Appendix 2: comparison with the model with variable utilization

This appendix presents some results from numerical simulations of two models: first, the model of the paper, and second the model of Gilchrist and Williams (2000) which has variable utilization. The overall flavor of the results is that the differences between the two models are not large. This justifies the simplifying assumption of the paper. Note that to conduct these simulations, I need to specify the preferences, the process for shocks, and to impose the resource constraints, while the stock market valuation formula does not require any of these. For comparison purpose, I stick to the parameters used by Gilchrist and Williams (2000).

Table 1 gives the standard deviations of the growth rates of consumption, investment and employment, relative to output, for each model, and for each of two shocks: TFP shocks, and embodied-technology shocks. The difference between the two model is relatively small. The largest difference regards employment: in the GW model it can respond instantaneously to a shock through increased utilization, while in my version of the model employment is predetermined and can increase only over time through investment in new capacity. Hence, employment is somewhat more volatile in the GW model.

This comparison is generous since I follow GW in incorporating idiosyncratic productivity shocks to their model. These generate more low productivity machines for which full utilization is not optimal. (In my model, given the full utilization assumption, the aggregate results are independent of the amount of idiosyncratic volatility.) If I reduce the amount of idiosyncratic productivity, I bring the implications of the GW model even closer to my model. Indeed, under certain conditions the two models become equivalent since the GW model will feature nearly constant utilization (for a depreciation rate high enough, small idiosyncratic shocks, and growth low enough).

Figure 1 presents the impulse responses of output, consumption, investment and employment following a TFP shock, and Figure 2 presents the responses to an embodied-technology shock. The autocorrelation of each shock is set to 0.95 as in the figure 2 of GW. The qualitative patterns are the same across the two models, and the quantitative differences are not large.[1]

Figure 3 presents the prices of different machines, as a function of productivity, in the GW model and in my model. These prices were obtained by simulations, using the expression:

$$P_t(X) = E_t \sum_{j\geq 1} (1-\delta)^j \frac{\beta^j \lambda_{t+j}}{\lambda_t} \max\left(A_{t+j}X - W_{t+j}, 0\right).$$

More precisely, for a given value of productivity X and a given state today, I simulate the evolution of the economy over the next 200 periods and compute the realized value of this sum (the present value of profits). I repeat this simulation 5000 times and I take the average of the sum across simulations. This yields an estimate of the expectation of the sum of future profits, i.e. an estimate of the price today.

There are two reasons why the prices are different in the GW economy and in my simplified version. First, machines have the option of shutting down (and possibly reopening later) in the GW model, while in my model they must remain open until they disappear due to depreciation. This makes the price an affine function of productivity in my model, whereas in GW due to the option value the price is a convex function of productivity, and is bounded below by zero.[2] The second reason is simply that the aggregates behave differently in the two models: the wage and the interest rate do not follow the same law of motions with full utilization and with variable utilization. Figure 3 shows that the differences in the mean level of prices are small

[1] These figures actually seem to overstate slightly the difference between the two models - consider the figures 2 and 3 of the GW paper instead.

[2] The fact that some machines can have negative value in my model is unimportant, since depending on parameter values there may be very few such machines. More precisely, if the depreciation rate is high enough, wages do not grow too fast, and idiosyncratic shocks are small enough, this will rarely happen.

for all but the very low productivity machines, for which the option of shutting down is an important component of their value.

Finally, Figure 4 gives the response of machine prices to a 1% TFP shock in the GW economy and in my economy, as a function of the productivity of the machine. The varying utilization creates an additional channel here too: the option value of opening becomes much more valuable following the TFP shock. This makes the value of low productivity machine increase more (or fall less) in the GW model than in my model. This channel works in my favor, since low productivity machines become more procyclical. For high-productivity machines, the option value of shutting down is nearly worthless and this channel is insignificant - in finance langage, the beta of an option is higher when the price is below the strike.[3]

While this option value channel deserves further exploration, the overall conclusion from these numerical results is that the assumption of full utilization does not affect the results greatly.

