

Life-Cycle Dynamics and the Expansion Strategies of U.S. Multinational Firms*

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September 16, 2017

Abstract

This paper examines how the activities performed by multinational firms change over the life cycle, in order to quantify the frictions to their expansion. Using a long panel of U.S. multinational firms, we document that: the ratio of affiliate-to-parent sales grow very little over the life cycle of the affiliate; affiliates are born specialized in their life-long main activity, which is overwhelmingly serving the host market of operations; and activities that start later in life, namely exports, are performed at a low intensity. Informed by these facts, we propose a quantitative dynamic model of multinational activity that features entry costs to both multinational activity and affiliate export markets, heterogenous firms, and persistent aggregate productivity shocks. The model delivers qualitative testable implications that are consistent with the data. Quantitatively, the calibrated model sheds light on the nature of the costs of multinational activity.

JEL Codes: F1. Key Words: Multinational firms, Foreign direct investment, Firm dynamics, Brownian motion, Sunk costs.

*We have benefited from comments from Costas Arkolakis, Javier Cravino, Jonathan Eaton, Oleg Itskhoki, Sam Kortum, Andrei Levchenko, Eduardo Morales, Ezra Oberfield, Andrès Rodríguez-Clare, Ana Maria Santacreu, Stephen Yeaple, as well as seminar participants at various conferences and institutions. The statistical analysis of firm-level data on U.S. multinational companies was conducted at the Bureau of Economic Analysis, U.S. Department of Commerce, under arrangements that maintain legal confidentiality requirements. The views expressed are those of the authors and do not reflect official positions of the U.S. Department of Commerce.

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1 Introduction

Multinational enterprises (henceforth, MNEs) are complex production structures engaged in different activities worldwide, and as such are the largest players in the global economy. In 2009, 75 percent of U.S. sales to foreign customers (nearly US\$5 trillions) was accounted for by the sales of foreign affiliates of U.S.-based multinationals, rather than by sales of domestically produced goods.¹ Additionally, affiliates' exports represent one third of world exports, and also one third of their total sales, according to UNCTAD (2013)'s estimates. Similar magnitudes are reported by the Bureau of Economic Analysis (BEA) for foreign affiliates of U.S. MNEs: in 2009, majority-owned foreign affiliates of U.S. MNEs accounted for US\$4,6 billions in sales, forty percent of which were exports, *i.e.* directed to customers outside the host market of operation.

A frequently overlooked aspect in the analyses of the MNE and its affiliates is their dynamic behavior, primarily because the data requirements are large. Consequently, questions about the activities of MNE affiliates and their evolution through time have been barely addressed in the literature. Yet, the answers to these questions are key to dissecting the nature of the costs of multinational activity: whether these costs are country and/or activity dependent, and whether variable, fixed, or sunk costs are relatively more important. In turn, understanding the nature of the costs of multinational activity can be crucial for the quantification of the gains from openness arising from MNEs operations.

This paper fills the gap in the literature first by documenting salient features of the life-cycle behavior of U.S.-based MNEs and of their affiliates, and second by presenting a tractable dynamic model of MNE activities amenable to quantitative analysis.

Using a panel of U.S. multinational firms over 25 years from the U.S. BEA, we classify MNE affiliates' activities as horizontal (directed to the host market) or exports (directed to other markets). We document three facts on the evolution of these types of sales over the life cycle of the affiliate. First, the life-cycle affiliate-to-parent sales ratio is relatively flat, particularly when compared with the same magnitude for new exporters. Second, affiliates of U.S. multinational firms tend to specialize in a core activity at birth, typically horizontal sales, which persists as the main activity during the life cycle. Third, some diversification from horizontal to export activities is observed later in life, but this new activity remains secondary in terms of its share of affiliate sales.

Motivated by the facts, we present a dynamic model of the multinational firm that builds on elements from Fillat and Garetto (2015) and Fillat et al. (2015). We model a set of Home-based firms that must decide whether, how, and when to serve foreign markets through affiliate sales.

¹See Antrás and Yeaple (2014) for a detailed survey of the main facts and theories about multinational firms.

Multinational activities are treated as a real option for the firm, which gets exercised once an affiliate opens abroad. Affiliate sales entail fixed and sunk costs of production, and the decisions of setting up an affiliate and of exporting from it are shaped by the interaction of firms' individual productivity, persistent aggregate productivity shocks, and demand conditions in foreign markets. Guided by the observation that almost all firms in our sample have horizontal activities, we assume that firms that decide to do Foreign Direct Investment (FDI) must first set up an affiliate and sell to the local market, and only then they can consider exporting from that affiliate. The model is set up in continuous time, so that those two decisions —opening an affiliate and exporting from it— can be made virtually simultaneously. In this way, the model is able to generate affiliates that are born as exporters. Additionally, the continuous time problem delivers closed-form solutions for the value functions, which are simple additive functions of the firm's realized profit flow plus the option value of further expansion. Crucially, we assume that the decision of opening an affiliate and eventually exporting from it is independent across markets (e.g., whether a firm decides to export to France from an affiliate located in Germany is independent of having an affiliate in France). In this way, we avoid having to solve the complex permutational problem present in settings that model these decisions as interdependent, such as in Tintelnot (2017), and achieve tractability in the dynamic setting.

While the dynamic component of the model is built to replicate qualitatively the motivating facts described above, the cross-sectional components of the model deliver additional testable implications which are confirmed by the data. First, affiliates that both serve the host market and export have larger horizontal sales than affiliates devoted exclusively to serving the host market. This fact mimics an analogous pattern about exporters that is documented in the literature.² In this regard, affiliates of multinational firms are not different from standard domestic firms. Second, affiliates that are exporters at birth have larger horizontal sales than affiliates that become exporters later in their life cycle. Third, there is a pecking order in the way the MNE chooses to open foreign affiliates: MNEs open first their largest affiliates, and subsequently their smaller affiliates. Finally, there is a pecking order in the way an MNE accesses markets: first affiliates are established in the largest host markets, and in markets which are easier to access, while later in life the MNE opens affiliates in markets that are smaller and costlier to access.

These cross-sectional patterns mimic the facts for non-MNE exporters documented in the literature. The main difference between exporting affiliates and non-MNE exporters is that MNE affiliates are more engaged in the exports, both on the extensive and on the intensive margin. Moreover, the evidence hints to MNEs facing an array of barriers to their expansion, in particular

²See Bernard and Jensen (1999), among others.

into export markets, that are different in nature to the barriers faced by non-MNE firms, and in particular, by non-MNE exporters.

The ability of the model to generate predictions that are qualitatively consistent with the data raises our confidence in using this framework to quantify the barriers to expansion that MNEs face. To this end, we extend the model to make it amenable to quantitative analysis, by allowing MNEs to set up affiliates in any country and from them export to any destination. Thanks to the assumption whereby an MNE's affiliate location choice is independent across locations, the quantitative model preserves most of the tractability of the simple framework, and can be used to shed light on the nature and magnitudes of the costs of multinational activity. Preliminary simulations show, for instance, that a small change in the per-period cost of multinational activities can induce large reallocations on the activities of affiliates, and the effects of these changes critically depend on which type of frictions are involved, whether variable, fixed, or sunk.

Most contributions in the literature have analyzed MNEs' complex choices in static settings. As evident in the models in Arkolakis et al. (2017) and Tintelnot (2017), allowing firms to set up affiliates in countries that might differ from the destinations of their sales results in a very complex combinatorial problem when fixed costs of production are taken into account. The sharp patterns that we document, arising from the observation of affiliates over time, help to simplify this problem by reducing the choice set of firms in a way that is consistent with the data. More precisely, given that most new affiliates in the data start out as entities partially or entirely specialized in horizontal FDI, and possibly start exporting later in life, we argue that decisions about performing complex foreign activities can be separated into simple choices that happen at different points in time. This significantly simplifies the dynamic problem of the firm.

There is a large literature on export dynamics which has been concerned primarily with quantifying fixed and sunk costs of export activities and studying their welfare implications. Earlier contributions by Baldwin and Krugman (1989), Roberts and Tybout (1997), Das et al. (2007), and Alessandria and Choi (2007) find substantial sunk costs of exporting, by focusing on observed patterns of export entry and exit. Subsequent analyses, such as Eaton et al. (2008) and Ruhl and Willis (2017), incorporate facts related to the life-cycle dynamics of new exporters and find that those costs are much lower. Alessandria et al. (2015) take a further step and also calculate the welfare gains from trade in a dynamic setting that matches well the life-cycle facts. Arkolakis (2016) presents rich micro evidence on firm selection and export growth that supports dynamic theories of endogenous entry costs *vis-a-vis* standard export sunk costs. By analyzing MNEs' dynamics and quantifying their frictions, our work complements the one on exporters and helps making useful comparisons between the barriers to the two modes of market penetration.

There is also a small, but growing, literature on analyzing different aspects of the dynamics of MNEs that uses rich firm-level data. Ramondo et al. (2013) study the implications of the proximity-concentration tradeoff under uncertainty using BEA data. Egger et al. (2014) and Conconi et al. (2016) use data for Germany and Belgium, respectively, and claim that their findings are consistent with substantial learning. Even though these papers have rich data on the MNE behavior, they do not focus on life cycle features. Conversely, Gumpert et al. (2016), using very rich data for several countries, focus on life-cycle patterns of both MNEs and exporters, in the context of the proximity-concentration tradeoff. Our paper complements theirs as it focuses on the life-cycle of affiliates' activities, for the first time separating them across locations and sales destinations.

Finally, our paper also makes contact with the large literature on domestic firms' life-cycle dynamics, which goes back to Davis et al. (1996), and more recently Decker et al. (2014, 2015). We show that affiliates of MNEs are starkly different from domestic firms. We interpret the difference between the behavior of new U.S. firms in the domestic and foreign markets as indicative of the fact that they face different sets of costs.

The rest of the paper is organized as follows. Section 2 documents the facts about affiliates' dynamics. Section 3 presents the simple version of the model and its testable implications. Section 4 shows empirical evidence in support of the model's qualitative predictions. In Section 5 we present the quantitative model and use it to pin down the magnitudes of the costs to MNE expansion. Section 6 concludes.

2 Establishing the Facts

We document three novel empirical regularities concerning the behavior of foreign affiliates of U.S. MNEs operating in the manufacturing sector. First, sales grow slowly over the life cycle of the affiliate. Second, affiliates are born specialized in their life-long main activity which, for the vast majority, consists of sales to the host market; only slowly affiliates incorporate a second activity, mainly export sales. Finally, those activities that are incorporated later in life are done at a relatively low intensity.

2.1 Data

Our descriptive empirical analysis is conducted using data from the U.S. Bureau of Economic Analysis (BEA). The BEA collects firm-level data on U.S. multinational companies' operations in its annual surveys of U.S. direct investment abroad. All U.S.-located firms that have at least one

foreign affiliate and that meet a minimum size threshold are required by law to respond to these surveys. The data include detailed information on the firms' operations both in the U.S. and at their foreign affiliates, for the period 1987-2011. Each foreign affiliate in the dataset is assigned an industry classification based on its primary activity according to the BEA International Surveys Industry (ISI) system, which closely follows the 3-digit Standard Industrial Classification (SIC) system.³ We include affiliates that list an activity in manufacturing as their primary activity and belong to a U.S. parent operating in any sector. We restrict our attention to majority-owned affiliates that do not operate in tax haven countries.⁴ We further consolidate affiliates belonging to the same parent and operating in the same country and four-digit industry.⁵ We also remove from our sample affiliates and parents with zero total sales, assuming that there is a reporting error. Finally, since we are interested in firms' life cycle behavior, we focus on *new* affiliates that open during our sample period and that survive for at least ten consecutive years in the market. We end up with a sample that covers 23.12 percent of all new affiliates in manufacturing as well as 38.27 percent of their total sales.⁶

Crucially, the BEA data break down affiliate sales by destination. Affiliate sales can be directed to the host market of operation (horizontal sales), or to other markets (exports).⁷ Table 1 shows how our sample is distributed between the two types of affiliates' activities. Almost 95 percent of our affiliate-year observations have some horizontal sales, while more than two thirds of them have some exports, indicating larger export market participation than for non-MNE firms. More than one third of the observations correspond to pure-type horizontal affiliates (*i.e.*, affiliates whose sales are all directed to the host market), while the share of pure-type exporting affiliates is only six percent. These shares go up to 52 and nine percent if one considers affiliates that are born with only horizontal and export sales, respectively.

Table 1 provides our first assessment of the fact that horizontal sales are pervasive, while pure exporting affiliates are few and account for a small share of total sales. Appendix A provides more

³The BEA data use 3-digit SIC-based ISI codes for years prior to 1999. From 1999 onward, they use 4-digit NAICS-based ISI codes. For consistency, we convert the NAICS-based codes to 3-digit SIC-based ISI codes for the relevant years.

⁴The list of tax havens is from Gravelle (2015), but we keep in our sample Hong Kong, Singapore, Ireland, and Switzerland, since these are important destinations of FDI. See Appendix A for more details.

⁵Based on the BEA definition, an affiliate is a business enterprise in a given industry operating in a particular host country; it thus could operate several plants in different locations within the host country. We discuss the rationale of this aggregation in Appendix A.

⁶The sample coverage may appear limited for a number of reasons. First, we drop tax havens. Second, we only include new affiliates that open during our sample and exist for 10 years. This implies excluding any affiliates that open in 2003 or later. We also drop observations for affiliates in their 11th year or greater, to have a balanced 10 year panel.

⁷The data further distinguish between exports to the United States and to third markets. This distinction does not make any substantial difference for the facts presented below. We exploit the distinction among different export destination markets in the calibration.

Table 1: Summary statistics: number of observations, by sale type.

	Horizontal sales	Export sales
No. of observations	38,088	38,088
with positive sales	36,127	25,950
	(94.9%)	(68.1%)
of pure type	14,030	2,423
	(36.8%)	(6.4%)
of pure type at birth	19,905	3,595
	(52.3%)	(9.4%)
Sales accounted by pure-type	15.62%	7.71%

Note: Observations are at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. A pure-type affiliate is an affiliate with either only horizontal or only export sales.

details on the BEA data, the construction and coverage of our sample, and summary statistics.

2.2 The life-cycle affiliate-to-parent sales ratio is flat

Figure 1 shows the ratio of affiliate-to-parent sales, by affiliate age, for all affiliates. Sales are broken down in horizontal and export sales.

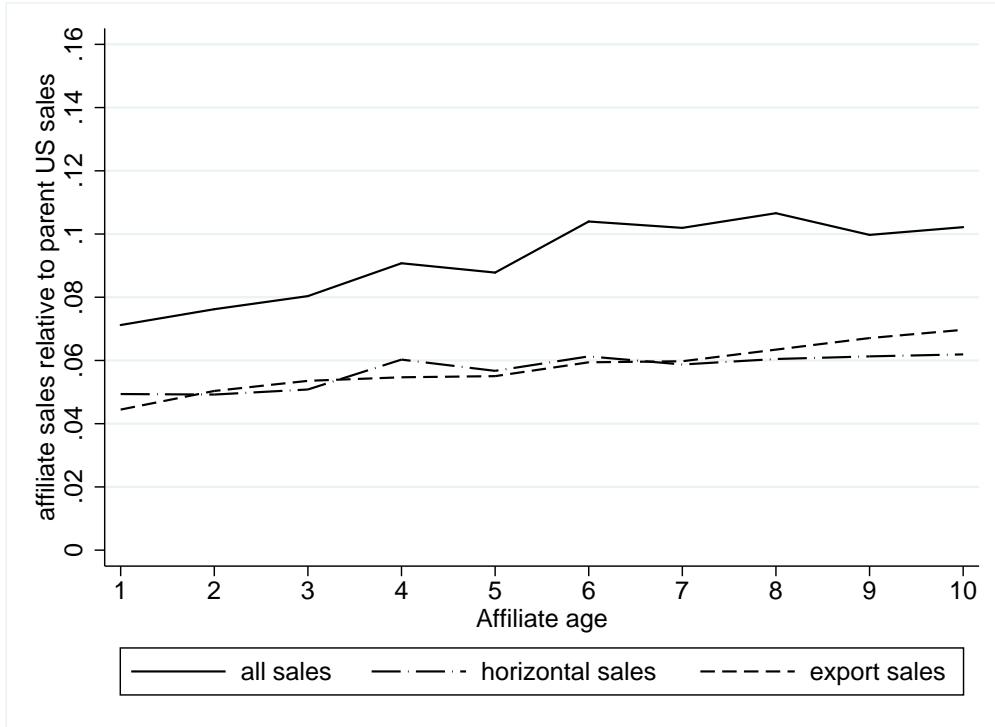
On average, new affiliates have sales volumes of about seven percent of their parent's sales. Over the first five years of life, this ratio goes up to about nine percent, reaching ten percent at age six and staying flat until at least the 10th year of the affiliate's life. Examining sales profiles by sale type reveals that horizontal and export sales, relative to the parent's sales, exhibit a very similar behavior in the first ten years of the affiliate's life: they grow from below five percent to more than six percent.

