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Positional Advantage in Networks

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Recent research in strategy has called attention to the fact that particular positions in inter-firm networks may serve as a source of competitive advantage for the firms occupying them. This empirical literature has nonetheless found it difficult to separate the effects of positions from those of firm capabilities and resources. We develop a general model for addressing this issue analytically. Our results suggest that agents can enjoy a competitive advantage due only to their positions, but only when several conditions hold, most notably: (i) the agent has relationships to at least three other firms; and (ii) the agent does not hold too strong a position. We also assess the stability of competitive advantages, finding that, while capabilities and resources can confer a stable competitive advantage, positional advantage is not robust to the activities that others might use to diffuse them.

Key words: social networks, strategy, biform games

1. Introduction

One of, if not the, central issue in strategy has been understanding the determinants of firm performance and, from a practical perspective, which of these determinants managers can influence. What gives a firm an edge in its engagements with other firms? When does one firm have a "competitive advantage" over rivals? Though answers to these questions abound, one important line of theory and research within this domain holds that firms in attractive "positions" can consistently earn economic profits (i.e. appropriate value in excess of the opportunity costs of creating it). This logic appears, for example, in the notion that differentiation offers a route to sustainable advantage (Hunt, 1972; Porter, 1980), in the theory that particular combinations of activities defy imitation (Porter & Rivkin, 1998; Rivkin, 2000), and even in the idea that geographic location might serve as a strategic factor (Marshall, 1920; Porter, 1990; Sorenson & Baum, 2003, provide a review).

Within this vein, a recent literature building on the sociological research on social networks has begun to examine whether a firm's location in a pattern of exchange might also serve as a potential source of competitive advantage. Indeed, this topic has become one of the most active domains of research in management today.[1] Consistent relations between buyers and suppliers, for example,

[1] Though we doubt that many would question this assertion, for evidence one needs look no further than the dozens of

have been found to influence the choice of exchange partners, the prices of the goods they exchange, and the viability of both parties (Baker, 1984; Uzzi, 1996, 1999; Sorenson & Waguespack, 2005). Research has shown that firms occupying central positions in syndication networks enjoy access to a wider range of potential deals, and receive better terms and earn higher rates of return in those transactions (Podolny, 1993; Sorenson & Stuart, 2000; Hsu, 2004; Hochberg, Ljungqvist & Lu, 2006). And firms' positions in strategic alliance—and other forms of collaboration—networks have been found to influence their innovativeness and performance (Stuart, Hoang & Hybels, 1999; Ahuja, 2000; Stuart, 2000). The list could go on and on.

Position, with respect to a network, nonetheless differs from positioning in the sense of location in a market or geographic space. It also differs from the conditions governing the sociological settings from which much of the theory it builds upon emerged. For one, even when a relationship exists, exchange only occurs when both parties agree to the trade and joint production only takes place when both parties agree to cooperate. One might therefore worry that partners would attempt to grab a larger share of the total earnings, potentially limiting the ability of those in "attractive" positions to appropriate value. Secondly, though any broad characterization of this large and dynamic literature risks severe oversimplification, we believe that researchers typically consider as a given the existing patterns of relations. In other words, they treat network structure, and the occupancy of positions within the network, as exogenous to the rewards that accrue to these positions. Though undoubtedly justified when examining the effects of friendship networks, in the realm of commerce, the assumption that firms do not engage in rational calculation when choosing their partners seems somewhat less reasonable. Inter-firm networks more likely evolve in response to actors' attempts to position themselves to advantage vis-à-vis both potential and realized partners and rivals.

Though a substantial and growing literature has attempted to understand these processes better through empirical analysis, it has had limited ability to establish with certainty whether particular positions within interfirm networks do, in fact, offer a competitive advantage. In particular, critics have reasonably questioned whether positions in themselves have value, or whether firms with unusually valuable capabilities or resources come to occupy these positions (producing a spurious correlation between position and performance).[2] This limitation does not in any way reflect the abilities of the researchers active in this field; indeed, research on exchange networks has attracted

papers appearing in the journals each year and the theme of the 2002 Academy of Management Meetings: "Building Effective Networks"

[2] Gould's (2002) model, for example, implies that centrality—the number of connections an actor has to others—must initially stem from actor-level heterogeneity in "quality."

a number of talented empiricists. Rather, it stems from the fundamental incompleteness of any empirical study. Even with numerous controls and panel methods, one cannot completely rule out the possibility that some unobserved heterogeneity in firm characteristics accounts for differences in network position, and that variation in firm performance reflects this heterogeneity rather than positional differences across actors.

Especially given the intrinsic difficulties involved in answering this question empirically, we see substantial value in coming to a deeper theoretical understanding of the underlying issues. We therefore develop a two-stage, general analytical framework to model positional advantage among agents operating within a network structure. In the first stage, agents invite others and accept invitations to form dyadic relationships based on their expectations regarding the probable value of these connections. The resulting pattern of relations then forms the basis for (i.e. restricts) the coalitions that firms might form in the second stage in their pursuit of economic profits. The second stage connects the production of value (through exchange and/or joint production), the structure of the networks linking actors, and the distribution of rewards (i.e. patterns of value appropriation). This approach allows us to examine the subtle interplay between actors in networks as they cooperate to produce value and compete (both within and across clusters of relations) to appropriate a share of it. Moreover, it crucially allows us to distinguish the rewards that accrue to network position—by exploring cases in which actors do not hold valuable capabilities or resources—from those that accrue to the control of scarce resources—by considering cases in which actors have identical network positions.

From a technical perspective, our model uses a biform game approach to model "co-opetition." Nalebuff and Brandenburger (1996) coined the term co-opetition to refer to situations in which agents must simultaneously cooperate to create value through joint activities while competing over who receives the greatest share of whatever value gets produced. The biform game approach, forwarded by Brandenburger and Stuart (2006), provides a framework for investigating how these dynamics interact with various types of activities in which an actor might invest. Despite substantial enthusiasm for the application of coalitional game theory in general and biform games in particular to strategy (Brandenburger and Stuart, 1996; Lippman and Rumelt, 2003; cf. MacDonald and Ryall, 2004 for a formal application), very few researchers have actually developed formal models using this approach to address core issues in strategy (for an exception, see Chatain and Zemsky, 2006). Our model builds on the existing theoretical literature in several ways. In addition to offering a rare application of the biform game approach, we extend the existing strategy literature that uses coaltional game theory to study value appropriation under competition by considering cases in which agents can invest in relationships, and in which the networks formed by those relationships then restrict agents' options for creating value. With respect to the emerging economics literature on endogenous network formation, we shift the research focus to considering the *distribution* of rewards in, rather than the social *efficiency* of, particular topological structures. (for an outstanding review of this literature, see Jackson, 2005).

Our model generates a number of propositions that usefully describe when an actor might enjoy a competitive advantage—defined as a guaranteed economic profit—as a consequence of its location in a network. We begin by noting that valuable capabilities and resources can serve as a source of competitive advantage even when no agent enjoys a positional advantage in their network. Here, we flip this issue around: In the absence of any firm-specific attributes, can a firm enjoy an advantage based exclusively on its position in the network? Yes, but only under a fairly restrictive set of conditions. Centrality, for example, does not convey a competitive advantage. A brokering position—one in which an actor sits between (sets of) actors who can only commence a valuable trade or production activity through it (also frequently referred to in Burt's language as a "structural hole")—on the other hand, can offer a sustainable competitive advantage, but only when three conditions hold: (i) the broker has connections with at least *three* other parties; (ii) the broker does not hold too strong of a position; and (iii) the intermediating position arises concomitantly from activities not related to expectations regarding their usefulness in appropriating value. Unlike its resource-based counterpart, pure positional advantage is not sustainable. Our model therefore suggests that while positions within exchange networks may account for some of the variation in firm performance, managers probably *cannot* pursue them as a primary path to securing a competitive advantage.

2. Competition and the distribution of rewards

Our full model incorporates two stages. In the first stage, agents form relationships with one another. One might think of these relationships as strategic investments; the rationale for such a labeling should become clear as the model unfolds. In the second stage, these same agents cooperate and compete to create and appropriate value. Our treatment uses these investments—the relationships built in the first stage—as a "feasible set" of connections available for agents to use in the creation of value in the second stage. Though we do not believe that it greatly limits the applicability of the model, our setup emerges from the idea (and hence assumes) that the establishment of new relationships requires time and effort. Once established, however, agents can freely access these relationships to produce value.

Because decisions in the first stage depend critically upon the expected outcomes of the creation and distribution of value in the second, we begin by detailing the mechanics of the latter. In Section 4.4, we extend the model to include the first-stage endogenous network formation.

2.1. Exchange networks

As with most research on networks, we define the social structure in terms of relationship-edges connecting actor-nodes. Begin with a set of agents—firms, groups or individuals—indexed by $M \equiv \{1, ..., m\}$.[3] Let Γ denote the (indexed) set of all possible patterns of relations on M. In other words, Γ contains all undirected graphs of the form (M, R) in which R describes an established set of (potential) exchange and/or production relationships between agents. We then define *potential structure* k as the undirected graph $\gamma^k \equiv (M, R^k)$ in Γ.[4] To preserve generality, we do not restrict the potential structure to being connected; any (M, R) ranging from completely unconnected to fully connected represents a valid potential structure.

A *group* denotes any subset of agents $G \subseteq M$, and $\gamma^k_G \equiv (G, R^k_G)$ represents the subgraph of γ^k induced by G—in other words, the graph that results from combining the elements of subset G with all of the edges in γ^k that directly connect agents in G. When a group corresponds to (has the same members as) a component of R^k (i.e., it forms a distinct, fully connected subgraph), we call it a *network* (any isolated individual also trivially forms a network). Thus, γ^k typically includes *multiple* independent networks that compete to produce value and, ultimately, to appropriate it to the benefit of their members. To emphasize the distinction between arbitrary groups and those that correspond to a network, we use the notation $N \subseteq M$ to indicate the latter (rather than the more generic G). When γ^k is clear from the context, we simply refer to N as a "network" rather than specifying "the network (N, R^k_N) corresponding to N."

