Putty-Clay: a Simple Example

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The *putty-clay* technology is rarely covered in classes or textbooks, yet it offers an interesting and, in some cases, more realistic, representation of technology. I present here a simple energy example as an introduction.

Modern references include Atkeson and Kehoe (AER 1999), Gilchrist and Williams (JPE 2000), Wei (AER 2003), and Gourio (mimeo 2007). This note is closest to Atkeson and Kehoe. Classic references include Johansen (Econometrica, 1959) and Solow (1960).

Putty-Clay Technology and Energy: a simple example

Consider a population of size 1. Each household must have a car, and each household drives m miles per year. The gas price is exogenous equal to $p(t)$ per gallon at time t. There is a constant-return-to-scale (CRS) supply of cars which are differentiated according to their gasoline mileage: a car of type v runs v miles per gallon. The average cost, which given the CRS assumption is also the marginal cost, of building a car of type v is $C(v)$. I assume that $C' > 0$ and $C'' > 0$: cars that consume less gas are more expensive to make, and this occurs at an increasing rate. Cars depreciate at rate δ independent of age and gasoline mileage.

These assumptions reflect the putty-clay hypothesis: car *services* are obtained in constant proportions (Leontief) from a car and gas, with no substitability between the car and the gas. Substitution occurs exclusively through new investment, i.e. new cars with different gas intensities v. The additional assumptions of inelastic mileage m and inelastic number of households having a car simplify the analysis but can be relaxed.

Let $f(v,t)$ be the number (probability distribution function) of cars of type v at time t. Let's first examine the investment decision. At time t, the δ households whose car broke down choose which type of car v_t to buy to replace it. They pick the type that minimizes their expected discounted expenditures from driving:

$$v_t \in \arg\min_{v\geq 0} \left\{ C(v) + \frac{m}{v} \int_t^\infty e^{-(r+\delta)(s-t)} p(s) ds \right\}, \tag{0.1}$$

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where the first component is the purchase cost of the car, and the second term is the expected present-value of gasoline costs: $\frac{m}{v}$ is the annual consumption of gas (miles/year divided by miles/gallon), which is multiplied by the gas price. Everybody thus buys the same type of cars at time t (there is a unique solution since this program is convex[1]). The first-order condition is

$$v_t^2 C'(v_t) = m \int_t^{\infty} e^{-(r+\delta)(s-t)} p(s) ds. \tag{0.2}$$

The left hand-side is increasing in v_t by our assumptions; hence more miles to drive, a higher gas price either now or in the future, lead to a more "economical" car, i.e. a higher v_t. A higher interest rate, or a higher depreciation rate of cars on the contrary leads to "gas-guzzlers" since the present-value cost of the gas falls, and it's not worth investing in a high v car.

The total consumption of gasoline at time t is obtained by aggregating over the existing car distribution:

$$D(t) = \int_0^{\infty} \frac{m}{v} f(v,t) dv. \tag{0.3}$$

The law of motion for the pdf $f(.,.)$ is:

$$\frac{\partial f(v,t)}{\partial t} = -\delta f(v,t) + \delta . \varepsilon_{\{v=v_t\}}^{dirac}, \tag{0.4}$$

where $\varepsilon_{\{v=v_t\}}^{dirac}$ means a degenerate distribution equal to 0 everywhere except for v_t where it is $+\infty$, and integrates to 1.[2] Plugging (0.4) into (0.5) yields the law of motion for the demand for gasoline:

$$\frac{dD_t}{dt} = -\delta D_t + \delta \frac{m}{v_t}. \tag{0.5}$$

Note that the demand is inelastic in the short-run - mathematically, D_t is continuous and won't respond to a price shock instantaneously. This is because we assume that households do not adjust their current gas consumption: there is no intensive margin. [A natural extension of this model to consider the intensive margin is that in each period, each household solves $\max_{m \geq 0} \left\{ u(m) - \frac{m}{v} p \right\}$.]

Given an initial cross-sectional distribution (or only its moment $\int_0^{\infty} \frac{f(v,0)}{v} dv$ which is all what we need) and given a path for expected future gas prices, we can run the two equations (0.2) and (0.5) model to give us the gas consumption at all future dates. The key feature that is obtained from the putty-clay hypothesis is a *slow adjustment* of the demand for gas in response to a price

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[1] Clearly, if households had different m, or different expectations regarding future gas prices, they would buy different types of cars. For our purpose, the lack of heterogeneity simplifies the analysis.

[2] This is a Dirac distribution (Except the support here is $(0, \infty)$ not $\mathbb{R}$). It has the property that $\int_0^{\infty} \varepsilon_{\{v=v_t\}}^{dirac} dv = 1$. Alternatively,one may assume that households that buy the car of type v actually get the car of type ηv where η is a mean 1 random variable. This leads to the same expressions in the aggregate.

shock. This property is attractive both because it can help fit the time series on gas consumption: as the oil price shocks of 1974, 1979, 1985 unfolded, the share spent on oil increased significantly as the oil price increased in the short-run, but a gradual adjustment also occured so that the share mean-reverted in the long-run.

