Is there a majority to support a capital tax cut?

François Gourio*

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Abstract

A capital income tax cut must in general be financed by increasing other taxes, and thus will have redistributive effects. This paper studies analytically the redistribution implied by a capital income tax cut in the Ramsey-Cass-Koopmans neoclassical growth model when agents differ in wealth and human capital and markets are frictionless. A few parameters affect the efficiency benefits and redistributive costs of capital taxation, and determine the set of agents who are in favor of a capital income tax cut. For plausible parameter values, a majority would lose from the tax cut, i.e. high capital taxes may be politically sustainable.

1 Introduction

Chamley (1986) and Judd (1985) discovered that in the neoclassical growth model, the optimal capital income tax rate is zero in steady-state. This result is robust to several extensions, as discussed in Atkeson, Chari and Kehoe (1999). However, in the real world, governments still levy substantial taxes on capital income: for instance, Lucas (1990) and Mulligan (2004) estimate a capital income tax rate between 30% and 40% for the United States today. This raises the question, why don't we have lower capital income tax rates? Possible explanations include that economies have not reached the steady-state, that governments have limited commitment, or that markets are incomplete.

This paper explores the hypothesis that capital income tax cuts benefit only a minority and are thus unlikely to arise in a median-voter economy. The main contribution of the paper is to present a simple analytical formula which determines which agents benefit from a capital income tax cut that is financed by increased labor income taxes. I use a version of the Ramsey-Cass-Koopmans neoclassical growth model, with agents heterogeneous in financial wealth and fixed human capital and perfect credit markets. The paper considers the following experiment: starting at time 0 in a steady-state, the government cuts infinitesimally the tax rate on capital - and keeps it constant thereafter - while increasing the tax rate on labor to maintain the present-value budget balance. The analytical results allow to conduct simple comparative statics exercises, and to use this formula as a theory of the tax rate that will prevail in a society where consumers vote to decide the tax structure: the median voter must be indifferent between

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*Boston University, Department of Economics, 270 Bay State Road, Boston MA 02215. Email: fgourio@bu.edu. I thank Fernando Alvarez, Robert E. Lucas Jr., Casey B. Mulligan, and anonymous referees for helpful comments.

marginal increases and decreases in the capital income tax rate. Because these changes in tax rates are small, we can find an explicit answer using linear approximations, taking into account the important transitional dynamics.

This formula shows that an agent will favor the tax reform if and only if the ratio of its physical to human capital exceeds a threshold, which depends on some structural parameters. This leads to some analytical results. First, the "average" agent -i.e., an agent whose relative endowments of physical and human capital are the same as the relative endowments of the aggregate economy- will always favor a capital tax cut. This reflects the efficiency effect of the tax cut, which decreases a distortive tax and increases a nondistortive one (since labor supply is assumed to be inelastic). Second, agents who have little capital may oppose the reform since the labor income tax rate increases. This is the equity effect of the reform. The efficiency effect of the reform stems from a higher capital stock in the long-run, while the redistribution is implied by short-run changes in prices: initially, the after-tax wage falls since the labor income tax increases, and the after-tax interest rate increases since the capital income tax decreases.

How many agents favor the reform thus depends on the distribution of the ratio of physical to human capital in the population, and on some structural parameters, which affect the equity-efficiency trade-off. First, I show that if the transition is fast enough, everybody will favor a tax cut, since the redistributive effects implied by the short-term changes in prices are unimportant. (A fast transition speed could be due, for instance, to a high intertemporal substitution of consumption.) Conversely, if the transition is very slow, the efficiency effects of the tax reform are very small, and the reform is purely redistributive: in this case, only agents who are richer than the average will favor the reform. Similarly, if capital is very inelastically demanded, or if the initial capital tax rate is very low, the efficiency effects are negligible, and only the agents who are richer than the average favor a tax cut.

Next, I perform some quantitative experiments. I show that a realistically calibrated model, taking as input the U.S. distributions of wealth and income, can generate capital income taxes of around 20%: there would not be a majority to support lower taxes. While previous work has shown similar results numerically in complicated models, the present analysis is more simple and the analytical results allow to deliver some general insights. However, the cost is that I abstract from many important issues, including nonlinear income taxes, elastic labor supply, life-cycle, and idiosyncratic risk.

An important empirical implication of this formula is that the set of people who favor a tax cut depends on how the budget is balanced. If one assumes (as in the main part of the paper) that labor income taxes are used to balance the budget, then what matters is the ratio of physical and human capital (or equivalently of income and wealth). In section 5, I consider other possible tax adjustments, and it may be that the labor-poor and the capital-poor both oppose the tax reform (e.g. if the lump-sum transfer is adjusted to balance the budget) . A large empirical literature tests the effect of inequality on taxes, but most measures are based on inequality in wealth or in income *alone*, rather the joint cross-sectional distribution. My model suggests these measures may be poor proxies for the true determinants of capital income taxation, because wealth and income are significantly correlated. Hence, the inequality in wealth may be very different than the inequality in the wealth-income ratio.

This paper also makes a methodological contribution by showing how the distributional effects can be studied in closed form. The methods I use are related to Bernheim (1983) and Judd (1985). These methods can be used to solve more general models or analyze fiscal reforms involving lump-sum transfers and government spending changes as well as capital, consumption or labor taxation.

The paper is organized as follows. The rest of the introduction discusses the relation to the literature. Section 2 presents the model and shows that it can be conveniently aggregated. Section 3 discusses the experiment that I carry out, presents my method, and derives the key formula of the paper. Section 4 discusses the theoretical and quantitative implications of this formula. Section 5 extends the analysis to general fiscal reforms involving changes in government spending, lump-sum transfers, and linear capital, consumption or labor taxes. Section 6 concludes. An appendix contains some tedious but simple computations.

Relation to the Literature

This paper is of course related to the classic contributions of Chamley (1986) and especially Judd (1985). The intuitive idea that taxes on capital income have undesirable redistributive implications is sometimes dismissed, because increased capital accumulation raises wages by increasing the marginal product of labor. Judd (1985) famously proved that even if some agents ("workers") are not allowed to save, any Pareto optimal policy will have a zero capital income tax rate in steady-state. This result might appear to contradict the findings of this paper. However, the experiments we consider is different: rather than computing the steady-state optimal rate, I ask who benefits from a small perturbation in fiscal policy whereby tax rates are changed by a small amount and held constant thereafter. This is also an interesting experiment.[1] By considering a somewhat different experiment, my results suggest that these models' implications may be less stark than is sometimes assumed.[2]

A maintained assumption in the paper is the class of policies that are considered. First, it may be that voters prefer large tax changes than small ones. Second, the new tax rates are assumed to be permanent. Finally, taxation is linear. These limitations are important for the political economy analysis, because there may be other reforms which would be more popular (see e.g. Bassetto and Benhabib (2006)). From a technical point of view, an issue that is not addressed is the existence of a median voter, which is assumed. Correia (1999) also studies welfare in an economy amenable to (Gorman) aggregation. She proves general results on welfare comparison for (possibly global, nonlinear) policy changes, including a median voter result. My contribution is the linear approximation, which yields an exact solution for small changes.[3]

My paper is also related to (and motivated by) a recent literature which explores numerically quantitative models with heterogeneity, uninsured idiosyncratic risk and borrowing constraints, to study the efficiency and distributional consequences of a tax reform (among others, Domeij and Heathcote (2003), Conesa, Kitao and Krueger (2006)). In these paper, fiscal policy has the usual negative disincentive

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[1] A somewhat undesirable property of the tax paths implied by the optimal policy in Judd's model is that the government raises a large revenue today, then use the interest to pay for spending later. It does not seem to happen in the real world, while small reforms which promise constant tax rates may be more reasonable.

[2] Judd (1985) actually also studies an experiment similar to the one I describe, but his setup is somewhat different (I allow all agents to save and work).

[3] See also Floden (2008) for a related analysis.

effects on savings and labor supply, but it also has two positive effects. First, fiscal policy can improve on the market allocation by providing insurance since markets are incomplete. Second, fiscal policy also has a "purely" redistributive effect by equalizing consumption across different types of agents, which increases welfare from an ex-ante point of view. These papers find that high level of taxation can be politically sustained (and may be efficient from an ex-ante point of view). However, in these papers it is hard to disentangle the insurance and redistributive effects of fiscal policies. This study focuses on the purely redistributive issues.

Mankiw and Weinzierl (2006) used similar approximations to consider the efficiency and government budget implications of a tax cut. I add to their work by considering the distributional effects. Finally, the political-economy argument is similar to Mulligan (2001), who considered the trade-off between lump-sum and labor taxation in a static model where agents differ in abilities.

2 A simple model with wealth and income heterogeneity

The first section describes the model setup. The second section uses an aggregation result to characterize the local dynamics of this economy around its steady-state.

2.1 Model Setup

Time is continuous and there is no uncertainty.

2.1.1 Preferences

There is a continuum of infinitely-lived agents -dynasties- indexed by $i \in [0,1]$. All dynasties have one member at time zero, and all grow in size at the same rate n. Each dynasty has constant elasticity of substitution preferences over the consumption flow:

$$U_i = \int_0^\infty e^{(n-\rho)t} \frac{c_{it}^{1-\sigma}}{1-\sigma} dt,$$

where c_{it} is per capita consumption of dynasty i. The dynasties all have the same preference parameters σ and ρ. Labor supply is inelastic: each agent of each dynasty provides one unit of labor at each point in time. Dynasty i is endowed at time 0 with initial assets a_{i0}, and the efficiency of each of its hours is $h_i e^{zt}$. In this expression h_i represents fixed differences in human capital, and the trend $Z_t = e^{zt}$ represents labor-augmenting technological progress, which is assumed to grow at a constant rate.

In principle, agents who have negative assets face the pre-tax interest rate. Hence, because of the capital income taxation, there is a kink in the budget constraint when assets are zero. To avoid this complication, I assume that all agents start with positive net assets; that is $a_{i0} > 0$.[4]

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[4] According to Budria et al. (2002), who use the U.S. Survey of Consumer Finances, 7.4% of households had negative wealth in 1998, and 2.5% had zero wealth.

2.1.2 Technology

There is an aggregate constant-return-to-scale production function:

$$Y_t = F(K_t, N_t Z_t H),$$

where K_t denotes capital, N_t is employment (physical hours), Z_t is labor-augmenting technological progress, and H reflects the fixed efficiency factors: $H = \int_0^1 h_i di$. Total population, grows at rate n and is normalized to one at time 0, thus the total effective population in efficiency units is $N_t Z_t H = e^{(n+z)t} \int_0^1 h_i di$. Capital depreciates at rate δ.