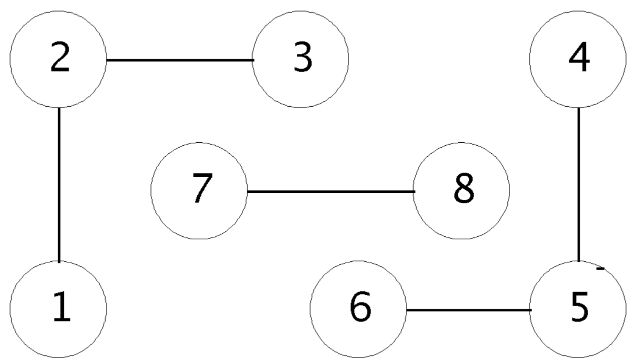
If the agents in a group G choose to produce value independently (of the other members of M), we assume they do so by restricting themselves to the relationships within their group when engaging in exchange to create value. This decision effectively alters the overall social structure by removing any relationships in R^s involving an agent in G and one in $M \backslash G$ (the subset of agents in M not in group G). In other words, such a decision eliminates the links connecting those inside the group to those outside it. By this assumption, we do not mean to imply that members of G dissolve their relations with those outside the group, but rather that they simply make a decision not to include them in their value-producing exchange and production activities.

[3] Unless otherwise indicated, we assume sets to be finite.

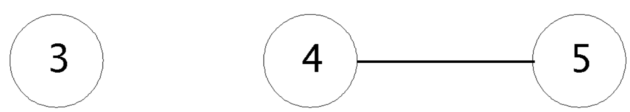
[4] See Appendix A for formal definitions of all graph-theoretic terminology.

To track these effects, let us introduce some additional notation. Any subset of relationships $R_j^k \subseteq R^k$ implies a partition of M—one corresponding to the node sets of the component (connected) graphs under R_j^k—which we label $\mathcal{P}_j^k$. Similarly, we label the partition of M resulting from the decision of group G to produce independently of others outside the group as $\mathcal{P}_G^k$ ($\mathcal{P}_M^s$ represents a special case where each $N \in \mathcal{P}_M^k$ is a network in γ^k). The decision of a group G to produce value on its own also creates a new potential structure γ^l for which $\mathcal{P}_M^l = \mathcal{P}_G^k$. Example 1 illustrates the application of this notation.

Example 1. (Simple network) Consider a potential structure γ^k consisting of 8 agents:



$\mathcal{P}_M^k = \{\{123\}, \{456\}, \{78\}\}$. Suppose the agents in $G = \{3, 4, 5\}$ decide to produce value independently of the other agents. γ_G^k is then



and $\mathcal{P}_G^k = \{\{12\}, \{3\}, \{45\}, \{6\}, \{78\}\}$.

2.2. Market environment

Networks in γ^k produce value by completing transactions desired by an exogenous "market." Though we will discuss them as transactions for ease of exposition, these interactions can include almost any activity in which individual agents or groups of agents might engage to produce value (the examples below showcase the wide variety of potential applications to which our results apply). We define the *market environment* as a pair $(\mathcal{T}, \mu)$ where $\mathcal{T}$ denotes a set with typical member T, which we refer to as a *transaction collection*, and $\mu(T)$ represents the probability that the networks in γ^k compete over the transaction collection T. Each of these transaction collections in turn include h *transactions*, activities on which the market places a positive value ($T \equiv \{t_1, ..., t_h\}$).

We describe each transaction with a triple, $t_\omega \equiv (X_\omega, c_\omega, u_\omega)$, in which: i) $X_\omega \subseteq M$ corresponds to the agents required to complete t_ω (i.e. the set of agents with the necessary capabilities and resources); ii) $c_\omega : M^2 \to \mathbb{R}_+$ is a function that summarizes link-related "processing" costs (any dyadic costs required to facilitate the transaction, including communication costs, coordination costs, contract writing costs, monitoring costs, transportation costs and transfer prices);[5] and, iii) $u_\omega > 0$ represents the gross market value of the completed transaction. If X_ω includes only a single agent, then we assume $u_\omega = 0$ (a normalization).

Before continuing, let us call attention to one particularly attractive feature of our model construction: its ability to distinguish analytically positions from agents' capabilities and resources. We describe the agents in X_ω as *essential* to t_ω. In other words, agent i possesses some firm-specific attribute, a capability or a resource, without which the network cannot complete the transaction (we can address the availability of substitutes through the existence of alternate t_ω, potentially with identical output, involving different sets of agents). The model thereby clearly distinguishes between the economic scarcity of a firm's position and its capabilities and/or resources. While the former depends only on the market's relationship structures (R^k), the latter stems solely from agent essentiality (determined by the X_ω's).

For any particular transaction, we can summarize costs by a complete graph with nodes M and edge weights $c_\omega(ij)$. Whenever the total value created by a transaction exceeds the cost of any particular i-j relationship (i.e. any $ij \in M^2$ such that $c_\omega(ij) \leq u_\omega$), we call that relationship *admissible*. (When depicting cost graphs below, we omit inadmissible relationships to prevent these graphs from becoming overly cluttered). We assume that processing a transaction costs the least when it only involves direct relationships between the essential agents: given t_ω, for all trees (Y, R^k_Y) such that $X_\omega \subseteq Y \subseteq M$, $\sum_{ij \in R^k_Y} c_\omega(ij)$ is minimized only if $X_\omega = Y$. Without such an assumption, agents not necessary for a transaction might nonetheless appear in the optimal solution simply because an indirect path through those actors lowers costs. Though such situations may occur—for example, imagine the owner of a railroad being able to reduce the costs of transporting iron ore to a steel producer—from the perspective of our model, we would consider such agents essential and therefore would include them in X_ω.

[5] We introduce a separate cost for forming and maintaining relationships in Section 4.4. For tractability, our setup excludes processing costs outside of the dyad. As a result, we cannot consider the potential effects of triadic interdependencies on our results. For example, Heider's (1958; Davis, 1967) balance theory suggests that an agent would find it difficult to exchange with both a friend and that friend's enemy. In such a case, the cost depends not just on each individual but on the fact that the agent would wish to interact with two incompatible parties. Though clearly these cases raise interesting issues, we must consider them outside the domain of our model.

We assume that relationships within a group must link all of the agents essential to a transaction in order for that group to consider completing the transaction. Additionally, we impose the restriction that any agent can participate in the completion of one and only one transaction. An analytical convenience, this assumption has little effect on the generality of the model. For example, one can easily construct collections T containing "basic/small" transactions for a set of agents (e.g., t_1 indicates the value and cost associated with IBM and Microsoft producing one personal computer for the market) as well as composite transactions built up from basic transactions (e.g., t_q indicates the value and cost associated with IBM and Microsoft producing q computers for the market). Market demand and the productive technologies of the associated agents therefore jointly determine the composition of $\mathcal{T}$.

Networks with more than one agent can process multiple transactions. To do so, for each transaction, the network must include a connected subset of members essential to the transaction. In formal terms, we consider a set of transactions $T_N = \{t_1, ..., t_l\}$ *feasible by* N under γ^k if a pairwise disjoint set of subsets of N, $\{G_1, ..., G_l\}$, exists such that, for $\omega = 1, ..., l$: i) $X_\omega \subseteq G_\omega$; and ii) for $R^k_{G_\omega} \subseteq R^k_N, \left(G_\omega, R^k_{G_\omega}\right)$ forms a tree.[6] Note that we do not require agents in X_ω to have direct relationships in N to complete t_ω; indirect connections via other agents—themselves not essential to the transaction—can also enable the completion of a transaction. Some examples should help to illustrate the flexibility of our setup and the generality of our upcoming results.

Example 2. (Bilateral trade) Let us begin with one of the most simple, stylized applications. To model bilateral trade, one could partition M into a set of buyers B and a set of venders V. Each vender wishes to sell and each buyer wishes purchase a single item. Each transaction t_ω has $N_\omega = \{i, j | i \in B, j \in V\}$. Here, u_ω is i's reservation price for the good offered by j, and $c_\omega(ij)$ is j's reservation price for her own item. The completion of t_ω generates economic surplus equal to $u_\omega - c_\omega(ij)$. In environments lacking uncertainty, T would contain one transaction for each $ij \in B \times V$, but one could easily introduce supply and/or demand uncertainty by setting $\mathcal{T} = 2^B \times 2^V$ (i.e. all possible combinations of buyers and venders, each pertaining to a particular possible state of the world) and choosing μ appropriately.

Example 3. (Multilateral trade) Though clearly more complex, the extension to multilateral trade is straightforward. Venders can sell more than one unit of product up to some capacity constraint. Buyers may also desire more than one unit or type of output. In addition to venders,

[6] Since our model requires agents to form efficient production networks, it precludes the formation of cycles as these structures would involve at least one redundant relationship.

M may also contain suppliers of various types. Transactions then describe the various vertical-chain/output-quantity combinations and their associated economic surpluses. In these chains, the c_ω's would account for differences in the efficacy of particular productive configurations, transportation costs, etc. A common situation in market economies, examples of multilateral trade would include the financing of fledgling firms by venture capitalists, the provision of IT consulting services and the production of titanium dioxide.

Example 4. (Trading circles) These cycles represent a special case of multilateral trade in which agents have a "location" (geographical, familial, etc.) and can only transact with "adjacent" agents (i.e. those close to them). Though no longer common, anthropological studies have found that many primitive cultures engaged in trade only with their neighbors and relied on chains of connections between neighbors to bring more distant goods to them. Modern day equivalents may nonetheless still exist in systems of exchange governed by bartering (e.g., trading favors).

Example 5. (Skilled collaboration) Suppose we wish to model movie production. Creating a movie requires collaboration between multiple agents with complementary sets of skills. In this case, we would partition M according to each agent's professional skill (director, screenwriter, actor, producer, grip, set designer, etc.).[7] Let us index the skills required to complete a movie with $K \equiv \{1, ..., k\}$, $k \geq 2$, and let $p(i) \in K$ indicate agent i's profession. Each transaction t_ω would then represent a movie and would identify the personnel required to create it (i.e. $p(X_\omega) = K$). Any group G such that $K \nsubseteq p(G)$—that is, lacking one or more of the requisite skills—would therefore fail to complete the transaction (movie), but the group *could* include members who facilitate the recruitment of the necessary personnel but who do not directly contribute to producing the film (e.g. a talent agent might usefully bring together a director and actors that do not know each other personally). In the simplest case, one would set $c_\omega(\cdot) = 0$, and u_ω would then become the net profit of the movie (given the agents involved). For transactions with certain returns, T would contain one transaction for each combination of agents holding the requisite skills. The model could also incorporate uncertainty—for example, in box office sales—by varying the u_ω's to create a distribution of possible states of the world in $\mathcal{T}$.

Example 6. (Partnerships) Partnerships look like skilled collaborations with, for our purposes, two key distinctions. In partnerships, agents may draw from a common set of skills ($m \geq k \geq 2$), have identical skills ($k = 1$), or contribute unique skills ($k = m$). Moreover, unlike collaborations, we assume that partners have the option of breaking off on their own (if $t_\omega \in T$ then, for

[7] Here we treat skills as synonymous with roles. We recognize that roles probably do not map in a one-to-one manner to skills, and the model could accommodate a treatment based on actual skills rather than role structures, but we nonetheless discuss roles for their intuitive appeal.

all $G \subset X_\omega$, a transaction t_κ exists such that $X_\kappa = G$). For example, any subset of the partners in a law firm (down to an individual lawyer) could depart the partnership and form their own competing firm.

