 0$, $E(Q^*)$ would be increasing over time and for $t \to \infty$ all firms would become multinationals. By setting $\mu = 0$, we are implicitly assuming that Q^* and b grow at a rate such that the distribution of firms in the three groups

of OECD countries relative to the U.S.:

$$\sigma = \frac{st.dev.(g_t^{oecd})}{st.dev.(g_t^{us})} = 0.033$$

where g_t^{oecd} (g_t^{us}) is the average growth rate of consumption across OECD countries (in the U.S.) in year t (t=1979-2006).[34] We set the risk-free rate r to match a long-run average of 3-month T-bills rate of 2%. For the calibration of σ_M, notice that – in a world with CRRA preferences with risk-aversion coefficient γ, the stochastic discount factor can be expressed as $M = -\gamma(C_{t+1}/C_t)$, where C_t is the aggregate consumption bundle at time t. Follows that $\sigma_M = \gamma\sigma_{\Delta C}$. Let S denote the Sharpe ratio: $S \equiv \frac{E(ret-r)}{\sigma_{ret}}$. It is possible to prove that $S \leq (1+r)(\gamma\sigma_{\Delta C})$.[35] Estimates of the Sharpe ratio for the U.S. found $S \approx 0.4$, which implies that 0.4 is a good lower bound for the value of σ_M.

It remains to calibrate the variable trade cost τ, fixed operating costs f_X, f_I, sunk costs F_X, F_I, domestic aggregate demand Q and the initial value for the foreign aggregate demand, $Q^*(0)$. We follow the methodology of the calibration in Alessandria and Choi (2007), and select values for these parameters to match a set of moments related to trade and FDI dynamics. We target data on firms' persistence in the same status, on size differentials across firms in different statuses, and on the relative presence of the three types of firms in the data.

We compute all the moments from Compustat data. In terms of status persistence, on average, 91.25% of domestic firms remain domestic the following year, while 4.38% of them become exporters, and the remaining become multinationals. 87.58% of exporters continue exporting the following year, while 7.82% of them become multinationals, and the remaining exit the foreign market to sell domestically only. Multinational firms exhibit even higher persistence, with 97.7% of them continuing being multinationals the following year, and only 1.05% of them becoming exporters the following year. Domestic firms' and exporters' persistence moments are close to the ones reported in Alessandria and Choi (2007), but we are unaware of other papers computing moments related

does not degenerate over time.

[34] Data source: OECD Statistics.

[35] Let $R = 1 + ret$, $R^f = 1 + r$. Expected returns can be expressed as:

$$\begin{aligned} E(R) &= R^f + R^f \cdot Cov(M, R) \\ E(R) &= R^f + R^f\gamma \cdot Cov(\Delta C, R) \\ E(R - R^f) &= R^f\gamma \cdot \rho(\Delta C, R) \cdot \sigma_{\Delta C} \cdot \sigma_R \\ \frac{E(R - R^f)}{\sigma_R} &\leq R^f\gamma \cdot \sigma_{\Delta C}. \end{aligned}$$

to persistence in multinational activity.

Next, we look at employment differences as a measure of size differences. In Compustat, multinational firms appear to employ about 6.69 times more workers than exporters, and exporters appear to employ about 1.19 times more workers than domestic firms. This last number is smaller than what reported by other studies, due o the selection problem associated with Compustat: since the data set includes only publicly traded firms, it is biased towards larger firms, and excludes most small firms, that tend to be the majority of domestic ones. For this reason, domestic firms in the sample do not look as dramatically different from exporters as other literature shows.[36]

We conclude the list of moments with the average share of firms in each status. In Compustat, the average share of firms selling only domestically is 33.26% of the sample, while exporters are 24.84% of the sample, and multinational firms account for the remaining 41.89% of the sample. As previously noted, also these numbers reflect selection of the largest firms in the data set.[37] Despite the divergence of our moments with the ones reported from other papers, to be internally consistent we decided to match Compustat data in this exercise. While not representative of the entire population of U.S. firms, Compustat offers a detailed portrait of the largest firms in the economy, which are the major actors when talking about volumes of trade and FDI.[38]

