Estimating Firm-Level Risk

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- Interest in models with firm heterogeneity, e.g.:
    1. Reallocation
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- Productivity shock process is a critical input in these models.

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- Implications: size distribution; quantitative implications for invt/CF

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- E.g. Blundell-Preston, Guvenen-Smith, Campbell-Abbring.

Outline

1. Example to illustrate the role of the persistence

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5. Implications in a GE framework
6. Application: evolution of idiosyncratic risk in the last 40y in the US.

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- No aggregate shocks, Representative consumer

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- Assume firms know their z_{it} when they pick capital:

$$\alpha z_{it} k_{it}^{\alpha-1} = r + \delta$$

$$k_{it} = \left(\frac{\alpha z_{it}}{r+\delta} \right)^{\frac{1}{1-\alpha}}$$

Example of the role of persistence: Reallocation (2)

- Aggregating:

$$Y_t = Y = \int z_{it} k_{it}^{\alpha} di,$$
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3. Ex: if z is lognormal, then

$$\log TFP = \log E(z) + \frac{Var(\log z)}{2} \frac{\alpha}{1-\alpha}$$

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  - hence **"micro persistence matters for macro"**

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- Marginal Q:

$$q_{it} \simeq E_t \sum_{k \geq 1} \beta^k (1-\delta)^{k-1} A_{it+k} = \frac{\beta \rho z_{it}}{1 - \beta(1-\delta)\rho}.$$

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Var\left(\frac{\Pi_{it}}{K_{it}}\right) &= Var\left(A_{it}\right) = \sigma_z^2 + \sigma_\varepsilon^2, \\
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- Assume we know β, δ, η.
- Then, given the 3 moments can recover $\sigma_z^2, \sigma_\varepsilon^2$ and ρ :

$$\frac{Cov\left(\frac{I_{it}}{K_{it}}, \frac{\Pi_{it}}{K_{it}}\right)}{Var\left(\frac{I_{it}}{K_{it}}\right)} \rightarrow \rho,$$

$$\text{Then, } Var\left(\frac{I_{it}}{K_{it}}\right) \rightarrow \sigma_z^2, \qquad Var\left(\frac{\Pi_{it}}{K_{it}}\right) \rightarrow \sigma_\varepsilon^2.$$

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- e.g. fixed costs, irreversibility, financing constraints, ...
- data of large US firms $\longrightarrow$ these constraints less likely to matter.
- Adjustment cost model is likely a good benchmark for these firms
  Eberly et al'08, Erickson-Whited '00, Gilchrist-Himmelberg'95, Philippon'08

Model

$$E \sum_{t=0}^{\infty} \beta^t \left(z_t K_t^{\alpha} - I_t - K_t c \left(\frac{I_t}{K_t} - \varepsilon_t^{AC} \right) \right)$$

$$\begin{aligned} c(\delta) &= 0, \quad c'(\delta) = 0, \\ c'' &> 0 \end{aligned}$$

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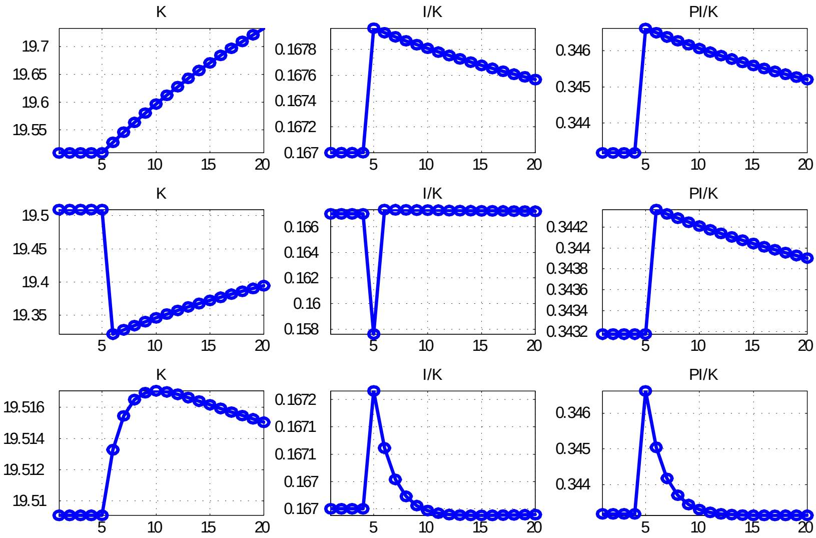
- Solve model using linear-quadratic approximation.