It is worth comparing the growth profile of horizontal sales of new affiliates with the growth profile of foreign sales for new exporters, being these two modes the predominant ones through which firms choose to enter foreign markets. For Colombia, Ruhl and Willis (2017) report that the export-to-domestic sales share goes from six to 14 percent in the first five years of the exporter; in contrast, horizontal sales, relative to the parent's sales increase, on average, from five to six percent.⁸

The pattern observed in Figure 1 is confirmed formally by an Ordinary Least Square (OLS)

⁸Ruhl and Willis (2017) consider exporters that survive at least four years in the export market, while we consider affiliates that survive at least ten years in their market of operations.

Figure 1: Affiliate-to-parent sales ratio, by sales type.



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Simple averages taken over affiliates with positive sales in each category.

regression of the affiliate-to-parent sales ratio on affiliate age. The regression also includes as other controls the size of the affiliate and the size of the corporation, measured as employment and global sales, respectively, and a set of country-year fixed effects. Results are shown in Table B.1 in Appendix B. Not only the affiliate-to-parent sales ratio is flat across affiliates of different age (i.e. including industry fixed effects), but also within affiliates (i.e. including affiliate fixed effects).

One could argue that the flatness in the life-cycle sales profiles of MNE affiliates may be due to the fact that the affiliate “inherits” the age of the parent so that, *de facto*, it is a much older firm and hence larger and growing more slowly. This may well be happening, as documented for multi-plant versus single-plant firms in the United States by Kueng et al. (2016).⁹ Unfortunately, the BEA data do not record the age of the parent firm. However, we can still look at affiliates’ position in the opening sequence of the parent —in particular, first affiliates versus subsequent affiliates. In this way, we compare affiliates belonging to younger MNEs with affiliates belonging to older MNEs (or to the same MNE when it is older). Figure B.2 in Appendix B shows that first

⁹They document a stark difference in the life-cycle employment profiles of establishments belonging to single-versus multi-unit firms in manufacturing: while establishments in the first group grow steeply, the ones in the second group do not grow.

affiliates are much larger than subsequent affiliates, but they do not appear to grow faster.

A second argument is that the flat sales profiles observed for new affiliates may be due to the fact that firms acquire experience and grow in a foreign market first through exports, and only subsequently open affiliates at their optimum long-run size. Unfortunately, the BEA data do not include information about parents' exports that can inform our analysis in this respect. Gumpert et al. (2016), however, report that for Norway and France, the difference in growth profiles for MNEs with previous export experience into a market and those without it is not significant.

A final concern is related to the mode of FDI entry. If MNEs establish foreign affiliates mostly through mergers and acquisitions (M&A), one could argue that "new" foreign affiliates are in reality pre-existing plants that likely grew previously and were acquired by the MNEs already at their maturity stage. This would explain the observed flat sales profiles. Again, Gumpert et al. (2016) show that, for Germany, new affiliates that were previously domestic firms (i.e. were established through M&A) have flatter life-cycle sales profiles than new affiliates created through greenfield FDI. The BEA data also contains information on whether affiliates are the result of M&A or of a greenfield investment. We plan to use this information to examine separately the growth profiles of these two types of affiliates.

The fact portrayed in Figure 1 suggests that MNEs' growth happens at the extensive margin (i.e. adding new markets), not at the intensive margin within a market. For this reason, the model we propose below features growth only at the extensive margin.

2.3 Affiliates are born specialized in their main life-long activity

We present here evidence on the specialization patterns of affiliates in terms of horizontal versus export activities. We show that affiliates are born specialized in a core activity, typically horizontal sales, that persists as the main activity later in life, even though affiliates may incorporate exports as a secondary activity.

Figure 2 shows the evolution of the intensive and extensive margins of horizontal and export sales shares, for all affiliates and for the sub-set of affiliates born with either exclusively horizontal or export sales (pure-type at birth). More precisely, Figure 2a shows the evolution of mean affiliate sales shares for affiliates with positive horizontal sales and for affiliates with positive export sales, respectively. On average, horizontal sales account for about 80 percent of affiliate sales and decrease by ten percentage points over the first ten years of life of the affiliate, while the export share is flat at 40 percent. To capture new affiliates that start exporting, Figure 2b plots the percentage of affiliates with non-zero horizontal or export sales, respectively. While the share of affiliates with

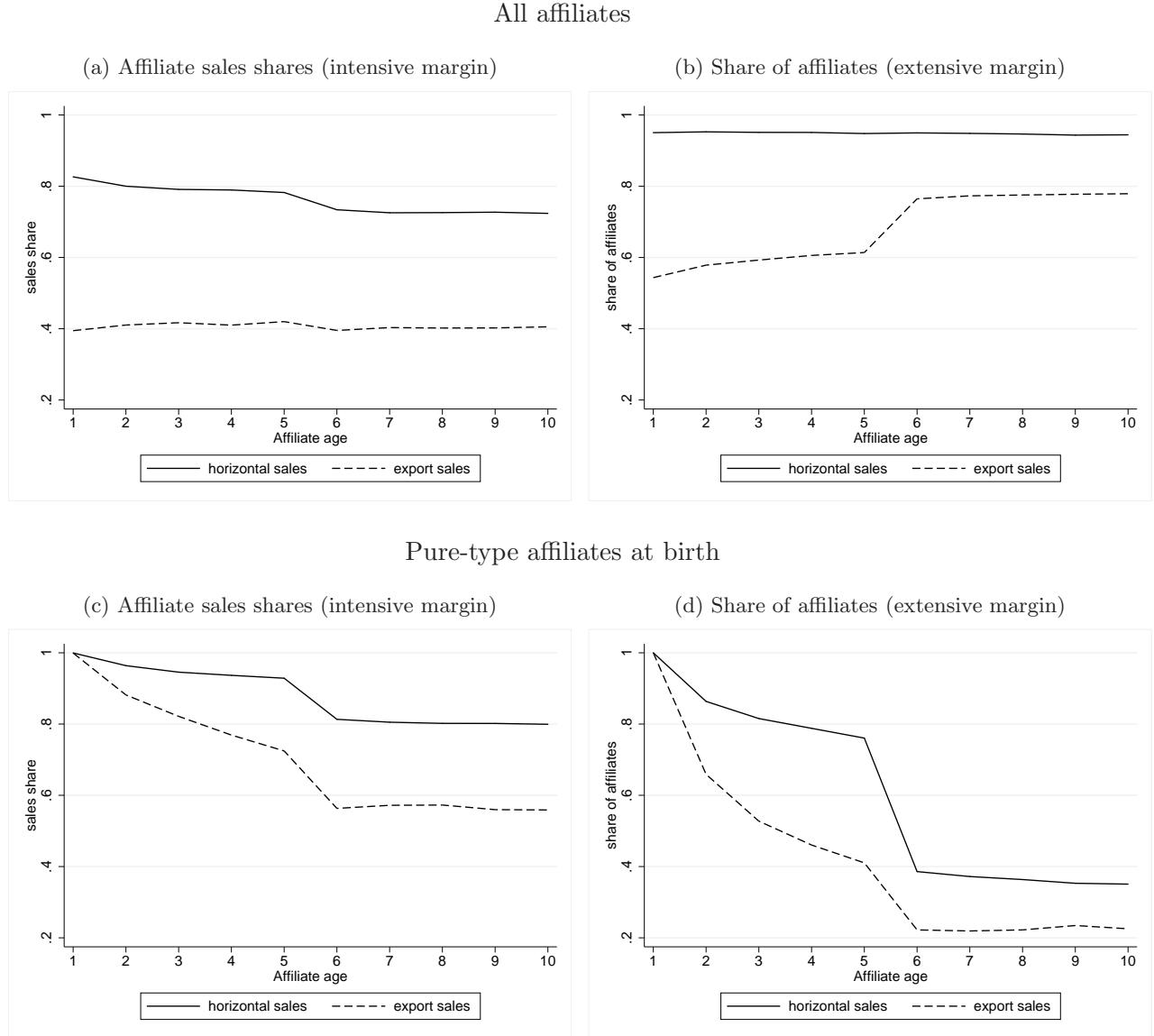
horizontal sales is stable at more than 95 percent, the share of exporting affiliates increases from 50 to 70 percent during the affiliates' life cycle. In other words, for horizontal activities, changes in sales shares are due to the intensive margin, while for export activities, affiliates with previously zero exports are the ones contributing to the increase in export shares. Hence the data suggest that, over time, affiliates incorporate export sales into their activities, but they never stop selling in their host market.

Figures 2c - 2d show the evolution of the intensive and extensive margins of sales shares for affiliates that are pure-type at birth. It is worth remembering that around 50 percent of affiliates are born with only horizontal sales, while affiliates born with only exports are less than five percent of observations. Figure 2c shows that, on average, sale shares of pure-type born affiliates decrease with age: the initial specialization is soon followed by more diversified sales patterns. One may get the misleading impression that horizontal sale shares decrease by less than export shares for affiliates born with only one type of sales. However, it is worth noting that by the tenth year of life, an affiliate born with exclusively export sales is still above the 75th percentile of the exports sale share distribution —and much above the average (0.28) and median (0.11) of the distribution, as indicated by Table A.2, while an affiliate born exclusively serving the host market, by age ten, is below the 50th percentile of the horizontal sale share distribution. As panel 2d shows, the set of pure horizontal affiliates at birth shrinks over the life cycle of affiliates indicating, once again, that these new affiliates start by serving their host market exclusively, and then they start exporting. By their sixth year of life, more than 60 percent of previously pure horizontal affiliates have started exporting.

The patterns in Figure 2 are confirmed by OLS regressions including a battery of fixed effects, as shown in Table B.2 in Appendix B: on average, horizontal sales shares in each activity type for younger affiliates are not significantly different from the shares for older affiliates, but the share of affiliates with exports is higher among older affiliates. Moreover, estimates that include affiliate fixed effects suggest that the patterns observed in Figure 2 are driven by within-firm variation: over time, young affiliates decrease the intensity of horizontal sales and add export activities. The OLS estimates also confirm that for pure-type affiliates at birth the decrease in sales shares is a result of within-affiliate changes (see Table B.3 in Appendix B): on average, affiliates of different ages have undistinguishable export shares, but a pure exporter at birth decreases its export shares as it gets older; for pure horizontal affiliates at birth, the effect is a mix of between- and within-firm effects.

The fact portrayed in Figure 2 motivates another convenient feature of the model presented below: we assume that all affiliates start foreign operations with some horizontal sales and may endogenously expand into export markets over time.

Figure 2: Intensive and extensive margins of sale shares, by activity type.



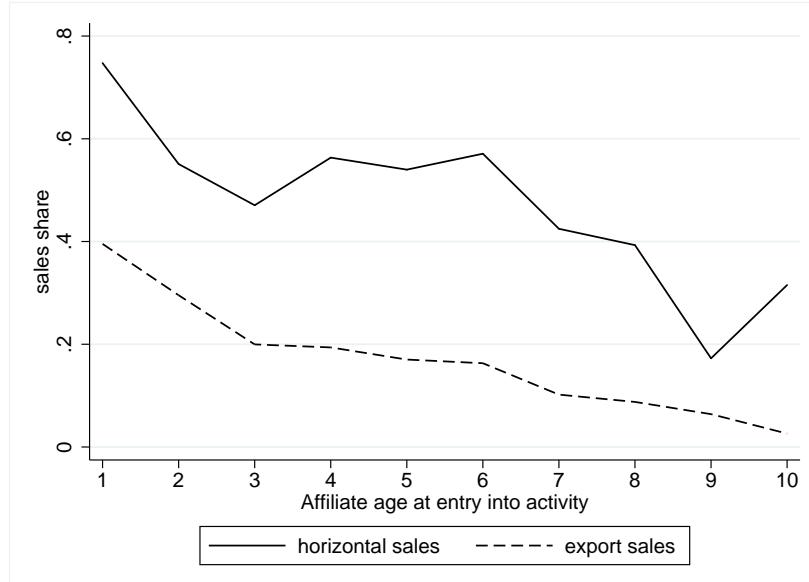
Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Horizontal and export sales refer, respectively, to sales to the market where the affiliate is located, and to sales to markets outside the local market. (2a): average sales, as a share of total affiliate sales, include affiliates with positive horizontal and export sales, respectively (same for 2c for the subset of affiliates with only horizontal and only export sales at birth, respectively). (2b): number of affiliates, as a share of the total number of affiliates, include affiliates with positive horizontal and export sales, respectively (same for 2d for the subset of affiliates with only horizontal and only export sales at birth, respectively).

2.4 Affiliate activities that start later in life have low intensity

Finally, we document that the older an affiliate is when it starts a new activity (e.g. exporting), the lower the intensity at which that activity is performed.

Figure 3 shows the average sales share, for horizontal and export sales, by the age at which the affiliate starts the activity. This figure makes clear that the primary activity affiliates are born doing remains their main activity for the remainder of their life. In contrast, if an the activity is incorporated later in life, it never overcomes the activity performed earlier in life. Affiliates that are born doing some horizontal sales have an average horizontal sale share of around 75 percent (around the 50th percentile of this variable's distribution), while affiliates that are born doing some exports have an average export share of around 40 percent, which correspond to almost the 90th percentile of that share's distribution. If an affiliate were to start doing horizontal sales in its tenth year of life, on average, it would dedicate only 35 percent of its sales to the local market, while an affiliate that starts exporting in its tenth year would dedicate at most three percent of its sales to exports.

Figure 3: Sales shares and entry age, by activity type.



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Average horizontal (export) sales shares, by affiliate first age with positive horizontal (export) sales.

3 Baseline Model

We present here a simple dynamic model of MNE activities that is designed to reproduce the facts documented in Section 2. In this section we put forward several simplifying assumptions to present the intuition and the mechanism in a transparent way. In fact, this version of the model can be solved entirely in closed form. In Section 5 we extend the model in order to use it for quantitative analysis.

The static components of our model follow the treatment of firms in Melitz (2003), while the dynamic choice of whether and when to enter a country with an affiliate is modeled as in Fillat and Garetto (2015) and Fillat et al. (2015), where the dynamic FDI decision is treated as a real option that the firm has for the future, and from which it derives value. The option is exercised when the MNE enters a host market with some form of FDI activity, horizontal FDI or a mix of horizontal sales and exports.

Guided by the observation of the time series data, we assume that MNEs need to first establish an affiliate in the host country and carry out some horizontal activity before eventually engaging in export activities. Because the model is specified in continuous time, these two decisions can happen almost simultaneously, allowing for the existence of affiliates that are born as exporters. The separation in time of the two decisions is a mere artifact to gain tractability.