2.1.3 Markets

Consumers can borrow and lend without restrictions in a risk-free loan market at pretax interest rate r_t.

2.1.4 Government

The government levies linear taxes on capital and labor income, at rate τ_k and τ_l respectively, on consumption at rate τ_c, and issues bonds $\{B_t\}_{t\geq 0}$ to finance an exogenous stream of public spending $\{G_t\}_{t\geq 0}$ and an exogenous stream of lump-sum transfers $\{F_t\}_{t\geq 0}$. These transfers and spending are both assumed to grow at a rate such that they represent a constant nontrivial fraction of GDP in the long-run: i.e., $G_t = ge^{(n+z)t}$, and $F_t = fe^{(n+z)t}$, all $t \geq 0$. F_t is rebated equally across dynasties. (This assumption is innocuous for most of the analysis since the distribution of lump-sum transfers does not affect the equilibrium determination of aggregates.) As usual, marginal utility of private consumption is unaffected by government spending.

2.1.5 Equilibrium

For a given initial level of debt B_0 and capital K_0, an equilibrium *with constant fiscal policy* is a collection of individual consumption and asset choices $\{c_{it}, a_{it}\}_{t\geq 0, i\in[0,1]}$, and aggregates $\{K_t, C_t, B_t, A_t\}_{t\geq 0}$, and five numbers $\{\tau_k, \tau_l, \tau_c, g, f\}$ such that:

(i) Each dynasty $i \in [0,1]$ maximizes utility, i.e. it solves:

$$\max_{\{c_{it}, a_{it}\}_{t\geq 0}} \int_0^\infty e^{(n-\rho)t} \frac{c_{it}^{1-\sigma}}{1-\sigma} dt, \tag{1}$$

$$s.t. \quad : \quad \frac{da_{it}}{dt} = r_t(1-\tau_k)a_{it} + h_i e^{(n+z)t} w_t (1-\tau_l) - (1+\tau_c)e^{nt} c_{it} + fe^{(n+z)t}, \tag{2}$$

a_{i0}, h_i given.

(ii) The resource constraints hold:

$$\forall t \geq 0 : F(K_t, N_t Z_t H) = C_t + \frac{dK_t}{dt} + \delta K_t + ge^{(n+z)t}, \tag{3}$$

$$\forall t \geq 0 : N_t Z_t H = e^{(n+z)t} \int_0^1 h_i di,$$

where C_t denotes aggregate consumption : $C_t = \int_0^1 e^{nt} c_{it} di$.

(iii) The government budget constraint holds:[5]

$$\begin{aligned}\forall t &\geq 0 : \frac{dB_t}{dt} = r_t B_t + (g+f)\, e^{(n+z)t} - \tau_k r_t A_t - \tau_l w_t N_t Z_t H - \tau_c C_t, \\ & B_0 \text{ given.}\end{aligned} \tag{4}$$

(iv) The capital market is in equilibrium:

$$\forall t \geq 0 : A_t = K_t + B_t,$$

where A_t denotes total assets: $A_t = \int_0^1 a_{it} di$.

(v) The representative firm is optimizing, i.e.

$$\begin{aligned}\forall t &\geq 0 : r_t = F_1(K_t, N_t Z_t H) - \delta, \\ \forall t &\geq 0 : w_t = F_2(K_t, N_t Z_t H),\end{aligned}$$

where w_t is the wage for one unit of efficiency of labor, and F_i denotes the $i-$th partial derivative of F. Naturally, the household dynamic budget constraint can be rewritten in present-value terms as:

$$\int_0^\infty p_t e^{nt}(1+\tau_c)c_{it} dt = \int_0^\infty p_t e^{(n+z)t} \left(h_i(1-\tau_l)w_t + f\right) dt + a_{i0},$$

where $p_t = e^{-\int_0^t r_s(1-\tau_k)ds}$ is the price of one unit of consumption at time t, in units of consumption at time zero. From now on I will denote by $\widetilde{r}_t = (1-\tau_k)r_t$ the after-tax interest rate, and by $\widetilde{w}_t = (1-\tau_l)w_t$ the after-tax wage.

2.2 Aggregation and Equilibrium Characterization

In this section, I show that the path for aggregate variables is characterized by a standard two-variable saddlepath system. Hence, the economy is equivalent to a representative agent economy (i.e., there is Gorman aggregation). This is a standard result under complete markets and power utility functions (see for instance Caselli and Ventura (2000)). To prove this result, start with the first-order conditions for each household:

$$\forall i \in [0,1], \forall t \geq 0 : \frac{dc_{it}/dt}{c_{it}} = \frac{\widetilde{r}_t - \rho}{\sigma}. \tag{5}$$

Summing across agents:

$$\frac{dC_t}{dt} - nC_t = \int_0^1 e^{nt}\frac{dc_{it}}{dt} di = \int_0^1 \left(\frac{\widetilde{r}_t - \rho}{\sigma}\right) c_{it} di = \left(\frac{\widetilde{r}_t - \rho}{\sigma}\right) C_t,$$

so that an Euler equation for aggregate consumption holds, once we adjust for population growth:

$$\frac{dC_t/dt}{C_t} = n + \frac{\widetilde{r}_t - \rho}{\sigma}. \tag{6}$$

Notice that the asset accumulation equation (2) is also easily aggregated:

$$\frac{dA_t}{dt} = \widetilde{r}_t A_t + \widetilde{w}_t N_t Z_t H - C_t + f e^{(n+z)t}.$$

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[5] Note that the government budget constraint (4) is of course equivalent to $\frac{dB_t}{dt} = r_t(1-\tau_k)B_t + (g+f)e^{(n+z)t} + \tau_k r_t K_t - \tau_l w_t N_t Z_t H$, i.e. the government can make interest on government bonds taxable or not.

This asset accumulation equation, together with the government budget constraint, is equivalent to the the market-clearing condition in the goods market:

$$\frac{dK_t}{dt} = F(K_t, N_t Z_t H) - g e^{(n+z)t} - C_t - \delta K_t. \tag{7}$$

Hence the aggregates of this economy can be generated by a representative agent economy with the same preference parameters as the agents and endowments equal to the sum of the endowments of the agents of the heterogeneous-agent economy.[6] Heterogeneity is still key for welfare analysis as different agents will have different views on the desirability of a given fiscal policy.

Thanks to this aggregation result, it is straightforward to analyze the dynamics of the economy around the steady-state. In particular, we know that this economy is saddlepath stable: given some constant fiscal policy $(\tau_k, \tau_l, \tau_c, f, g)$, for any initial capital stock K_0, the detrended consumption $C_t/e^{(n+z)t}$ and capital $K_t/e^{(n+z)t}$ both converge to steady-state values. More precisely, define $c_t = C_t e^{-(n+z)t}$ and $k_t = K_t e^{-(n+z)t}$, then we can rewrite the system of equations (6)-(7) as:

$$\frac{dk_t}{dt} = F(k_t, H) - g - c_t - (n + \delta + z)k_t, \tag{8}$$

$$\frac{dc_t/dt}{c_t} = \frac{\widetilde{r}_t - \rho - \sigma z}{\sigma}. \tag{9}$$

For a given initial value of $k_0 = K_0$, we can solve the system of equations (8)-(9) for the unique saddlepath stable path. Given these paths for quantities, we can deduce prices using the firm's first order conditions, and we can then solve each individual's problem to obtain the paths for individual consumption and assets.

3 A Fiscal Reform

I first discuss the particular experiment that I study and outline my method for obtaining an analytical solution for small tax changes. I then apply this method and perform the computations.

The experiment and the method

Suppose we start at time zero in the steady-state of this economy for given values of $(\tau_k, \tau_l, \tau_c, f, g)$. Consider now a one-time unexpected change in these tax parameters: for instance, a decrease in τ_k, which is compensated by a change in another parameter, say τ_l, so as to keep the intertemporal budget constraint of the government satisfied. Note that I impose taxes to be constant over time, at a new level, from time zero on. The goal is to determine the set of agents who benefit from this reform. This will depend on initial assets a_{i0} and initial (and fixed) human capital h_i.

To obtain an analytical result, I consider an infinitesimal tax reform: the change in τ_k and in τ_l are both very small. This implies that prices and quantities will change only by a very small amount, for any time $t \geq 0$. To ascertain if a given agent is better off, I use a method from basic price theory.

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[6] Under some technical conditions (interversion of limits and integrals), if the transversality conditions hold for each agent, i.e.

$$\lim_{t\to\infty} e^{-\int_0^t r_s(1-\tau_k)ds} a_{it} = 0,$$

then the transversality condition holds for the representative agent economy.

Consider an agent with endowment vector $\mathbf{e}$ who maximizes utility, subject to a linear budget constraint with price vector $\mathbf{p}$:

$$\begin{aligned} V(\mathbf{p},\mathbf{e}) &= \max_{\mathbf{x}\geq 0} u(\mathbf{x}), \\ s.t. &: \mathbf{p}'\mathbf{x} \leq \mathbf{p}'\mathbf{e}, \end{aligned}$$

where a prime denotes a transpose. We have the following result for small price changes:

$$\Delta V = \mu \times \Delta \mathbf{p}'(\mathbf{e}-\mathbf{x}), \tag{10}$$

where Δ stands for a first-order small change and μ is the marginal utility of wealth, i.e. the multiplier on the budget constraint. Hence, the agent is better off if and only if the value of his endowment increases by more than the value of the bundle he was consuming initially. The intuition is that if this condition is satisfied, the consumer can still afford to buy the bundle he was consuming initially, so he must be no worse off.

In the case at hand, of course, the endowment $\mathbf{e}$ and bundle $\mathbf{x}$, as well as the price vector $\mathbf{p}$, are infinite dimensional (i.e. are functions of $t \in \mathbb{R}^+$). Because the tax rates changes are small, the evolution of the economy, which is given by the system of ODEs (8)-(9) can be approximated by linearizing the system around the steady-state. In particular, after-tax prices $\{\widetilde{r}_t, \widetilde{w}_t\}$ change by a small amount for all $t \geq 0$. I then evaluate the expression (10) to find which consumers enjoy a utility gain, and which include a utility loss.

The analysis thus involves the following four steps:

(1) For given changes in capital income and labor income tax rates $\Delta\tau_k$ and $\Delta\tau_l$, determine the path of the aggregate economy; it starts in the old steady-state and converges to the new steady-state, and the transition is obtained through the linear approximation.

(2) Deduce the path for prices from the path of the aggregate economy.

(3) Find the change in labor tax $\Delta\tau_l$ required to satisfy intertemporal budget balance.

(4) Compute (10) for each individual, as a function of his assets and human capital.