[3]Note that in this figure, prices of machines go down, because interest rates increase; this is the GW calibration, which does not generate the key condition $\partial i / \partial A < 0$ that I require for my results. To get this condition to arise in the full DSGE model, I need a very elastic labor supply (so that the cost of labor is not very cyclical) and a low intertemporal elasticity of substitution of consumption.

		$\frac{\sigma(\Delta C_t)}{\sigma(\Delta y_t)}$	$\frac{\sigma(\Delta I_t)}{\sigma(\Delta y_t)}$	$\frac{\sigma(\Delta N_t)}{\sigma(\Delta y_t)}$
TFP shocks	GW	0.70	1.76	0.09
	This paper	0.76	1.61	0.05
Embodied shocks	GW	2.87	8.14	0.91
	This paper	3.33	7.36	0.56

Table 1. Business cycle statistics for consumption, investment and employment in the full Gilchrist-Williams (GW) model and for the model of this paper. Results based on 500 simulations of 200 periods each. The parameters are set as in Gilchrist and Williams (2000).

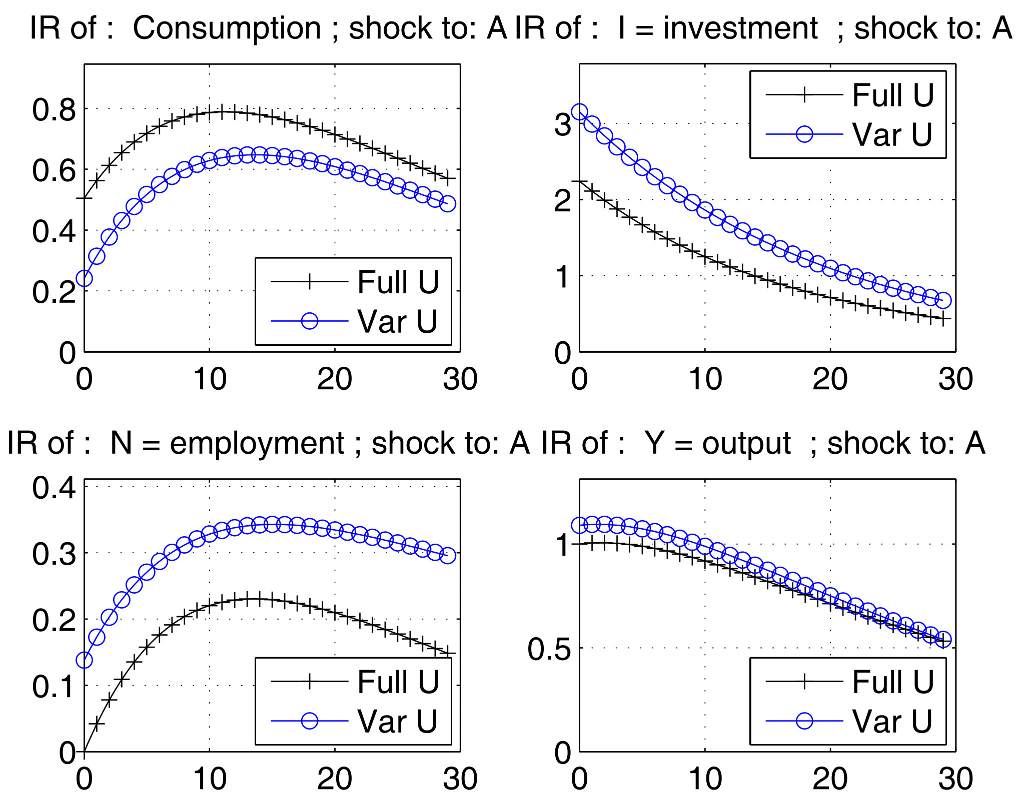

FIG. 1. This figure plots the response of consumption, investment, output and employment to a persistent shock to TFP, in the full utilization model (this paper), and in the variable utilization model (Gilchrist and Williams). Units: quarters after the shock on the x-axis, and % deviation from the steady-state on the y-axis.