Also key for tractability is an independence assumption. We assume that the choice of whether to open an affiliate in a country—and export from there—is independent for each host country. In other words, the profits of an affiliate are independent on the number of affiliates that the firm has, so that—for example—whether an affiliate in Germany exports to France is independent of having an affiliate in France. We can interpret this assumption as indicating that the demand for a variety that a firm produces depends both on its source and on its location of production. For example, consumers perceive differently Möet Chandon champagne produced in France and Chandon sparkling wine produced by the same firm in Napa, California. This assumption implies that there is no within-firm cannibalization associated with the production decisions of a MNE because the decision of how much to produce is independent for each host-destination country pair in which the firm operates.¹⁰ Both the independence in decisions across markets as well as the sequential choice of affiliate activities allow us to avoid the difficult computational problem faced by Tintelnot (2017) in a static setup, which would be even harder to solve in a dynamic environment.

In the baseline version of the model we assume that exit is exogenous, we do not distinguish among different affiliate export destinations, and we take aggregate prices and quantities as given. We remove all these simplifying assumptions in the extended model in Section 5.

3.1 Preferences and technology

The economy is composed by $N + 1$ countries: the Home country (the U.S. in our data) and N possibly asymmetric foreign countries. The Home country is populated by a mass of domestic firms

¹⁰Notice that, as a consequence of this treatment of variety, our model does not feature the proximity-concentration tradeoff by virtue of which exports and FDI are substitutes, as in Helpman et al. (2004).

that decide whether to operate only in their home market or to establish foreign affiliates in other countries.

Time is continuous. In each country k , consumers have linear intertemporal preferences over an aggregate good Q ,

$$U_k = \int_0^\infty e^{-\rho t} Q_k(t) dt, \quad (1)$$

where ρ is the subjective time discount rate. $Q_k(t)$ aggregates a continuum of varieties, indexed by v , with constant elasticity of substitution (CES) $\eta > 1$:

$$Q_k(t) = \left[\sum_i \sum_j \int_{\Omega_{ijk}(t)} q_{ijk}(v, t)^{\frac{\eta-1}{\eta}} dv \right]^{\frac{\eta}{\eta-1}} \quad (2)$$

where $q_{ijk}(v, t)$ denotes consumption of variety v produced by a firm from country i via an affiliate located in j and sold to country k at time t , and $\Omega_{ijk}(t)$ denotes the set of varieties produced by firms from country i via affiliates located in j and sold to k at time t .

Labor is the only factor of production. Each country is populated by a continuum of firms. Each firm produces with a linear technology and operates under monopolistic competition. As in Melitz (2003), each firm is endowed with a productivity parameter φ that determines the labor cost of one unit of its output. Each firm sets prices to maximize profits from sales to each destination, so that prices are given by a constant mark-up over marginal cost, $p_{ijk}(\varphi) = \frac{\eta}{\eta-1} MC_{ijk}(\varphi)$, and the marginal cost depends on the location of production: $MC_{ijk}(\varphi) = w_j \tau_{jk} / \varphi$, where w_j denotes the wage of the country where affiliate production takes place, and τ_{jk} is a variable iceberg export cost.¹¹

When a firm establishes an affiliate in a foreign country, it starts by selling there, so engaging in horizontal FDI. We assume that an affiliate has to pay a sunk entry cost, $F_j^h > 0$, to start producing in country j , and a per-period fixed cost, $f_j^h > 0$, and , to .¹² Once the affiliate is in place, the firm can expand its operations to serve other markets, so engaging in export activities. We assume that an affiliate located in j has to pay an entry cost $F_{jk}^e > 0$ to start exporting to country k , and a per-period fixed cost $f_{jk}^e > 0$. In the baseline model, firms and affiliates exogenously die at a constant rate δ . We will remove this assumption in Section 5, where we introduce endogenous exit to match data on exit rates.

Since in our data there is only one Home market (the U.S.) and the baseline version of the

¹¹ $\tau_{jk} > 1 \forall j \neq k$ and $\tau_{jj} = 1$.

¹²For simplicity, we assume that there are no per-period fixed costs associated with domestic production, so that all firms produce in their Home market.

model does not distinguish different affiliate export markets, in the remainder of this section we simplify the notation to reflect only the location of an affiliate.

Let $\pi_d(\varphi)$ denote a firm's domestic profits, $\pi_j^h(\varphi)$ denote the profits from local sales of an affiliate located in country j (horizontal sales), and $\pi_j^e(\varphi)$ denote the profits from exports of an affiliate located in country j :

$$\pi_d(\varphi) = H \left(\frac{w_d}{\varphi} \right)^{1-\eta} E_d, \quad (3)$$

$$\pi_j^h(\varphi) = H \left(\frac{w_j}{\varphi} \right)^{1-\eta} E_j - f_j^h \equiv \bar{\pi}_j^h(\varphi) - f_j^h \quad (4)$$

$$\pi_j^e(\varphi) = H \left(\frac{\tau_j w_j}{\varphi} \right)^{1-\eta} E_{\sim j} - f_j^e \equiv \bar{\pi}_j^e(\varphi) - f_j^e, \quad (5)$$

where $H \equiv \eta^{-\eta}(\eta-1)^{\eta-1}$, $E_j \equiv P_j^\eta Q_j$ is the size of market j , and $E_{\sim j} \sim P_{\sim j}^\eta Q_{\sim j}$ is the total market size for the exports of affiliates located in j .

Following Ghironi and Melitz (2005), we define the firm-level productivity φ to be the product of a constant firm-specific component, z , and of a stochastic Home country-specific component, Z $\varphi \equiv z \cdot Z$. The term z is a firm-specific draw from a distribution $G(z)$ (e.g., Pareto), as in Melitz (2003). As in Impullitti et al. (2013), we assume that $Z = e^X$, where X is a Brownian motion with drift,

$$dX = \mu dt + \sigma dW, \quad (6)$$

for $\mu \in \Re$ and $\sigma > 0$, and dW denoting a standard Wiener process. This specification is equivalent to assume that aggregate productivity behaves like a random walk, and that productivity growth is independently and identically distributed. This is a convenient functional form assumption, which guarantees tractability to the model.

We assume that when a firm operates an affiliate in a foreign country, it transfers both the aggregate and the idiosyncratic components of the productivity shocks to the host market, so that MNEs operations contribute to the transmission of productivity shocks across countries, in the spirit of Cravino and Levchenko (2017).¹³ In Section 4, we present evidence suggesting that the structure of shocks we impose in the model explains a significant amount of variation in the data.

¹³Our shock structure shares with Cravino and Levchenko (2017) the fact that both home country- and host country-specific shocks affect affiliate sales in industry equilibrium. Exogenous home country shocks get transferred to the host country, while host country shocks affect affiliate operations through their impact on aggregate demand in industry equilibrium. Third country shocks also matter through their effect on the price indexes (see section ??).

3.2 The MNE dynamic problem

Firms take decisions about whether, when and how to enter a market j by computing their expected profits net of entry and continuation costs, which depend on their productivity z and on market specific variables (here the aggregate productivity shock X), which represent the aggregate state of the economy.

Bellman Equations. Let $\mathcal{V}(z, X)$ denote the expected net present value of a Home country firm with productivity z , when the state of the economy is described by X , and following optimal policy. The value of the firm is given by the value of its domestic operations, $V_d(z, X)$, and by the value of its operations abroad:

$$\mathcal{V}(z, X) = V_d(z, X) + \sum_{j=1}^N \max \left\{ V_j^o(z, X), V_j^h(z, X), V_j^e(z, X) \right\}. \quad (7)$$

$V_j^o(z, X)$ denotes the option value of opening an affiliate in country j , $V_j^h(z, X)$ denotes the value of a pure horizontal affiliate in country j , and $V_j^e(z, X)$ denotes the value of an affiliate in country j that also exports.

Since all firms operate in the domestic market, the value of domestic operations is simply given by the evolution of domestic profits over time. Over a generic time interval Δt ,

$$V_d(z, X) = \frac{1}{1 + (\rho + \delta)\Delta t} [\pi_d(z, X)\Delta t + E[V_d(z, X')|X]], \quad (8)$$

where X' denotes the next period realization of the aggregate state.

We assume that a firm must open first a horizontal affiliate and then has the choice of starting exporting from it. In this simple version of the model we also don't allow for endogenous exit, so the only decisions that a firm can take in a host country are whether or not to open an affiliate and whether or not to start exporting from an existing affiliate. These possibilities are reflected in the Bellman equations.

If a domestic firm does not have an affiliate in country j , all the value from operations in j is an option value, *i.e.*, the value of the possibility of entering j in the future, described by:

$$V_j^o(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} E[V_j^o(z, X')|X]; V_j^h(z, X) - F_j^h \right\}. \quad (9)$$

Equation (9) describes the fact that a firm may keep the option of entering market j , or may enter country j by opening a horizontal affiliate there, in which case it pays the entry cost F_j^h and

gets the value of a horizontal affiliate in j , $V_j^h(z, X)$.

Alternately, a domestic firm may already have an affiliate located in country j . In this case, it gets value from horizontal sales in j . Once the affiliate is set up, it may decide to export from j to other markets. This option is reflected in the Bellman equation:

$$V_j^h(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} \left[\pi_j^h(z, X)\Delta t + E[V_j^h(z, X')|X] \right]; V_j^e(z, X) - F_j^e \right\}, \quad (10)$$

where $V_j^e(z, X)$ is the value of sales of an affiliate in country j which also exports, and F_j^e is the sunk cost of starting exporting from an affiliate in j .

Lastly, the value of an affiliate that both serves the host market and exports is simply given by its flow profit over time:

$$V_j^e(z, X) = \frac{1}{1 + (\rho + \delta)\Delta t} \left[(\pi_j^h(z, X) + \pi_j^e(z, X))\Delta t + E[V_j^e(z, X')|X] \right]. \quad (11)$$

Value Functions. The structure that we impose on the profit functions and the shock process implies that, by evaluating the Bellman equations in their continuation regions and applying Ito's lemma, we can solve for the value functions in closed form up to multiplicative parameters. All the derivations are contained in Appendix C.

The value of domestic sales is simply given by the present discounted value of profits from domestic sales,

$$V_d(z, X) = \frac{\pi_d(z, X)}{\rho + \delta - \hat{\mu}}, \quad (12)$$

where $\hat{\mu} = \mu(\eta - 1) - \frac{1}{2}\sigma^2(\eta - 1)^2$ is the drift of the stochastic process for the profit flow, and the discount rate $(\rho + \delta - \hat{\mu})$ takes into account the exogenous exit rate and the effect of the evolution of aggregate productivity on profits.

The option value of opening an affiliate in country j is

$$V_j^o(z, X) = B_j^o(z)e^{\beta X}, \quad (13)$$

where $B_j^o(z) > 0$ is a firm-specific parameter yet to be determined, and $\beta > 1$ is the positive root of $\frac{1}{2}\sigma^2\beta^2 + \mu\beta - (\rho + \delta) = 0$. The option value is increasing in the realization of the aggregate productivity shock, indicating that there is a higher value to be obtained from opening an affiliate when aggregate productivity is high.

The value of an affiliate with only horizontal sales in country j is

$$V_j^h(z, X) = \frac{\bar{\pi}_j^h(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta} + B_j^h(z)e^{\beta X}, \quad (14)$$

where $B_j^h(z) > 0$ is a firm-specific parameter yet to be determined. The value of a horizontal affiliate is the sum of discounted profits from sales in the host country plus the option value of expanding to export markets. Additionally, the option value of exporting is increasing in the realization of the aggregate productivity shock, indicating that the value of exporting is higher when aggregate productivity is high.

Finally, the value of an affiliate located in country j who sells locally and exports is given by the present discounted value of its profits,

$$V_j^e(z, X) = \frac{\bar{\pi}_j^h(z, X) + \bar{\pi}_j^e(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h + f_j^e}{\rho + \delta}. \quad (15)$$

Solution: Parameters and Thresholds. To completely characterize the problem of the MNE, it remains to solve for the two firm-specific parameters $B_j^o(z)$, $B_j^h(z)$ and for the aggregate productivity thresholds that induce a firm with productivity z to open an affiliate in j or to start exporting from it, which we denote by $X_j^h(z)$ and $X_j^e(z)$, respectively. For each firm and each foreign market, these four variables are identified by the following system of value matching and smooth pasting conditions:

$$V_j^o(z, X_j^h) = V_j^h(z, X_j^h) - F_j^h \quad (16)$$

$$V_j^h(z, X_j^e) = V_j^e(z, X_j^e) - F_j^e \quad (17)$$

$$V_j^{o'}(z, X_j^h) = V_j^{h'}(z, X_j^h) \quad (18)$$

$$V_j^{h'}(z, X_j^e) = V_j^{e'}(z, X_j^e). \quad (19)$$

The value function parameters $B_j^h(z)$ and $B_j^o(z)$ are given by:

$$B_j^h(z) = k_B \cdot \left(\frac{k_j^e(z)}{\beta(\rho + \delta - \hat{\mu})} \right)^{\frac{\beta}{\eta-1}} \cdot \left(\frac{f_j^e + (\rho + \delta)F_j^e}{\rho + \delta} \right)^{\frac{\eta-1-\beta}{\eta-1}} \quad (20)$$

$$B_j^o(z) = k_B \cdot \left(\frac{k_j^h(z)}{\beta(\rho + \delta - \hat{\mu})} \right)^{\frac{\beta}{\eta-1}} \cdot \left(\frac{f_j^h + (\rho + \delta)F_j^h}{\rho + \delta} \right)^{\frac{\eta-1-\beta}{\eta-1}} + B_j^h(z) \quad (21)$$

where k_B is a constant, and $k_j^h(z)$ and $k_j^e(z)$ are firm-specific revenue terms.¹⁴

Under the parameter restriction that $\beta > \eta - 1$, equation (21) shows that the option value of opening an affiliate is decreasing in both the fixed and sunk costs of opening an affiliate and of exporting from the affiliate. In other words, the less costly are an affiliate's operations in a country, the more appealing it is to open an affiliate there. Similarly, equation (20) shows that the option value of exporting from an affiliate is decreasing in both the fixed and sunk costs of exporting from the affiliate. Finally, both option value parameters are increasing in the firm productivity z , indicating that affiliate operations are more appealing for more productive firms.

The aggregate productivity thresholds $X_j^h(z)$ and $X_j^e(z)$ are given by:

$$X_j^h(z) = \frac{1}{\eta-1} \log \left[\left(\frac{\beta}{\beta-\eta+1} \right) \cdot \left(\frac{\rho+\delta-\hat{\mu}}{k_j^h(z)} \right) \cdot \left(\frac{f_j^h + (\rho+\delta)F_j^h}{\rho+\delta} \right) \right] \quad (22)$$

$$X_j^e(z) = \frac{1}{\eta-1} \log \left[\left(\frac{\beta}{\beta-\eta+1} \right) \cdot \left(\frac{\rho+\delta-\hat{\mu}}{k_j^e(z)} \right) \cdot \left(\frac{f_j^e + (\rho+\delta)F_j^e}{\rho+\delta} \right) \right]. \quad (23)$$

From (22), it is clear that the aggregate productivity threshold to open an affiliate in country j is increasing in the fixed and sunk costs of opening the affiliate. Similarly, from (23), the aggregate productivity threshold to export from an affiliate in country j is increasing in the fixed and sunk costs of exporting from the affiliate. Moreover, both thresholds are decreasing in the firm productivity z , indicating that more productive firms need smaller positive aggregate productivity shocks to start and expand affiliate operations compared to less productive firms.