Table 8 summarizes the calibrated parameters. The calibrated iceberg cost is 24%, consistent with a low-range estimate in Eaton and Kortum (2002). Sunk costs of export and FDI equal to 0.9 and 3.5, respectively, indicate that a domestic firm must spend on average about 93% of its per-period revenue to enter the foreign market as an exporter, and about 3.6 times its per-period revenue to start FDI operations there. Fixed costs of export and FDI equal to 0.35 and 0.6, respectively, indicate that an exporter must spend on average about 11% of its per-period revenue in operating costs, and that a multinational firms must spend on average about 2% of its per-period revenue in operating costs. Aggregate demand parameters are set at $Q = 8$, $Q^*(0) = 2.2$, indicating that the domestic market must be larger than the foreign market to generate relative size and employment across the three groups as close as possible to the data.

[36]For example, Alessandria and Choi (2007) report that exporters tend to hire between 77 and 95% more workers than domestic firms.

[37]Bernard et al. (2007), among others, report that the average share of manufacturing firms that export is about 18%, while Bernard and Jensen (2007) report that multinational firms represent only 1% of manufacturing firms.

[38]The choice of a Pareto distribution for firms' productivity is robust to the selection problem associated with dealing with Compustat data. The Pareto distribution is invariant to lower truncations, hence if we assume that the entire productivity distribution is Pareto with parameters (b, k), the distribution of firms in Compustat will also be Pareto, with parameters (b', k), $b' > b$. As the lower bound of the distribution does not enter the computation of the moments, we normalize it to one.

Table 8: **Summary of Calibrated Parameters.**

Parameter	Definition	Value	Source
Brownian motions			
μ	drift of Q^*	0	no productivity growth
σ	variance of Q^*	0.033	st.dev. of relative cons. growth
r	risk-free rate	0.02	3-month T-bills rate
σ_M	variance of s.d.f.	0.4	lower bound of Sharpe ratio
Pareto distribution			
b	lower bound	1	normalization
k	shape parameter	1.63	Luttmer (2007)
Preferences			
η	el. of substitution	2.54	Broda and Weinstein (2006)
Trade and FDI costs			
w,w^*	wages	1	homogeneous sector
τ	iceberg export cost	1.24	to match data
f_X	fixed export cost	0.35	to match data
F_X	sunk export cost	0.9	to match data
f_I	fixed FDI cost	0.6	to match data
F_I	sunk FDI cost	3.5	to match data
Aggregate demand			
Q	domestic demand	8	to match data
$Q^*(0)$	initial foreign demand	2.2	to match data

Table 9: **Moments.** Comparison of the moments, model *versus* data. (Source: Compustat)

	Data	Model		Data	Model
$D \rightarrow D$ (%)	91.25	91.47	X (%)	24.85	40.42
$D \rightarrow X$ (%)	4.38	8.53	I (%)	41.98	37.16
$X \rightarrow X$ (%)	87.58	93.62	empl. X/D	1.19	9.46
$X \rightarrow I$ (%)	7.82	0.52	empl. I/X	6.69	16.3
$I \rightarrow I$ (%)	97.7	94.78			
$I \rightarrow X$ (%)	1.05	5.22			

Table 9 displays jointly the moments computed from the data and the moments generated by the calibrated model.[39] The model does a good job in matching the persistence parameters. The share of exporters is higher than in the data, and the model predicts a much larger size advantage of exporters and multinationals with respect to what we observe in our sample.

Despite these discrepancies, the fit of the model with the trade data is good. We are choosing seven parameters to match ten relevant moments, so it is not surprising that the matching is not extremely accurate. Moreover, the stylized features of our model are subject to the criticisms raised by Armenter and Koren (2009), who point out how Melitz-type models are not suited to match quantitatively both the share of exporting firms and their size advantage.