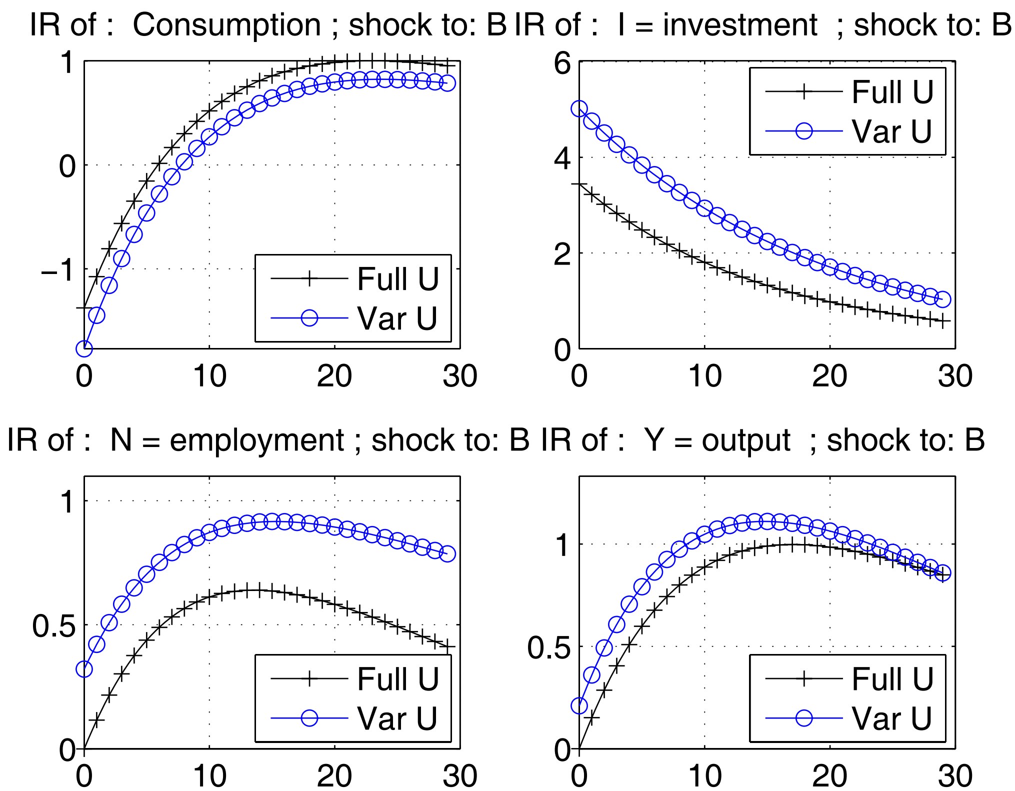

FIG. 2. This figure plots the response of consumption, investment, output and employment to a persistent shock to embodied technology, in the full utilization model (this paper), and in the variable utilization model (Gilchrist and Williams). Units: quarters after the shock on the x-axis, and % deviation from the steady-state on the y-axis.