Notice that if $\frac{f_j^h + (\rho+\delta)F_j^h}{P_j^\eta Q_j} < \frac{(f_j^e + (\rho+\delta)F_j^e)\tau^{\eta-1}}{P_{\sim j}^\eta Q_{\sim j}}$, i.e., if the overall cost of a horizontal affiliate relative to its host market size is lower than the overall cost of a diversified affiliate relative to its destination market size, then $X_j^h(z) < X_j^e(z)$. We assume that this restriction holds to illustrate the predictions of the model that follow.

3.3 Testable implications

The baseline model is designed to capture *qualitatively* the facts presented in Section 2. First, we have shown that the affiliate-to-parent life-cycle sales ratio is virtually flat (Figure 1). The presence of only aggregate Home country shocks, together with the assumption that the MNE transfers its productivity to its affiliates abroad, imply that a firm's domestic and foreign sales

¹⁴ $k_B \equiv (\eta-1)(1+\beta-\eta)^{\frac{\eta-1-\beta}{1-\eta}}$, $k_j^h(z) \equiv H(w_j/z)^{1-\eta} E_j$, and $k_j^e(z) \equiv H(\tau_j w_j/z)^{1-\eta} E_{\sim j}$.

perfectly co-move, conditional on entry:¹⁵

$$\frac{\text{sales}_j(z)}{\text{sales}_d(z)} = \left(\frac{w_j/\varphi}{w_d/\varphi} \right)^{1-\eta} \frac{E_j}{E_d}. \quad (24)$$

Second, we documented that the vast majority of affiliates have some horizontal sales at birth and that a negligible share of affiliates are pure exporters. The assumptions we put forward are consistent with these observations: in the model, affiliates are born either with only horizontal sales or with some horizontal sales and some exports. The specification of the aggregate productivity shock as a unit root process drives persistence in the affiliate's type, as observed in the data. Moreover, if aggregate productivity grows over time (i.e., $\mu > 0$), firms tend to expand internationally giving rise to the diversification pattern that we document.

Next, we illustrate the implications of the cross-sectional Melitz-style component of the model. The relationship between firm-level productivity, host market characteristics, and entry and exporting thresholds, $X_j^h(z)$, $X_j^e(z)$, has two sets of implications: within a host market, across affiliates of different MNEs; and within an MNE, across host markets.

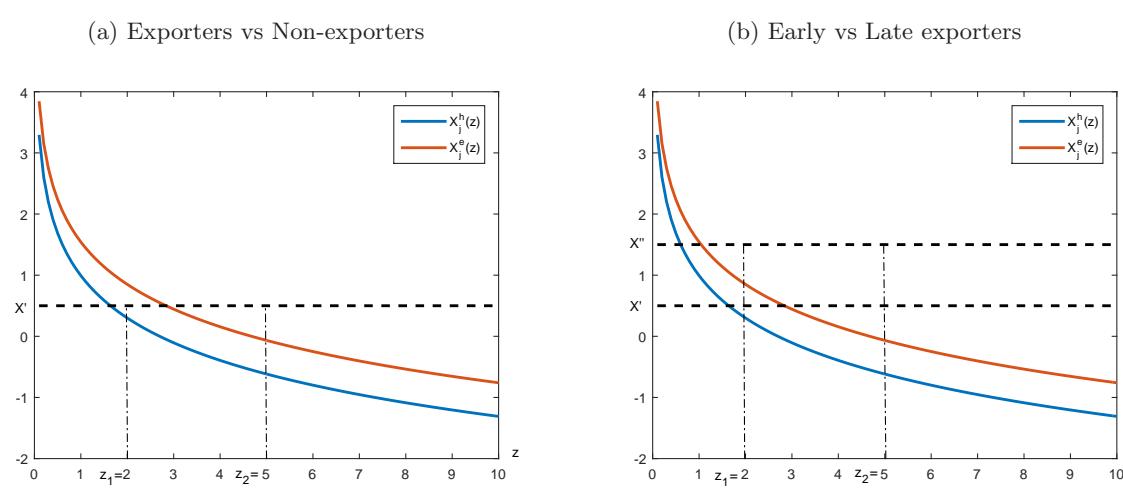
Given a host market j , the model has two clear predictions relating affiliate size in the host market with export status and timing of exports. Since more productive firms have lower entry thresholds ($\partial X_j^h(z)/\partial z \leq 0$ and $\partial X_j^e(z)/\partial z \leq 0$): 1) affiliates that are exporters from birth have larger horizontal sales than affiliates born with exclusively horizontal sales; 2) conditional on aggregate productivity increasing over time, affiliates that start exporting later in life have smaller horizontal sales than affiliates that start exporting earlier in their life cycle.

Figure 4 illustrates these predictions. Suppose the realization of the aggregate shock is X' and we observe two firms having affiliates in the same host country j . Firm 1 (with productivity z_1) has a pure type horizontal affiliate in j , while firm 2 (with productivity z_2) has a diversified affiliate in j . Since the thresholds $X_j^h(z)$, $X_j^e(z)$ are decreasing functions of z , they are invertible. The observed selection pattern of affiliates indicates that $z_2 \geq z_j^e(X') \geq z_1$, hence (since $z_2 \geq z_1$) the horizontal sales of the diversified affiliate of firm 2 must be larger than the horizontal sales of the pure horizontal affiliate of firm 1 (panel 4a). To illustrate the prediction about the timing of exports, suppose now that as aggregate productivity grows on average, the realization of the aggregate shock becomes $X'' > X'$. Now, as illustrated in panel 4b, $z_1 \geq z_j^e(X'')$ and also firm 1 starts exporting from its foreign affiliate. Hence, keeping the host country fixed, early exporting

¹⁵This result is exact in partial equilibrium. In an industry equilibrium, E_j/E_d fluctuates driving fluctuations in the affiliate-to-parent sales ratio. However, the fluctuations induced by productivity shocks on aggregate variables are typically small in this class of models, so we expect that the sales ratio will be relatively stable over time.

affiliates are more productive and exhibit larger horizontal sales than late exporters.

Figure 4: Affiliate size in the host market, export status, and timing of exports.



Regarding the implications about the expansion strategies of an MNE across countries, the model predicts that: 1) since entry thresholds are decreasing in the size of the host market ($\partial X_j^h(z)/\partial E_j \leq 0$), MNEs open first affiliates located in larger countries and subsequently affiliates located in smaller countries; and 2) MNEs open first their largest affiliates and subsequently their smaller affiliates. Moreover, since entry thresholds are increasing in entry costs ($\partial X_j^h(z)/\partial F_j^h \geq 0$), 3) MNEs open first affiliates in markets with lower entry costs.

Figure 5: Market characteristics and timing of entry.

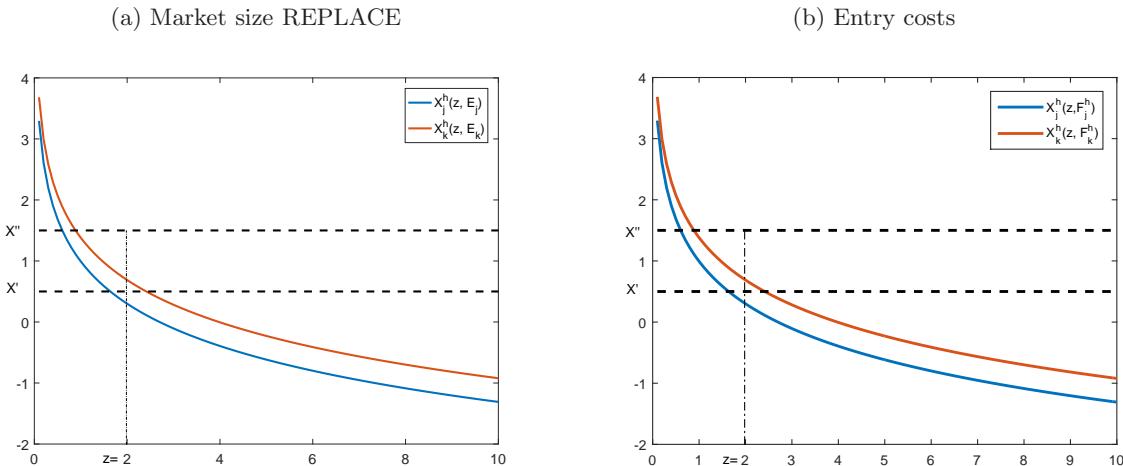


Figure 5 illustrates these predictions. Panel 5a plots affiliate entry thresholds for two host

countries j, k of different size: $E_k < E_j$, so that $X_k^h(z, E_k) \geq X_j^h(z, E_j)$. As illustrated in the figure, firm z only opens an affiliate in country j when the realization of the aggregate shock is X' . When aggregate productivity grows to $X'' > X'$, the firm can afford to open an affiliate also in country k . Since affiliate sales are positively correlated with host country size, the same figure also illustrates the fact that, controlling for factor costs, an MNEs opens first its largest affiliates. Panel 5b plots affiliate entry thresholds for two host countries j, k with different entry costs: $F_k^h > F_j^h$, so that $X_k^h(z, F_k^h) \geq X_j^h(z, F_j^h)$. As illustrated in the figure, firm z only opens an affiliate in country j when the realization of the aggregate shock is X' . When aggregate productivity grows to $X'' > X'$, the firm can afford to open an affiliate also in country k . Notice that these predictions hold within a corporation across affiliates located in different countries.

4 Back to the Data: Testing the Model's Predictions

In this section we go back to the data to test the qualitative implications of the baseline model. The empirical evidence strongly supports the model's predictions about affiliate size, export status, and timing of exports, as well as those about MNEs' expansion across countries.

4.1 Affiliate size, export status, and timing of export

We start by testing the model's predictions that exploit variation within a market across affiliates of different MNEs.

Prediction 1. Exporters at birth have larger horizontal sales than pure-horizontal affiliates at birth. Figure 6a plots the distribution of log horizontal sales for two subsets of affiliates in our sample: affiliates that are born with only horizontal sales, and affiliates that are born also with exports. The figure clearly shows that affiliates that export are on average larger than affiliates with only horizontal sales at birth, consistent with the model's prediction.

Formally, we regress affiliate horizontal sales on a dummy variable equal one if the affiliate is pure horizontal at birth, and zero if the affiliate is also an exporter. Column 1 in Table 2 shows that the negative correlation between size in the host market and being a pure horizontal affiliate at birth survives the inclusion of an age control and of country-year and industry fixed effects.

Mimicking well-documented facts on domestic exporters, this result shows that affiliates that export are larger in their host market than affiliates whose sales are limited to their host country.

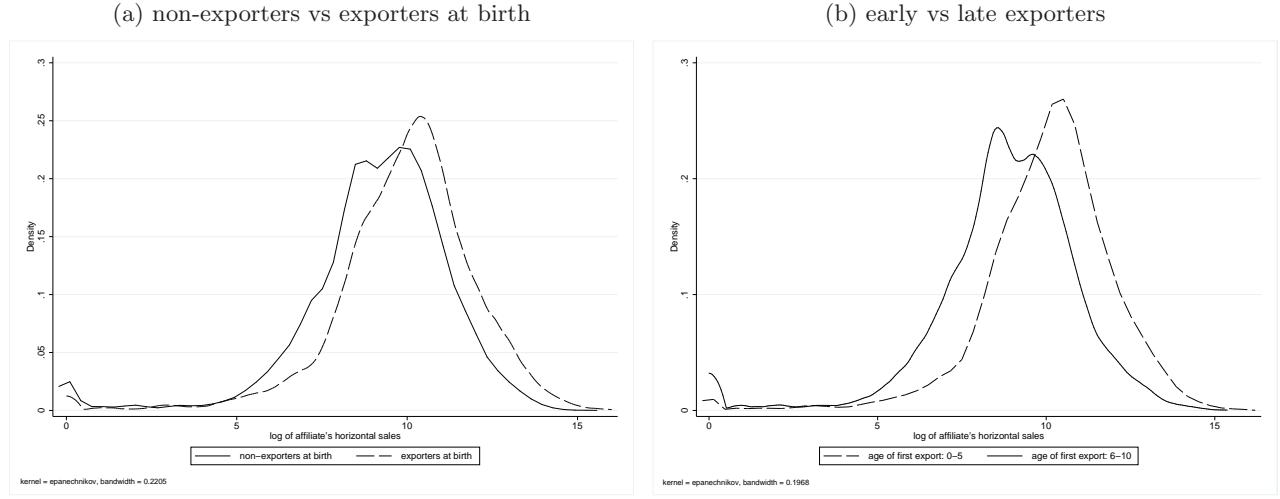
Prediction 2. Affiliates that start exporting earlier in life have larger horizontal sales

Table 2: Affiliate size, by export status, timing of entry, and order in the affiliate opening sequence.
OLS.

Dep var	log of horizontal sales			
	(1)	(2)	(3)	(4)
D(pure horizontal at birth)	-0.979*** (0.085)		-0.698*** (0.079)	
Age at first export		-0.135*** (0.015)	-0.066*** (0.018)	
D(first affiliate)				0.418*** (0.111)
Age	0.069*** (0.012)	0.072*** (0.018)	0.065*** (0.013)	0.080*** (0.010)
D(first affiliate) \times Age				-0.029*** (0.010)
log global employment				0.421*** (0.090)
Parent FE	no	no	no	yes
Obs	33,939	30,117	30,117	33,939
R-sq	0.12	0.11	0.12	0.03

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. $D(\text{pure } H \text{ at birth})$ is equal to one if the affiliate is born with only horizontal sales; and zero otherwise. $D(1st \text{ affiliate})$ is equal to one if the affiliate is first in the opening sequence of the MNE; and zero otherwise. Global employment refers to the aggregate employment of the MNE, both in the United States and abroad. Pure exporters are excluded from the sample. All specifications include country-year and industry fixed effects. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

Figure 6: Affiliate size, export status, and the timing of export entry.



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Kernel density of log horizontal sales for affiliates that: are born with exclusively horizontal sales (non-exporters) and those with exports (exporters), in (6a); start exporting in their first five years of life and those that start after five years of life, in (6b).

than late starters. Figure 6b illustrates the relationship between the size of an affiliate in its host country, measured by the log of sales in the host country, and the time in the affiliate's life cycle at which it decides to export. We split the sample between affiliates that start exporting in the first five years of life and affiliates that start exporting later in their life cycle. As the figure shows, affiliates that start exporting earlier in life are larger in their host country compared to affiliates that start exporting later. The pattern is robust to the choice of the age cutoff for first exports (see Figure B.3 in Appendix B). Moreover, as column 2 in Table 2 shows, the negative relationship between size in the host market and age at first export survives the inclusion of affiliate age, country-year and industry fixed effects.

Column 3 in Table 2 combines Predictions 1 and 2 and shows that they jointly hold in the data.

4.2 The expansion strategies of MNEs across countries

The second set of model's predictions refer to the expansion of individual MNEs across different host markets; that is, we exploit within-MNE variation across countries.

Prediction 3. MNEs open their largest affiliates first. Column 4 in Table 2 confirms this prediction: horizontal sales are systematically larger for the first affiliate of a MNE, controlling for affiliate age and size of the corporation. Notice that this specification includes country-industry and parent fixed effects, so that we exploit variation across affiliates of the same MNE. Table B.4

Table 3: Affiliate size and market characteristics, by affiliate position in the MNE opening sequence.

Avg.	first affiliates	subsequent affiliates
GDP (in bill of U\$)	970	868
number of biz procedures	6.4	7.6
number of days to start biz	20.2	24.6
cost of start biz (% of GDPpc)	7.0	11.8
Min K needed to start biz (U\$)	4,634	6,017

Note: Observations at the affiliate level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Variables related to entry costs are from the World Bank, Doing Business. GDP is from the Penn World Tables (8.0).

in Appendix B provides more statistics differentiating first affiliates from subsequent ones: first affiliates tend to have larger sales and employment than subsequent affiliates, and are more likely to be exporters. Figure B.2 in Appendix B illustrates some of these size differences over the life cycle.