I now details these four steps. The next four subsections are algebra-intensive, so the reader might want to skip some intermediate computations and concentrate on the comments which discuss the intermediate results. I stick to the example of a change in capital and labor taxes, but the method can be applied to consider changes in lump-sum transfers or in government spending, and these will be explored in Section 5.

It is important to note that the method can be applied for general utility and production functions. I maintain the assumption of a constant elasticity of substitution utility function because it is standard in macroeconomics, but this can be easily relaxed.

3.1 Step 1: Path of the Aggregate Economy Following the Tax Reform

Let $(\overline{k},\overline{c})$ be the initial steady-state and (k^*,c^*) be the new steady-state after the small change in tax rates. The old and new steady-state are each characterized by the equations:

$$F(\overline{k},H) - (\delta+n+z)\,\overline{k} = g + \overline{c}, \tag{11}$$

$$F_1(\overline{k},H)=\delta+\frac{\rho+\sigma z}{1-\tau_k}. \tag{12}$$

In steady-state, the real after-tax interest rate is $\rho+\sigma z$ and the wage per efficiency unit is $w=F_2\left(k,H\right).$ Linearizing the system (8)-(9) around the new steady-state yields:

$$\begin{pmatrix} dc/dt \\ dk/dt \end{pmatrix} = \begin{pmatrix} 0 & \frac{c^*F_{11}}{\sigma}(1-\tau_k^*) \\ -1 & \frac{\rho+\sigma z}{1-\tau_k^*}-(n+z) \end{pmatrix} \begin{pmatrix} c-c^* \\ k-k^* \end{pmatrix},$$

where F_{11} is the second derivative of F with respect to capital, evaluated at the new steady-state.[7] This approximation will be valid for us because we change taxes infinitesimally so that the new steady-state is close to the one from which we start. Hence the path for aggregate quantities:

$$\begin{aligned} k_t &= k^*+\left(k_0-k^*\right)e^{-\lambda t}=k^*+\left(\overline{k}-k^*\right)e^{-\lambda t}, \\ c_t &= c^*+\left(c_0-c^*\right)e^{-\lambda t}, \end{aligned}$$

where $\lambda>0$ is the absolute value of the eigenvalue that governs the speed of convergence to the steady-state:

$$\lambda=\frac{-\frac{\rho+\sigma z}{1-\tau_k^*}+n+z+\sqrt{\left(\frac{\rho+\sigma z}{1-\tau_k*}-(n+z)\right)^2-4\frac{c^*F_{11}}{\sigma}(1-\tau_k)}}{2}.$$

Finally, c_0 is determined from the $t=0$ resource constraint:

$$c_0=F(\overline{k},H)-(\delta+n+z)\,\overline{k}-g-\left(\frac{dk}{dt}\right)_{t=0}=\overline{c}+\lambda\left(\overline{k}-k^*\right).$$

This is the standard analysis of saddlepath stability in the neoclassical growth model. The change in the quantity of capital in the economy at time t after the tax reform, compared to before the tax reform, is:

$$\Delta k_t=k_t-\overline{k}=\left(k^*-\overline{k}\right)\left(1-e^{-\lambda t}\right). \tag{13}$$

This number is small for all $t\geq 0$, because $k^*-\overline{k}$ is small, since the tax change is small. Moreover, since the steady-state capital stock is determined by the condition $(1-\tau_k)\left(F_1(k,H)-\delta\right)=\rho+\sigma z$, we know that the steady-state change in capital stock $k^*-\overline{k}=\Delta k_\infty$ following the reform is:

$$\frac{\Delta k_\infty}{\Delta\tau_k}=\frac{F_1-\delta}{(1-\tau_k^*)F_{11}}. \tag{14}$$

Because labor supply is inelastic, the level of the labor income tax τ_l does not have any effect on the aggregate economy - only changes in capital income taxes create transitional dynamics and a change in the steady-state. (This can be seen from the system (11)-(12) where τ_l does not appear.)

Similarly, the change in consumption from before to after the tax reform is easily calculated:

$$\begin{aligned} \Delta c_t &= c_t-\overline{c}, \\ &= \left(k^*-\overline{k}\right)\left((\overline{r}-n-z)\left(1-e^{-\lambda t}\right)-\lambda e^{-\lambda t}\right). \end{aligned}$$

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[7] Here and elsewhere, it is irrelevant if the expression F_{11} is evaluated at the initial or final steady-state, since it creates only a second-order error and this is a first-order approximation. Mathematically, for any function which is twice continuously differentiable, to a first order approximation, $f(x_1)\simeq f(x_0)+f'(x_0)(x_1-x_0)$, but also $f(x_0)\simeq f(x_1)+f'(x_1)(x_0-x_1)$.

3.2 Step 2: Computing the changes in prices

There are two sequence of prices in this economy: $p_t = e^{-\int_0^t r_s(1-\tau_k)ds}$, is the price at time zero of a unit of good of consumption at time t, and $p_t\widetilde{w}_t$ is the (after-tax) price of one unit of labor delivered at time t, where $\widetilde{w}_t$ is the after-tax wage: $\widetilde{w}_t = (1-\tau_l)F_2(k_t, H)$. In the initial steady-state, $p_t = \overline{p}_t = e^{-(\rho+\sigma z)t}$. Using the path known for the state variable k_t, we can obtain the first-order approximation to price changes between before and after the tax reform:

$$\Delta p_t = e^{-(\rho+\sigma z)t}(1-\tau_k)\Delta k_t \frac{F_{11}}{\lambda}. \tag{15}$$

The algebra is relegated to appendix A. This formula shows that, following a capital income tax cut, the after-tax interest rate increases, so that the price of future consumption decreases. By the same logic,

$$\Delta\left(\widetilde{w}_t p_t\right) = e^{-(\rho+\sigma z)t}\left\{-\overline{w}\Delta\tau_l + (1-\tau_l)F_{21}\Delta k_t + \overline{w}(1-\tau_l)(1-\tau_k)F_{11}\frac{\Delta k_t}{\lambda}\right\}. \tag{16}$$

The first term in this formula, $-\overline{w}\Delta\tau_l$, reflects the direct effect of a change in the labor income tax rate on the after-tax price. The second term, $(1-\tau_l)F_{21}\Delta k_t$, represents the increase in the wage due to additional capital accumulation: at time t, an extra amount of capital Δk_t has been accumulated, leading to an increase in the wage of $(1-\tau_l)F_{21}\Delta k_t$. The third term is the effect on interest rates, i.e. on p_t.

Given these approximations, it is easy to compute the following present-values of price changes, which will be used later. Define $\eta = \rho - n + (\sigma - 1)z$, then[8]:

$$\begin{aligned}\int_0^\infty e^{(n+z)t}\Delta p_t dt &= \frac{(1-\tau_k)(k^* - \overline{k})F_{11}}{\eta(\eta+\lambda)}, & (17)\\ \int_0^\infty e^{(n+z)t}\Delta\left(\widetilde{w}_t p_t\right)dt &= \frac{-\overline{w}\Delta\tau_l}{\eta} + \frac{(1-\tau_l)(k^*-\overline{k})\lambda F_{21}}{\eta(\eta+\lambda)} + \frac{\overline{w}(1-\tau_l)(k^*-\overline{k})(1-\tau_k)F_{11}}{\eta(\eta+\lambda)}. & (18)\end{aligned}$$

3.3 Step 3: Balancing the Budget

We know that before and after the reform, the government budget constraint must hold:

$$B_0 = \overline{B} = \int_0^\infty e^{-\int_0^t r_s(1-\tau_k)ds} e^{(n+z)t}\left(r_t\tau_k k_t + w_t\tau_l H + \tau_c c_t - g - f\right)dt = \int_0^\infty e^{(n+z)t} p_t s_t dt,$$

where I defined the (detrended) primary surplus $s_t = r_t\tau_k k_t + w_t\tau_l H + \tau_c c_t - g - f$. Government budget balance requires to pick $\Delta\tau_l$ and $\Delta\tau_k$ such that: $\int_0^\infty e^{(n+z)t}\Delta\left(p_t s_t\right)dt = 0$. To find the implied relation between $\Delta\tau_l$ and $\Delta\tau_k$, we can simply use first-order approximations, keeping in mind that the infinitely small terms are $\Delta\tau_k, \Delta\tau_l$, and the ensuing Δk_t, for all $t \geq 0$.

More precisely, the condition $\int_0^\infty e^{(n+z)t}\Delta\left(p_t s_t\right)dt = 0$ can be rewritten as: $\int_0^\infty e^{(n+z)t}\left(\Delta p_t \times \overline{s}\right)dt + \int_0^\infty e^{(n+z)t}\left(\Delta s_t \times \overline{p}_t\right)dt = 0$. The first term is easy to obtain, since we know $\int_0^\infty e^{(n+z)t}\Delta p_t$ from step 2 and initial government budget balance requires that $\overline{s} = \eta\overline{B}$. To obtain the second term, I compute a first-order approximation to Δs_t for each $t \geq 0$, and then integrate it:

$$\begin{aligned}\Delta s_t \quad &\simeq \quad \Delta r_t \times \tau_k \times \overline{k} + \overline{r} \times \Delta\tau_k \times \overline{k} + \overline{r} \times \tau_k \times \Delta k_t & (19)\\ &\quad\; + \Delta w_t \times \tau_l \times H + \overline{w} \times \Delta\tau_l \times H + \tau_c \times \Delta c_t.\end{aligned}$$

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[8] I assume that $\eta > 0$. This is the standard requirement that the interest rate is greater than the growth rate of the economy, which is a necessary condition for wealth to be finite.