IRF to shocks



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- Set $\beta = 0.95$.
- Estimate $\alpha, \delta, \eta, \sigma_p, \sigma_t, \rho, \sigma_m, \sigma_{ac}$ [or σ_{mi}] by SMM using the following 12 moments:

$$E\left(\frac{I_{it}}{K_{it}}\right),\ E\left(\frac{\pi_{it}}{K_{it}}\right)$$

$$Var\left(\frac{I_{it}}{K_{it}}\right), Var\left(\frac{\pi_{it}}{K_{it}}\right), Cov\left(\frac{I_{it}}{K_{it}}, \frac{\pi_{it}}{K_{it}}\right),$$

$$Cov\left(\frac{I_{it}}{K_{it}}, \frac{I_{it-1}}{K_{it-1}}\right), Cov\left(\frac{I_{it}}{K_{it}}, \frac{I_{it-2}}{K_{it-2}}\right), Cov\left(\frac{I_{it}}{K_{it}}, \frac{I_{it-3}}{K_{it-3}}\right),$$

$$Cov\left(\frac{\pi_{it}}{K_{it}}, \frac{\pi_{it-1}}{K_{it-1}}\right), Cov\left(\frac{\pi_{it}}{K_{it}}, \frac{\pi_{it-2}}{K_{it-2}}\right), Cov\left(\frac{\pi_{it}}{K_{it}}, \frac{\pi_{it-3}}{K_{it-3}}\right),$$

$$Cov\left(\log\frac{K_{it}}{K_{it-3}}, \frac{\pi_{it-3}}{K_{it-3}}\right).$$

Monte Carlo Experiments

- Concern: ρ poorly identified?
- Simulate 100 panels of artificial data from the model
- Does the estimation method recover the true parameters?

Specification		η	σ_p	σ_t	ρ	σ_m	σ_{mi}	α	δ
Perm.	Truth	1	.15	–	–	.3	.6	.64	.17
only	Mean	1.00	.148	–	–	.302	.602	.641	.17
	SD	.038	.008	–	–	.017	.024	.012	.006
AR(1)	Truth	1	–	.4	.93	.3	.6	.64	.16
only	Mean	.997	–	.402	.934	.304	.673	.641	.158
	SD	.067	–	.028	.013	.024	.021	.0210	.005
Perm	Truth	1	.15	.35	.7	.3	.6	.64	.16
+ AR(1)	Mean	.984	.145	.360	.696	.292	.600	.641	.160
	SD	.088	.015	.311	.086	.033	.020	.017	.005

Data Moments

- Compustat, drop mergers, outliers for $\frac{\pi}{K}$, $\frac{I}{K}$.
- Balanced panel 1980-2006. $N = 128$.
- Unbalanced panel 1972-2006. $N = 18,309$, total obs $= 131,448$.

	Ei	$E\pi$	σ_i	σ_π	$\rho_{i\pi}$	ρ_{1i}	ρ_{2i}	$\rho_{1\pi}$	$\rho_{2\pi}$	$\rho(\log \frac{K}{K_{-3}}, \frac{\pi_{-3}}{K_{-3}})$
UB	.167	.35	.15	.48	.19	.43	.19	.69	.49	.31
s.e.	.00	.01	.00	.01	.01	.01	.01	.01	.01	.01
Bal.	.128	.32	.09	.28	.29	.52	.27	.79	.66	.34
s.e.	.00	.02	.01	.02	.05	.04	.05	.03	.04	.04
TE	–	–	.15	.48	.18	.50	.33	.71	.53	.30
s.e.	–	–	.00	.01	.01	.01	.01	.01	.01	.01

Estimation Results: Unbalanced panel

	ρ	η	σ_m	σ_t	σ_p	α	δ	σ_{mi}
Perm.	—	1.943	.962	—	.245	.637	.167	.790
only	—	0.074	.015	—	.005	.010	.001	.011
AR(1)	.780	1.123	.465	0.890	—	.647	.169	.788
only	.007	0.055	.018	0.016	—	.010	.001	.010
Perm.	.550	1.486	.030	1.004	.207	.645	.169	.773
+ AR(1)	.020	0.068	.628	0.026	.006	.010	.001	.0112

Interpretation

- Adjustment cost parameter: if $c\left(\frac{I}{K}\right) = \frac{\psi}{2}\left(\frac{I}{K} - \delta\right)^2$ then $\eta = \delta\psi \rightarrow \psi \simeq 8$, middle range of estimates

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- If $\upsilon = \frac{2}{3}$, divide size of shocks by 3 $\rightarrow \sigma_p = 7\%$.