Prediction 4. MNEs open affiliates first in larger markets. The model predicts sorting in the order in which a MNE opens affiliates over time. The first row of Table B.4 illustrates that, on average, affiliates located in a large host market open first.

Prediction 5. MNEs open affiliates first in markets with lower entry costs. Table 3 provides suggestive evidence supporting this prediction. As commonly done in the literature, we proxy for entry costs using indicators from the World Bank's Doing Business Database: the number of administrative procedures required to open a business, the average number of days it takes to open, the cost of starting a business as a percent of GDP per capita, and the minimum capital requirement in US dollars. Table 3 makes clear that, under various measures, MNEs choose, on average, to open affiliates first in markets that are less costly to enter. Countries in which MNEs open their first affiliate have around a 20 percent lower number of business procedures, take 20, rather than 25, days to open a business, and have a cost of starting a business that is two thirds of the cost faced in subsequent markets. Additionally, on average, affiliates are first opened in markets for which the minimum capital needed to start a business is 20 percent lower.

We conclude this section with some additional evidence in support of two of the model's assumptions: the independence assumption and the assumptions on the shock structure.

4.3 Extended gravity and the independence assumption

The tractability of the model hinges on the independence assumption, which postulates that both the decisions of opening an affiliate and of exporting from it are independent across countries. We find support for this assumption in the data.

The independence assumption implies that it is possible for a firm to have an affiliate in a country and at the same time to have affiliates elsewhere that export to that same country. This is different from the models in Arkolakis et al. (2017) and Tintelnot (2017), in which each firm has only one lowest-cost location to reach final consumers in a country.

Even though the BEA data contains limited information about the destination of affiliates' exports, we are able to examine the coexistence of affiliates' exports to three countries (Canada, the United Kingdom, and Japan) with the presence of affiliates owned by the same parents in those countries, for 2004.¹⁶ Our calculations imply that of the 20,359 affiliates that export to Canada, 64 percent belong to a U.S. parent that also has affiliates located in Canada. Similarly, of the 5,017 affiliates that export to the United Kingdom, 70 percent belong to a U.S. parent that also has affiliates located in that country. Finally, of the 5,224 affiliates that export to Japan, 47 percent belong to a U.S. parent that also has affiliates located in Japan.

A more direct test of the independence assumption is, for a given U.S. parent, the comparison between the probability of owning an affiliate in a country and the same probability conditional on already having an affiliate in a “neighboring” country (*i.e.* in a country belonging to the same region). We follow Morales et al. (2015) and call the difference in these probabilities “extended gravity”. Of course, the comparison is only possible for U.S. parents with at least two foreign affiliates. Table B.5 in Appendix B shows the results for MNEs with at least two, five, and ten affiliates worldwide, respectively. Countries are restricted to the ten most popular destinations for U.S. MNEs that belong to four regions: North America, Europe, Latin America, and Asia (Central, South, and East Asia, plus the Pacific). Conditional and unconditional probabilities are strikingly similar the larger the MNE in terms of number of affiliates worldwide. The largest differences are observed when we include smaller MNEs (at least 2 affiliates) in countries –such as China– that are typically part of global supply chains. Compared with the evidence for exporters in Morales et al. (2015), extended gravity is much less pronounced for MNEs opening affiliates than for domestic firms entering export destinations.

¹⁶Benchmark year surveys contain more information about affiliate export destinations than non-benchmark year ones.

4.4 The structure of MNE shocks in the data

Our model assumes that the shock structure of MNEs is composed of a MNE-wide time-invariant component and of a time-varying source-country component. Time-varying host-country components also matter in industry equilibrium. How much of the variation in affiliates sales is captured in the data by these shocks? Table B.6 in Appendix B illustrates that, while country-level time-varying shocks and parent fixed effects explain almost a third of affiliate sales variation in the data, parent-level time-varying variables (i.e. parent sales) contribute little to explain affiliate sales variation in the data. We interpret this evidence as support for our assumptions on the MNE shock which does not rely in any time-varying firm-level component.

To conclude, the evidence in this section suggests that while in the cross-section, affiliates of U.S. MNEs broadly behave like domestic exporters, in the time series, they display much flatter growth profiles, more persistence, and less extended gravity than domestic exporters. The evidence lends further support to modeling the cross-section of MNEs' affiliates along the lines of a Melitz model, and the time series with a productivity process that grows over time due to its aggregate component, and not due to the idiosyncratic component.

5 Quantitative Analysis

We extend the model to make it amenable to quantitative analysis. To this end, firms can endogenously decide to shut down affiliates, chose the destination of affiliate exports, and endogenously decide to exit any export market. We close the model in industry equilibrium with aggregation and determination of the country-level price indexes.

Incorporating endogenous exit is important for two reasons. Empirically, exit rates decline with affiliate age and size, but are independent of whether the affiliate is an exporter or not (see Table B.7 in Appendix B). Quantitatively, incorporating endogenous exit rates in the model allows to separately identify the sunk and fixed costs of affiliate opening by matching moments on entry and exit rates, respectively. In turn, incorporating the choice of the destination of exports for affiliates will allow us to separately identify costs related to the Home market and to third markets, and possibly to separate vertical investment motives distinguishing sales back to the U.S. market.

5.1 Quantitative model

Extending the model to endogenous exit, from a host country and from export markets, is straightforward. The value functions include an extra term, which is the option value of exit, as well as additional value-matching and smooth pasting conditions that deliver the exit thresholds.

Modeling the choice of the destination of affiliate exports is more complex. An affiliate located in any country j can in principle export to any subset of the set of potential export destinations, and the value of an exporting affiliate depends on the set of countries in which the affiliate exports. To maintain the problem tractable, we resort again to an independence assumption. Precisely, we assume that the decision of an affiliate located in country j to export to a country $k \neq j$ is independent from the decision to export to any other country. Relying on this assumption, we can write the problem of the firm as a compound option and solve it backwards, as suggested by Dixit and Pindyck (1994, chap. 10). In other words, conditional on the firm having an affiliate in country j , we solve for the value of exports and of horizontal sales, and determine the thresholds that induce the affiliate to export or stop exporting to each country $k \neq j$. Once determined the value of an affiliate in country j , we solve for the thresholds that induce the firm to open or shut down that affiliate.

The value of a firm with productivity z when the state of the economy is X is:

$$\mathcal{V}(z, X) = V_d(z, X) + \sum_{j=1}^N \max \{V_j^o(z, X), V_j^a(z, X)\}, \quad (25)$$

where $V_d(z, X)$ is the value of domestic sales, $V_j^o(z, X)$ is the option value of opening an affiliate in country j , and $V_j^a(z, X)$ is the value of an affiliate in country j , regardless of the destination of its sales. In turn, we can define $V_j^a(z, X)$ as

$$V_j^a(z, X) = V_j^h(z, X) + \sum_{k \neq j} \max \{V_{jk}^o(z, X), V_{jk}^e(z, X)\}, \quad (26)$$

where $V_j^h(z, X)$ is the value of horizontal sales, $V_{jk}^o(z, X)$ is the option value of exporting to country k for an affiliate located in j , and $V_{jk}^e(z, X)$ is the value of exports to country k for an affiliate located in j . This formulation of the problem is analogous to a compound option because opening an affiliate in a country is equivalent to exercising an option that gives access to another set of options: the options to export to any other country.

We start by solving for $V_{jk}^o(z, X)$ and $V_{jk}^e(z, X)$, conditional on the firm having an affiliate in country j . This is a simple case of interlinked options (see Dixit and Pindyck 1994, chap. 7), that

gives as solution:

$$V_{jk}^o(z, X) = B_{jk}^o(z)e^{\beta X} \quad (27)$$

$$V_{jk}^e(z, X) = \frac{\bar{\pi}_{jk}^e(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_{jk}^e}{\rho + \delta} + A_{jk}^e(z)e^{\alpha X} \quad (28)$$

where $B_{jk}^o(z) > 0$ and $A_{jk}^e(z) > 0$ are firm-specific parameters, and $\alpha < 0$, $\beta > 1$ are the roots of $\frac{1}{2}\sigma^2\beta^2 + \mu\beta - (\rho + \delta) = 0$. As $B_{jk}^o(z)e^{\beta X}$ represents the option value of exporting to country k , and is increasing in the realization of the aggregate productivity shock, similarly $A_{jk}^e(z)e^{\alpha X}$ is the option value of quitting the export market k , and is decreasing in the realization of the aggregate productivity shock, indicating that the option of exiting an export market has a larger value in “bad times”.

For each country pair (j, k) and for each firm with productivity z , the parameters $B_{jk}^o(z) > 0$, $A_{jk}^e(z) > 0$, and the aggregate productivity thresholds that induce the affiliate to start and stop exporting, denoted by X_{jk}^{OE} and X_{jk}^{EO} , respectively, can be recovered from the following system of value-matching and smooth pasting conditions,

$$V_{jk}^o(z, X_{jk}^{OE}) = V_{jk}^e(z, X_{jk}^{OE}) - F_{jk}^e \quad (29)$$

$$V_{jk}^o(z, X_{jk}^{EO}) = V_{jk}^e(z, X_{jk}^{EO}) \quad (30)$$

$$V'_{jk}^o(z, X_{jk}^{OE}) = V'_{jk}^e(z, X_{jk}^{OE}) \quad (31)$$

$$V'_{jk}^o(z, X_{jk}^{EO}) = V'_{jk}^e(z, X_{jk}^{EO}). \quad (32)$$

The value of horizontal sales, conditional on having an affiliate, is given by the present discounted value of profits from horizontal sales plus the option value of shutting down the affiliate,

$$V_j^h(z, X) = \frac{\bar{\pi}_j^h(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta} + A_j^h(z)e^{\alpha X}, \quad (33)$$

where $A_j^h(z) > 0$ is a firm-specific parameter.

The value of an affiliate in country j can then be written as

$$V_j^a(z, X) = \frac{\bar{\pi}_j^h(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta} + A_j^h(z)e^{\alpha X} + \sum_{k \in \mathcal{A}_j(z)} \left[\frac{\bar{\pi}_{jk}^e(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_{jk}^e}{\rho + \delta} + A_{jk}^e(z)e^{\alpha X} \right] + \sum_{k \notin \mathcal{A}_j(z)} \left[B_{jk}^o(z)e^{\beta X} \right] \quad (34)$$

where $\mathcal{A}_j(z)$ denotes the set of export markets to which the affiliate in j with productivity z exports. The independence assumption is clearly shown in (34): the value of an affiliate does not depend on

the sales or on the value of the firm's other affiliates in other countries, but it does depend on the set of potential export destinations from the affiliate's host country.

It remains to solve for the decision of a firm to set up an affiliate in country j . The option value of opening an affiliate in j is

$$V_j^o(z, X) = B_j^o(z)e^{\beta X}. \quad (35)$$

Hence, for each host country j and for each firm with productivity z , the parameters $B_j^o(z) > 0$, $A_j^h(z) > 0$, and the aggregate productivity thresholds that induce the firm to open and shut down an affiliate, denoted by X_j^{OH} , X_j^{HO} , respectively, can be recovered from the following system of value-matching and smooth pasting conditions,

$$V_j^o(z, X_j^{OH}) = V_j^a(z, X_j^{OH}) - F_j^h \quad (36)$$

$$V_j^o(z, X_j^{HO}) = V_j^a(z, X_j^{HO}) \quad (37)$$

$$V'_j^o(z, X_j^{OH}) = V'_j^a(z, X_j^{OH}) \quad (38)$$

$$V'_j^o(z, X_j^{HO}) = V'_j^a(z, X_j^{HO}). \quad (39)$$

Lastly, the value of domestic sales is simply given by the present discounted value of profits from domestic sales,

$$V_d(z, X) = \frac{\pi_d(z, X)}{\rho + \delta - \hat{\mu}}. \quad (40)$$

Details about the solution of the model are contained in Appendix C.

5.2 Industry equilibrium

The industry equilibrium in this economy is defined by a vector of price indexes $\{P_k\}$, for $k = 1, \dots, N$, and by laws of motion ruling the evolution of affiliate operations over time across countries. The price index in country k at time k is

$$P_k^{1-\eta} = \sum_{i=1}^N \sum_{j=1}^N P_{ijk}^{1-\eta} \quad (41)$$

where $P_{jk}(t)$ denotes the price index of varieties produced by affiliates of Home firms located in country j and selling to country k , at time t ,

$$P_{jk}^{1-\eta}(t) = \int_{\Omega_{jk}} \left(\frac{\tau_{jk} w_j}{z Z} \right)^{1-\eta} dz, \quad (42)$$

where Ω_{jk} is the set of Home firms having affiliates in j that export to k .

Let M_i denote the (exogenous) mass of firms from country i . The endogenous mass of affiliates of firms from i located in j , M_{ij} , is given by continuing affiliates plus new affiliates,

$$M'_{ij} = M_{ij} \cdot (1 - G_i(z_{ij}^{HO})) + (M_i - M_{ij}) \cdot (1 - G_i(z_{ij}^{OH})), \quad (43)$$

while the mass of affiliates of firms from i located in j that export to k is given by continuing exporting affiliates to k plus new exporters to k ,

$$M'_{ijk} = M_{ijk} \cdot (1 - G_i(z_{ijk}^{EO})) + (M_{ij} - M_{ijk}) \cdot (1 - G_i(z_{ijk}^{OE})). \quad (44)$$

The variable z_{ij}^{OH} (z_{ij}^{HO}) is the productivity threshold that induces a firm from i to open (shut down) an affiliate in j , while z_{ijk}^{OE} (z_{ijk}^{EO}) is the productivity threshold that induces an affiliate of a firm from i in j to start (stop) exporting from j to k .¹⁷ Finally, $G_i(z)$ denotes the c.d.f. of the (exogenous) firm-level productivity distribution in country i .

5.3 Numerical example: the rise of China

We illustrate the mechanisms of the quantitative model with a numerical example. Assume that there are three countries: the United States, Japan, and China. As in our data, multinational firms are headquartered in the United States. They have to decide whether and where to open affiliates and whether the affiliates will sell only to the host market or also export. The purpose of this numerical exercise is to show that different changes in the cost structure of multinational activity across host countries generate differences in the number of affiliates, sales, and the share of affiliates that exports, by age.

We start with a scenario where China is identical to Japan. In this case, we set $w = P = Q = 1$, $F^h = 10$, $F^e = 8$, $\tau = 2$, and $f^e = f^h = 0.3$. A more realistic scenario is one where China is poorer, but larger, than Japan. In this case we set $w_{chn} = P_{chn} = 0.6$ and $Q_{chn} = 2$; we will refer to this as our “baseline” scenario.

Keeping the baseline values for wages and expenditure unchanged, we perform three comparative statics exercises, meant to depict the “rise of China” in the different ways that our model allows: a drop in iceberg trade costs from China to Japan; a drop in the sunk cost of entering China; and a drop in the sunk cost of exporting from China to Japan. To explore the effect of a reduction in

¹⁷Theorem 1 in Fillat and Garett (2015) assures that these firm-level productivity thresholds are well-defined.

Table 4: Number of U.S. affiliates abroad, by country. Three-country simulation.

	symmetry	baseline	low $\tau_{chn,jpn}$	low $F_{chn,jpn}^e$	low F_{chn}^h
Japan	191	191	191	191	191
China	191	352	352	352	403

Notes: Affiliates that survive at least ten periods in the model, in each country.