From Steps 1 and 2 we know that:

$$\begin{aligned}\Delta r_t &= F_{11} \times \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right), \\ \Delta k_t &= k_t - \overline{k} = \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right), \\ \Delta w_t &= F_{21} \times \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right), \\ \Delta c_t &= \left(k^* - \overline{k}\right)\left(\left(\overline{r} - n - z\right)\left(1 - e^{-\lambda t}\right) - \lambda e^{-\lambda t}\right).\end{aligned}$$

Plugging these expressions into (19) yields:

$$\begin{aligned}\Delta s_t &\simeq F_{11} \times \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right) \times \tau_k \overline{k} + \overline{r}\left(\Delta\tau_k \times \overline{k} + \tau_k\left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right)\right) \\ &\quad + \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right) F_{21}\tau_l H + F_2 \times \Delta\tau_l \times H + \tau_c\left(k^* - \overline{k}\right)\left(\left(\overline{r} - n - z\right)\left(1 - e^{-\lambda t}\right) - \lambda e^{-\lambda t}\right), \\ &\simeq \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right)\left[F_{11}\tau_k\overline{k} + \tau_k\overline{r} + F_{21}\tau_l H + \tau_c(\overline{r} - n - z)\right] + F_2\Delta\tau_l H + \overline{r}\Delta\tau_k\overline{k} - \lambda\tau_c\left(k^* - \overline{k}\right)e^{-\lambda t}.\end{aligned}$$

Putting the two pieces together yields, after some algebra (which is again relegated to Appendix A):

$$-\frac{\Delta\tau_l}{\Delta\tau_k} = \frac{\overline{r}\overline{k}}{\overline{w}H}\left(1 + \frac{\lambda}{\eta + \lambda}\frac{\tau_k(1 - \zeta) - \tau_l}{(1 - \tau_k)} + \frac{\eta}{\eta + \lambda}\frac{\overline{B}}{\overline{k}}\right), \tag{20}$$

where $\zeta = -\frac{\overline{r}}{\overline{k}F_{11}} > 0$ is a measure of the elasticity of capital demand.[9]

This formula is actually quite revealing. For a given decrease in capital income tax rate (e.g. $\Delta\tau_k = -0.01$), we need to increase the labor tax rate to balance the budget.[10] The first term simply reflects the relative size of the tax bases: for a given change in capital income tax, the more capital income there is, the larger the increase in labor tax required to balance the budget; and the larger the labor income, the smaller the increase in tax required. The second term reflects the increase in the tax bases following the capital income tax rate cut: capital is accumulated, and wages increase. When the capital demand elasticity parameter ζ is large, a capital tax cut implies a large increase in the tax base and a smaller required increase in the labor income tax. When the labor income tax τ_l is large, the government appropriates a large share of the increase in wages, so that the required increase in tax rate to balance the budget is smaller. Finally, the third term $\overline{B}/\overline{k}$ reflects that government's liabilities increase if there is a cut in the capital income tax rate, since after-tax interest rates increase.

Interestingly, this formula also reveals the role of the convergence speed λ. If the transition is very fast ($\lambda \to \infty$) the third term becomes irrelevant since after-tax interest rates return quickly to the steady-state value $\rho + \sigma z$. On the other hand, the second term becomes more important: capital is accumulated quickly, raising the tax base for both capital and labor income taxes.

3.4 Step 4: Welfare comparison

We are now in position to perform the welfare analysis outlined above. Consider an agent who has human capital h_i and has initially assets a_{i0}. The tax reform induces a small variation in the prices

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[9] As $\delta \to 0$ (or if we define the production function as net output), $\zeta = \sigma/s_L$ where σ is the elasticity of substitution between capital and labor and s_L is the labor share.

[10] Unless we are on the 'wrong side' of the Laffer curve, in which case we need to decrease the labor tax. While the formula incorporates this possibility, my discussion assumes that we are on the left side of the Laffer curve.

$\{p_t, \widetilde{w_t}\}_{t\geq 0}$, i.e. prices move by $\{\Delta p_t, \Delta \widetilde{w}_t\}_{t\geq 0}$ as compared to the initial steady-state. Does the present value of the consumption or of the income of agent i grow more? We need to compare

$$(1+\tau_c)\int_0^\infty e^{(n+z)t}\Delta p_t \times c_{it}dt,$$

and

$$h_i \int_0^\infty e^{(n+z)t}\Delta\left(\widetilde{w}_t p_t\right)dt + f\int_0^\infty e^{(n+z)t}\Delta p_t dt.$$

A key simplification is that $\int_0^\infty e^{(n+z)t}\Delta p_t c_{it}dt = \overline{c_i}\int_0^\infty e^{(n+z)t}\Delta p_t dt$ since this is evaluated at the initial bundle. Hence these three integrals have been computed before. The agent is better off from the tax reform if and only if:

$$(1+\tau_c)\overline{c}_i\frac{(1-\tau_k)\Delta k_\infty F_{11}}{\eta(\eta+\lambda)} < h_i\left(\frac{-\Delta\tau_l\times\overline{w}}{\eta} + \frac{(1-\tau_l)\Delta k_\infty\lambda F_{21}}{\eta(\eta+\lambda)} + \frac{\overline{w}(1-\tau_l)\Delta k_\infty(1-\tau_k)F_{11}}{\eta(\eta+\lambda)}\right) + f\frac{(1-\tau_k)\Delta k_\infty F_{11}}{\eta(\eta+\lambda)}.$$

Moreover, from the agent's budget constraint, his consumption is initially $(1+\tau_c)\overline{c}_i = \eta a_{i0} + h_i\overline{w}(1-\tau_l)+f$. Using this condition, and after a bit of algebra (see Appendix A), we see that the agent is better off if and only if:

$$\frac{a_{i0}}{h_i} > \frac{-\frac{\Delta\tau_l}{\Delta k_\infty}\times\overline{w}(\eta+\lambda)+(1-\tau_l)\lambda F_{21}}{\eta(1-\tau_k)F_{11}}.$$

Hence, the consumer is favorable to the fiscal reform if he is sufficiently endowed in capital, relative to his labor endowment. Using our result from Section 3, we can get $\frac{\Delta\tau_l}{\Delta k_\infty} = \frac{\Delta\tau_l}{\Delta\tau_k}\frac{\Delta\tau_k}{\Delta k_\infty}$. Finally, the agent is better off if and only if:

$$\frac{a_{i0}}{h_i} > \frac{\overline{k}}{H}\left(1-\frac{\tau_k\lambda\zeta}{\eta(1-\tau_k)}\right)+\frac{\overline{B}}{H}. \tag{21}$$

This is the key result of the paper: agents benefit from the tax reform if the ratio of their financial wealth to human capital is greater than a threshold, which depends on the aggregate economy's ratio of physical capital to human capital, the ratio of government debt to physical capital, and some structural parameters: the elasticity of capital demand ζ, the transition speed λ, the initial capital tax rate τ_k, and the difference η between the interest rate and the growth rate. Note that this formula naturally "nets out" from agents' assets government debt, since it is in zero net supply in the economy - e.g., if there is only one agent, only capital matters.

This formula reveals of course that capital-rich agents are more likely to favor the tax reform. To understand the formula, note that when the term $\frac{\tau_k\lambda\zeta}{\eta(1-\tau_k)}$ is close to zero, only agents with an a/h ratio above the average favor the reform. In this case, the reform's main effect is to redistribute income from the labor-rich to the capital-rich. But in general, this term is positive, reflecting that the reform also increases the capital stock, wages, and efficiency; as a result, even some agents whose a/h ratio is below the average may favor the reform. Hence, the crucial term $\frac{\tau_k\lambda\zeta}{\eta(1-\tau_k)}$ measures the size of the efficiency gains of the reform. The next section develops this intuition and states some formal results.

4 Results

I first discuss the formula (21) by stating some simple results. Next I consider its quantitative implications.

4.1 Analytical Results

The tax reform has both an efficiency effect, which stems from a higher capital stock in the long-run (leading, inter alia, to a higher marginal product of labor). On the other hand, there is a redistributive effect, implied by short-run changes in prices: initially, the after-tax wage falls since the labor income tax increases, and the after-tax interest rate increases since the capital income tax decreases. These two effects will play a role in our results.

Result 1: The average agent - i.e., an agent whose relative holdings of physical and human capital equals the ratio of aggregate physical to human capital - is always favorable to a tax cut.

The proof is simply obtained by checking that the threshold for benefiting from a tax cut is less than total assets over total human capital. The average agent has a ratio equal to $\frac{A}{H} = \frac{\overline{k}+\overline{B}}{H}$. Since $\lambda > 0$, and $\zeta > 0$, we have $\frac{\overline{k}}{H}\left(1 - \frac{\tau_k \lambda \zeta}{\eta(1-\tau_k)}\right) + \frac{\overline{B}}{H} \leq \frac{\overline{k}+\overline{B}}{H}$, hence the inequality is true.

An agent with relative holdings of capital and labor equal to the economy-wide will always favor a capital cut. This is because this agents effectively weights the alternative as the representative agent (who is well-defined in this economy) would. This result was much expected. We know from Chamley and Judd that *in steady-state* the average agent will want a zero capital income tax rate. We learn here that the average agent always want to decrease the tax rate. This is not surprising in our setup: since labor supply is inelastic, the labor tax is effectively a lump-sum tax, hence it is always better to reduce the distortive capital income tax.[11] Interestingly, the homogeneity implies that the level of physical or human capital is irrelevant, i.e. in no sense do poor people benefit less from a capital income tax cut: what matters is the *relative* endowments of physical and human capital.

As noted above, only agents who are richer than the average will favor the reform if $\frac{\tau_k \lambda \zeta}{\eta(1-\tau_k)} \to 0$. Hence, this parameter measures the efficiency gains of the reform. If they are very low, the reform has mostly redistributive effects and only agents who are more capital-rich than average will favor the reform. For instance, if capital hardly reacts to the reform, there are no changes to aggregates or to pretax prices. The only effect of the reform is to change after-tax prices. Hence, the effects are purely redistributive, and there are no efficiency effects, so only agents whose income derives mainly from capital are satisfied with the reform. This leads to several interesting results.

Result 2: If the elasticity of demand for capital is very low, or if the speed of transition is very low, then only agents whose a/h is greater than the aggregate will favor a capital income tax cut.

This point is obvious by taking the limit $\zeta \to 0$ or $\lambda \to 0$ in equation (21).

The opposite happens when $\lambda \to \infty$, as can easily be checked: since we assumed that all agents have nonnegative assets, all agents will satisfy (21). Hence the following result.

Result 3: If the transition to the new steady-state is fast enough, all agents will favor a capital tax cut.

The intuition is clear and important: the new steady-state is preferred to the old steady-state by everybody, because the after-tax interest rate is unchanged and wages are higher. Hence, only the transition makes the trade-off between capital taxation and labor taxation nontrivial. For instance, if

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[11] Notice that this results hold for sure only if $\tau_k \geq 0$: the average agent will not want to lower the capital income tax rate to arbitrarily low negative values.

the economy is a small open economy, there are no adjustment costs, and capital taxes apply equally to foreigners and residents, then lowering capital taxes necessarily improves the welfare of all the residents.

These results are of course, partial since λ (and ζ in general) are not structural parameters. But the intuition is clear, and the effect of other structural parameters can be worked out through their effect on λ and ζ. For instance, one can check that if $\frac{1}{\sigma}$, the intertemporal elasticity of substitution in consumption, is high enough, all agents will favor a capital tax cut, since the transition will be very fast. Moreover, if agents are more patient, so that ρ is lower, more agents will (ceteris paribus) favor the tax cut since η is lower: agents give more weight to the (long-run) efficiency gains over the (short-term) redistributive effects. Last, the level of the tax rate τ_k determines also the efficiency cost of taxation.