5.3 Quantitative Results: Earnings Yields and Returns

With the calibrated version of the model, we compute earnings yields and returns across the three groups of firms. In our calculations, we follow the construction of the portfolios we used in the data analysis presented in Section 2. We run 20 Monte Carlo simulations of an artificial dataset of 100 firms (with productivities drawn from a Pareto distribution) for 10 years.[40] For each firm, year, and Monte Carlo simulation we compute earnings, equilibrium value (our model-based measure of the market price), and variation in the equilibrium value of the firm. For each year and Monte Carlo simulation, we create the three portfolios of domestic firms, exporters, and multinationals, and we compute portfolio earnings, prices, value changes, earnings yields (earnings-to-price ratios),

[39] We are currently working on a simulated method of moments estimation of the parameters of the model, which will improve the fit of the model with the data.

[40] The small number of Monte Carlo simulations is justified by the fact that for $\mu = 0$ there are small differences across simulations.

Table 10: **Earnings Yields**. Summary statistics of earnings yields computed from simulated data, and comparison with real data. All values are in percentage terms.

	Mean (model)	Std. Dev. (model)	Mean (data)	Std. Dev. (data)
DOM	0.5626	0.0006	2.12	3.54
EXP	0.661	0.0003	3.7	2.85
MN	0.6895	0.0001	4.85	2.76

Table 11: **Returns**. Summary statistics of returns computed from simulated data, and comparison with real data. All values are in percentage terms.

	Mean (model)	Std. Dev. (model)	Mean (data)	Std. Dev. (data)
DOM	3.9406	0.0049	7.37	15.03
EXP	4.1398	0.0112	10.69	15.34
MN	5.334	0.0003	11.62	12.86

and returns. For each simulation, we compute the mean and standard deviation of earning yields and returns over time. Finally, we average our results across simulations. Table 10 reports the results for earnings yields.

The model generates average earnings yields of 0.69% for multinational firms, 0.66% for exporters, and 0.56% for firms selling only domestically, which are consistent with the ordering we found in the data. Notice that both in the model and in the data there is no ranking in the volatility of earnings yields. Table 11 reports the results for the returns. The model generates average returns of 5.33% for multinational firms, 4.14% for exporters, and 3.94% for firms selling only domestically, which are consistent with the ordering we found in the data. While computed returns exhibit the expected ranking, there is no ranking in their volatility.[41]

6 Robustness: Symmetric Case

In this section we lay out the symmetric model, where firms from both countries produce differentiated goods and the aggregate quantity demanded evolves stochastically in both countries. We use the calibration of Section 5.2 to show that our results continue to hold when both countries are hit by shocks.

[41]The small predicted volatilities are a consequence of the Monte-Carlo simulation procedure, as they are infact averages of standard deviations over simulations.

Since firms operate under monopolistic competition, and there are no strategic considerations to be made in the price setting or in the choice of status, the problem of the firm is unchanged. Since there are no fixed costs to sell in the domestic market, firms from both countries always make positive profits in their respective domestic markets, and we can still write the value functions net of domestic profits. Domestic profits evolve stochastically, but they are always positive and do not affect the decision to enter the foreign market.

Each country has firms producing in both sectors, hence there are six quantity/productivity thresholds for each country: a_{RS} for firms from the domestic country and a^*_{RS} for firms from the foreign country ($R, S, \in \{D, X, I\}$). We denote with n (n^*) the mass of firms in the home (foreign) country. Notice that if the two countries are identical the thresholds are the same in the two countries and we can normalize $n = n^* = 1$.