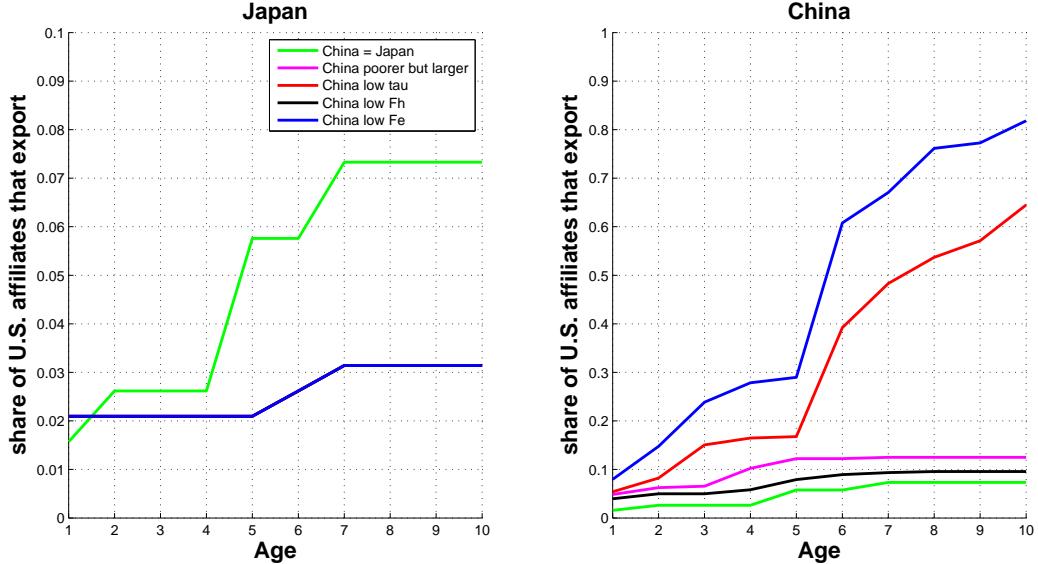
the affiliate entry cost, we lower the fixed cost F_{chn}^h such that the per-period cost of horizontal FDI ($f_{chn}^h + (\rho + \delta)F_{chn}^h$) drops by ten percent. Similarly, to explore the differential effects of reductions in iceberg versus fixed costs of exports, we lower either $\tau_{chn,jpn}$ or $F_{chn,jpn}^e$ in such a way that the per-period cost of affiliate exports ($(f_{chn}^e + (\rho + \delta)F_{chn}^e)\tau_{chn,jpn}$) drops by ten percent. Notice that in this way, a static model would generate exactly the same comparative statics following a change in F_e and a change in τ , so that this exercise is well suited to illustrate the different predictions of our model compared to static ones.¹⁸

Table 4 reports the number of U.S. affiliates located in China and Japan, respectively, under each scenario. Of course, in the symmetric case, the United States opens the same number of plants in the two host countries. As expected, in the baseline scenario where China is larger and poorer than Japan, the number of affiliates located in China increases. Starting from the baseline scenario, a decrease in the sunk cost of opening an affiliate increases the numbers of affiliates located in China. However, changes in the costs of affiliate exports from China to Japan do not change the number of U.S. affiliates that choose to operate in China and survive at least than ten periods. The fact that the number of U.S. affiliates located in Japan does not change with shocks to China's MNE costs is just a consequence of our independence assumption on the MNE location choices and of the partial equilibrium nature of this example.

Figure 7 shows the share of exporting affiliates located in Japan (left panel) and China (right panel) in each of the described scenarios, by age. As expected, in the symmetric case those shares are the same. In the baseline case, where China is larger and poorer, more U.S. firms open affiliates in China than in Japan, and those affiliates have lower costs than the ones in Japan, so that they are more likely to export. Notice that quantitatively, the cost savings of U.S. affiliates in China dominate the fact that U.S. affiliates located in Japan have a larger export market.

¹⁸The remaining parameters of the model are: the time preference parameter, $\rho = 0.02$; the exogenous death rate, $\delta = 0.01$; the elasticity of substitution, $\eta = 2$; the drift for the productivity process, $\mu = 0.02$; the standard deviation for the productivity process $\sigma = 0.06$; the Pareto-shape parameter, $\theta = 3$; and the lower bound of the

Figure 7: Share of exporters, by age. Three-country simulation

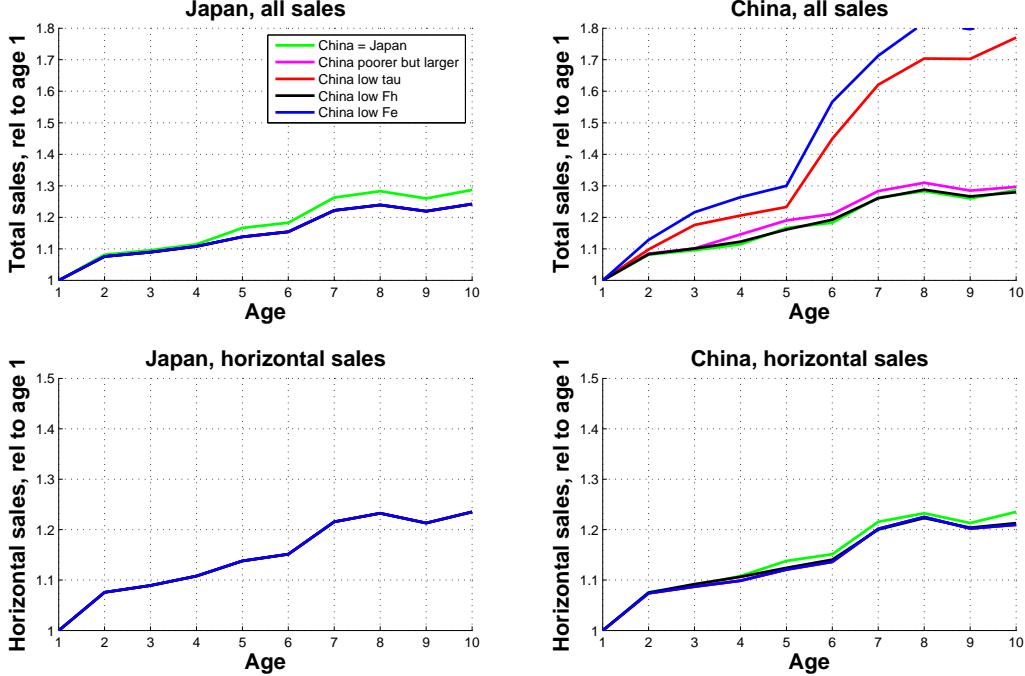


The decrease in the sunk cost F_h significantly increases the number of affiliates in China, but not their likelihood to export, which actually decreases as less productive affiliates enter. In contrast, changes in the variable and the sunk cost of affiliate exports do not affect the number of affiliates, but increase their probability to export. It is interesting how the reduction in the sunk cost of export generates a larger effect than the reduction in the variable cost, despite the numbers being chosen so that the change in per-period costs is the same. The reason for this difference is that when F_e decreases, not only the per period cost decreases, but also the band of inaction shrinks. In other words, affiliates are more likely to start exporting because now their optimal choices are more flexible. The overall effect of F_e is larger than the one of τ because it is a combination of a static and a dynamic effect.

Finally, Figure 8 shows average horizontal sales and total sales of affiliates located in Japan (left panel) and China (right panel) in each of the described scenarios. The “rise of China” has interesting effects on the affiliates’ sales in the countries where the affiliates locate and export to. As the variable cost $\tau_{chn,jpn}$ declines, Chinese affiliates become more profitable because exporting is now less costly. This decline induces more entry in China and increases exports to Japan. Lower entry costs into China barely have an effect on horizontal and total sales of Chinese affiliates, while lower sunk costs of exports for Chinese affiliates have the effect of increasing total sales of Chinese affiliates via exports.

Pareto distribution, $b = 1$. We simulate 500 affiliates and 50 periods.

Figure 8: Affiliate sales, by age. Three-country simulation.



Notes: Affiliates that survive at least ten periods in the model. Averages across affiliates.

5.4 Calibration

[TO BE COMPLETED]

6 Conclusions

This paper is a theoretical and empirical investigation of how the activities of multinational corporations evolve over the life cycle. We establish three novel facts: first, MNEs' affiliates grow little over their life cycle; second, MNEs affiliates' are born specialized in their life-long activity; and third, they diversify a small share of their activities over their life cycle.

These facts guide us in the construction of a simple dynamic model of multinational activity which, albeit stylized, delivers rich testable implications for which we find strong support in the data.

The quantitative model sheds light on the implications of dynamic features of the data for the magnitudes and characteristics of the costs to the MNE, an essential ingredient to study the welfare

gains from multinational activity.

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Table A.1: BEA minimum survey exemptions levels

survey year	Minimum exemption levels (in U\$ millions)	survey year	Minimum exemption levels (in U\$ millions)
1987-88	10	2000-03	30
1989	3	2004	25
1990 -93	15	2005-07	40
1994	3	2008	60
1995-98	20	2009	25
1999	7	2010-11	60

Note: Exemption levels are for majority-owned foreign affiliates. Benchmark survey years are highlighted.

Roberts, M. J. and J. R. Tybout (1997). The decision to export in colombia: An empirical model of entry with sunk costs. *The American Economic Review* 87(4), 545–564.

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Appendix

A Data Description and Summary Statistics

To describe the construction of our sample, it is important to notice that the BEA surveys affiliates of U.S. parents applying minimum survey exemption levels, in terms of affiliate sales, which differ over time. In general, reporting thresholds increased in recent years, reaching US\$60 millions for 2011. Additionally, benchmark survey years (i.e., years in which the survey is more comprehensive), which occur every 4 years, have lower reporting thresholds. Table A.1 shows the reporting thresholds for the years in our sample.

Since the goal of this paper is to understand the evolution of the global structure of production, and not the establishment of headquarters due to profit shifting motives, we restrict our attention to affiliates that do not operate in tax haven countries. Affiliates in tax haven countries are likely to open for different reasons and be subject to different cost structures than those in non-tax haven countries, confounding our analysis. We compile our list of tax havens using information from

Gravelle (2015). We omit countries that meet some of the criteria for tax haven status but that also have a substantial amount of real FDI production from the list. If a country is in the top ten percent of U.S. FDI destinations measured by total U.S. MNE affiliate employment, we consider it to be a location for actual production rather than a strict tax haven. Based on this definition, Ireland, Switzerland, Hong Kong, and Singapore are the only countries from the Gravelle (2015) list that we do not classify as tax havens. A full list of tax haven countries is reported below (TBA).

When a firm has more than one enterprise operating in the same country and industry, we group these enterprises' activities together and refer to them as a single affiliate. We do this for two reasons. First, the firms themselves are permitted to report combined data in this way, making it difficult to isolate individual plants. Based on the BEA definition, an affiliate is a business enterprise in a given industry operating in a particular host country; it thus could operate several plants in different locations within the host country. The BEA rules permit consolidated reporting for distinct enterprises located in the same country that operate in the same narrowly defined industry or otherwise are integral parts of the same business operation. Second, to the extent that the costs of opening a new affiliate are incurred at the country-industry level, this is the appropriate level of aggregation for our analysis. In a robustness exercise, we use reported openings of new enterprises in a country-industry in which the firm already had existing affiliates to check whether costs are incurred at the enterprise or country-industry level. This exercise, though based on noisy data, confirms that focusing on the country-industry-firm-, rather than plant-level, is appropriate.

Table A.2 shows in more detail the distribution of horizontal sales and exports as a share of total affiliate sales. On average, around 72 percent of the sales of an affiliate are destined to the host market, while the remaining 28 percent are exports. The distribution of export sales is extremely skewed: while the 25th percentile of the horizontal shares distribution is above 50 percent and reaches 100 percent at the 75th percentile, the 25th percentile of the export shares distribution is zero, and is still below 50 percent at the 75th percentile. Exports appear to be concentrated within the largest affiliates: the top-ten percent largest affiliates in our sample, in terms of sales, account for 75 percent of all exports, and the top-one percent account for 35 percent of all exports, as shown in Figure A.1a.¹ Finally, around 25 percent of exporting affiliates export less than ten percent of their total output, while around 15 percent export more than 90 percent of their output, as shown in Figure A.1b.

¹Related, affiliates that export have, on average, three times as many sales and twice as many employees as affiliates that do not export. SOURCE?

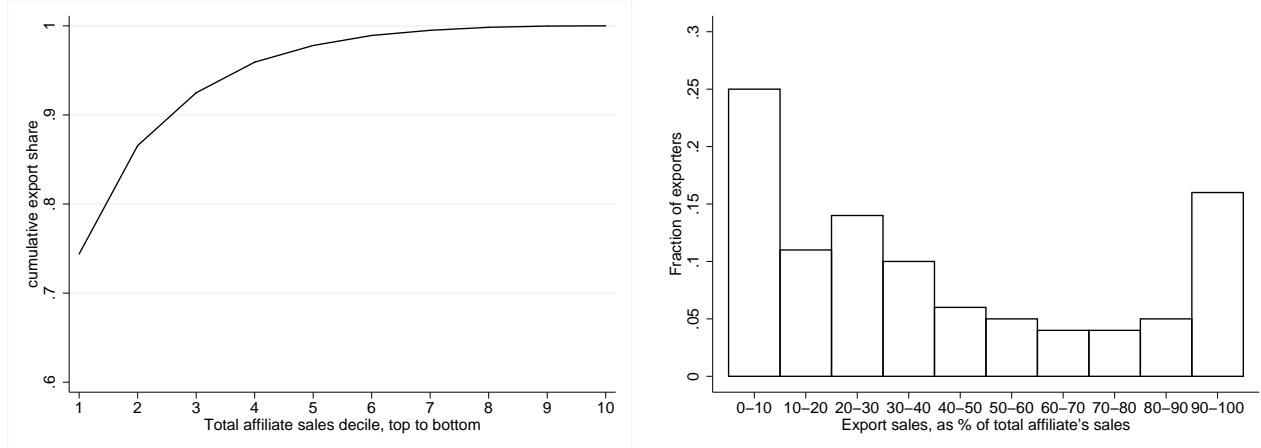
Table A.2: Affiliate sales distribution, by sale type.

	Horizontal sales (as a share of total affiliate sales)	Export sales (as a share of total affiliate sales)
average	0.723	0.277
std dev	0.339	0.339
25 pc	0.539	0.000
50 pc	0.887	0.113
75 pc	1.000	0.461
90 pc	1.000	0.915
95 pc	1.000	1.000

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Percentiles are taken with respect to the variable of interest so that the X-percentile affiliates change as the sorting variable changes. Averages of the 11 firms around the indicated percentile are reported to preserve confidentiality.

Figure A.1: Exporting affiliates, summary statistics.

(a) Size-export distribution (b) Export intensity distribution



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing.

Table B.1: Affiliate-to-parent sales ratio, by sales type. OLS.