Result 4: Holding λ, ζ, η and $\frac{\overline{k}}{H}$ constant, if the tax rate τ_k high enough, all agents favor a capital tax cut, and if $\tau_k = 0$, only agents who are more endowed in physical capital will favor it.

This is again an obvious implication of (21), by considering the cases $\tau_k \to 1$ or $\tau_k \to 0$. The intuition is that the distortion is increasing in the level of taxes; as the capital tax goes to 1, capital goes to zero, and everyone would benefit from lower taxes. On the other hand, for very low tax rates, the distortion is second-order and there are, to a first-order approximation, only redistributive effect of the reform.

Finally, we state the obvious political economy implication:

Result 5: Let Φ be the cross-sectional cumulative distribution function of the ratio a/h (physical wealth over human capital). The median agent favors a capital income tax rate cut iff

$$\Phi\left(\frac{\overline{B}}{H} + \frac{\overline{k}}{H}\left(1 - \frac{\tau_k \lambda \zeta}{\eta(1-\tau_k)}\right)\right) < \frac{1}{2}.$$

The proof is obvious since the median agent has assets $\Phi^{-1}(1/2)$. Of course, by symmetry, the median agent will favor an increase in the tax rate if $\Phi\left(\frac{\overline{B}}{H} + \frac{\overline{k}}{H}\left(1 - \frac{\tau_k \lambda \zeta}{\eta(1-\tau_k)}\right)\right) > \frac{1}{2}$. Hence, the median agent will be in favor of keeping the current tax rate if and only if:

$$\Phi\left(\frac{\overline{B}}{H} + \frac{\overline{k}}{H}\left(1 - \frac{\tau_k \lambda \zeta}{\eta(1-\tau_k)}\right)\right) = \frac{1}{2}.$$

This formula is a theory of the capital income tax rate. Conditional on a wealth distribution, a level of public debt and some structural parameters, this formula gives the tax rate which will make the median voter indifferent between an infinitesimal increase and decrease in the capital tax rate.[12] The next section examines quantitatively if this formula can explain the observed level of capital tax rates. Interestingly, the formula highlights the role of the *joint* distribution of physical and human capital: an agent with low human and physical capital would exhibit the same preferences over taxes as an agent with high human and physical capital. Empirical work typically proxies the median voter with the median income or the median wealth, which may be different. It would be interesting to examine empirically if this formula can account for variation in tax rates over time and across countries, but this is beyond the scope of this paper.

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[12] Of course, the wealth distribution is itself in general endogenous to capital tax rates. But the Ramsey model offers no theory of the wealth distribution: any initial distribution of wealth is an equilibrium, and inequality is constant.

4.2 Quantitative Results

The formula (21) gives the capital tax rate that will prevail in a median-voter economy, given the median of the distribution of a/h and some structural parameters. It is interesting to compute what this formula implies for the U.S. given some estimate of this median and the structural parameters.

Table 1 reports the parameter value that I use, which are standard in the macroeconomic and public finance literature. I consider a Cobb-Douglas production function, and I assume a power utility function with an elasticity of substitution equal to half. The debt-GDP ratio is taken to be 38%[13] and the government spending-output ratio is 19%.[14]

The other input in the model is the median of a/h is in the data. It is simpler to measure and match the median of a/y, i.e. the ratio of wealth to pretax and pretransfer income. Even this number is not readily available, because it requires knowledge of the joint distribution of wealth and labor earnings. Hence, I use the 2004 Survey of Consumer Finances. I find that the median of net worth over total income is 2.12.[15] It is also possible to infer a rough estimate using the study by Budria et al. (2002). These authors used the 1998 Survey of Consumer Finances. Their Table 7 shows that if we rank households by wealth, households with a wealth in the median quintile have an average wealth of 72,600$ and an average income of 40,700$. This yields an estimate of 1.78, which is not very different. Of course, the average a/y for the entire economy is essentially the capital-output ratio, which is higher, around 3.31.[16] The higher value reflects the inequality in the distribution of assets in the economy, even when they are scaled by income.

Given these parameters, I can solve for the tax rate τ_k which will make the median household indifferent between a tax cut and a tax increase. The limiting τ_k is the one that solves

$$2.12 = \frac{a_i}{wh_i + ra_i} = \frac{\frac{a_i}{h_i}}{w + r\frac{a_i}{h_i}} = \frac{T}{w + rT},$$

where $T = \frac{\overline{B}}{H} + \frac{\overline{k}}{H}\left(1 - \frac{\tau_k \lambda \zeta}{\eta(1-\tau_k)}\right)$.[17]

The capital tax rate is 17.4%. This number is lower than most estimates for the U.S.: Domeij and Heathcote (2004), Lucas (1990), or Mulligan (2004) all use estimates above 30%. However, it is still sizeable. Hence, the simple presence of heterogeneity can explain why capital taxes persist.

Interestingly, this number turns out to be fairly sensitive to parameter changes. If the elasticity of substitution is higher, then the sustainable tax rate is lower: for instance if $\sigma = 1$, the tax rate is

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[13] This is the ratio of the government debt held by the public at the end of 2006 (5.0 trillions of dollars, see www.gpoaccess.gov/usbudget) to U.S. GDP in 2006 (13.2 trillions of dollars).

[14] This corresponds to the ratio of line 20 to line 1 of Table 1.1.5 from the National Income and Product Accounts for 2006.

[15] The data I used is available on www.federalreserve.gov/pubs/oss/oss2/2004/scfp2004.zip
The number I give is the weighted median of net worth over income.

[16] This number is the ratio of line 1 of Table 1.1 from the Fixed Assets Table of the Bureau of Economic Analysis, to nominal GDP, for the year 2005.

[17] To solve this equation, we solve the steady-state of the aggregate model and then compute T and w.

Parameter	Meaning	Value
α	capital share	0.3
δ	depreciation	0.06
ρ	discount factor	0.04
σ	1/IES	2
z	labor-augmenting technology growth	0.015
n	population growth	0.01
$\overline{B}$/Y	public debt/GDP	0.38
G/Y	gov't spending (excl. transfers)	0.19

Table 1: Parameter values.

13.8%. The benefits to cutting capital taxes seem to be higher since the transition will be faster. If the government public debt is higher, then higher capital taxes are sustainable, apparently because cutting capital taxes is costly for the government who will face high interest rates. Hence if $\overline{B}/Y = 0.7$, I find that $\tau_k = 22.5\%$. But of course the most interesting parameter is the median of a/y. If this number is one (rather than 2.12), the tax rate jumps to 32.0%: with more inequality, higher capital taxes become politically sustainable. On the other hand, if a/y is three, the tax rate falls to only 0.6%.

5 More general fiscal reforms

In this section, I study fiscal reforms that change not only τ_k and τ_l, but also f and g, the lump-sum transfer and government spending parameters, or the tax rate on consumption τ_c. Hence, I consider infinitesimal changes $\Delta\tau_k, \Delta\tau_l, \Delta f$,Δg, and $\Delta\tau_c$ which occur unexpectedly and permanently at time zero. The standard results of the Ramsey model apply: a change in f, τ_l, or τ_c in itself has no effect on any aggregate quantities, and a change in g has no effect on the capital stock, but it reduces consumption one-for-one. The same method can be applied: first, we find the evolution of the economy using the linear approximation around the steady-state, and deduce the change in prices; then, we find the conditions for intertemporal budget balance to be satisfied. Finally we compute the effect of price changes on welfare. The computations, which are very similar to those of Section 3, are relegated to Appendix B. There it is shown that agent i benefits from the reform if and only if:

$$a_{i0}\left(\Delta\tau_k + \frac{\Delta\tau_c}{1+\tau_c}\frac{\eta+\lambda}{F_1-\delta}\right) < h_i\left(-\Delta\tau_l\frac{\overline{w}(\eta+\lambda)}{(F_1-\delta)\,\eta} - \frac{\Delta\tau_c(1-\tau_l)\overline{w}(\eta+\lambda)}{(1+\tau_c)\,\eta\,(F_1-\delta)} - \frac{(1-\tau_l)\lambda\overline{k}}{\eta(1-\tau_k)H}\Delta\tau_k\right) + \frac{(\eta+\lambda)}{\eta(F_1-\delta)}\Delta f, \tag{22}$$

and that budget balances requires that the changes in taxes, transfers and the level of government spending be linked by the following equation:

$$\Delta\tau_l \times \overline{w}H + \Delta\tau_k \times \pi + \Delta\tau_c \times c = \Delta f + \Delta g,$$

where $\pi = \overline{r}\overline{k}\left(1 + \frac{\lambda}{\eta+\lambda}\frac{\tau_k(1-\zeta)-\tau_l}{(1-\tau_k)} + \frac{\eta}{\eta+\lambda}\frac{\overline{B}}{\overline{k}}\right)$ is the same expression as in Section 3. Of course these two equations nest the special case studied in Section 3 ($\Delta\tau_c = \Delta f = \Delta g = 0$).

There are ten possible cases, depending on which combination of taxes are adjusted. (Of course the signs can be switched, since this analysis is symmetric.) However, the first four can be joined together, and the case of $\Delta\tau_k > 0$ and $\Delta\tau_l < 0$ was analyzed in Section 4.

Case 1: an increase in g compensated by a decrease in f or an increase in τ_k,τ_l or τ_c or a combination of the four (hence $\Delta g > 0, \Delta f \leq 0, \Delta\tau_l \geq 0, \Delta\tau_k \geq 0, \Delta\tau_c \geq 0$). Inspecting equation (22) reveals that the inequality is never satisfied. Because government spending brings no utility, all agents are worse off after this reform.

Case 2: an increase in τ_l compensated by an increase in $f : \Delta f > 0, \Delta\tau_l > 0, \Delta g = \Delta\tau_k = \Delta\tau_c = 0,$. In this case, rearranging (22) shows that agent i will be better off from the reform if and only if:

$$\Delta\tau_l \times \frac{\overline{w}(\eta+\lambda)}{(F_1-\delta)\,\eta} h_i < \frac{(\eta+\lambda)}{\eta(F_1-\delta)} \Delta f,$$

and given the government budget constraint this is equivalent to:

$$h_i < H = \int_0^1 h_j dj,$$

i.e. if the agent has a human capital less than the average. This simply reflects that these tax changes have no effect on the aggregate economy given that labor supply is inelastic. Hence, this is purely a redistributive policy, and agents prefer the labor income tax when they have little human capital, regardless of how much physical capital they have.