The price indexes in the two countries are the solution of the following system of two equations, where each price index is an aggregate of prices of domestic sales, prices of imports, and prices of FDI sales of multinational firms from the other country:

$$P^{1-\eta} = \quad n \int \left(\frac{\eta a w}{\eta - 1}\right)^{1-\eta} dG(a) + n^* \left[\int_{\Omega^*_X(Q)} \left(\frac{\eta \tau a w^*}{\eta - 1}\right)^{1-\eta} dG^*(a) + \int_{\Omega^*_I(Q)} \left(\frac{\eta a w}{\eta - 1}\right)^{1-\eta} dG^*(a)\right] \tag{41}$$

$$(P^*)^{1-\eta} = \quad n^* \int \left(\frac{\eta a w^*}{\eta - 1}\right)^{1-\eta} dG^*(a) + n \left[\int_{\Omega_X(Q^*)} \left(\frac{\eta \tau a w}{\eta - 1}\right)^{1-\eta} dG(a) + \int_{\Omega_I(Q^*)} \left(\frac{\eta a w^*}{\eta - 1}\right)^{1-\eta} dG(a)\right]. \tag{42}$$

[TO BE COMPLETED]

7 Conclusions

This paper started by presenting a novel fact distinguishing multinational firms from exporters and from firms selling only domestically. Multinational corporations tend to have higher earnings yields and returns than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings yields than firms selling only in their domestic market. To explain this fact, we presented a real option value model where firms choose optimally whether to produce only domestically, export, or serve the foreign market through FDI. In equilibrium, firms imperfectly sort into the three statuses according to their productivity and to the realization of the shocks, and the option value – or the option of changing status – introduces a wedge between the firm's profits and its valuation, which generates the sought ranking in the financial variables.

While being consistent with a number of facts about trade and FDI dynamics, the model provides a complementary explanation for the cross section of returns by exploiting the production side from an international point of view. Firms selling in foreign markets are more exposed to aggregate risk since – in case of a negative shock – by exiting they would forgo the sunk cost that they paid to enter. This generates status persistence, and the risk of negative profits. Moreover, exiting is a more expensive strategy for multinationals than for exporters, who paid a lower premium and are hence less exposed and less risky: the difference in sunk costs generates a difference in exposure and in excess reutns.

The solution of the model delivers a series of predictions relating firms' productivity and the realization of the shocks to the pattern of status changes. These predictions find support in the switching pattern in the data. We calibrate the model to match quantitatively the persistence moments and other relevant moments on firm size and selection into export/FDI. With the parameterized model we compute moments of the financial variables from simulated data. We show that while matching fairly well the overall aggregate dynamics of trade and FDI, the model is also able to reproduce the ordering in earnings yields and returns we observe in the data.

We see this paper as the first step in a novel research agenda linking trade and FDI dynamics to asset pricing. Interesting extensions could include a careful analysis of the evolution of financial variables at the times of status switches, and the study of differential exit patterns of exporters *versus* multinational firms. We think this is a promising avenue for research in finance and international trade, that we plan to pursue in future work.

Appendix

A Accounting Standards and Data Selection

We obtain our data from the Standard & Poors Compustat Database, in particular from the Compustat Segments data. The Financial Accounting Standards Board (FASB), in its Statement No. 131, sets the standards for the way in which public businesses report information about operating segments in their annual financial statements. Operating segments are defined by the FASB as *"components of an enterprise about which separate financial information is available that is evaluated regularly by the chief operating decision maker in deciding how to allocate resources and in assessing performance"*.