Dep var	affiliate-to-parent sales ratio					
	all sales		horizontal sales		export sales	
	(1)	(2)	(3)	(4)	(5)	(6)
affiliate age	0.019 (0.014)	0.005 (0.003)	0.011 (0.007)	0.002 (0.001)	0.011 (0.012)	0.004* (0.002)
industry fe	yes	no	yes	no	yes	no
country-year fe	yes	yes	yes	yes	yes	yes
affiliate fe	no	yes	no	yes	no	yes
Observations	38,088	38,088	36,127	36,127	25,950	25,950
R-square	0.01	0.0002	0.04	0.0003	0.01	0.0001

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. The dependent variable *affiliate to parent sales* refers to affiliate sales in each type, relative to the domestic sales of the U.S. parent. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

B New Facts on MNE Dynamics: Robustness

B.1 Flat Profiles and Affiliate Expansion over Time

Table B.1 reports the results of the following regression:

$$\text{affiliate sales}_a^i / \text{parent sales} = \beta_a \text{age} + FE + \varepsilon_a^i. \quad (\text{B.1})$$

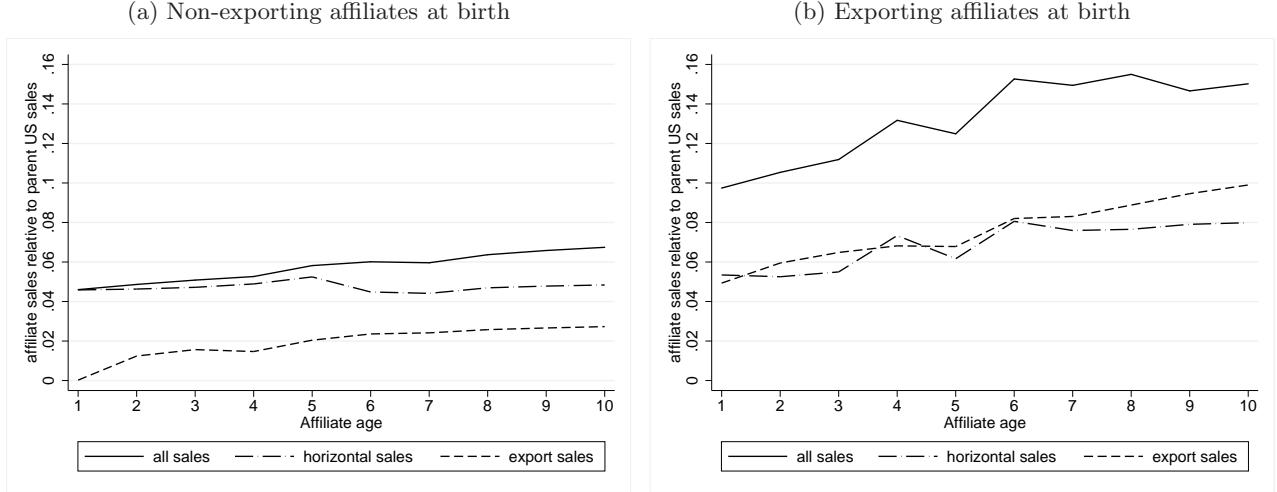
The left-hand side variable is the ratio of sales of type $i = all, H, E$, for a new affiliate of age a , to sales of the parent, age is the age of the affiliate (from 1 to 10), and ε_a^i is the error term. We include country-year and industry fixed effects, and alternatively, country-year and affiliate fixed effects.² The flat profiles observed in Figure 1 are confirmed by the regression analysis: the ratio of affiliate to parent sales is not significantly correlated with affiliate age, controlling for country-year and industry fixed effects. The result holds both with and without affiliate fixed effects. As a robustness check, we also include as controls the size of the affiliate and the size of the corporation, measured as employment and global sales, respectively; results are unchanged.

The average across all affiliates shown in Figure 1 masks some heterogeneity between affiliates born with exclusively horizontal sales and the ones born with some exports. Figure B.1 shows the

²We drop outliers which turn out to be five percent of observations.

ratio of affiliate-to-parent sales, by affiliate age, for affiliates born with exclusively horizontal sales (B.1a), and affiliates born with both horizontal and export sales (B.1b). Each figure, in turn, shows all, horizontal, and export affiliate sales, relative to the parent's sales.

Figure B.1: Affiliate-to-parent sales ratio, by sales and affiliate type .



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Pure horizontal affiliates at birth refers to affiliates born with sales to the market of operation exclusively. Exporting affiliates at birth refers to affiliates born with both sales to the market of operation and other markets. Simple averages taken over affiliates with positive sales in each category.

While exporters at birth are born with sales of about ten percent their parent's and reach 15 percent by age ten, non-exporting affiliates at birth are much smaller, with sales around five percent their parent's. This pattern is similar if we consider, for each group of affiliates, sales to the market of operations or exports.

REGRESSIONS TBA

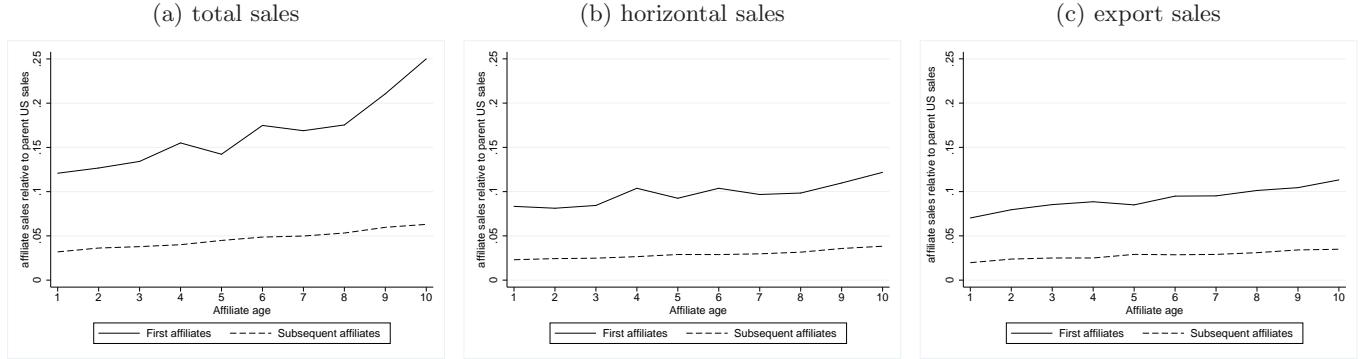
As a further step to describe sales growth profiles of different affiliates, Figure B.2 shows that first affiliates are much larger than subsequent affiliates, but they do not appear to grow faster.

To illustrate the evolution of horizontal and export sales shares over time, Tables B.2 and B.3 report the results of the following regression:

$$\text{affiliate sales}_a^i / \text{total affiliate sales}_a = \beta_a \text{age} + FE + \varepsilon_a^i \quad (\text{B.2})$$

where the left-hand side variable the share of horizontal or export sales of the affiliate, age is the age of the affiliate (from 1 to 10), and ε_a^i is the error term. We include country-year and industry fixed effects, and alternately, country-year and affiliate fixed effects. In a robustness exercise, we

Figure B.2: Affiliate size, by activity type, age, and position in the MNE opening sequence.



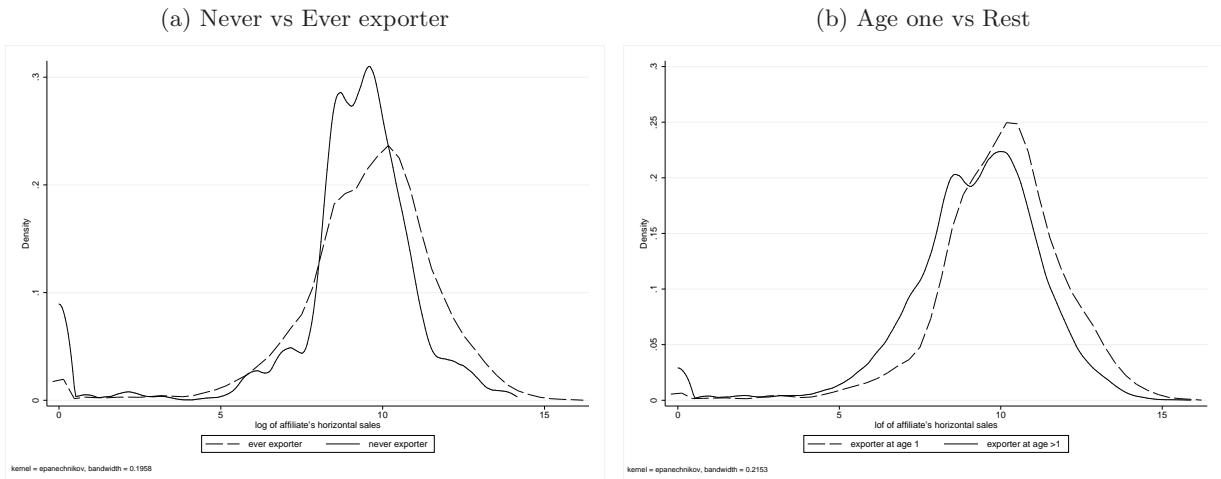
Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. First affiliates refer to the first affiliates opened by the MNE, while subsequent affiliates refer to the rest (2nd, 3rd, ..., nth).

also include the size of the affiliate and the size of the corporation measured as employment and global sales, respectively; results are unchanged.

B.2 Testing the Predictions of the Model

Figure B.3 illustrates the relationship between affiliate size and timing of export for different time cutoffs.

Figure B.3: Affiliate size, export status, and timing of export entry.



Notes: Sample of new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Pure exporters are not included. Kernel density of (log) horizontal sales for affiliates that: never export versus the ones that export at some point in their life, in (B.3a); first export at age one versus the ones that export at any age older than one, (B.3b).

Table B.4 illustrates systematic differences between the first affiliate and subsequent affiliates

Table B.2: Intensive and extensive margins of sale shares. OLS.

Dep var	Intensive margin of sale shares				Extensive margin of sale shares			
	horizontal sales		export sales		horizontal sales		export sale share	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
affiliate age	-0.002 (0.002)	-0.012*** (0.001)	-0.005** (0.002)	0.005*** (0.001)	0.00003 (0.001)	-0.001 (0.0006)	0.014*** (0.003)	0.029 (0.002)
industry fe	yes	no	yes	no	yes	no	yes	no
country-year fe	yes	yes	yes	yes	yes	yes	yes	yes
affiliate fe	no	yes	no	yes	no	yes	no	yes
Observations	36,127	36,127	25,950	25,950	38,088	38,088	38,088	38,088
R-square	0.079	0.013	0.092	0.000	0.042	0.0001	0.081	0.036

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. In columns (1)-(4), the dependent variable is horizontal (export) sales, as a share of total affiliate's sales, for affiliates with positive horizontal (export) sales; in columns (5)-(8), the dependent variable is the share of affiliates with positive horizontal (export) sales. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

Table B.3: Intensive and extensive margins of sale shares, pure-type affiliates at birth. OLS.

Dep var	Intensive margin of sale shares				Extensive margin of sale shares			
	horizontal sales		export sales		horizontal sales		export sale share	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
affiliate age	-0.014*** (0.001)	-0.096*** (0.001)	-0.036** (0.004)	-0.021*** (0.003)	-0.046*** (0.004)	-0.044*** (0.002)	-0.059*** (0.005)	-0.038*** (0.004)
industry fe	yes	no	yes	no	yes	no	yes	no
country-year fe	yes	yes	yes	yes	yes	yes	yes	yes
affiliate fe	no	yes	no	yes	no	yes	no	yes
Observations	19,463	19,463	3,032	3,032	19,905	19,905	3,595	3,595
R-square	0.147	0.020	0.288	0.133	0.245	0.099	0.268	0.125

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. In columns (1)-(4), the dependent variable is horizontal (export) sales, as a share of total affiliate's sales, for affiliates born with only horizontal (export) sales; in columns (5)-(8), the dependent variable is the share of affiliates born with only horizontal (export) sales. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

Table B.4: Affiliate size, by affiliate position in the MNE opening sequence.

Avg.	first affiliates	subsequent affiliates
affiliate employment	709	526
affiliate sales, as % of parent's	20	6.5
affiliate H-sales, as % of parent's	10	3.8
diversified affiliates (%)	65.7	59.3

Note: Observations at the affiliate level, for new majority-owned affiliates that survive for at least 10 consecutive years, in manufacturing.

of a MNE in terms of size (defined as sales and employment) and likelihood of being an exporter.

Table B.5 provides support for the independence assumption by comparing the unconditional probability that a MNE opens an affiliate in a country with the probability of opening conditional on already having an affiliate in a “neighboring” country (*i.e.*, a country located in the same continent).

Finally, table B.6 provides support for the structure of shocks that we assumed in the model by illustrating that aggregate country-specific shocks and parent fixed effects account about 30 percent of the variation in affiliate sales in our sample.

Table B.5: Probabilities of observing affiliates in top-ten most popular destinations.

MNEs with:	Probability of observing an affiliate in the same region					
	at least 10 affiliates		at least 5 affiliates		at least 2 affiliates	
	unconditional	conditional	unconditional	conditional	unconditional	conditional
1 Canada	0.61	0.70	0.51	0.62	0.42	0.54
2 United Kingdom	0.68	0.64	0.62	0.64	0.57	0.59
3 Germany	0.71	0.72	0.60	0.62	0.40	0.51
4 Ireland	0.30	0.30	0.21	0.22	0.13	0.16
5 China	0.66	0.68	0.48	0.54	0.29	0.44
6 France	0.70	0.71	0.57	0.59	0.36	0.45
7 Brazil	0.60	0.60	0.50	0.47	0.26	0.31
8 Singapore	0.45	0.46	0.46	0.36	0.19	0.28
9 Mexico	0.61	0.70	0.50	0.54	0.32	0.40
10 Japan	0.31	0.32	0.25	0.22	0.21	0.21

Note: Countries in the list belong to four regions (Europe, Asia, Latin America, and North America). Conditional probabilities refer to the probability of observing an affiliate country i given that the parent has an affiliate in a different country of the same region. The number of parents with at least X affiliates worldwide is: 2,896 (X=10), 6,562 (X=5), and 14,974 (X=2).

Table B.6: MNE shock structure. OLS.

Dep var	log affiliate horizontal sales							
	Country-industry fe				Country-year fe			
parent fe	no	yes	yes	no	no	yes	yes	no
affiliate fe	no	no	no	yes	no	no	no	yes
US GDP	yes	yes	yes	yes	no	no	no	no
Host country GDP	yes	yes	yes	yes	no	no	no	no
parent sales	no	no	yes	yes	no	no	yes	yes
R-sq	0.257	0.285	0.307	0.829	0.088	0.321	0.348	0.854
R-sq adj	0.243	0.268	0.290	0.794	0.072	0.305	0.332	0.824
Observations	153,773				155,962			

Notes: Sample of all affiliates born during the sample period. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

Table B.7: Affiliate exit, by affiliate type. OLS.

Dep var	D(affiliate exit)		
	all affiliates	pure horizontal affiliates	exporting affiliates
age	-0.001* (0.0003)	-0.001 (0.001)	-0.001** (0.0003)
log affiliate employment	-0.016*** (0.0007)	-0.014*** (0.001)	-0.015*** (0.001)
log global employment	0.001 (0.001)	0.001 (0.002)	0.001 (0.001)
Observations	156,446	47,772	102,518
R-square	0.031	0.033	0.031

Note: Observations at the affiliate-year level, for new majority-owned affiliates that survive for at least ten consecutive years, in manufacturing. Pure exporters are not included. The variable D(affiliate exit) equals one if the affiliate exits the following year. The variable "global employment" refers to the aggregate MNE employment, both in the United States and abroad. All specifications with country-year and industry fixed effects. Standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

C Derivation of the Solution of the Model

In this Appendix we provide details about the solution procedure of the model presented in Section 3 and of its quantitative extension in Section 5.

C.1 Solution of the value functions in the baseline model

In Section 3 we specified the problem of the firm by defining the Bellman equations. We derive here the solution of the value functions.

We start by writing equation (8) in the continuation region:

$$[1 + (\rho + \delta)\Delta t]V_d(z, X) = \pi_d(z, X)\Delta t + E[V_d(z, X')|X].$$

Taking the limit for $\Delta t \rightarrow 0$:

$$\begin{aligned} (\rho + \delta)V_d(z, X)dt &= \pi_d(z, X)dt + E[V_d(z, X')|X] - V_d(z, X) \\ (\rho + \delta)V_d(z, X) &= \pi_d(z, X) + E\left[\frac{dV_d(z, X)}{dt}\right]. \end{aligned} \tag{C.1}$$

From Ito's Lemma:

$$E[dV_d(z, X)] = \mu V'_d(z, X) + \frac{\sigma^2}{2} V''_d(z, X), \quad (\text{C.2})$$

where $V'_d(z, X)$ and $V''_d(z, X)$ denote the first and second derivative of the value function with respect to X . By substituting (C.2) into (C.1), we obtain the non-arbitrage condition:

$$(\rho + \delta)V_d(z, X) = \pi_d(z, X) + \mu V'_d(z, X) + \frac{\sigma^2}{2} V''_d(z, X) \quad (\text{C.3})$$

which states that the expected value of the asset (profit flow plus expected change) must be equal to its normal return.