Case 3: an increase in τ_k compensated by an increase in $f : \Delta f > 0, \Delta\tau_k > 0, \Delta g = \Delta\tau_l = \Delta\tau_c = 0$. Rearranging (22) yields, after using the budget constraint:

$$\frac{a_{i0}}{\overline{k}} + \frac{h_i}{H} \frac{(1-\tau_l)\lambda}{(1-\tau_k)\eta} < \left(1 + \frac{\lambda}{\eta}\left(\frac{1-\tau_k\zeta-\tau_l}{1-\tau_k}\right)\right) + \frac{\overline{B}}{\overline{k}}.$$

Interestingly, agents with either a lot of physical capital or a lot of human capital will favor this reform. Both are likely to gain if the capital tax is cut: agents with a lot of financial wealth benefit from the lower capital tax, and agents with a lot of human wealth benefit from the higher wages caused by capital accumulation. For agents who are relatively wealthy (in human or financial terms), the lump-sum transfer is only a small share of their wealth, hence a reduction in the transfer is acceptable. Hence, this kind of reform would yield very different political-economy implications than the reform considered in Section 3. Again, the joint cross-sectional distribution of (a, h) is necessary to evaluate the welfare effects properly.

Case 4: An increase in τ_c compensated by an increase in $f : \Delta\tau_c > 0$, $\Delta f > 0$, and $\Delta g = \Delta\tau_l = \Delta\tau_k = 0$. In this case equation (22) simplifies to:

$$\frac{\Delta\tau_c}{1+\tau_c} \left(a_{i0}\eta + h_i(1-\tau_l)\overline{w}\right) < \Delta f,$$

and since government budget balance requires that $\Delta\tau_c \times c = \Delta f$, we see that the agents in favor of the tax change are those with

$$c_i < c,$$

i.e. those who are poorer than the average, either because they have little labor income or because they have little capital income.

Case 5: An increase in τ_c compensated by a decrease in τ_l : $\Delta\tau_c > 0$, $\Delta\tau_l < 0$, and $\Delta g = \Delta f = \Delta\tau_k = 0$. In this case equation (22) simplifies to:

$$\frac{\Delta\tau_c}{1+\tau_c} \times (a_{i0}\eta + h_i(1-\tau_l)\overline{w}) < -h_i \Delta\tau_l \overline{w},$$

and since present value budget balance requires that $\Delta\tau_l \times \overline{w}H = -\Delta\tau_c \times c$, we obtain that the agents in favor of the reform are those that satisfy

$$\frac{c_i}{c} < \frac{h_i}{H},$$

i.e. they are relatively more endowed in human capital than in total wealth (and thus in physical capital).

Case 6: An increase in τ_c compensated by a decrease in τ_k : $\Delta g = \Delta f = \Delta\tau_l = 0$; $\Delta\tau_c > 0$ and $\Delta\tau_k < 0$. In this case, the condition for an agent to favor this is:

$$\frac{a_{i0}}{\overline{k}}\eta + \frac{h_i}{H}\frac{(1-\tau_l)\lambda}{(1-\tau_k)} > \frac{c_i}{c}\left(\eta + \lambda + \lambda\frac{\tau_k(1-\zeta)-\tau_l}{(1-\tau_k)} + \eta\frac{\overline{B}}{\overline{k}}\right).$$

This is condition is more complicated, because on the one hand the consumption tax hits agents according to their total wealth, while the capital tax has effect on agents depending on their endowment of physical capital as well as human capital. Hence, in this case (as in cases 3, 4 and 5) both dimensions of heterogeneity matter. The winners also depend on aggregate parameters; for instance, if the transition is very fast, labor-rich people will favor the reform; but if it is very slow, the capital-rich agent will like it.

Overall, these experiments reveal that which taxes are adjusted to maintain government budget balance matters crucially. change the political economy implications significantly. These experiments also speak to the usefulness of the method, which after an initial setup yields many analytical results.

6 Conclusion

This paper characterizes analytically the set of winners from simple tax reforms. In the case of a capital tax cut compensated by higher labor income taxes, the analysis shows that agents with a high wealth-income ratio will favor the tax cut, but others may not, depending on some simple parameters which measure the efficiency effect of the capital tax cut (i.e. the elasticity of demand for capital, the speed of convergence to the new steady-state, the difference between the interest rate and the growth rate, and the initial tax rates). Numerical experiments suggest that there may be a majority of people to oppose a capital income tax cut even if the tax rate is as high as about 20%. The precise set of winners depends of course on which taxes are adjusted to maintain budget balance, and this set depends in general on both wealth and income, so that the joint cross-sectional distribution of wealth and income is needed to determine which reforms are politically sustainable.

There are several extensions worthy of future research. It seems possible to use the methodology of this paper to study the transitional dynamics of the income distribution in more detail. It may also be possible to incorporate some market incompleteness (i.e. uninsurable idiosyncratic shocks) in this analysis.

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7 Appendix A: Computations for Section 3

7.1 Computations of Step 2

Let $x(t,\tau) = \int_0^t r_s(1-\tau)ds$. If $\tau = \tau_k$ is the initial tax rate, interest rates net of taxes are constant equal to $\rho + \sigma z$ for all t as we stay in the initial steady-state: $x(t,\tau_k) = x(t) = (\rho + \sigma z)\, t$, all t. For small changes in τ, the economy converges to a new steady-state. If the change is small enough, we can apply first-order approximation as we stay near the initial steady-state at all dates.

We apply this first-order approximation to $\phi(u) = e^{-u}$, around $x(t)$ i.e. around the path of the old steady-state:

$$\begin{aligned} \phi\left(x(t,\tau)\right) &\simeq \phi\left(x(t)\right) + \phi'\left(x(t)\right)\left(x(t,\tau) - x(t)\right) \\ e^{-\int_0^t \widetilde{r}_s ds} &\simeq e^{-(\rho+\sigma z)t} - e^{-(\rho+\sigma z)t}\left(\int_0^t \left(\widetilde{r}_s - \rho - \sigma z\right) ds\right). \end{aligned}$$

Using a first-order approximation around the new steady-state:

$$\begin{aligned} \widetilde{r}_t &= \left(1 - \tau_k^*\right)\left(F_1(k_t, H) - \delta\right) \\ &= \left(1 - \tau_k^*\right)\left(F_1(k^*, H) - \delta\right) + \left(1 - \tau_k^*\right) F_{11}\left(k_t - k^*\right) \\ &= \rho + \sigma z + (1 - \tau_k^*) F_{11}\left(\overline{k} - k^*\right) e^{-\lambda t}. \end{aligned}$$

Putting the two pieces together:

$$\begin{aligned} e^{-\int_0^t \widetilde{r}_s ds} &\simeq e^{-(\rho+\sigma z)t} - e^{-(\rho+\sigma z)t}(1 - \tau_k^*) F_{11}\left(\overline{k} - k^*\right) e^{-\lambda t}\left(\int_0^t e^{-\lambda s} ds\right) \\ &\simeq e^{-(\rho+\sigma z)t} - e^{-(\rho+\sigma z)t}(1 - \tau_k^*) F_{11}\left(\overline{k} - k^*\right) e^{-\lambda t}\frac{1 - e^{-\lambda t}}{\lambda}. \end{aligned}$$

Hence,

$$\begin{aligned} \Delta p_t &= e^{-\int_0^t \widetilde{r}_s ds} - e^{-(\rho+\sigma z)t} \\ &= e^{-(\rho+\sigma z)t}(1 - \tau_k^*) F_{11}\left(k^* - \overline{k}\right)\frac{1 - e^{-\lambda t}}{\lambda} \\ &= e^{-(\rho+\sigma z)t}(1 - \tau_k) F_{11}\left(k^* - \overline{k}\right)\frac{1 - e^{-\lambda t}}{\lambda}, \end{aligned}$$

since the term $e^{-(\rho+\sigma z)t}\left(\tau_k - \tau_k^*\right) F_{11}\left(k^* - \overline{k}\right)\frac{1-e^{-\lambda t}}{\lambda}$ is second order.

Let's turn now to factor prices. Recall that $\Delta k_t = k_t - \overline{k} = \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right)$. We can deduce the change in interest rates:

$$\Delta r_t = F_1\left(k_t, H\right) - \delta - \left(F_1\left(\overline{k}, H\right) - \delta\right) \simeq F_{11} \times \left(k_t - \overline{k}\right) = F_{11} \times \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right),$$

where again it is irrelevant if F_{11} is evaluated at the initial or final steady-state, because this creates only a second-order error. Similarly the change in the wage at time t is:

$$\Delta w_t = F_2\left(k_t, H\right) - F_2\left(\overline{k}, H\right) = F_{21}\left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right)$$

and for after-tax wage:

$$\Delta \widetilde{w}_t = \Delta((1 - \tau_l) w_t) = -\Delta\tau_l \times \overline{w} + \Delta w_t \times \tau_l.$$

Putting these results together:

$$\begin{aligned}
\Delta(\widetilde{w}_t p_t) &= \Delta\widetilde{w}_t \times \overline{p}_t + \Delta p_t \times \overline{\widetilde{w}}_t \\
&= -\Delta\tau_l \times w_t \times \overline{p}_t + \Delta w_t \times \tau_l \times \overline{p}_t + \Delta p_t \times \overline{\widetilde{w}}_t \\
&= -\Delta\tau_l \times \overline{w} e^{-(\rho+\sigma z)t} + F_{21}\left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right)\tau_l e^{-(\rho+\sigma z)t} \\
&\quad + e^{-(\rho+\sigma z)t}(1-\tau_k)F_{11}\left(k^* - \overline{k}\right)\frac{1 - e^{-\lambda t}}{\lambda}(1-\tau_l)\overline{w}.
\end{aligned}$$

We can also compute the present values of price changes; define $\eta = \rho + \sigma z - (n+z)$, then:

$$\int_0^\infty \Delta p_t e^{(n+z)t} dt = \frac{(1-\tau_k)F_{11}\left(k^* - \overline{k}\right)}{\eta\left(\eta + \lambda\right)},$$

and

$$\int_0^\infty e^{(n+z)t}\Delta\left(\widetilde{w}_t p_t\right) dt = \frac{-\overline{w}\Delta\tau_l}{\eta} + \frac{(1-\tau_l)(k^* - \overline{k})\lambda F_{21}}{\eta\left(\eta+\lambda\right)} + \frac{\overline{w}(1-\tau_l)(k^* - \overline{k})(1-\tau_k)F_{11}}{\eta\left(\eta+\lambda\right)}.$$