FAS 131 establishes standards for the way firms should disclose data about products and services, geographic areas, and major customers. The FAS 131 determines that firms should report data about revenues derived from the firm's *products or services*, *countries* in which they earn revenues and hold assets, and about major *customers* regardless of whether that information is used in making operating decisions. However, the statement does not require firms to disclose the information on all the different segment types if it is not prepared for internal use and reporting would be impracticable. Therefore, the firms decide how to report the data, disaggregated in several different ways: either by product, geography, legal entity, or by customer, but they do not necessarily have to report all of them. This method is referred to as the management approach. The statement establishes a minimum threshold to report separately information about an operating segment: either revenues of the segment are 10% or more of the combined revenue of all operating segments, or profits or losses are 10% or more of the combined reported profit or losses, or its assets are 10% or more of the combined assets of all operating segments. Hence, if a given firm considers best practice to aggregate the information upstream to the management level is by customer, they may elect not to disclose geographical segments information. That contrasts with the previous FAS No. 14, superseded by FAS No. 131 in 1998, in which firms were required that the financial statements of a business include information about the enterprise's operations in different industries, its foreign operations and export sales, and its major customers.

Faced with the potential measurements problems associated with the loose reporting requirements of the Compustat Segments, we had two options to select our dataset: 1) include in the dataset only those firms included in the Compustat Segments data, and drop all the others, or 2) include all firms in Compustat and impute as Domestic the status for those firms that are not included in the Segments data. The data analysis reported in Section 2 corresponds to the first selection criterium, which we prefer, because it generates a cleaner, albeit smaller, dataset. For robustness, we run all the regressions also using the dataset constructed with the second selection criterium, and the results on the ranking of earnings yields and returns are unchanged.

Another dimension of selecting the data involves which criteria to use to establish the unit of observation and to eliminate outliers. The data span 28 years, from 1979 to 2006, but many firms have observations only for a part of this time interval. In the results reported in the paper, we do not impose any lower bound on the number of years a firm must be present in the dataset to consider it an observation. For robustness though we run the data analysis of Section 2 also with lower limits of 2 and 5 years of presence in Compustat for a firm to be included in our sample. The results are unchanged. Finally, as reported in the tables in Section 2, we excluded the observations in the top and bottom 1% of earnings-to-price ratios by group, and in the top and bottom 5% of returns by group.

B Proofs

B.1 Derivation of the Value Functions

In this section we illustrate the steps that from equation (11) bring to equations (12), (14), (15). Starting from equation (11):

$$
\begin{aligned}
E[d(M \cdot V_D(a,Q^*))] &= 0 \\
E[dM \cdot V_D(a,Q^*) + M \cdot dV_D(a,Q^*) + dM \cdot dV_D(a,Q^*)] &= 0 \\
E\left[\frac{dM}{M} + \frac{dV_D(a,Q^*)}{V_D(a,Q^*)} + \frac{dM}{M} \cdot \frac{dV_D(a,Q^*)}{V_D(a,Q^*)}\right] &= 0 \\
-rdt + E\left[\frac{dV_D(a,Q^*)}{V_D(a,Q^*)}\right] + E\left[\frac{dM}{M} \cdot \frac{dV_D(a,Q^*)}{V_D(a,Q^*)}\right] &= 0 \\
-rV_D(a,Q^*) + E\left[\frac{dV_D(a,Q^*)}{dt}\right] + E\left[\frac{dM}{M} \cdot \frac{dV_D(a,Q^*)}{dt}\right] &= 0.
\end{aligned}
$$

By applying Ito's Lemma to the terms $\frac{dV_D(a,Q^*)}{dt}$:

$$
\begin{aligned}
-rV_D(a,Q^*) + \left[V_D'(a,Q^*)\mu Q^* + \frac{1}{2}V_D''(a,Q^*)\sigma^2(Q^*)^2\right] + ... \\
...\ E\left[(-rdt + \sigma_M dz) \cdot \left(V_D'(a,Q^*)\mu Q^* + V_D'(a,Q^*)\sigma Q^* \frac{dz}{dt} + \frac{1}{2}V_D''(a,Q^*)\sigma^2(Q^*)^2\right)\right] = 0.
\end{aligned}
$$