Example 7. (Hierarchies) With respect to social networks, a particularly interesting example concerns hierarchies, situations in which we can order the members of the competitive network on some dimension—including, potentially, status (Chase, 1980; Podolny, 2005). Suppose that $(M, <)$ represents a partial order. For example, $j < i$ if i has a higher professional stature than j (or "more" experience, "higher" ability, etc.). We can then model certain types of hierarchy effects by defining t_ω appropriately (if, for all feasible pairs $ij \in X_\omega^2, j < i$ or $i < j$).

2.3. Value creation

Next, we define how agents choose among the available transactions in their pursuit of value. Given γ^k and a set of transactions $T \in \mathcal{T}$, let $\mathcal{T}_N \subseteq 2^T$ denote the set of all transaction sets feasible by N. If N cannot feasibly complete any transactions, then N produces zero value. Otherwise, N produces value (φ_N^k) according to:

$$\varphi_N^k(T) \equiv \max_{T_N \in \mathcal{T}_N} \left[\sum_{t_\omega \in T_N} \left(u_\omega - \sum_{ij \in R^k_{G_\omega}} c_\omega(ij) \right) \right]. \tag{1}$$

In other words, we assume that N chooses to complete the most valuable set of transactions available to it. When we refer to the transactions *completed by* N, we mean the feasible set of transactions that maximizes (1).

We also extend this definition to the value created by an arbitrary group. Note that, by the definition of a transaction, no network outside of N can complete the transactions in $\mathcal{T}_N$. Given $T \in \mathcal{T}$ and $\gamma^k \in \Gamma$, if $G \subseteq M$ does not represent a network, it generates:

$$\varphi_G^k(T) \equiv \sum_{N \in \left\{ G \cap X \mid X \in \mathcal{P}_G^k \right\}} \varphi_N^k(T). \tag{2}$$

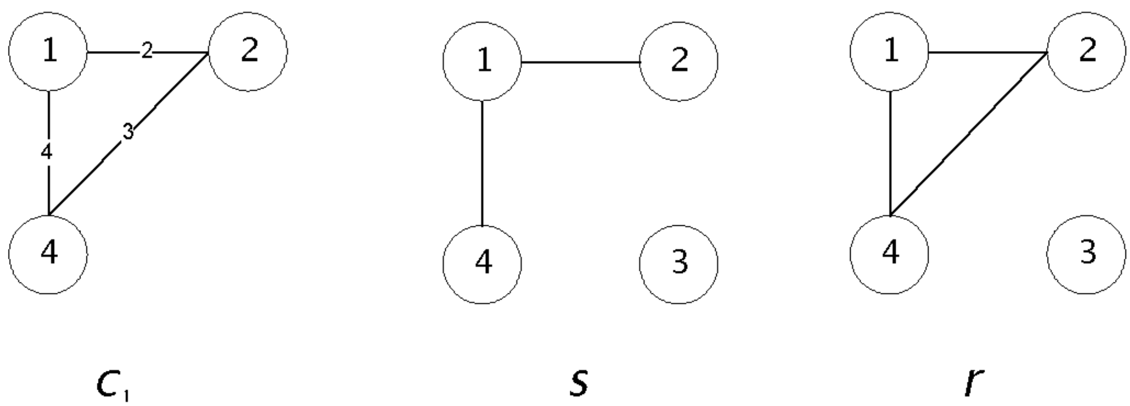
Simply put, the group produces value equal to the sum of the values created by its constituent networks.

Given market environment $(\mathcal{T}, \mu)$ and structure $\gamma^k \in \Gamma$, we define the *value available to G under* γ^k, denoted v_G^k, as:

$$v_G^k \equiv \sum_{T \in \mathcal{T}} \varphi_G^k(T)\, \mu(T), \tag{3}$$

when γ^k is obvious from the context, we simply write v_G. It follows almost immediately that, for all G, $G' \subseteq M$ such that $R^k_{G'} \subseteq R^k_G$ implies $v_G \geq v_{G'}$; therefore, for all $G \subseteq M$, $v_M \geq v_G$. In other words, the exchange network as a whole must have at least as much value available to it as any subgroup of that network. Moreover, adding relationships never reduces the value of any group (i.e., for all $\gamma^k, \gamma^l \in \Gamma$ such that $R^k \subseteq R^l$ and for all $G \subseteq M$, $v_G^l \geq v_G^k$).[8]

Example 8. (Value creation) Consider the following situation with four agents. Let $T = \{t_1\}$ with probability $= 1$, where $u_1 = 6$. The left-most pane of the graph below reports the c_1 for the potential links between each member of X_1. Consider the potential structures γ^s and γ^r depicted in the center and right panes.



In both cases, the group $G = \{1, 2, 4\}$ completes t_1. Potential structure γ^s would generate value:

$$v_G^s = \varphi_G^s(t_1) = u_1 - (c_1(12) + c_1(14)) = 6 - (2 + 4) = 0.$$

In potential structure γ^r, however, the group would produce $v_G^r = 1$ (because 2-4 belongs to the feasible set of relations, structure γ^r can use it, instead of the more costly 1-4 link, to complete the transaction). The trivial group $G = \{3\}$ cannot complete t_1 in either case.

2.4. Value appropriation

Groups create value over which individual agents contend for their own share. Let $v \equiv (v_G)_{G \subseteq M}$ and notice that (M, v) then defines a coalitional game. In this framework, individual payoffs depend jointly upon the *actual* value generated by the completion of transactions, the attributes of *alternative* transactions forgone, and agents' *extra-competitive* skills in getting others to part with value. To adress the question of who gets what, define a *pattern of appropriation* as a profile $\pi \equiv (\pi_1, ..., \pi_m)$ in which π_i represents the quantity of value appropriated by agent i. We often find it useful to refer to the value that accrues jointly to all members of a group G, which we call the *appropriation by G under* π (denoted $\Pi_G \equiv \sum_{i \in G} \pi_i$).

[8] In technical terms, the value function v is superadditive both in terms of relationships and in the sense that adding agents to a group never reduces the value produced.

Recall from Section 2.3 that—holding the potential structure constant—v_M represents the maximum value that M can generate through any combination of trade and/or joint production (i.e. the decision of any subgroup to produce on its own cannot increase the aggregate value produced by M). Though one might easily imagine a central authority attempting to impose this value-maximizing outcome, our interest concerns understanding the conditions under which self-interested agents—bargaining in unstructured negotiations—would agree to contribute to the creation of v_M. In such situations, the completion of the transactions required to produce v_M should only occur when the essential agents believe that completing the transaction would yield more value to them than completing some other transaction(s) on their own. Formally, this condition holds if, for every possible group $G \subset M$, the total value allocated to the group under v_M (i.e. Π_G) meets or exceeds v_G, the amount that the group could produce on its own (and divide amongst its members). We call π a *competitive pattern of appropriation* (CPA) if: (i) $\Pi_M = v_M$; and, (ii) for all $G \subset M$, $\Pi_G \geq v_G$.[9] Although one could construct degenerate value functions v that fail to satisfy these conditions, we exclude them from consideration because our results do not extend to situations where CPAs do not exist.[10]

Of particular interest to us are situations in which competition alone guarantees that an agent will earn an economic profit (strictly positive appropriation). In any coalitional game, each agent faces a range of outcomes $[\pi_i^{\min}, \pi_i^{\max}]$ consistent with the requirements for a CPA.[11] At least one CPA corresponds to the minimum level, at least one other to the maximum, and still others to every level of appropriation in-between. We have normalized the value of i's "outside alternative"—$v_{\{i\}}$, the amount that i could earn independent of any other agents—to zero; $\pi_i^{\min} > 0$ therefore implies that i has a sufficiently attractive set of alternatives to guarantee that it earns an economic profit, independent of any other considerations (e.g., bargaining ability). Following MacDonald and Ryall (2004), if $\pi_i^{\min} > 0$ we say that i has an *absolute competitive advantage* (hereafter, ACA).

[9] Although our definition is equivalent to the so-called "core" in the modern economics literature, we have adopted this nomenclature because we believe that it provides better intuition into the meaning of this term. We label such patterns of appropriation as "competitive" because they could arise purely from agents individually competing for rewards.

[10] Bondavera (1962) and Shapley (1967) characterize the conditions governing the existence of the core (CPAs). Though we have nothing to add to their results, we would nonetheless note that from our experience, one rarely finds cases where the core does not exist in situations of interest to strategy researchers.

[11] We see this multiplicity of potential outcomes as a strength of the analytical framework because it allows for a clear line of demarcation between the effects of competitive forces and other determinants of appropriation. Competition determines the agent's *range* of appropriation possibilities. The actual level of appropriation within that range may still depend on a variety of factors not explicitly included in our model, such as norms of fairness or negotiating skills. In some extreme cases, $\pi_i^{\min} = \pi_i^{\max}$, competition alone precisely determines agent i's appropriation. When true for every agent, we call such market environments *perfectly competitive*.

In our setup, an agent's position within his exchange network as well as the extent to which the completion of transactions requires access to his capabilities/resources *jointly* determine whether that agent has an ACA. In any real-world situation, both positions and capabilities/resources come into play. Our model nonetheless allows us to isolate these effects and consider situations in which an agent's advantage (or lack thereof) depends entirely on one or the other.

3. Value creation and appropriation in a network

Now that we have set up the second-stage model, we examine some of its implications with respect to the effects of network structure on the production and appropriation of value. We begin by reviewing some general results.

3.1. Maximum value production

One point that falls immediately out of our setup – one that appears obvious, but rarely attracts explicit attentio – is that the efficacy of any particular network depends strongly upon its environment. For example, social structures that prove effective in the production and delivery of tangible products to end-users probably underperform other structures in technology development. Different productive contexts therefore should require different resources and exchange network structures.

PROPOSITION 1. *For all potential structures γ^k, groups G, and transactions t_ω, $\varphi_G^k(t_\omega)$ is maximal if and only if G contains a minimal spanning tree (MST) with respect to c_ω.*[12]

Proof. G completes t_ω, by the definition of feasibility, if and only if it spans it. Because superfluous network links add no cost, a MST with respect to c_ω minimizes the completion cost. The conclusion then follows because u_ω does not vary with network structure.