7.2 Computations of Step 3

First, note that the change in the detrended surplus is:

$$\begin{aligned}
\Delta s_t \simeq{}& \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right)\left[F_{11}\tau_k\overline{k} + \tau_k\overline{r} + F_{21}\tau_l H\right] + F_2\Delta\tau_l H + \overline{r}\Delta\tau_k\overline{k} \\
& + \tau_c\left(k^* - \overline{k}\right)\left(F_1 - (\delta + n + z)\right)\left(1 - e^{-\lambda t}\right) + \tau_c\lambda\left(\overline{k} - k^*\right)e^{-\lambda t}.
\end{aligned}$$

The present value of the change in detrended surplus is thus:

$$\begin{aligned}
& \int_0^\infty e^{(n+z)t}\left(p_t \times \Delta s_t\right) dt \\
={}& \int_0^\infty e^{(n+z-\rho-\sigma z)t}\Delta s_t dt \\
={}& \frac{\Delta\tau_l F_2 H + \Delta\tau_k\overline{r}\overline{k}}{\eta} + \frac{\lambda\left(k^* - \overline{k}\right)\left[\tau_k F_{11}\overline{k} + \tau_k\overline{r} + \tau_l F_{21}H + \tau_c(\overline{r} - n - z - \eta)\right]}{\eta(\eta+\lambda)}.
\end{aligned}$$

But note that $\eta = \overline{r} - n - z$, so the two terms multiplied by τ_c cancel out. Moreover,

$$\int_0^\infty e^{(n+z)t}\left(\Delta p_t \times \overline{s}\right) dt = \frac{(1-\tau_k)F_{11}\left(k^* - \overline{k}\right)}{\eta\left(\eta+\lambda\right)}\overline{s}.$$

To ensure budget balance we require that

$$\int_0^\infty e^{(n+z)t}\left(\Delta s_t \times \overline{p}_t\right) dt = -\int_0^\infty e^{(n+z)t}\left(\Delta p_t \times \overline{s}\right) dt,$$

which given the simplifications above is:

$$\frac{\Delta\tau_l F_2 H + \Delta\tau_k\overline{r}\overline{k}}{\eta} + \frac{\lambda\left(k^* - \overline{k}\right)\left[\tau_k F_{11}\overline{k} + \tau_k\overline{r} + \tau_l F_{21}H\right]}{\eta(\eta+\lambda)} = -\frac{(1-\tau_k)F_{11}\left(k^* - \overline{k}\right)}{\eta\left(\eta+\lambda\right)}\overline{s}$$

Using (14), we obtain:

$$\Delta\tau_l F_2 H + \Delta\tau_k \overline{r}\overline{k} = -\frac{\Delta\tau_k}{\eta+\lambda}\frac{F_1-\delta}{(1-\tau_k)F_{11}}\lambda\left[\tau_k F_{11}\overline{k} + \tau_k\overline{r} + \tau_l F_{21}H\right] - \frac{(1-\tau_k)F_{11}}{(\eta+\lambda)}\frac{F_1-\delta}{(1-\tau_k)F_{11}}\Delta\tau_k\overline{s}$$

$$\frac{\Delta\tau_l}{\Delta\tau_k} = -\frac{\overline{r}\overline{k}}{F_2H} - \frac{1}{\eta+\lambda}\left(\frac{F_1-\delta}{(1-\tau_k)F_{11}}\frac{\lambda\left[\tau_k F_{11}\overline{k} + \tau_k\overline{r} + \tau_l F_{21}H\right]}{F_2H} - \frac{\overline{s}\,(F_1-\delta)}{F_2H}\right)$$

$$-\frac{\Delta\tau_l}{\Delta\tau_k} = \frac{\overline{r}\overline{k}}{\overline{w}H}\left(1 + \frac{1}{\eta+\lambda}\left(\frac{F_1-\delta}{(1-\tau_k)F_{11}}\frac{\lambda\left[\tau_k F_{11}\overline{k} + \tau_k\overline{r} + \tau_l F_{21}H\right]}{\overline{r}\overline{k}} + \frac{\overline{s}\,(F_1-\delta)}{\overline{r}\overline{k}}\right)\right)$$

$$-\frac{\Delta\tau_l}{\Delta\tau_k} = \frac{\overline{r}\overline{k}}{\overline{w}H}\left(1 + \frac{\lambda}{\eta+\lambda}\frac{1}{(1-\tau_k)}\left(\tau_k + \tau_k\frac{\overline{r}}{\overline{k}F_{11}} + \tau_l\frac{F_{21}H}{F_{11}\overline{k}}\right) + \frac{\eta}{\eta+\lambda}\frac{\overline{B}}{k}\right)$$

$$-\frac{\Delta\tau_l}{\Delta\tau_k} = \frac{\overline{r}\overline{k}}{\overline{w}H}\left(1 + \frac{\lambda}{\eta+\lambda}\frac{\tau_k(1-\zeta)-\tau_l}{(1-\tau_k)} + \frac{\eta}{\eta+\lambda}\frac{\overline{B}}{\overline{k}}\right),$$

where $\zeta = -\frac{r}{\overline{k}F_{11}}$.(This computation uses the fact that for any CRS production function, $F_{21}H = -F_{11}k$.)

7.3 Computations of Step 4

The condition for the agent to be better off is:

$$((1+\tau_c)\overline{c}_i - f)\,\frac{(1-\tau_k)\Delta k_\infty F_{11}}{\eta\,(\eta+\lambda)} < h_i\left(\frac{-\Delta\tau_l\times\overline{w}}{\eta} + \frac{(1-\tau_l)\Delta k_\infty\lambda F_{21}}{\eta\,(\eta+\lambda)} + \frac{\overline{w}(1-\tau_l)\Delta k_\infty(1-\tau_k)F_{11}}{\eta\,(\eta+\lambda)}\right).$$

The budget constraint in the initial steady-state is:

$$(1+\tau_c)\overline{c}_i = (h_i\overline{w}(1-\tau_l)+f) + \eta a_{i0}.$$

Hence we can simplify the condition:

$$(h_i\overline{w}(1-\tau_l)+a_{i0}\eta)\,\frac{(1-\tau_k)\Delta k_\infty F_{11}}{\eta\,(\eta+\lambda)} < h_i\left(\frac{-\Delta\tau_l\times\overline{w}}{\eta} + \frac{(1-\tau_l)\Delta k_\infty\lambda F_{21}}{\eta\,(\eta+\lambda)} + \frac{\overline{w}(1-\tau_l)\Delta k_\infty(1-\tau_k)F_{11}}{\eta\,(\eta+\lambda)}\right)$$

$$a_{i0}\frac{(1-\tau_k)\Delta k_\infty F_{11}}{(\eta+\lambda)} < h_i\left(\frac{-\Delta\tau_l\times\overline{w}}{\eta} + \frac{(1-\overline{\tau_l})\Delta k_\infty\lambda F_{21}}{\eta\,(\eta+\lambda)} + \frac{\overline{w}(1-\overline{\tau_l})\Delta k_\infty(1-\tau_k)F_{11}}{\eta\,(\eta+\lambda)} - \frac{(1-\tau_k)\Delta k_\infty F_{11}}{\eta\,(\eta+\lambda)}\overline{w}(1-\tau_l)\right)$$

$$a_{i0} > \frac{h_i}{\eta}\frac{(-\Delta\tau_l\times\overline{w}(\eta+\lambda) + (1-\overline{\tau_l})\Delta k_\infty\lambda F_{21} + \overline{w}(1-\overline{\tau_l})\Delta k_\infty(1-\tau_k)F_{11} - (1-\tau_k)\Delta k_\infty F_{11}\overline{w}(1-\tau_l))}{(1-\tau_k)\Delta k_\infty F_{11}},$$

whence the formula of the text:

$$\frac{a_{i0}}{h_i} > \frac{-\frac{\Delta\tau_l}{\Delta k_\infty}\times\overline{w}(\eta+\lambda) + (1-\tau_l)\lambda F_{21}}{\eta(1-\tau_k)F_{11}}.$$

Using our formulas for $\frac{\Delta\tau_l}{\Delta\tau_k}$ and $\frac{\Delta\tau_k}{\Delta k_\infty}$:

$$\begin{aligned}
\frac{a_{i0}}{h_i} &> \frac{1}{\eta}\frac{\left(-\frac{\Delta\tau_l}{\Delta\tau_k}\frac{\Delta\tau_k}{\Delta k_\infty}\overline{w}(\eta+\lambda)+(1-\tau_l)\lambda F_{21}\right)}{(1-\tau_k)F_{11}} \\
&> \frac{1}{\eta}\frac{\left(\frac{\overline{r}\overline{k}}{\overline{w}H}\left(1+\frac{\lambda}{\eta+\lambda}\frac{\tau_k(1-\zeta)-\tau_l}{(1-\tau_k)}+\frac{\eta}{\eta+\lambda}\frac{\overline{B}}{\overline{k}}\right)\frac{(1-\tau_k)F_{11}}{F_1-\delta}\overline{w}(\eta+\lambda)+(1-\tau_l)\lambda F_{21}\right)}{(1-\tau_k)F_{11}} \\
&> \frac{1}{\eta}\left(\frac{\overline{r}\overline{k}}{\overline{w}H}\left(1+\frac{\lambda}{\eta+\lambda}\frac{\tau_k(1-\zeta)-\tau_l}{(1-\tau_k)}+\frac{\eta}{\eta+\lambda}\frac{\overline{B}}{\overline{k}}\right)\frac{\overline{w}(\eta+\lambda)}{F_1-\delta}+\frac{(1-\tau_l)\lambda F_{21}}{(1-\tau_k)F_{11}}\right) \\
&> \frac{\overline{k}}{H}\left(\frac{\eta+\lambda}{\eta}+\frac{\lambda}{\eta}\frac{\tau_k(1-\zeta)-\tau_l}{(1-\tau_k)}+\frac{\overline{B}}{\overline{k}}\right)+\frac{1}{\eta}\frac{(1-\tau_l)\lambda F_{21}}{(1-\tau_k)F_{11}} \\
&> \frac{\overline{k}}{H}\left(\frac{\eta+\lambda}{\eta}+\frac{\lambda}{\eta}\frac{\tau_k(1-\zeta)-\tau_l}{(1-\tau_k)}+\frac{\overline{B}}{\overline{k}}-\frac{1}{\eta}\frac{(1-\tau_l)\lambda}{(1-\tau_k)}\right) \\
&> \frac{\overline{k}}{H}\left(1+\frac{\lambda}{\eta}\left(1+\frac{\tau_k(1-\zeta)-\tau_l-(1-\tau_l)}{(1-\tau_k)}\right)+\frac{\overline{B}}{\overline{k}}\right) \\
&> \frac{\overline{k}}{H}\left(1-\frac{\tau_k\lambda\zeta}{\eta(1-\tau_k)}+\frac{\overline{B}}{\overline{k}}\right) \\
&> \frac{\overline{k}}{H}\left(1-\frac{\tau_k\lambda\zeta}{\eta(1-\tau_k)}\right)+\frac{\overline{B}}{H},
\end{aligned}$$

which is the formula of the main text.