Moments Fit

	Ei	$E\pi$	σ_i^2	σ_π^2	$\gamma(i,\pi)$	$\gamma_1(i)$	$\gamma_1(\pi)$	$\gamma_2(i)$	$\gamma_2(\pi$
Data	.169	.343	2.37	24.16	1.35	1.01	16.6	.42	11.8
Models:									
1	.167	.345	2.37	24.16	2.88	.62	12.8	.60	12.4
2	.169	.343	2.37	24.16	3.49	.44	16.3	.32	12.1
3	.169	.343	2.37	24.16	3.16	.61	16.3	.58	11.8

- Second moments mutplied by 100

Variance Decomposition

		σ_p	σ_t	σ_m	σ_{mi}	Total
Model 1a	i	26.4	0	0	73.6	100
	π	54.0	0	46.0	0	100
Model 2a	i	0	24.1	0	75.9	100
	π	0	88.6	11.4	0	100
Model 3a	i	25.3	2.8	0	71.8	100
	π	30.6	69.3	0.1	0	100

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- Important temporary shocks to profits
- **But they matter much less for investment**
- Measurement error is important ("failure of q-theory")

Unbalanced panel with time Effects

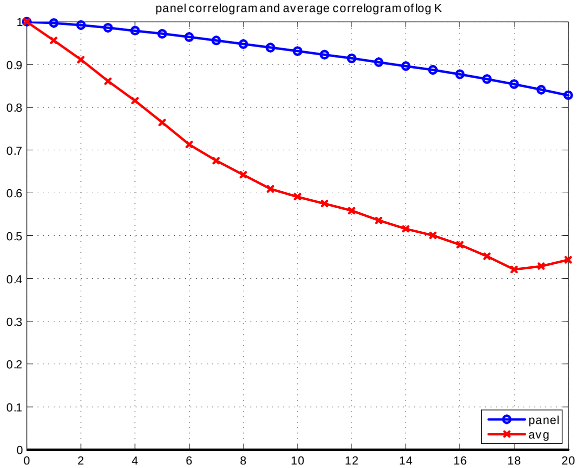
		ρ	η	σ_m	σ_t	σ_p	σ_{mi}
Perm.	Est.	—	2.142	.927	—	.242	.793
only	s.e.	—	0.078	.013	—	.004	.011
AR(1)	Est.	.820	1.308	.519	.809	—	.800
only	s.e.	.005	0.060	.014	.015	—	.011
Perm	Est.	.525	1.728	.000	.959	.218	.787
+AR(1)	s.e.	.022	0.072	na	.027	.004	.012

Balanced Panel

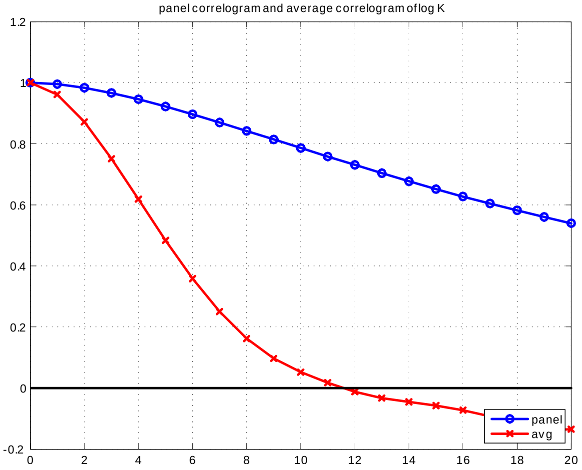
		ρ	η	σ_m	σ_t	σ_p	α	δ	σ_{mi}
Perm.	Est.	—	1.668	.482	—	.176	.566	.127	.577
only	s.e.	—	0.314	.041	—	.015	.030	.004	.032
AR(1)	Est.	.890	1.079	.275	.433	—	.567	.127	.575
only	s.e.	.019	0.220	.053	.046	—	.030	.004	.034
Perm	Est.	.600	1.388	.000	.499	.163	.567	.127	.569
+AR(1)	s.e.	.105	0.252	na	.053	.017	0.030	.004	.034