With respect to the empirical literature on the effects of social networks, this proposition highlights the need for these studies to consider the market context not only when trying to understand what types of positions might prove valuable but also when evaluating the generalizability of these results. Though some notable studies have highlighted the contingent nature of the value of particular relationship structures—for example, Podolny and Baron (1997) demonstrate that employees benefit from brokering positions in their information networks but from tightly connected cliques in their supports networks—the vast majority of studies investigate social networks within a single setting and therefore cannot address the crucial issue of how the value of these structures might depend on the environment.

COROLLARY 1. *Given t_ω, start with an arbitrary agent $i_1 \in X_\omega$ and follow these steps:*

[12] More precisely: if and only if $\mathcal{P}_G^s$ contains a network corresponding to a graph containing a MST of t_ω.

1. *Pick* $i_2 \in X_\omega \backslash i_1$ *such that* $i_2 = \arg\min_{j \in N_\omega \backslash i_1} c_\omega(i_1 j)$*.Let* $G_W = \{i_1, i_2\}$, $G_W^c = X_\omega \backslash G_W$ *and* $R_W = \{i_1 i_2\}$.
2. *Pick* $i_3 \in G_W^c$ *such that* $i_3 = \arg\min_{g \in G_W^c}(\min_{y \in G_W} c_\omega(yg))$. *Let* $G_W = \{i_1, i_2, i_3\}$, $G_W^c = X_\omega \backslash G_W$ *and* $R_W = \{i_1 i_2, y i_3\}$ *where* $c_\omega(y i_3)$ *is the minimum above.*
3. *Continue inductively until* $G_W = X_\omega$.

For $G \subseteq M$ *and* $\gamma^k \in \Gamma$, *if* $ij \in R_W$ *implies* $ij \in R_G^k$, *then* $\varphi_G^k(t_\omega)$ *is maximal.*[13]

We derive this corollary from a well-known algorithm for solving MST problems (see, e.g., Papadimitriou and Steiglitz, 1998, p. 271-8).

Corollary 2. *Given* $(\mathcal{T}, \mu)$ *and* $T \in \mathcal{T}$, $\varphi_G^k(T)$ *is maximal if and only if* G *contains a MST of every transaction in* T.

Corollary 3. *Given* $(\mathcal{T}, \mu)$ *and* $\gamma^k \in \Gamma$, v_M *is maximal if and only if* M *contains a MST for every transaction that arises with positive probability; i.e., all elements of* $\{t_\omega \in T | \forall T \in \mathcal{T}, \mu(T) > 0\}$.

By Proposition 1, we can see that our transaction objects identify not only which networks can supply the specific value-generating product, service or outcome, but also which ones can do so most effectively. Corollaries 2 and 3 extend this point to multiple transactions and multiple groups. Corollary 1 demonstrates that a relatively simple inductive procedure can find the value-maximizing network.

3.2. Value-added in the network context

In the literature on games, one often finds it useful to assess the value that an agent brings to the game (Makowski & Ostroy, 1995; Nalebuff & Brandenburger, 1996). Our setup allows us to extend this notion to groups and relations: We define group G's *value-added under* $\gamma^k \in \Gamma$ as $ava_G^k \equiv v_M^k - v_{M \backslash G}^k$ (i.e. the value generated by the all agents using the full potential structure versus the value generated by everyone when G chooses to isolate itself from other members of M). We define the *value-added of* $R_l^k \subseteq R^k$ as $rva_l^k \equiv v_M^k - v_M^l$, where $\gamma^l \in \Gamma$ represents the potential structure implied by the relationships $R^k \backslash R_r^k$.[14] Our setup requires both groups and relationships to have non-negative value-added.

Proposition 2. *For all* $\gamma^k \in \Gamma$ *and* $i \in M$, $ava_i^k = rva_i^k$.

[13] See Appendix B for an example.

[14] Through this definition, one can also relate our model to Bloch & Jackson. Positive relationship value-added in our context is a necessary condition for positive edge marginal utility in their terminology (Bloch & Jackson, 2005, p.6).

Proof. For an individual agent, ava_i^k and rva_i^k are equivalent because producing value on one's own amounts to ignoring one's established relationships.

In other words, individual value-added represents a special case of edge value-added. Under any competitive pattern of appropriation, $ava_G^k = 0$ implies $\Pi_G = 0$. If the links between the members in group G and those outside of it do not contribute to the value produced, then that group cannot expect to extract any positive economic surplus (though agents within that group still might). We call a potential structure γ^k *minimally efficient* if v_M^k is maximal and, for all $R_l^k \subset R^k$, $rva_l^k > 0$ (i.e., the elimination of any relationship reduces the aggregate value produced).

3.3. Perfect competition

When does competition *fully* determine value appropriation by the agents and networks competing in the market? In other words, when, for a network N, does

$$\Pi_N^{\min} = \Pi_N^{\max}? \tag{4}$$

When (4) holds, extra-competitive forces (such as negotiating skills or perceived "bargaining power") play no role in the value captured by the network; rather, what the network receives (as a whole) depends entirely on the tension between the total value generated in the market, v_M, and the productive alternatives available to the network. If, given $(\mathcal{T}, \mu)$ and γ^k, (4) holds for all $N \in \mathcal{P}_M^k$, then we designate γ^k as *perfectly competitive at the network level under* $(\mathcal{T}, \mu)$. Note that, even if (4) holds, extra-competitive factors may still play a role in determining individual appropriation by the individuals *within* N. If, on the other hand, for all $i \in M$, $\pi_i^{\min} = \pi_i^{\max}$ (i.e. if competition determines the amount appropriated by every agent), then we call γ^k *perfectly competitive.*

It turns out that the potential structures that we analyze always have the property of being perfectly competitive at the network level. To see why, notice that, by (3),

$$v_M^k = \sum_{T \in \mathcal{T}} \varphi_M^k(T)\, \mu(T). \tag{5}$$

and by (2),

$$\begin{aligned} \sum_{T \in \mathcal{T}} \varphi_M^k(T)\, \mu(T) &= \sum_{N \in \mathcal{P}^k} \sum_{T \in \mathcal{T}} \varphi_N^k(T)\, \mu(T) \\ &= \sum_{N \in \mathcal{P}^k} v_N^k. \end{aligned}$$

Therefore,

$$v_M^k = \sum_{N \in \mathcal{P}^k} v_N^k. \tag{6}$$

Proposition 3 follows almost immediately.

Proposition 3. *For every environment* $(\mathcal{T},\mu)$ *and potential structure* γ^k*, the market is perfectly competitive at the network level.*

Corollary 4. *Given environment* $(\mathcal{T},\mu)$ *and potential structure* γ^k*, every network appropriates its own added value (i.e., for all* $N\in\mathcal{P}^k$, $\Pi_N = ava_N^k$*).*

We can actually strengthen Proposition 3 considerably. In our setup, value appropriation depends *entirely* on within-network interactions (Proposition 4). This fact usefully means that our analysis of individual appropriation under network competition need only consider within-network factors. Given $(\mathcal{T},\mu)$, $\gamma^k\in\Gamma$ and $N\in\mathcal{P}_M^k$, let $(N,\hat{v})$ represent the coalitional game with agents N, and for which all $G\subseteq N$, $\hat{v}_G = v_G$. Then, for all $i\in N$, let $\hat{\pi}_i^{\min}$ and $\hat{\pi}_i^{\max}$ denote the agents' minimum and maximum competitive appropriation levels in $(N,\hat{v})$.

Proposition 4. *For all* $(\mathcal{T},\mu)$, $\gamma^k\in\Gamma$*,and* $N\in\mathcal{P}^k$*, any CPA for the agents in* N *under* $(N,\hat{v})$ *belongs to some CPA for the agents in* M *under* (M,v)*.*

Longer proofs appear in Appendix C.

Corollary 5. *For all* $(\mathcal{T},\mu)$, $\gamma^k\in\Gamma$, $N\in\mathcal{P}^k$*, and* $i\in N$, $\pi_i^{\min}=\hat{\pi}_i^{\min}$ *and* $\pi_i^{\max}=\hat{\pi}_i^{\max}$.

Given these results, we now simplify our analysis by shifting our focus from competition across networks to competition within a particular network. This allows us to disregard the larger potential structure and market environment outside of a focal network and its feasible transactions. From this point forward we adjust our notation as follows: N now denotes the focal network, $\mathcal{T}_N$ its set of feasible transaction collections, μ the probability on $\mathcal{T}_N$ inherited from the market environment, and $\gamma^k=(N,R^k)$ the network relationships inherited from the larger potential structure. Accordingly, we now refer to γ^k as a *network structure*.

4. Absolute competitive advantage

Recall that an agent's position within its network as well as the qualities of the resources it controls (and, hence, its essentiality with respect to the transactions demanded by the market) jointly determine whether the agent enjoys a absolute competitive advantage (ACA). We nonetheless wish to isolate the effect of position from those of capabilities and resources in determining the firm's ability to achieve an ACA. We do so in four steps. First, we characterize the necessary and sufficient conditions for capability/resource scarcity—independently—to generate a competitive advantage. We proceed by examining some special cases in which agents both hold scarce resources and occupy scarce positions. Third, we characterize the necessary and sufficient conditions for positional scarcity—independently—to generate a competitive advantage. Our results will show that both position and resources can form an independent basis for ACA. Finally, we analyze whether strategies designed to establish a competitive advantage entirely via one route or the other would likely to succeed.

4.1. Resource-based advantage

To isolate the effects of resources, we begin by analyzing a case in which all firms have equivalent positions—that is, one in which no local measure of network structure can distinguish one agent from another. Perhaps the most obvious case that meets this criterion is the complete network (i.e. one including every possible dyadic relation). Since all firms occupy identical positions in a complete network, any competitive advantage would have to stem entirely from the relative scarcity of the agent's capabilities and/or resources (as determined by the market environment). Hence, given $(\mathcal{T}_N, \mu)$, we say that agent i has a *pure resource-based advantage* if it enjoys an ACA in the complete network.

In a complete network, agents' relationships place no constraints (i.e., have no effect) on the production of value. As a result, the question of whether a firm has a resource-based advantage is equivalent to the question of whether it has an ACA in a general coalitional game. Consequently, we can draw on prior research on general coalitional games; we briefly restate an existing result.

Define agent i's *minimum total value under network structure* γ^k as

$$mv_i^k \equiv \min\left\{\Pi_{N_{-i}} \mid \text{for all } G \subseteq N \text{ including } i, \Pi_{G_{-i}} \geq v_G^k\right\}, \tag{7}$$

where N_{-i} indicates the set of all agents in N except i (similarly, G_{-i} denotes the set of all agents in grouping G except i). Given γ^k, mv_i^k represents the minimum quantity of value required to ensure that all agents receive compensation sufficient to prevent i from enticing them to join it and then to isolate themselves from the network and to participate in some alternative set of transactions, *even though* $\pi_i = 0$. If $v_N^k \geq mv_i^k$, then there exists at least one CPA in which $\pi_i = 0$ and, hence, i does not have an absolute competitive advantage. If, on the other hand, $v_N^k < mv_i^k$, then no matter how carefully one constructs it, any pattern of appropriation in which $\pi_i = 0$ leaves some group of agents preferring to join i in producing value on their own. To assure the completion of the value-maximizing set of transactions, v_N^k, agent i must receive a positive economic profit (i.e. $\pi_i > 0$).