8 Appendix B: Computations for Section 5

Note that the steady-state is determined by the following two equations:

$$F(\overline{k},H)-(\delta+n+z)\,\overline{k}=g+\overline{c},$$

$$F_1(\overline{k},H)-\delta=\frac{\rho+\sigma z}{1-\tau_k},$$

so that τ_l, f, and τ_c have no effect on the steady-state. Moreover, the steady-state level of capital is only affected by τ_k. A permanent unexpected increase in g has only the effect of reducing consumption, without changing capital (and thus) prices.

Hence we always have:

$$\Delta k_t=\left(k^*-\overline{k}\right)\left(1-e^{-\lambda t}\right)=\frac{(F_1-\delta)}{(1-\tau_k)F_{11}}\left(1-e^{-\lambda t}\right)$$

and the results of Steps 1 and 2 apply to all of the cases of Section 5.

Now let's replicate the computations of step 3 for arbitrary changes in g, f, τ_l, τ_k, or τ_c. The surplus is defined as:

$$s_t=r_t\tau_k k_t+w_t\tau_l H+\tau_c c_t-g-f.$$

Initial government budget balance implies that $\overline{s}=\eta\overline{B}$. Keeping the government budget balanced requires that:

$$\int_0^\infty e^{(n+z)t}\left(\Delta p_t\times\overline{s}\right)dt=-\int_0^\infty e^{(n+z)t}\left(\Delta s_t\times\overline{p}_t\right)dt.$$

As before, the change in surplus is:

$$\begin{aligned}\Delta s_t &= \Delta r_t \times \tau_k \times \overline{k} + \overline{r} \times \Delta \tau_k \times \overline{k} + \overline{r} \times \tau_k \times \Delta k_t + \Delta w_t \times \tau_l \times H + \overline{w} \times \Delta \tau_l \times H \\ &\quad + \overline{c}\Delta \tau_c + \tau_c \Delta \overline{c} - \Delta f - \Delta g.\end{aligned}$$

Given the first-order approximations:

$$\begin{aligned}\Delta s_t &\simeq F_{11} \times \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right) \times \tau_k \overline{k} + \overline{r}\left(\Delta \tau_k \times \overline{k} + \tau_k \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right)\right) \\ &\quad + \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right) F_{21} \tau_l H + F_2 \times \Delta \tau_l \times H + \tau_c \left(k^* - \overline{k}\right)\left(\left(\overline{r} - n - z\right)\left(1 - e^{-\lambda t}\right) - \lambda e^{-\lambda t}\right), \\ &\simeq \Delta \tau_k \overline{r}\overline{k} + \Delta \tau_l F_2 \times H - \Delta f - \Delta g + \left(k^* - \overline{k}\right)\left(1 - e^{-\lambda t}\right)\left[\tau_k F_{11} \overline{k} + \tau_k \overline{r} + \tau_l F_{21} H\right] \\ &\quad + \tau_c \left(k^* - \overline{k}\right)\left(F_1 - (\delta + n + z)\right)\left(1 - e^{-\lambda t}\right) + \tau_c \lambda \left(\overline{k} - k^*\right) e^{-\lambda t} + \overline{c}\Delta \tau_c.\end{aligned}$$

Using the same simplification as in Appendix A, we have:

$$\int_0^\infty e^{(n+z)t}\left(p_t \Delta s_t\right) dt = \int_0^\infty e^{(n+z-\rho-\sigma z)t} \Delta s_t dt$$

$$= \frac{\Delta \tau_l F_2 \times H + \Delta \tau_k \overline{r}\overline{k} - \Delta f - \Delta g + \overline{c}\Delta \tau_c}{\eta} + \frac{\lambda \left(k^* - \overline{k}\right)\left[\tau_k F_{11} \overline{k} + \tau_k \overline{r} + \tau_l F_{21} H\right]}{\eta(\eta + \lambda)}.$$

To ensure budget balance we need $\int_0^\infty e^{(n+z)t} \Delta s_t \times p_t dt = - \int_0^\infty e^{(n+z)t} \Delta p_t \times \overline{s} dt$ i.e.:

$$\begin{aligned}&\frac{\Delta \tau_l F_2 H + \Delta \tau_k \overline{r}\overline{k} - \Delta f - \Delta g + \overline{c}\Delta \tau_c}{\eta} + \frac{\lambda \left(k^* - \overline{k}\right)\left[\tau_k F_{11} \overline{k} + \tau_k \overline{r} + \tau_l F_{21} H\right]}{\eta(\eta + \lambda)} \\ &= -\frac{(1 - \tau_k) F_{11} \left(k^* - \overline{k}\right)}{\eta\left(\eta + \lambda\right)} \overline{s},\end{aligned}$$

Or:

$$\Delta \tau_l F_2 H + \Delta \tau_k \overline{r}\overline{k} - \Delta f - \Delta g + \overline{c}\Delta \tau_c = -\frac{\left(k^* - \overline{k}\right)}{\eta + \lambda}\left(\lambda\left[\tau_k F_{11} \overline{k} + \tau_k \overline{r} + \tau_l F_{21} H\right]\right) - \frac{(1 - \tau_k) F_{11} \left(k^* - \overline{k}\right)}{\eta\left(\eta + \lambda\right)} \overline{s}.$$

$$\Delta \tau_l F_2 H + \Delta \tau_k \overline{r}\overline{k} - \Delta f - \Delta g + \overline{c}\Delta \tau_c = -\frac{\Delta \tau_k}{\eta + \lambda}\frac{F_1 - \delta}{(1 - \tau_k) F_{11}}\lambda\left[\tau_k F_{11} \overline{k} + \tau_k \overline{r} + \tau_l F_{21} H\right] - \frac{(1 - \tau_k) F_{11}}{\eta\left(\eta + \lambda\right)}\frac{F_1 - \delta}{(1 - \tau_k) F_{11}} \Delta \tau_k \overline{s}.$$

Hence we have a relation between changes in tax rates, which tells us what is consistent with the budget balance:

$$\Delta \tau_l \times \overline{w} H + \Delta \tau_k \times \pi + \overline{c}\Delta \tau_c = \Delta f + \Delta g,$$

where

$$\pi = \overline{r}\overline{k}\left(1 + \frac{\lambda}{\eta + \lambda}\frac{\tau_k(1 - \zeta) - \tau_l}{(1 - \tau_k)} + \frac{\eta}{\eta + \lambda}\frac{\overline{B}}{\overline{k}}\right).$$

For instance, an increase in f or g that is unexpected and permanent has no effect on behavior, since labor supply is lump-sum. Hence, the present-value consequences on government revenue are obvious.

Finally we can make the welfare comparison. As in appendix A, the agent is better off if and only if:

$$(1 + \tau_c)\overline{c}_i \int_0^\infty e^{(n+z)t} \Delta p_t dt + \Delta \tau_c \overline{c}_i \frac{1}{\eta} < h_i \int_0^\infty e^{(n+z)t} \Delta\left(\widetilde{w}_t p_t\right) dt + \int_0^\infty e^{(n+z)t} \Delta\left(f \times p_t\right) dt,$$

which after some simplifications yields:

$$\left((1+\tau_c)\overline{c}_i - f\right)\frac{(1-\tau_k)\Delta k_\infty F_{11}}{\eta\left(\eta+\lambda\right)} + \Delta\tau_c\overline{c}_i\frac{1}{\eta} < h_i\left(\frac{-\Delta\tau_l\times\overline{w}}{\eta} + \frac{(1-\tau_l)\Delta k_\infty\lambda F_{21}}{\eta\left(\eta+\lambda\right)} + \frac{\overline{w}(1-\tau_l)\Delta k_\infty(1-\tau_k)F_{11}}{\eta\left(\eta+\lambda\right)}\right) + \frac{\Delta f}{\eta}$$

$$\begin{aligned}
& \left(h_i\overline{w}(1-\tau_l)+\eta a_{i0}\right)\frac{(1-\tau_k)F_{11}}{\eta\left(\eta+\lambda\right)}\Delta\tau_k\frac{F_1-\delta}{(1-\tau_k)F_{11}} + \frac{\Delta\tau_c}{1+\tau_c}\left(h_i\overline{w}(1-\tau_l)+\eta a_{i0}\right)\frac{1}{\eta} \\
< \quad & h_i\left(\frac{-\Delta\tau_l\times\overline{w}}{\eta} + \frac{(1-\tau_l)\lambda F_{21}}{\eta\left(\eta+\lambda\right)}\frac{F_1-\delta}{(1-\tau_k)F_{11}}\Delta\tau_k + \frac{\overline{w}(1-\tau_l)(1-\tau_k)F_{11}}{\eta\left(\eta+\lambda\right)}\frac{F_1-\delta}{(1-\tau_k)F_{11}}\Delta\tau_k\right) + \frac{\Delta f}{\eta}
\end{aligned}$$

$$\begin{aligned}
& \left(h_i\overline{w}(1-\tau_l)+\eta a_{i0}\right)\left(F_1-\delta\right)\Delta\tau_k + \frac{\Delta\tau_c}{1+\tau_c}\left(h_i\overline{w}(1-\tau_l)+\eta a_{i0}\right)\left(\eta+\lambda\right) \\
< \quad & h_i\left(-\Delta\tau_l\times\overline{w}(\eta+\lambda) + (1-\tau_l)\lambda F_{21}\frac{F_1-\delta}{(1-\tau_k)F_{11}}\Delta\tau_k + \overline{w}(1-\tau_l)\left(F_1-\delta\right)\Delta\tau_k\right) + (\eta+\lambda)\Delta f
\end{aligned}$$

$$a_{i0}\left(\Delta\tau_k + \frac{\Delta\tau_c}{1+\tau_c}\frac{\eta+\lambda}{F_1-\delta}\right) < h_i\left(-\Delta\tau_l\frac{\overline{w}(\eta+\lambda)}{\left(F_1-\delta\right)\eta} - \frac{\Delta\tau_c(1-\tau_l)\overline{w}(\eta+\lambda)}{\left(1+\tau_c\right)\eta\left(F_1-\delta\right)} - \frac{(1-\tau_l)\lambda\overline{k}}{\eta(1-\tau_k)H}\Delta\tau_k\right) + \frac{(\eta+\lambda)}{\eta(F_1-\delta)}\Delta f,$$

which is the formula of the main text.