- Also tried industry effects; calibrating vs. estimating α, δ; changing β
- σ_{ac} specification very similar to. σ_{mi} specification

Correlogram of log(K) w and w/o fixed effects: Data



Correlogram of log(K) w and w/o fixed effects: AR(1) model



Correlogram of log(K) w and w/o fixed effects: Permanent shocks + AR(1) model

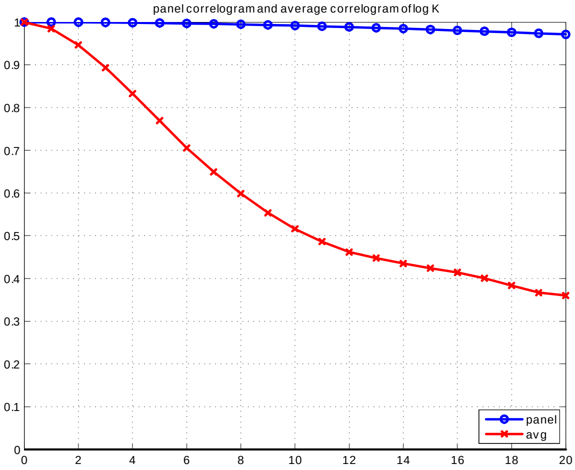
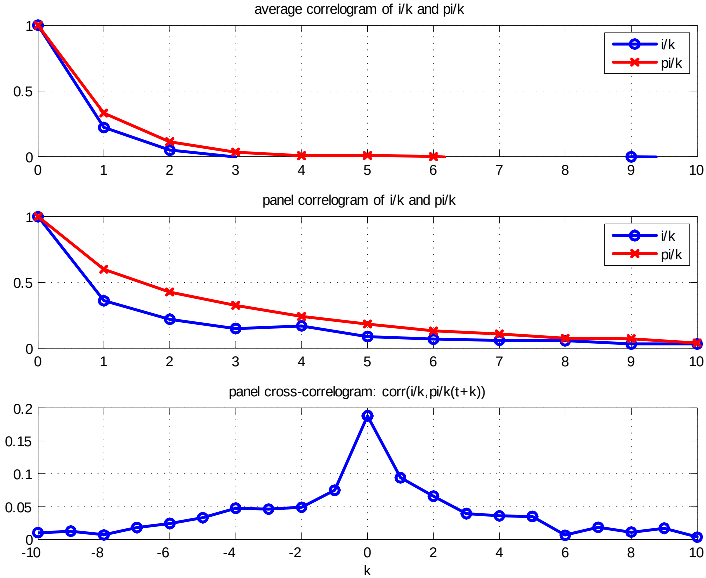
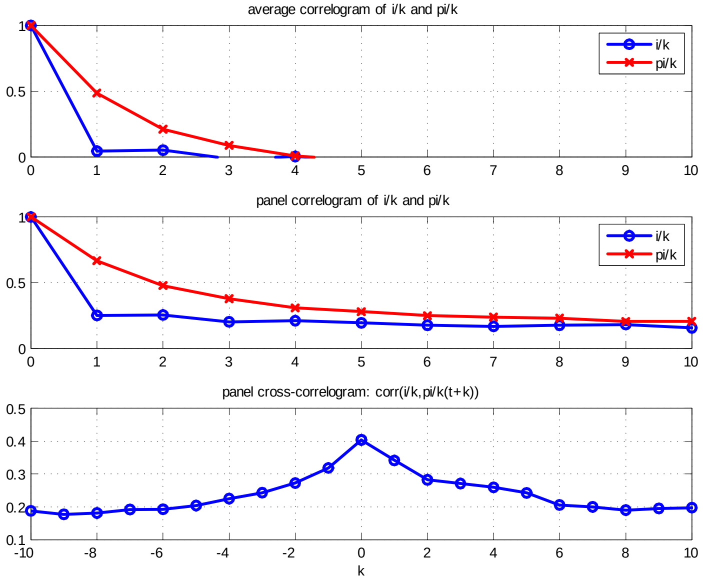