We guess the following form for the value function:

$$V_d(z, X) = W_d(z, X) + e^{\xi X}$$

and substitute it into (C.3):

$$(\rho + \delta)[W_d(z, X) + e^{\xi X}] = \pi_d(z, X) + \mu[W'_d(z, X) + \xi e^{\xi X}] + \frac{\sigma^2}{2}[W''_d(z, X) + \xi^2 e^{\xi X}].$$

We solve using the method of undetermined coefficients. Collecting the homogeneous terms we obtain:

$$\begin{aligned} (\rho + \delta)e^{\xi X} &= \mu\xi e^{\xi X} + \frac{\sigma^2}{2}\xi^2 e^{\xi X} \\ (\rho + \delta) &= \mu\xi + \frac{\sigma^2}{2}\xi^2. \end{aligned}$$

Hence ξ is given by the solution of the quadratic equation:

$$\xi = \frac{-\mu \pm \sqrt{\mu^2 + 2\sigma^2(\rho + \delta)}}{\sigma^2}. \quad (\text{C.4})$$

Collecting the nonhomogeneous term we obtain:

$$(\rho + \delta)W_d(z, X) = \pi_d(z, X) + \mu W'_d(z, X) + \frac{\sigma^2}{2} W''_d(z, X). \quad (\text{C.5})$$

We guess the following form for the nonhomogeneous term:

$$W_d(z, X) = \frac{\pi_d(z, X)}{\kappa} = \frac{H \left(\frac{w_d}{z} \right)^{1-\eta} e^{(\eta-1)X} P_d^\eta Q_d}{\kappa}.$$

Substituting it into C.5:

$$\begin{aligned}
(\rho + \delta) \frac{H \left(\frac{w_d}{z} \right)^{1-\eta} e^{(\eta-1)X} P_d^\eta Q_d}{\kappa} &= H \left(\frac{w_d}{z} \right)^{1-\eta} e^{(\eta-1)X} P_d^\eta Q_d + \mu \frac{H \left(\frac{w_d}{z} \right)^{1-\eta} (\eta-1) e^{(\eta-1)X} P_d^\eta Q_d}{\kappa} + \dots \\
&\quad \dots \frac{\sigma^2 H \left(\frac{w_d}{z} \right)^{1-\eta} (\eta-1)^2 e^{(\eta-1)X} P_d^\eta Q_d}{\kappa} \\
(\rho + \delta) \frac{1}{\kappa} &= 1 + \frac{\mu(\eta-1)}{\kappa} + \frac{\sigma^2}{2} \frac{(\eta-1)^2}{\kappa} \\
\kappa &= (\rho + \delta) - \mu(\eta-1) - \frac{\sigma^2}{2} (\eta-1)^2
\end{aligned}$$

so that:

$$W_d(z, X) = \frac{\pi_d(z, X)}{(\rho + \delta) - \mu(\eta-1) - \frac{\sigma^2}{2} (\eta-1)^2}.$$

Hence the general solution for the value of domestic sales is given by:

$$V_d(z, X) = \frac{\pi_d(z, X)}{(\rho + \delta) - \mu(\eta-1) - \frac{\sigma^2}{2} (\eta-1)^2} + A_d(z) e^{\alpha X} + B_d(z) e^{\beta X},$$

where $\alpha < 0$ and $\beta > 1$ are the two values of ξ . Since all firms always have domestic sales, there is no option value of domestic sales, so $A_d(z) = B_d(z) = 0$, and the value of domestic sales is simply given by the present discounted value of domestic profits.

We proceed in an analogous way to solve for the other value functions. By following the same steps as above, we obtain the following no-arbitrage condition for the option value of an affiliate $V_j^o(z, X)$.

$$(\rho + \delta) V_j^o(z, X) = \mu V'_j(z, X) + \frac{\sigma^2}{2} V''_j(z, X). \quad (\text{C.6})$$

We guess the following form for the value function:

$$V_j^o(z, X) = e^{\xi X}$$

and following the procedure outlined above, we obtain the general solution:

$$V_j^o(z, X) = A_j^o(z) e^{\alpha X} + B_j^o(z) e^{\beta X}$$

where $\alpha < 0$ and $\beta > 1$ are the two values of ξ . Notice that there are no profit flows associated with an option value function. Finally, as $X \rightarrow 0$, the option of opening an affiliate becomes worthless, so it must be that $A_j^o(z) = 0$. Conversely, the option of opening an affiliate becomes more attractive as X increases, so it must be that $B_j^o(z) > 0$.

The no-arbitrage condition for the value of a pure-type horizontal affiliate $V_j^h(z, X)$ is:

$$(\rho + \delta)V_j^h(z, X) = \pi_j^h(z, X) + \mu V'_j(z, X) + \frac{\sigma^2}{2}V''_j(z, X). \quad (\text{C.7})$$

We guess the following form for the value function:

$$V_j^h(z, X) = W_j^h(z, X) + e^{\xi X}.$$

The homogeneous term has the same functional form as in the previous cases. We then guess the following form for the non-homogeneous term:

$$W_j^h(z, X) = \frac{\bar{\pi}_j^h(z, X)}{\kappa_1} + \frac{f_j^h}{\kappa_2}$$

and following again the method of undetermined coefficients we find:

$$\begin{aligned} \kappa_1 &= (\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2 \\ \kappa_2 &= (\rho + \delta) \end{aligned}$$

so that the general solution for the value of a pure-type horizontal affiliate is:

$$V_j^h(z, X) = A_j^h(z)e^{\alpha X} + B_j^h(z)e^{\beta X} + \frac{\bar{\pi}_j^h(z, X)}{(\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2} - \frac{f_j^h}{(\rho + \delta)}.$$

Notice that, as $X \rightarrow 0$, the value of horizontal sales also goes to zero (exit is only exogenous and random in this version of the model), so it must be that $A_j^h(z) = 0$. Conversely, as X increases, the value of the affiliate also increases because of the option of starting to export, so it must be that $B_j^h(z) > 0$.

Finally, the no-arbitrage condition for the value of a diversified affiliate $V_j^e(z, X)$ is:

$$(\rho + \delta)V_j^e(z, X) = \pi_j^h(z, X) + \pi_j^e(z, X) + \mu V'_j(z, X) + \frac{\sigma^2}{2}V''_j(z, X). \quad (\text{C.8})$$

Also here, we guess the following form for the value function:

$$V_j^e(z, X) = W_j^e(z, X) + e^{\xi X}.$$

The homogeneous term has the same functional form as in the previous cases. We then guess the

following form for the non-homogeneous term:

$$W_j^e(z, X) = \frac{\bar{\pi}_j^h(z, X) + \bar{\pi}_j^e(z, X)}{\kappa_1} - \frac{f_j^h + f_j^e}{\kappa_2}$$

and following again the method of undetermined coefficients we find:

$$\begin{aligned}\kappa_1 &= (\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2 \\ \kappa_2 &= (\rho + \delta)\end{aligned}$$

so that the general solution for the value of a diversified affiliate is:

$$V_j^e(z, X) = A_j^e(z)e^{\alpha X} + B_j^e(z)e^{\beta X} + \frac{\bar{\pi}_j^h(z, X) + \bar{\pi}_j^e(z, X)}{(\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2} - \frac{f_j^h + f_j^e}{(\rho + \delta)}.$$

Notice that, as $X \rightarrow 0$, the value of the affiliate also goes to zero (exit is only exogenous and random in this version of the model), so it must be that $A_j^e(z) = 0$. Also, as X increases, the value of the affiliate converges to the discounted profit flow (there is no further expansion option), so it must be that $B_j^e(z) = 0$.

C.2 Solution of the value functions in the quantitative model

In Section 5 we outlined the solution of the model with endogenous affiliate exit, choice of affiliate export destinations, and endogenous exit from each export market. We provide here some details about the derivation of the solution of the value functions in this more general case.

Since the model has the structure of a compound option, we solve it backwards, starting from the problem of a firm that already has an affiliate in country j and has to decide whether to export to any country $k \neq j$.

The Bellman equation describing the value of the option to export to country k for a firm with an affiliate in country j is:

$$V_{jk}^o(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} E[V_{jk}^o(z, X')|X]; V_{jk}^e(z, X) - F_{jk}^e \right\}, \quad (\text{C.9})$$

which describes the fact that the affiliate may keep the option of exporting to country k (and get the continuation value of that option), or may start exporting to country k , in which case it pays the entry cost F_{jk}^e and gets the value of exporting to k from j , $V_{jk}^e(z, X)$. Writing equation (C.9) in the continuation region, taking the limit for $\Delta t \rightarrow 0$, and applying Ito's Lemma, we obtain the

non-arbitrage condition:

$$(\rho + \delta)V_{jk}^o(z, X) = \mu V'_{jk}(z, X) + \frac{\sigma^2}{2}V''_{jk}(z, X). \quad (\text{C.10})$$

Following the same procedure outlined in the previous section, we can conclude that the value of the option of exporting to country k for an affiliate in country j has the following general solution:

$$V_{jk}^o(z, X) = A_{jk}^o(z)e^{\alpha X} + B_{jk}^o(z)e^{\beta X}$$

where $\alpha < 0$ and $\beta > 1$ are the two values of ξ . As $X \rightarrow 0$, the option of exporting becomes worthless, so it must be that $A_{jk}^o(z) = 0$. Conversely, the option of exporting becomes more attractive as X increases, so it must be that $B_{jk}^o(z) > 0$.

Similarly, the Bellman equation describing the value of exporting to country k from an affiliate in country j is:

$$V_{jk}^e(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} [\pi_{jk}^e(z, X)\Delta t + E[V_{jk}^e(z, X')|X]] ; V_{jk}^o(z, X) \right\}, \quad (\text{C.11})$$

which describes the fact that the affiliate may keep exporting to country k (and get the continuation value of that option), or may stop exporting to country k , in which case it gets the value of the option of exporting to k from j , $V_{jk}^o(z, X)$. Writing equation (C.11) in the continuation region, taking the limit for $\Delta t \rightarrow 0$, and applying Ito's Lemma, we obtain the non-arbitrage condition:

$$(\rho + \delta)V_{jk}^e(z, X) = \pi_{jk}^e(z, X) + \mu V'_{jk}(z, X) + \frac{\sigma^2}{2}V''_{jk}(z, X). \quad (\text{C.12})$$

Following the same procedure outlined in the previous section, we can conclude that the value of the option of exporting to country k for an affiliate in country j has the following general solution:

$$V_{jk}^e(z, X) = A_{jk}^e(z)e^{\alpha X} + B_{jk}^e(z)e^{\beta X} + \frac{\bar{\pi}_j^e(z, X)}{(\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2} - \frac{f_j^e}{(\rho + \delta)}.$$

Notice that, as $X \rightarrow 0$, there is value from the possibility of endogenously stopping to export, so it must be that $A_{jk}^e(z) > 0$. Also, as X increases, the value of exports converges to the discounted profit flow (there is no further expansion option), so it must be that $B_{jk}^e(z) = 0$.

The Bellman equation describing the value of horizontal sales for an affiliate in country j is:

$$V_j^h(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} [\pi_j^h(z, X)\Delta t + E[V_j^h(z, X')|X]] ; V_j^o(z, X) \right\}, \quad (\text{C.13})$$

which describes the fact that the affiliate may keep surviving and have horizontal sales in j , or may shut down, in which case the firm gets the value of the option of opening an affiliate in j , $V_j^o(z, X)$. Writing equation (C.13) in the continuation region, taking the limit for $\Delta t \rightarrow 0$, and applying Ito's Lemma, we obtain the non-arbitrage condition:

$$(\rho + \delta)V_j^h(z, X) = \pi_j^h(z, X) + \mu V'_j(z, X) + \frac{\sigma^2}{2}V''_j(z, X). \quad (\text{C.14})$$

Following the same procedure outlined in the previous section, we can conclude that the value of horizontal sales for an affiliate in country j has the following general solution:

$$V_j^h(z, X) = A_j^h(z)e^{\alpha X} + B_j^h(z)e^{\beta X} + \frac{\bar{\pi}_j^h(z, X)}{(\rho + \delta) - \mu(\eta - 1) - \frac{\sigma^2}{2}(\eta - 1)^2} - \frac{f_j^h}{(\rho + \delta)}.$$

Notice that, as $X \rightarrow 0$, there is value from the possibility of shutting down the affiliate, so it must be that $A_j^h(z) > 0$. Also, as X increases, the value of horizontal sales converges to the discounted profit flow (the option value of exports has been determined already above), so it must be that $B_j^h(z) = 0$.

At this point, the value of an affiliate in country j , $V_j^a(z, X)$ is completely characterized up to the option value parameter $A_j^h(z)$:

$$V_j^a(z, X) = A_j^h(z)e^{\alpha X} + \frac{\bar{\pi}_j^h(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta} + \sum_{k \in \mathcal{A}_j(z)} \left[\frac{\bar{\pi}_{jk}^e(z, X)}{\rho + \delta - \hat{\mu}} - \frac{f_{jk}^e}{\rho + \delta} + A_{jk}^e(z)e^{\alpha X} \right] + \sum_{k \notin \mathcal{A}_j(z)} \left[B_{jk}^o(z)e^{\beta X} \right] \quad (\text{C.15})$$

where $\mathcal{A}_j(z)$ denotes the set of export markets in which an affiliate of a firm with productivity z located in country j exports.

Lastly, the Bellman equation describing the value of the option to open an affiliate in country j is:

$$V_j^o(z, X) = \max \left\{ \frac{1}{1 + (\rho + \delta)\Delta t} E[V_j^o(z, X')|X]; V_j^a(z, X) - F_j^h \right\}, \quad (\text{C.16})$$

which describes the fact that the affiliate may keep the option of opening an affiliate in country j (and get the continuation value of that option), or may open an affiliate in country k , in which case it pays the entry cost F_j^h and gets the value of an affiliate in country j , $V_j^a(z, X)$. Writing equation (C.16) in the continuation region, taking the limit for $\Delta t \rightarrow 0$, and applying Ito's Lemma, we obtain the non-arbitrage condition:

$$(\rho + \delta)V_j^o(z, X) = \mu V'_j(z, X) + \frac{\sigma^2}{2}V''_j(z, X). \quad (\text{C.17})$$

Following the same procedure outlined in the previous section, we can conclude that the value of the option of exporting to country k for an affiliate in country k has the following general solution:

$$V_j^o(z, X) = A_j^o(z)e^{\alpha X} + B_j^o(z)e^{\beta X}$$

where $\alpha < 0$ and $\beta > 1$ are the two values of ξ . As $X \rightarrow 0$, the option of opening an affiliate becomes worthless, so it must be that $A_j^o(z) = 0$. Conversely, the option of opening an affiliate becomes more attractive as X increases, so it must be that $B_j^o(z) > 0$.

Finally, the determination of the option value parameters in the more general case needs attention: when the firm decides to open an affiliate in a country, it considers not only the value of its horizontal sales, but also the option value of potential export to any destination country. For this reason, the value-matching and smooth-pasting conditions that deliver the parameters $A_j^h(z)$, $B_j^o(z)$ together with the aggregate productivity thresholds that induce the firm to open or shut down the affiliate (X_j^{OH} and X_j^{HO}) are computed using the option value function $V_j^o(z, X)$ and the total value of the affiliate $V_j^a(z, X)$.