Theorem 1. *(MacDonald and Ryall, 2004) Given a network structure γ^k and $(\mathcal{T}_N, \mu)$, i has a competitive advantage if and only if mv_i^k exceeds v_N^k: $\pi_i^{\min} > 0 \Leftrightarrow mv_i^k > v_N^k$.*

Corollary 6. *Agent i has a pure resource-based advantage if and only if γ^k is the complete graph and $mv_i^k > v_N^k$.*

This result has been discussed extensively elsewhere, so we limit our comments to a few basic points. First, an agent can clearly enjoy a capability- or resource-based competitive advantage independent of its position in the network. Second, the definition of mv_i^k implies that to check for the existence of an ACA, one need only examine the alternatives available to firm i (i.e. the values that groups that include i can generate under γ^k); the values of within-network coalitions that do not include i do not affect the value of mv_i^k. Finally, we would note that, according to Proposition 1, a competitive advantage *always* depends upon a tension between the aggregate value generated via actual transactions (v_N^k) and the values of an agent's alternatives forgone in order to participate in those transactions (summarized by mv_i^k). Networks sometimes impose enough additional structure on the ways in which agents can create value that, as we will see in the next section, we can obtain much stronger results.

4.2. Resource-based advantage with network constraints

To begin to see how network structure affects the pattern of value appropriation, let us consider a case that we refer to as an *efficiently specialized* network. Specifically, N is efficiently specialized if it can complete one particular type of transaction $\mathcal{T}_N = \{t_\omega\}$ and is a MST with respect to c_ω. Such situations fall into the class of "skilled collaboration" environments (described in Example 5). In addition to film production, other examples might include the founding team of a startup, or a one-off strategic R&D alliance created to advance some technology, in which each member brings a necessary resource to the group for jointly achieving a specific outcome.

PROPOSITION 5. *If N is efficiently specialized, then no agent in the network has an ACA (i.e. for all $i \in N$, $\pi_i^{\min} = 0$).*

Proof. Under the premise, $v_N > 0$ but, for all $G \subset N$, $v_G = 0$. Because N is an MST with respect to c_ω and $\mathcal{T}_N$ contains only one element: removing any relationship to any member of N results in a failure to produce value. As a result, *any* pattern of appropriation such that $\Pi_N = v_N$ is competitive. Hence, for all $i \in N$, $\pi_i \in [0, v_N]$

Proposition 5 offers a simple, but far-reaching result. It implies, for example, that in situations in which the network is optimally configured to achieve some unique outcome, *no* member has an absolute competitive advantage—regardless of its capabilities and resources, and regardless of its position in the network. Agents involved in such networks have no alternatives for producing value outside of the one which the network fits. No agent or subset of agents, therefore, can credibly threaten to leave the network to produce value on their own. Consequently, the distribution of value across the agents in specialized networks depends entirely on non-competitive factors (e.g., bargaining skill).

We should point out that this outcome holds even if N does not contain a MST and/or a minimal set of agents: the agents in the *most* efficient spanning tree split up v_N (which is not the maximum if N does not contain a MST) as they like; superfluous agents (those not in the most efficient spanning tree) add no value, and, consequently, can expect to appropriate none of the available surplus.

Proposition 5 actually states a special case of a more general phenomenon that we term *independent decomposability*. We define N as independently decomposable (with respect to $(\mathcal{T}_N, \mu)$) if some partition of N, denoted $\mathcal{G}$, exists such that, for all $G \in N$ such that $v_G > 0$, there exists some $\mathcal{G}_G \subseteq \mathcal{G}$ such that $v_{G'} = v_G$ where $G' \equiv \cup_{G'' \in \mathcal{G}_G} G''$. The definition says that all the value-producing groups comprise some combination of the sub-networks identified in $\mathcal{G}$; that is, disjoint (hence, "independent") subgraphs in the network.generate value. Note, however, that this definition does not rule out positive externalities (i.e., the value generated by N as a whole may well exceed the value generated by the sum of its independent parts).

PROPOSITION 6. *If N is independently decomposable, then no agent in the network has an ACA.*

When the set of transactions requires fixed, nontrivial sub-networks to produce value, no agent has a competitive advantage. In essence, each agent faces a bargaining problem with the other agents in her constituent network. These propositions begin to reveal some of the subtleties by which network structure interacts with resource scarcity in the determination of competitive advantage. We now turn to isolating the effects of position entirely to determine whether position alone can convey any advantage.

4.3. Positional advantage

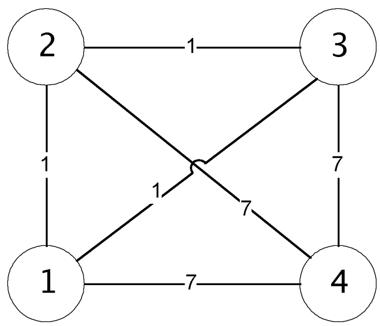
To proceed, we construct a situation in which the agent itself is not essential for *any* transaction (i.e. does not appear in the X_ω of any t_ω in $\mathcal{T}_N$), but whose position in the network facilitates the production of value by bringing together agents that do possess essential capabilities and/or resources to produce one or more transactions that the network could not complete without going through agent i. We view this case as highly consistent with the definitions of the brokerage position found in the literature (particularly along the lines developed by Burt, 1992). Under such conditions can i have a competitive advantage based entirely on its position (i.e. $\pi_i^{\min} > 0$)?

More formally, let us assume that a nonempty collection of feasible transactions, T_N, arises with probability 1. Let us further assume that i is not essential for any of the transactions in T_N. We define an agent i as a *broker* if, for every transaction in T_N, every tree capable of completing that transaction—that makes it feasible—includes i.[15] The next propositions follows immediately from the definition of a broker.

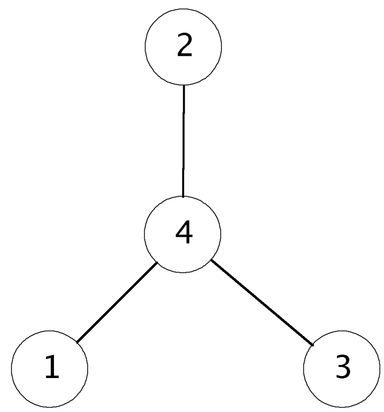
Proposition 7. *If more than one broker exists, then none has an ACA.*

Proposition 8. *If i is a broker in a complete network, then $\pi_i^{\max} = 0$.*

Now, consider the following situation.

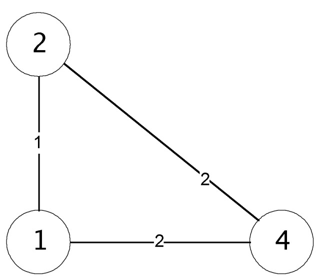


$t_\omega : u_\omega = 25, X_\omega = \{1,2,3\}$

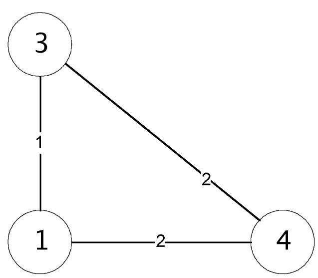


N

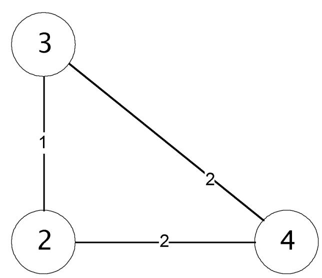
Here, N completes t_ω for total value $v_N = 4$. By Proposition 5, if t_ω is the only transaction then none of the agents has an ACA. Therefore, extend the example by adding the following transactions (recall that we do not depict inadmissible relations):



$t_{12} : u_{12} = 8, N_{12} = \{1,2\}$



$t_{13} : u_{13} = 8, N_{13} = \{1,3\}$



$t_{23} : u_{23} = 8, N_{23} = \{2,3\}$

Presented with feasible transaction set $T_N = \{t_\omega, t_{12}, t_{13}, t_{23}\}$, one can easily see that $\pi_4^{\min} = 4$. In this example, the broker appropriates *all* the available value! Why? Because the added value of any agent other than the broker (agent 4) is zero. For example, the group $G = \{1,2,4\}$ can complete t_{12}, thereby producing value of $v_{\{1,2,4\}} = 4$ via spanning tree $1 - 4 - 2$; thus, the added value of agent 3, $ava_3 = v_N - v_{\{1,2,4\}} = 0$. As the example illustrates, brokers can indeed enjoy strong competitive advantages even when they contribute nothing—beyond their ability to connect other agents—to the transaction.

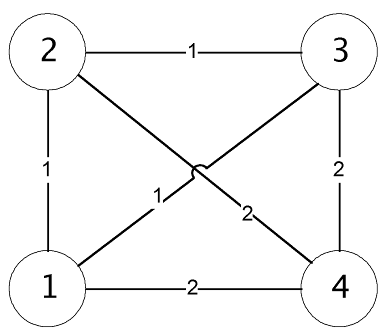
[15] Because we require i to be inessential to these transactions, this condition can only hold if i does not occupy a terminal position on any of these trees.

We can also say something about the minimum number of outside connections the broker must form and maintain in order to hold a positional advantage. If i is a broker in N, let d_i denote its degree in the network (i.e., the number of agents with whom i has a direct relationship in the network).

PROPOSITION 9. *Broker i has an ACA only if $d_i \geq 3$.*

This result strikes us as quite interesting in the context of the intellectual history of the theory of the broker's positional advantage. Simmel (1923) by reasoning through how three individuals might interact argued that the individual connecting two others could enjoy an advantage by bargaining them off against each other. That conjecture, however, would only hold if the broker also had some valuable capability or resource that it could combine independently with each of the other two parties to complete independent transactions. In the absence of such factors, this situation would not connote a positional advantage to the broker. Why? Because, without a third agent, the pure broker—who adds nothing essential to the transaction—does not have an alternative path for creating value, and hence no credible threat for leaving to produce value outside the network.