$$-rV_D(a,Q^*)dt + V_D'(a,Q^*)\mu Q^* dt + \frac{1}{2}V_D''(a,Q^*)\sigma^2(Q^*)^2 dt + ...$$
$$... \; E\left[(-rdt + \sigma_M dz)\cdot\left(V_D'(a,Q^*)\mu Q^* dt + V_D'(a,Q^*)\sigma Q^* dz + \frac{1}{2}V_D''(a,Q^*)\sigma^2(Q^*)^2 dt\right)\right] = 0$$

where the expectation term can be rewritten as:

$$E\left[(-rdt + \sigma_M dz)\cdot\left(V_D'(a,Q^*)\mu Q^* dt + V_D'(a,Q^*)\sigma Q^* dz + \frac{1}{2}V_D''(a,Q^*)\sigma^2(Q^*)^2 dt\right)\right] = ...$$
$$... \; E\Bigg[-r\left(V_D'(a,Q^*)\mu Q^* + \frac{1}{2}V_D''(a,Q^*)\sigma^2(Q^*)^2\right)dt^2 - rV_D'(a,Q^*)\sigma Q^* dtdz + ...$$
$$... \; \sigma_M\left(V_D'(a,Q^*)\mu Q^* + \frac{1}{2}V_D''(a,Q^*)\sigma^2(Q^*)^2\right)dtdz + \sigma\sigma_M V_D'(a,Q^*)Q^* dz^2\Bigg].$$

The terms in $dtdz$ have zero expectation, and the term in dt^2 is negligible, hence only the last term (in $dz^2 = dt$) remains and our equation can be written as:

$$\begin{aligned} rV_D(a,Q^*)dt &= V_D'(a,Q^*)\mu Q^* + \frac{1}{2}V_D''(a,Q^*)\sigma^2(Q^*)^2 + V_D'(a,Q^*)\sigma\sigma_M Q^* dt \\ rV_D(a,Q^*) &= (\mu + \sigma\sigma_M)V_D'(a,Q^*)Q^* + \frac{1}{2}V_D''(a,Q^*)\sigma^2(Q^*)^2. \end{aligned}$$

The derivation of the equations in $V_X(a,Q^*)$, $V_I'(a,Q^*)$ is analogous.

B.2 Hysteresis

In this section we extend the analytical results about the nature of hysteresis in Dixit (1989) to our model, where firms also choose the mode of entry in a (foreign) market. Hysteresis is defined as the non-zero difference between a switching threshold of the model (Q_{RS}) and the analogous threshold in a deterministic model (that we denote as W_{RS}).

Let $G_{SR}(Q) \equiv V_S(Q) - V_R(Q)$, for $S, R \in \{D, X, I\}$. We omit the dependence of the value functions on the productivity parameter a, as the proof carries over $\forall$ a.

Let us consider the value matching and smooth pasting conditions for a firm switching from being domestic only to also export and vice versa. Value matching implies: $G_{DX}(Q_{DX}) = F_X$ and $G_{DX}(Q_{XD}) = 0$. Similarly, smooth pasting implies: $G'_{DX}(Q_{DX}) = G'_{DX}(Q_{XD}) = 0$.

By using the expressions for the value functions in equations (13), (16), we can compute the limits of $G_{DX}(\cdot)$:

$$G_{DX}(Q) = A_X Q^\alpha + B_X Q^\beta + \frac{B(\tau aw)^{1-\eta} P^\eta Q}{r - \tilde{\mu}} - \frac{f_X}{r} - B_D Q^\beta$$

which implies:

$$\lim_{Q \to 0} G_{DX}(Q) = +\infty$$

since $\alpha < 0$ and $\beta > 1$. Similarly:

$$\lim_{Q \to +\infty} G_{DX}(Q) = \pm\infty$$

(according to whether $B_X \geq B_D$ or $B_X < B_D$). Hence $G''_{DX}(Q_{DX}) \leq 0$ and $G''_{DX}(Q_{XD}) > 0$. From Ito's Lemma (combining equations (12) and (14)):