Figure:

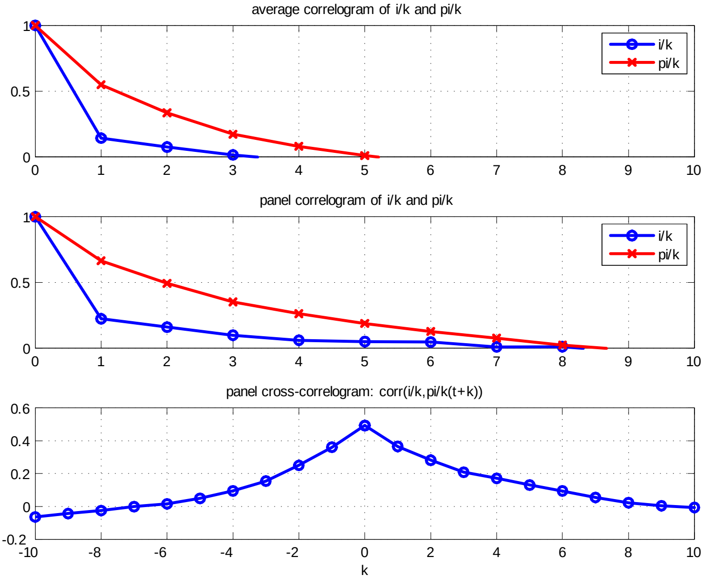
Correlogram of I/K, PI/K: Data



Correlogram of I/K, PI/K: AR(1) shock model



Correlogram of I/K, PI/K: AR(1) + permanent shocks



Implications of the estimates in a GE framework

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- Inelastic labor supply, no aggregate shocks.

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- Entry: cost c_{in}, get capital K_{in} and draw (z_p, z_t) from cdf υ.

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- Stationary measure:

$$\begin{aligned} & \mu(A \times B \times C) \\ = \; & (1-\delta) \int 1_{g(K, z_p, z_t) \in A} \Pr(\varepsilon_p', z_t' \in ...) \mu(dK, dz_p, dz_t) \\ & + M \times \upsilon(z_p, z_t) 1_{K_{in} \in A}, \end{aligned}$$

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- Important error.

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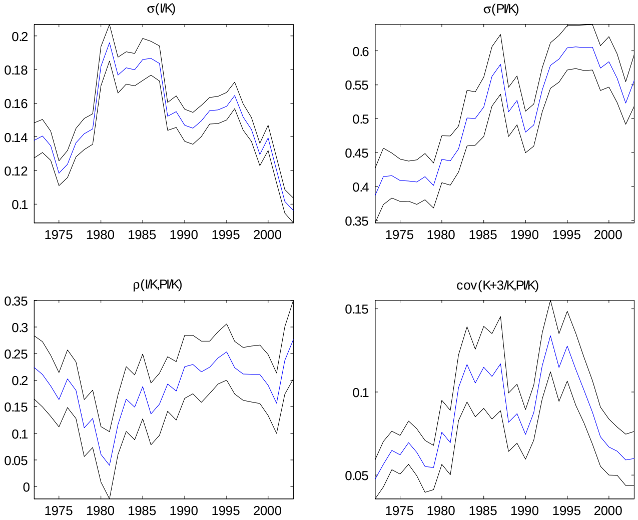
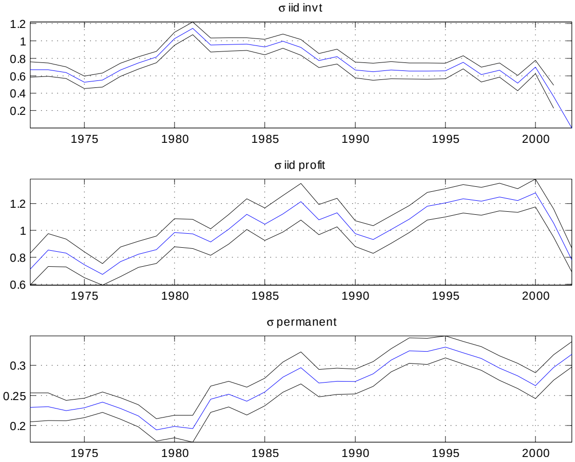
- Consistent with optimization since variances do not affect firms' decision rules.
- Fix α, δ, η.