Interestingly, other factors also constrain the broker's ability to gain a positional advantage. Suppose that the broker could improve his productivity within the network. Would that increase strength of his competitive advantage? Not necessarily. To see why, suppose the cost function for t_ω becomes:



$$t_\omega : u_\omega = 25, X_\omega = \{1, 2, 3\}$$

In this case, still assuming $T_N = \{t_\omega, t_{12}, t_{13}, t_{23}\}$, $\pi_4^{\min} = 0$. Counterintuitively, agent 4 lost his competitive advantage because he became *too* productive. This result actually stems from a general, though often-overlooked, feature of value appropriation under competition. If the value available to the network as a whole becomes too large, then the broker can no longer entice any of his co-opetitors to join him in producing value under some alternative transaction. In other words, his productivity effectively eliminates the leverage value of his outside options. As we know, in order for an agent to have an absolute competitive advantage, sufficient tension must exist between the aggregate value available under t_ω and the value that agent can produce via the available alternatives. (Of course, in this example aggregate appropriation *does* increase in the new situation; agent 4 is simply no longer guaranteed a share of it).

Let i represent a broker in N, a network containing n agents. To generalize the preceding example, let $\mathcal{G}_i^F$ be the set of smallest groups in N that make the transactions in T_N feasible. That is, $G \in \mathcal{G}_i^F$ if there exists some $\{t_\omega\} \in T_N$ such that G makes t_ω feasible and that the removal of any node from G would result in a violation of the condition. Then, we define the *spanning value of* i as:

$$sv_i \equiv \min_{\pi_{-i} \in \mathbb{R}^{n-1}} \left\{ \Pi_{N_{-i}} \middle| \forall G \in \mathcal{G}_i^F, \Pi_{G_{-i}} \geq v_G \right\}. \tag{8}$$

Similar to (7), (8) captures the minimum quantity of value required to satisfy the competitive constraints implied by the network under the entire set of transaction demands T_N (still assumed to occur with certainty) without any appropriation by i. The value of (8) stems from the fact that it depends only upon a potentially much smaller number of groups: those that contain the network subtrees capable of completing attainable transactions.

Proposition 10. *If i is a broker, then i has an ACA if and only if $sv_i > v_N$.*

4.4. Stability of advantage types

The preceding sections demonstrate that an absolute competitive advantage can accrue from either capability/resource or positional scarcity. The question nonetheless remains whether such advantages can arise as stable outcomes of strategic investments. To assess the question of strategic stability, we now elaborate the first stage of the model, in which agents decide on how they wish to invest their scarce resources to form the relationships that they can then potentially use in the second stage to create and appropriate value. With this addition, our model becomes a "biform" game as introduce by Brandenburger & Stuart (2006).

To proceed, assume that every agent in N announces a vector of transfers $s^i \in \mathbb{R}^{n-1}$ in which s^i_{ij} denotes the transfer that player i proposes in order to form relationship ij. This relationship forms if $s^i_{ij} + s^j_{ij} \geq 0$. Formally, we can write the network that forms, given the *strategy profile* $s \equiv (s^1, ..., s^m)$, as $\gamma^s = (N, R^s)$, where

$$R^s \equiv \left\{ ij | s^i_{ij} + s^j_{ij} \geq 0 \right\}.$$

Note that agent i can always refuse a relationship with agent j by demanding more than j would pay for the relationship (s^i_{ij} sufficiently negative with respect to s^j_{ij}). Also, in equilibrium, it is always the case that $ij \in R^s$ implies $s^i_{ij} + s^j_{ij} = 0$.

We assume that every market structure γ^s maps to a specific, competitive pattern of appropriation $\pi(\gamma^s)$.[16] The payoff to i in the bi-form game is:

$$\xi_i(s) \equiv \pi_i(\gamma^s) - \sum_{ij \in R^s} s_{ij}^i.$$

We say that environment $(\mathcal{T}_N, \mu)$ *supports* a network structure γ^s if s forms a *neighborhood equilbrium.* A strategy profile $s \in S$ forms a *neighborhood equilibrium* if it is a pure strategy Nash equilibrium of the biform game and, for all $i \in N$, all $G \subseteq N$ including i and all $s' \in S$, $\xi_i(s) \geq \xi_i(s'_G, s_{-G})$. This special condition says that agent i can adjust its relationship status with any group of consenting agents (i.e., via changes to the s_{ij}^is and s_{ij}^js). Our equilibrium concept refines Bloch and Jackson's (2004) "pairwise" equilibrium. Pairwise equilibrium rules out strategy profiles in which some pair of agents is prevented from forming a relation not included in the profile (this concept, itself, refines Nash equilibrium, which has some undesirable properties in network formation games). We introduce neighborhood equilibrium because pairwise equilibrium seems too restrictive: If agent i can coordinate with one potential neighbor (in the graph-theoretic sense), why should we not allow it to coordinate with all potential neighbors? Combinations of relationships may prove attractive to an agent even when any individual pair-wise relationship within its neighborhood does not. What our definition rules out (as does pairwise equilibrium) is coordination across independent groups—an individual agent sits at the locus of every decision (hence, neighborhood equilibrium is weaker than strong Nash equilibrium which requires that no arbitrary group would change its strategy).

Let us begin by focusing on the stability of resource-based advantage. We can demonstrate our first result easily through an example. Consider three agents facing a set of symmetric transaction demands. In all cases, assume all relationship-related processing costs, c_ω's, to be zero. Specifically, assume $T = \{t_{123}, t_{12}, t_{13}, t_{23}\}$ arises with probability 1 with $t_{123} \equiv (N, 0, 120)$ and $t_{ij} \equiv (\{i,j\}, 0, 80)$ for all $\{i,j\} \subset N$. Let $s_{ij}^i = 0$ for all $\{i,j\} \subset N$. Under this strategy profile, the complete graph forms. Is this profile in equilibrium? Since s results in a complete network, the question boils down to whether any agent i could improve its situation by deviating to $s_{ij}^i < 0$ for $j \in N_{-i}$, and therefore would prefer to refuse one or more of its relationships. Note that, in the complete network, $\pi_i^{\min} = \pi_i^{\max} = 40$. If agent i refuses relationships with both of his co-opetitors, it isolates itself and faces $\pi_i^{\min} = \pi_i^{\max} = 0$. Suppose i refuses its relationship with j. Then, $\pi_i^{\min} = 0$ and $\pi_i^{\max} = 40$. Hence, i can never receive more than it would in the complete network. We summarize this point in the following proposition:

[16] Note that π depends upon network structure only and not, for example, the specific strategy profile that induces it.

PROPOSITION 11. *There exist market environments that support pure resource-based advantage.*

Proposition 11 obviously states a fairly weak condition; it merely points to the possibility of stable resource-based advantages. To say more would require additional assumptions about what agents believe they would appropriate under each network structure (and hence would lack generality). Still, as we will see, this proposition does provide a useful counterpoint to the case of positional advantage which cannot even meet this weak level of stability.

PROPOSITION 12. *Given* $(\mathcal{T}_N, \mu)$, *let* γ^s *be a market structure in which* i *is a broker that enjoys ACA. Then,* γ^s *is not supported in* $(\mathcal{T}, \mu)$.

To see the problem for the broker, let us return to the example above in which $v_N = 4$ and the broker appropriates all the surplus. Let γ denote the pictured network structure and s be any strategy profile that generates it. In order for s to be a Nash equilibrium, it must be that, for all $i \in N$, $\xi_i(s) \geq 0$. Thus, $4 \geq \sum_{i \in \{1,2,3\}} s_{4i}^4$ and, for $i \in \{1,2,3\}$, $s_{4i}^i \leq 0$. Now, consider the situation of agent 1. Agent 1 can enter into bilateral negotiations with both agents 2 and 3 in an attempt to exclude agent 4, the broker, from the value pool. Notice that, under s, agents $i \in \{1,2,3\}$ have $\xi_i(s) = s_{4i}^i = -s_{4i}^4$. Suppose agent 1 induces agents 2 and 3 to form relationships (in addition to the ones in γ). Denote the resulting network $\hat{\gamma}$. Under this network, $v_N = 23$. More importantly, the broker's added value drops to zero; hence, $\pi_4^{\max} = 0$. Not only does the broker lose its ACA, but it also loses any hope of appropriating any value whatsoever. Thus, under $\hat{\gamma}$,

$$\Pi_{\{1,2,3\}}(\hat{\gamma}) = 23.$$

Since $\Pi_{\{1,2,3\}}(\gamma) = 0$, the agents can arrange transfers that split up the difference. For each $i \in \{2,3\}$, let $\hat{s}_{i1}^i = \pi_i(\hat{\gamma}) - \frac{23}{3}$ and $\hat{s}_{i1}^1 = -\hat{s}_{i1}^i$. The transfers between $\{1,2,3\}$ and 4 remain as under s. Call this new profile $\hat{s}$. Then, for $i \in \{1,2,3\}$, $\xi_i(\hat{s}) = \frac{23}{3} + s_{4i}^4 > \xi_i(s)$. Thus, s fails the neighborhood rationality condition of our equilrium concept. Essentially, agent 1 has the power to organize a sequence of bilateral relationships that cut 4 completely out of the picture.

Our final proposition demonstrates the fragility of the positional advantage, and illustrates the fact that researchers can probably only apply it to a limited range of strategic situations. On the one hand, we do find that brokers can gain a competitive advantage entirely on the basis of their positions in their networks. In this sense, our results fit well with the frequent finding that, at the level of an individual, researchers frequently find that tremendous rewards accrue to those in brokering positions (e.g., Burt, 1992; Podolny & Baron, 1997). If the structure of the network arises exogenously to the rewards associated with particular positions within it, then we see no reason why actors could not exploit these fortuitous circumstances.

On the other hand, our results raise serious questions as to whether brokers should ever appear, and if they do whether they can maintain their positions, in situations where actors may form their relationships strategically. Hence, when examining whether firms—that presumably at least attempt to maximize their profitability—can benefit from positional advantages, we find ourselves considerably more skeptical. Proposition 12 implies that firms most likely cannot . Perhaps not surprisingly then, empirical research at the firm level has been somewhat more equivocal in its findings. Ahuja (2000) and Bae and Gargiulo (2004), for example, both fail to find a positive relationship between brokering and performance.

5. Conclusion

Two primary issues in strategy concern: (1) What accounts for variation in firm performance? and (2) To what extent can managers control these factors? Of course, the answers to these questions abound, and we would not attempt to address such broad issues within the scope of a single paper. We nonetheless *do* consider these questions carefully within the context of a particular potential *source* of competitive advantage: a firm's (or other economic actor's) position within a network of relations, whether those relations represent sourcing relations, alliances, syndicated investments, joint ventures, or some other form of collaboration or exchange.