Experiment: an unexpected permanent increase in the price of gas p.

Assume that the price was constant at p and increases unexpectedly by Δp permanently. Then the present value of fuel costs increases by $\frac{m}{v}\frac{\Delta p}{r+\delta}$. The new optimal v jumps immediatly to its new steady-state value $v + \Delta v$ where

$$\Delta v = \frac{m}{2vC'(v) + C''(v)v^2}\left(\frac{\Delta p}{r+\delta}\right).$$

However, the aggregate demand for gas adjusts slowly towards its new value. From $\frac{dD}{dt} = -\delta D + \delta\frac{m}{v_t}$, one obtains that

$$D(t) - D^{new} = e^{-\delta t}\left(D^{old} - D^{new}\right).$$

The speed of adjustment is given by the depreciation rate, which governs how many new units are put in place each period. Note that the car fuel intensity jumps at time 0 then stays constant.

Pricing used cars

I now turn to the pricing implications (see Wei and my own paper for more). The retailer price of a new car is $C(v_t)$ which is just one part of the full cost $C(v_t) + \frac{m}{v_t}\int_t^\infty e^{-(r+\delta)(s-t)}p(s)ds$. Consider the market for used cars with gas mileage v, and let $q(v,t)$ be the market-clearing price in that market. Since there is positive car construction at all times, households must be indifferent between selling their cars at price $q(v,t)$ and buying a new one, i.e.

$$\forall t, v \geq 0 : q(v,t) + \frac{m}{v}\int_t^\infty e^{-(r+\delta)(s-t)}p(s)ds = C(v_t) + \frac{m}{v_t}\int_t^\infty e^{-(r+\delta)(s-t)}p(s)ds.$$

Thus,

$$q(v,t) = C(v_t) + \left(\frac{m}{v_t} - \frac{m}{v}\right)\int_t^\infty e^{-(r+\delta)(s-t)}p(s)ds.$$

This is just a compensating differential: all cars, yielding the same services, must have the same total costs (the buying price plus the discounted cost of gasoline).

Clearly $\frac{\partial q}{\partial v} > 0$ and $\frac{\partial^2 q}{\partial v^2} < 0$: more efficient cars are more expensive to buy, at a decreasing rate. If $v \neq v_t$, the price of the car will move if the present-value of future gas prices moves. A car with a gas mileage v smaller than the current optimal one v_t will see its price fall if the present value of gas costs increase, and inversely a gas-guzzler will gain value when the price of gas goes down. If $v = v_t$, the car has marginally no exposure to gas price risk. More precisely, consider an increase in a $x_t = \int_t^\infty e^{-(r+\delta)(s-t)} p(s) ds$ by Δx, then

$$\Delta q(v) = C'(v_t)\Delta v_t + \left(\frac{m}{v_t} - \frac{m}{v}\right)\Delta x - \frac{m}{v_t^2} x_t \Delta v_t.$$

But optimality of v_t requires $C'(v_t) = \frac{m}{v_t^2} x_t$, hence

$$\Delta q(v) = \left(\frac{m}{v_t} - \frac{m}{v}\right)\Delta x.$$

which justifies the intuition given above.

Note that I implicitely assumed risk-neutrality in deriving the compensating differential: households are indifferent between cars of same equal expected costs. However, because cars have different risk exposures, if households are not risk-neutral, a risk adjustment is required. The empirically relevant case is as follows: when the price of oil increases, aggregate consumption falls, and the value of gas-guzzlers falls: these cars, as assets, have the “wrong” insurance property, hence owners will require a risk premium to hold them, which will make them cheaper. In my paper, I go into detail, for the case of capital and labor, into the mechanics of risk premia that are generated by different capital intensities, or different labor productivities.

Adding a utilization margin

Variation in m (miles driven per year) over time and across cars would lead to a ‘utilization’ channel: demand for gas would vary not only with the gas mileage of new cars, but also because current demand would react instantaneously. (More precisely, it does not matter much if we have $m = m_t$ varying over time, or $m = m(v)$ varying across cars, but if $m = m_t(v)$ varies both across cars and over time, we have variable utilization.) An extreme case is that m becomes zero, i.e. the car is not used, because it is too gas-intensive. (This actually seems to happen for some aircrafts, e.g. the Boeing 727 was largely retired after the oil shocks, and nowadays the Airbus A340 is also being phased out because of its higher fuel requirements.)

The role of the cross-sectional distribution

With full utilization, i.e. a constant m, the cross-sectional distribution affects the economy only through its impact on the total demand $D(t)$, as reflected in the equation $D(t) - D^{new} = e^{-\delta t}\left(D^{old} - D^{new}\right)$. When utilization is variable, the shape of f may play a role in demand dynamics since demand is now $\int_{v \geq v^*(t)} \frac{m}{v} f(v,t) dv$, where $v^*(t)$ is the utilization cutoff.