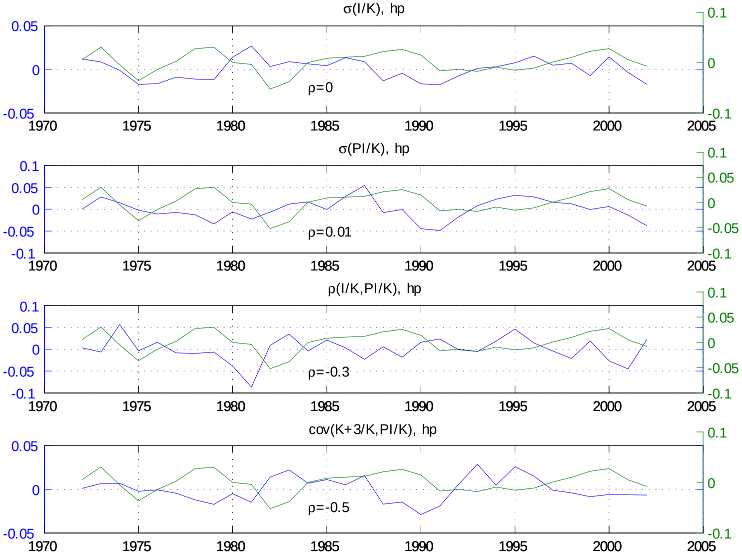
Time variation in data moments



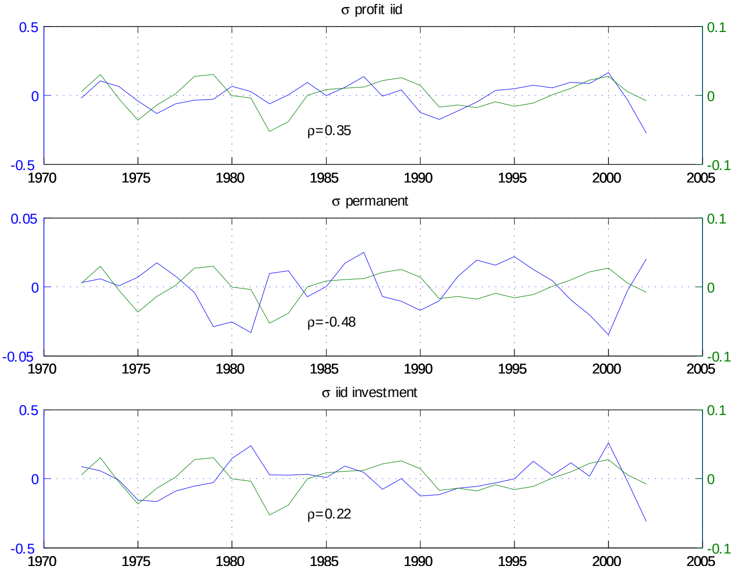
Estimated Parameters: model with no transitory shock



Business cycles variation in data moments



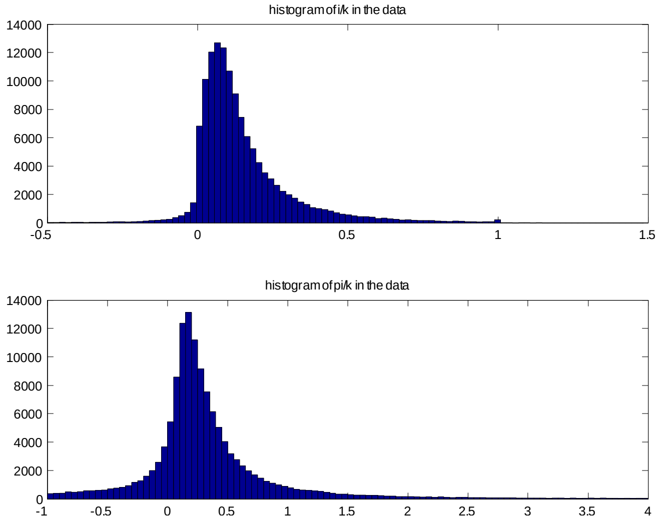
Business cycle variation in fundamental shocks