Though this subject has garnered ample attention—papers appear on the value of "social capital" and the benefits of "embeddedness" in nearly every issue of the major journals in management—these studies have often raised as many questions as they answer. For example, on the one hand, critics question whether network positions themselves can convey a competitive advantage on the actors occupying those positions or whether they simply reflect the valuable capabilities and/or resources that these actors control. Similarly, even if rewards do accrue to these positions, these (frequently cross-sectional) studies leave open the question of the durability of these positional advantages. Though some studies have investigated these issues with longitudinal data, they have generally used the panel structure more as a means of addressing the issue of observed heterogeneity described above than to investigate the temporal stability of positional advantages.

To address these issues, we have developed a biform game approach to analyze the dynamics of value creation and value appropriation in contexts where the topology of the network restricts the options available to actors. We see substantial need for such a formal model for three reasons. First, whereas disentangling positions from capabilities and resources in any empirical setting can prove daunting if not impossible, our framework allows us to separate these concepts analytically. Hence, we can assess whether a capability/resource-based advantage can exist independently from a valuable position and *vice versa*. Second, even when simply trying to assess the affects of network structure on the development of coalitions, the subtle interplay of competitive forces proves quite complex. The issues become even more involved when we allow networks to evolve endogenously. Attempts to think through these dynamics without resorting to analytical tools exceeded our combined cognitive capacity. Finally, simply the process of translating the existing verbal theory into a formal model alerted us to several points where progress required clarification on the existing concepts.

Although we will not review every proposition here, we will highlight those that relate specifically to the questions of interest. First, a scarce capability or resource can, in general, serve as a source of competitive advantage. Interestingly, however, such an advantage is not necessarily guaranteed under every network structure. Crucial to the existence of a competitive advantage in any coalitional game is the availability of attractive options outside the coalition. Network structures can effectively destroy an actor's competitive advantage by eliminating its options to create value elsewhere.

Second, positional scarcity can, itself, offer a competitive advantage to the actor occupying that position. Our notion of scarcity here accords with the imagery of a broker, bridge, or boundary-spanner used in the existing literature on networks. Hence, our result appears consistent with much of the empirical literature on the benefits that accrue to these positions in social networks. Our model nonetheless identifies at least two important scope conditions to such a positional advantage: (1) A broker must have connections to at least three other actors to enjoy a purely positional advantage; and (2) If the broker's position becomes *too strong*, its competitive advantage can disappear. Both of these non-obvious results stem from the need for outside options. If the broker links only two actors, then it has no options for producing value (through brokering) outside of the pair and therefore cannot credibly threaten to exit the coalition. Similarly, if the broker's position becomes too strong, its interest in seeing the "deal" go through become too strong for it to negotiate effectively vis-à-vis the other parties (though it could still capture value through negotiating acumen, a force outside our model).

Finally, positional scarcity *cannot* generate a stable competitive advantage under endogenous network formation—when actors actively form relations in response to their expectations of how those relations will influence their abilities to create, and to capture, value. Though we recognize that this condition does not hold in a variety of situations—for example, few form friends on the basis of such rational calculation—the recognition that it does form a scope condition to the availability of positional advantages has important consequences. Most notably, relationships formed between firms, or even between individuals for purely commercial purposes, may well violate this condition. Moreover, any strategic attempt to attain an attractive position clearly breaches. It therefore appears unlikely that managers could achieve a competitive advantage purely through the pursuit of a scarce position.

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Appendix A: Graph Theory

An undirected graph is a pair (M,R) where M is a finite set of *nodes* and $R \subseteq M^2$ is a set of *edges* with typical element $\{i,j\}$ said to *relate* i and j. For simplicity, we write $ij \in R$ (and note that, since the graph is undirected, ij refers to exactly the same edge as ji).[17] A *path* is an ordered set of nodes

$$P \equiv \{i_0,...,i_k\} \tag{9}$$

such that, for $l=0,...,k-1$, $\{i_l,i_{l+1}\} \in R$. The *length* of (9) is the number of edges between its nodes, i.e., k. An *edge* is a path of length 1. Nodes i and j are *connected* in (M,R) if there exists a path P such that $\{i,j\} \subset P$.

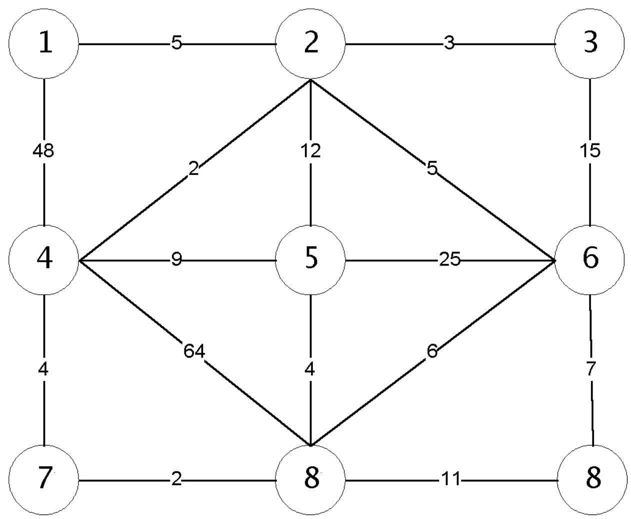
(M,R) is *connected* if, for all $\{i,j\} \subset M$, i and j are connected. (M,R) is *complete* if every node is connected to every other node. If any two nodes in (M,R) are connected by exactly one path, then (M,R) is a *tree*. Here, singleton graphs of the form $(\{i\},\emptyset)$ are (trivial) trees. If $M' \subseteq M$ and $R' \subseteq R$, then (M',R') is a *subgraph* of (M,R). If R' contains all the edges in R that join two vertices in M', then (M',R') is the subgraph *induced by* M'. If (M',R') is the subgraph induced by M' and, for all $i \in M', j \in M$, $ij \in R$ if and only if $ij \in R'$, then (M',R') is a *component* of (M,R). A component is a maximally connected subgraph. In our analysis, isolated nodes are considered components.

A *spanning tree* of (M,R) is any tree (M,R'). If (M,R) is associated with an edge cost function $c: R \to \mathbb{R}_+$, then a minimal spanning tree is a spanning tree (M,R') that minimizes $\sum_{ij \in R'} c(ij)$.

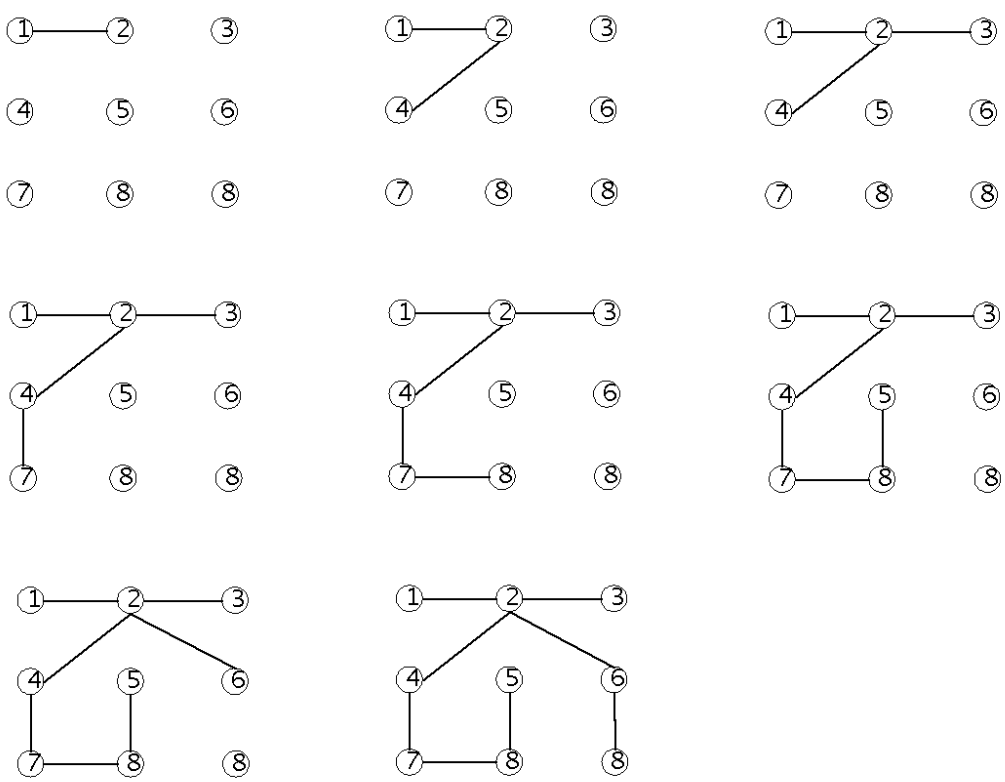
[17] Here, we adopt the notational convention for edges used, e.g., in Jackson & Wolinsky (1996).

Appendix B: MST Algorithm

To see how this works, consider the following transaction, t_1, in a market with $|M| = 9$ firms. Assume $X_1 = M$, $u_1 = 100$ and c_1 is as summarized below:



Start with agent 1 and proceed as shown below. Any network N containing a subgraph with agent resources linked as in the final graph delivers the maximum value: $v_N(t_1) = 100 - 34 = 66$.



Appendix C: Proofs of the Propositions

C.1. Proposition 4

First, consider an arbitrary group $G \subseteq M$. Structure s implies a partition of this group,

$$\mathcal{P}^s_{M|G} \equiv \{G \cap N | N \in \mathcal{P}^s_M\}.$$

Using the same logic as in the derivation of (6), it can be shown that

$$v_G = \sum_{X \in \mathcal{P}^s_{M|G}} v_X. \tag{10}$$

Now, recall, π is a CPA under (M, v) if it satisfies:

$$\begin{array}{r} \sum_{i \in M} \pi_i = v_M, \\ \forall G \subset M, \ \sum_{i \in G} \pi_i \geq v_G. \end{array} \tag{11}$$

By equation (10), for all $G \subseteq M$, condition (11) is implied by, for all $X \in \mathcal{P}^s_{M|G}$,

$$\sum_{i \in X} \pi_i \geq v_X.$$

In other words, constraints in (11) associated with groups containing agents from more than one network are redundant. This combined with Proposition 3 imply that constraints (11) can be rewritten and grouped: for all $N \in \mathcal{P}^s_M$

$$\begin{array}{r} \sum_{i \in N} \pi_i = v_N, \\ \forall G \subset N, \ \sum_{i \in G} \pi_i \geq v_G. \end{array} \tag{12}$$

For any $N \in \mathcal{P}^s_M$, let $\hat{\pi}$ be any CPA under $(N, \hat{v})$. By definition, $\hat{\pi}$ satisfies

$$\begin{array}{r} \sum_{i \in N} \pi_i = v_N, \\ \forall G \subset N, \ \sum_{i \in G} \pi_i \geq v_G. \end{array} \tag{13}$$

Given any CPA π under (M, v), construct a new CPA as follows: i) for all $i \in N$, $\breve{\pi}_i = \hat{\pi}_i$; and, ii) for all $j \notin N$, $\breve{\pi}_j = \pi_j$. Then, because $\hat{\pi}$ is a CPA under $(N, \hat{v})$, $\breve{\pi}$ satisfies all the constraints associated with $N \in \mathcal{P}^s_M$. Since π is already a CPA under (M, v), $\breve{\pi}$ satisfies all the constraints associated with $N' \in \mathcal{P}^s_M \backslash N$. Therefore, $\breve{\pi}$ is also a CPA under (M, v).