$$rG_{DX}(Q) = G'_{DX}(Q)\tilde{\mu}Q + \frac{1}{2}G''_{DX}(Q)\sigma^2 Q^2 + B(\tau aw)^{1-\eta} P^\eta Q - f_X$$

which, evaluated at Q_{DX} reduces to:

$$rF_X - B(\tau aw)^{1-\eta} P^\eta Q_{DX} + f_X \leq 0$$

since $G'_{DX}(Q_{DX}) = 0$ and $G''_{DX}(Q_{DX}) \leq 0$. Hence:

$$Q_{DX} \geq \frac{rF_X + f_X}{B(\tau aw)^{1-\eta} P^\eta} \equiv W_{DX}$$

where W_{DX} is the corresponding threshold in absence of uncertainty. By evaluating the Ito's lemma condition at Q_{XD} one can also show that:

$$Q_{XD} < \frac{f_X}{B(\tau aw)^{1-\eta} P^\eta} \equiv W_{XD}.$$

With the same procedure, the result follows for the other four thresholds as well:

$$
\begin{aligned}
Q_{DI} &> \frac{rF_I + f_I}{B(aw^*)^{1-\eta}P^\eta} \equiv W_{DI} \\
Q_{ID} &< \frac{f_I}{B(aw^*)^{1-\eta}P^\eta} \equiv W_{ID} \\
Q_{XI} &> \frac{rF_I + (f_I - f_X)}{B(aw^*)^{1-\eta}P^\eta - B(\tau aw)^{1-\eta}P^\eta} \equiv W_{XI} \\
Q_{IX} &\leq \frac{-rF_X + (f_I - f_X)}{B(aw^*)^{1-\eta}P^\eta - B(\tau aw)^{1-\eta}P^\eta} \equiv W_{IX}.
\end{aligned}
$$

B.3 Proof of Theorem 1: Ordering of the Quantity Thresholds

It is easy to prove that, if (7) holds, the following order of the deterministic thresholds holds:

$$W_{IX} < W_{ID} < W_{XD} < W_{DX} < W_{DI} < W_{XI}.$$

Dixit and Pindyck (1994) show that hysteresis, here defined as the difference $Q_{RS} - W_{RS}$, for $R, S, \in \{D, X, I\}$ is increasing in sunk costs. Hence: $Q_{DI} - W_{DI} > Q_{DX} - W_{DX}$. Since $W_{DI} > W_{DX}$, we have $Q_{DI} > Q_{DX}$. By applying the same reasoning, one can also show that $Q_{XI} > Q_{DI}$. Symmetrically, one can also show the ordering of the exit thresholds, so that:

$$Q_{IX} < Q_{ID} < Q_{XD} < Q_{DX} < Q_{DI} < Q_{XI}.$$

B.4 Proof of Theorem 2: Monotonicity of the Quantity Thresholds

Everything else constant, a higher productivity $1/a$ is equivalent to a lower variable operating cost. Dixit and Pindyck (1994), pp. 221-223, show that entry and exit thresholds are increasing in operating costs, hence $\frac{\partial Q_{RS}}{\partial(a)} \geq 0$.

B.5 Proof of Theorem 3: Hysteresis and Productivity

From the results in Section B.2 we know that:

$$
\begin{aligned}
Q_{DX}(a) &> \frac{rF_X + f_X}{B(\tau a w)^{1-\eta} P^{\eta}} \ \ \forall\ a \\
Q_{XD}(a) &< \frac{f_X}{B(\tau a w)^{1-\eta} P^{\eta}} \ \ \forall\ a.
\end{aligned}
$$

Hence:

$$
Q_{DX}(a) - Q_{XD}(a) > \frac{rF_X}{B(\tau a w)^{1-\eta} P^{\eta}} \ \ \forall\ a
$$

where the right hand side is increasing in a. Similarly one can prove that also $Q_{DI}(a) - Q_{ID}(a)$ and $Q_{XI}(a) - Q_{IX}(a)$ are increasing in a.

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