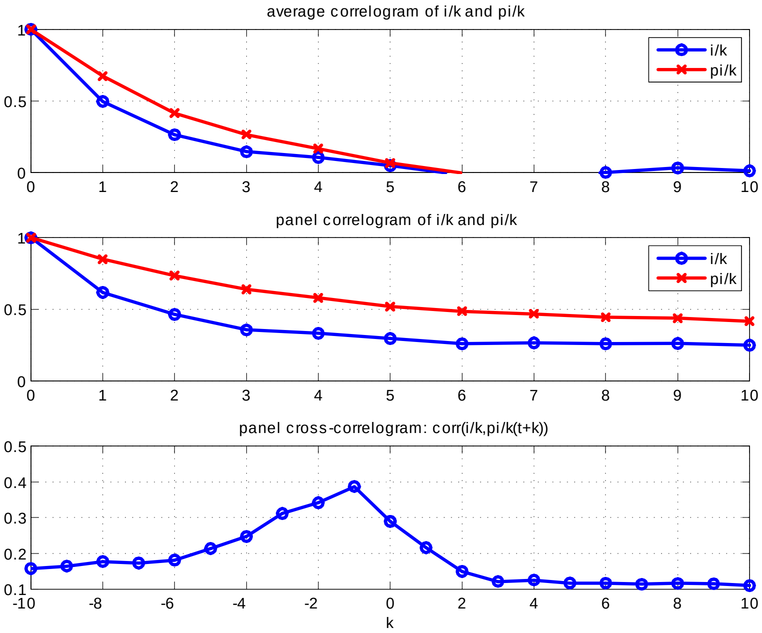


Conclusions and work in progress

- Use investment choices to estimate persistence of shocks
- Permanent shocks appear to be important.
- They may lead to different conclusions in some of our models.
- Next:
  1. News shocks / Persistent shocks to growth rates
  2. Estimation by Maximum Likelihood
  3. Robustness: how does the estimation behave if there are fixed costs / irreversibility / financing constraints...?

BACKUP


histogram of i/k in the data
14000
12000
10000
8000
6000
4000
2000
0
-0.5
0
0.5
1
1.5
histogram of pi/k in the data
14000
12000
10000
8000
6000
4000
2000
0
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4



average correlogram of i/k and pi/k
i/k
pi/k
panel correlogram of i/k and pi/k
i/k
pi/k
panel cross-correlogram: corr(i/k,pi/k(t+k))
k

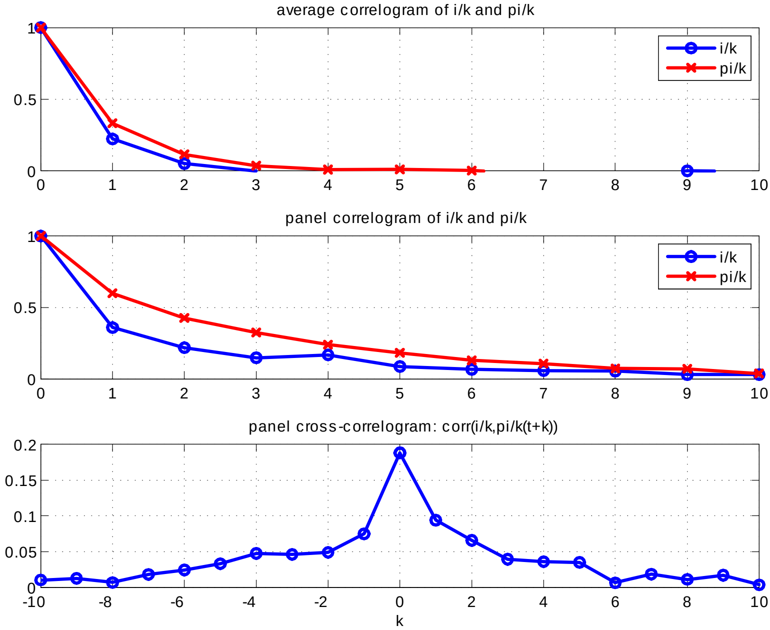


Figure: Correlogram of investment rate and profit rate in the unbalanced panel