Proposition

C.2. Proposition 6

Suppose N is independently decomposable and that $\pi^{\min}_i > 0$. If $\{i\} \in \mathcal{G}$ then $\pi^{\min}_i = 0$ because $v_{\{i\}} = 0$. If $\{i\} \notin \mathcal{G}$, then $i \in G \in \mathcal{G}$ for some G with $|G| \geq 2$. Suppose $j \neq i$ and $j \in G$. Let π be a CPA such that $\pi_i = \pi^{\min}_i$. Then, by the definition of a CPA, π satisfies $\Pi_N = v_N$ and, for all $G \in 2^N$, $\Pi_G \geq v_G$. Let: (i) $\mathcal{N}^{+i}_{++}$ be the set of groups $G \subset N$ such that $i \in G$ and $v_G > 0$, (ii) $\mathcal{N}^{-i}_{++}$ be those $G \subset N$ such that $i \notin G$ and $v_G > 0$, and (iii) $\mathcal{N}_0$ be the set of all $G \subset N$ such that $v_G = 0$. Note that $N \cup \mathcal{N}^{+i}_{++} \cup \mathcal{N}^{-i}_{++} \cup \mathcal{N}_0$ is a partition of 2^N.

Now, construct $\hat{\pi}$ as follows: for all $k \in \{i, j\}$, $\hat{\pi}_k = \pi_k$, $\hat{\pi}_i = 0$ and $\hat{\pi}_j = \pi_j + \pi^{\min}_i$. Then, $\Pi_N = v_N$ and, clearly, for all $G \in \mathcal{N}_0$, $\hat{\Pi}_G \geq v_G$. If $v_G > 0$ either $\{i, j\} \subseteq G$ or $\{i, j\} \nsubseteq G$. Hence, for all $G \in \mathcal{N}^{-i}_{++}$, $\hat{\Pi}_G \geq v_G$ as $\hat{\Pi}_G = \Pi_G$ because no changes were made to the payoffs to these agents. Finally, for all $G \in \mathcal{N}^{+i}_{++}$, $\hat{\Pi}_G \geq v_G$ as $\hat{\Pi}_G = \Pi_G$ because $\{i, j\} \subseteq G$ and the only change was to transfer i's appropriation to j.

C.3. Proposition ??

The logic is identical to the preceding proof, this time letting G_i be the smallest subtree containing i.

C.4. Proposition 7

This follows from the observation that if i and j are both brokers, then both are always required to complete transactions. The method of proof is then identical to that of Proposition 6.

C.5. Proposition 10

Assume $i \in N$ is a broker. Let $\mathcal{N}_{++}^{+i}$ be the set of groups $G \subset N$ such that $i \in G$ and $v_G > 0$ and $\mathcal{N}_0^{+i}$ be the set of groups $G \subset N$ such that $i \in G$ and $v_G = 0$. By Proposition 4 (and the fact that zero value groups are not binding), $mv_i > v_M$ if and only if

$$\min\left\{\sum_{j\in N_{-i}} \pi_j \mid \text{for all } G \in \mathcal{N}_{++}^{+i}, \sum_{j\in G_{-i}} \pi_j \geq v_G\right\} > v_N. \tag{14}$$

Then, we wish to show that the constraints in (8) imply those in (14) and, hence, $sv_i = mv_i$. Clearly, for all $G \in \mathcal{G}_i^F \cap \mathcal{N}_{++}^{+i}$, the constraints are identical. Note that if $G \in \mathcal{G}_i^F$ and $G \notin \mathcal{N}_{++}^{+i}$ then $G \in \mathcal{N}_0^{+i}$. So, consider $G \in \mathcal{N}_{++}^{+i}$ such that $G \notin \mathcal{G}_i^F$. Since i is required to complete every transaction, any arbitrary group in N completes at most 1 transaction. Thus, $\mathcal{T}_N$ is a collection of singleton sets. By the definition of a broker, $v_G > 0$ implies that there is some $\{t_\omega\} \in \mathcal{T}_N$ such that $v_G = \varphi_G(\{t_\omega\})$. Let G' be the smallest group that completes t_ω. By definition, $G' \in \mathcal{G}_i^F$. Moreover, $v_{G'} = v_G$. Since it must be the case that $G' \subset G$, for all π,

$$\sum_{j\in G'_{-i}} \pi_j \geq v_{G'} \Rightarrow \sum_{j\in G_{-i}} \pi_j \geq v_G.$$

Therefore, if i is broker, $sv_i = mv_i$.

C.6. Proposition 12

Let γ be a market structure in which i is a broker that enjoys ACA and assume that s is a neighborhood equilibrium that generates γ. If, for any $j \in N$, $\xi_j(s) < 0$, then s is not a Nash because it is not individually rational for j, who can always assure payoff of at least zero. Suppose this is not the case. In particular, agent i earns $\xi_i(s) = \pi_i(\gamma) - \sum_{ij\in R^s} s_{ij}^i$ where $\pi_i(\gamma) \geq \pi_i^{\min}(\gamma) > 0$.

Consider some $j \in X_\omega$. Let R_j^s be j's relationships that form under s and define $G^* \equiv \{k \in N | jk \notin R_j^s, k \in X_\omega\}$; i.e., the set of agent in X_ω with whom j has unformed relations under s. Let $G_{+j}^* \equiv G^* \cup \{j\}$. Suppose j and the agents in G^* pick new strategies such all jk relations, $k \in G^*$, are established. Let $\hat{\gamma}$ be the new network structure so generated (i.e., that includes all the relationships in γ as well).

Let t_ω indicate the transaction completed by N that produces v_N^s. Recall the assumption that, for all trees (Y, R_Y^k) such that $X_\omega \subseteq Y \subseteq N$, $\sum_{ij\in R_Y^k} c_\omega(ij)$ is minimized *only* if $X_\omega = Y$. Thus, $v_N^{\hat{\gamma}} > v_N^s$ (where $v_N^{\hat{\gamma}}$ is unambigous since v only depends upon network structure and not the strategy that generates it). Moreover, for all $l \notin G_{+j}^*$, $ava_l^{\hat{\gamma}} = 0$ implying $\pi_l(\hat{\gamma}) = \pi_l^{\min}(\hat{\gamma}) = 0$. (Note that broker $i \notin G_{+j}^*$.) Together, these imply that

$\Pi_{G^*_{+j}}(\hat{\gamma}) - \Pi_{G^*_{+j}}(\gamma) > \Pi_{-G^*_{+j}}(\gamma) \geq 0$. Let $\lambda \equiv \frac{\Pi_{G^*_{+j}}(\hat{\gamma}) - \Pi_{G^*_{+j}}(\gamma)}{\left|G^*_{+j}\right|}$ be the share of the additional value achieved under $\hat{\gamma}$ distributed to each agent in G^*_{+j}.

Under s, each agent $l \in G^*$ gets

$$\begin{aligned}\xi_l(s) &= \pi_l(\gamma) - \sum_{lk \in R^s} s^l_{lk}, \\ &= \pi_l(\gamma) - s^l_{lj} - \sum_{lk \in R^s \setminus \{lj\}} s^l_{lk}.\end{aligned}$$

Agent j's payoff is

$$\begin{aligned}\xi_j(s) &= \pi_j(\gamma) - \sum_{jk \in R^s} s^j_{jk}, \\ &= \pi_j(\gamma) - \sum_{l \in G^*} s^j_{lj} - \sum_{k \notin G^*} s^j_{jk}.\end{aligned}$$

For each agent $l \in G^*$ set

$$\hat{s}^l_{lj} = \pi_l(\hat{\gamma}) - \pi_l(\gamma) - \lambda.$$

Then, for agent j and all $l \in G^*$ set $\hat{s}^j_{lj} = -\hat{s}^l_{lj}$. Adopting these and maintaining all other transfers as under s results in a new strategy profile $\hat{s}$ that generates $\hat{\gamma}$. The payoffs to $l \in G^*$ are

$$\begin{aligned}\xi_l(\hat{s}) &= \pi_l(\hat{\gamma}) - \hat{s}^l_{lj} - \sum_{lk \in R^s \setminus \{lj\}} s^l_{lk}, \\ &= \pi_l(\gamma) + \lambda - \sum_{lk \in R^s \setminus \{lj\}} s^l_{lk} \\ &= \xi_l(s) + \lambda.\end{aligned}$$

For agent j,

$$\xi_j(\hat{s}) = \pi_j(\hat{\gamma}) - \sum_{l \in G^*} \hat{s}^j_{lj} - \sum_{k \notin G^*} s^j_{jk}.$$

Substituting,

$$\begin{aligned}\xi_j(\hat{s}) &= \pi_j(\hat{\gamma}) + \sum_{l \in G^*} \left(\pi_l(\hat{\gamma}) - \pi_l(\gamma) - \lambda\right) - \sum_{k \notin G^*} s^j_{jk}, \\ &= \sum_{l \in G^*_{+j}} \pi_j(\hat{\gamma}) - \sum_{l \in G^*} \pi_l(\gamma) - |G^*|\lambda - \sum_{k \notin G^*} s^j_{jk}, \\ &= \Pi_{G^*_{+j}}(\hat{\gamma}) - \Pi_{G^*_{+j}}(\gamma) - |G^*|\lambda + \pi_j(\gamma) - \sum_{k \notin G^*} s^j_{jk}.\end{aligned}$$

Substituting $\lambda \equiv \frac{\Pi_{G^*_{+j}}(\hat{\gamma}) - \Pi_{G^*_{+j}}(\gamma)}{\left|G^*_{+j}\right|}$ and noting that $\left|G^*_{+j}\right| - |G^*| = 1$ yields

$$\begin{aligned}\xi_j(\hat{s}) &= \pi_j(\gamma) + \lambda - \sum_{k \notin G^*} s^j_{jk}, \\ &= \xi_j(s) + \lambda.\end{aligned}$$

Since $\lambda > 0$, this contradicts the assumption that s satisfies neighborhood rationality.

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