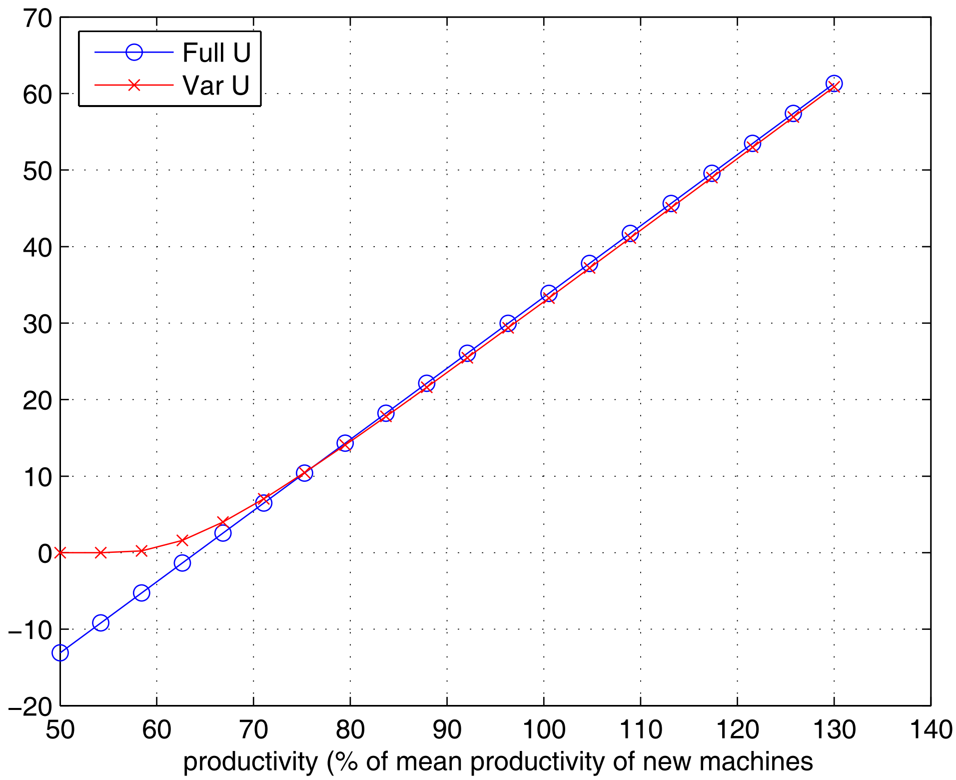

FIG. 3. This figure plots the price of a machine, as a function of its productivity, in the full utilization model (this paper) and in the variable utilization model (Gilchrist-Williams).

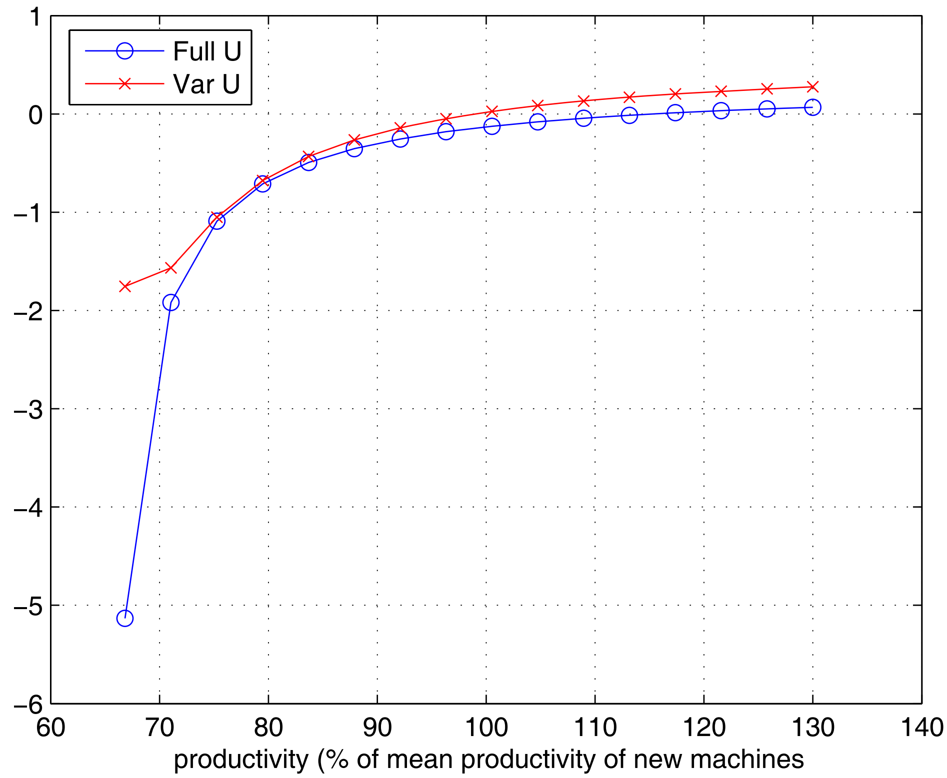

FIG. 4. This figure plots the % change in the price of a machine following a TFP shock, as a function of its productivity, in the full utilization model (this paper) and in the variable utilization model (Gilchrist-Williams). Calibration as in Gilchrist and Williams (2000). Results based on 5000 simulations of length 200 before and after the shock.