Ex Ante Collusion and Design of Supervisory Institutions[1]

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Abstract

We consider the problem of designing a mechanism with *ex ante* collusion, where a supervisor and agent collude on *both* participation and reporting decisions. The agent's unit cost has a continuous distribution; the supervisor receives a noisy signal of the agent's cost; agent and supervisor play non-cooperatively if they fail to agree on a side contract. Delegation is always dominated by eliminating the supervisor. With centralized contracting it is typically valuable to hire a supervisor despite the costs of deterring collusion. Changes in bargaining power within the coalition have no effect, while altruism of the supervisor towards the agent makes the Principal worse off.

KEYWORDS: mechanism design, intermediation, supervision, collusion, delegation

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1 Introduction

The potential for collusion is widely acknowledged to be a serious problem for a Principal (P) who relies on information provided by an expert intermediary or supervisor (S) to design a contract for a productive agent (A). There is considerable evidence for such collusion in many contexts, e.g. between company Directors and CEOs, management and workers, credit-rating agencies and firms, auditors and clients, and 'revolving doors' between regulators and firms.[5] Early literature on the mechanism design problem with collusion was based on the assumption of hard information (where S cannot lie, and can only withhold information), and exogenous transaction costs of collusion (Tirole (1986), Laffont-Tirole (1993)). Subsequent literature has considered contexts where the collusion problem is harder to control, owing to soft information (which allows S to report anything) and absence of exogenous transaction costs of collusion (Faure-Grimaud, Laffont and Martimort (2003, denoted FLM hereafter), Celik (2009)). However, these papers consider only the possibility of *interim* collusion, where S and A can collude over reporting decisions, but not whether to participate in the mechanism.

In this paper we consider the additional consequences of collusion in participation decisions, called *ex ante* collusion. This is relevant in many contexts where S and A know and communicate with each other before they are approached by P. Examples include the relationship between credit rating agencies and firms, regulators and regulated entities, company Directors and CEOs, or prime contractors and subcontractors. We examine consequences for optimal organization of contracting relationships, e.g., whether authority to contract with A should be delegated to S, the value of supervision, costs of collusion and how they are affected by relative bargaining power or altruism between S and A. Apart from collusion in participation, our model is similar to existing collusion literature: A observes the signal observed by S and enters into a side-contract with S which coordinates their responses to P in exchange for a side-payment. If they fail to agree on a side contract

[5] For evidence on collusion between outside Directors and CEOs, see Hallock (1997), Hwang and Kim (2009), Fracassi and Tate (2012), Kramarz and Thesmar (2013) and Schmidt (2015). Bertrand and Mullainathan (1999, 2003), Atanassov and Kim (2009), Cronqvist et al. (2009) focus on collusion between management and workers; de Haan et al. (2015) and Cornaggia et al. (2016) show evidence of 'revolving doors' between credit-rating agencies and firms, while Lennox (2005), Lennox and Park (2007), Firth et al. (2012) show evidence on collusion between auditors and their clients.

they play noncooperatively thereafter. They enter into a deviating side contract only if it results in an interim Pareto improving allocation for the coalition.

Apart from addressing the substantive issue of collusion in participation, we avoid exogenous message space restrictions implicitly employed in previous analyses of interim collusion. In the absence of any such restrictions, it is trivially possible for the Principal (P) to completely overcome the problem of interim collusion by requiring the supervisor (S) or agent (A) to communicate their private information to P at this stage (besides participation decisions).[6] Such restrictions are non-standard and invoke additional unmodelled frictions beyond private information and collusion *per se*.

An additional contribution of this paper at the technical level is that we consider a continuum type space for A's cost, and show how classical Myersonian mechanism design methods based on 'virtual' types can be extended (using techniques in Jullien (2000)) to incorporate collusion constraints, while allowing for general information structures for S. This is in contrast to previous analyses of supervision and collusion with adverse selection which have focused on discrete (two or three) type cases.

Our first main result is that collusion in participation has important implications for the optimality of delegating authority to S. Some authors have shown that delegation can be optimal in the presence of collusion in some specific settings, e.g., in a moral hazard setting by Baliga and Sjostrom (1998), and in an adverse selection setting with interim collusion and a specific information structure by FLM. Celik (2009) on the other hand shows that the FLM result does not hold under alternative information structures. Hence results in the interim collusion setting are sensitive to details of the information structure. By contrast in the ex ante setting we show that delegation is *never* optimal, irrespective of the information structure. Indeed, delegation to S is inferior to P not hiring S at all. Intuitively, collusion in participation results in an additional constraint on P's mechanism design problem is akin to (but not the same as) a limited liability constraint on S which results in double marginalization of rents (DMR).[7]

[6] P may offer each a menu of contracts, from which S and A are required to respectively select at the same time that they communicate their participation decision. This would obviate the need for any subsequent communication after both have accepted and selected their respective contracts. If S and A cannot collude at the participation stage, such a design would eliminate any scope for collusion that undermines P's payoff (Motta (2009)).

[7] A similar result is obtained by Mookherjee and Tsumagari (2004) in a team production setting involving

To illustrate resulting implications for optimal mechanisms, we start by considering the simple case of an indivisible good, with a continuum type space and two possible signals observed by S resulting in posterior beliefs with full support. This setting permits a detailed analysis of optimal contracts under ex ante collusion (EAC), when and how they differ from the interim collusion (IC) context. For low valuations of the good being procured, incentives are low-powered and optimal contracts in the two contexts coincide. For higher valuations, they diverge: the IC setting is characterized by high powered contracts and delegation to S with P charging a fixed upfront fee. Such contracts are infeasible in EAC: S has an incentive to refuse P's contract to avoid paying the upfront fee in states where A experiences a high cost and does not want to deliver the good at the offered price. Collusion in participation then necessitates centralized contracting, low powered incentives, and low sensitivity of prices with respect to cost information. When P's valuation of the good is sufficiently high, S is no longer worthwhile under ex ante collusion, but always remains valuable in interim collusion.

We then turn to the context of divisible good procurement, where relatively little structure is imposed on the information structure. Here we show that hiring S is *always* valuable. This helps explain the widespread prevalence of supervision in organizations and contracting networks, despite the presence of prior connections between supervisors and agents. This difference from the indivisible good context arises because there is greater scope for S's information to affect P's procurement decisions (e.g., regarding quantity of the good to be supplied). The proof is based on showing that small variations can be constructed around the optimal non-supervision contract which move it in the direction of the optimal no-collusion (second-best) contract with a supervisor, without giving rise to any collusion or changing S's payoff. The variation entails raising the output procured over some range of cost types, and lowering it over another range (corresponding to an arbitrary cost signal state). Differences in beliefs of P and S regarding A's cost ensures the existence of 'mutual gains from trade' from such a variation, enabling P to earn higher profits while preserving S's payoff.

We derive the following additional results concerning optimal mechanisms in the ex ante collusion setting:

- Ex ante collusion imposes a welfare cost for P in specific contexts. This contrasts with

two agents privately informed about their respective costs.

the results obtained by Che and Kim (2006) or Motta (2009) for interim collusion settings where the second-best welfare can generally be achieved by P.

- Altruism of S with respect to A always hurts P, thereby providing support for the common idea of importance of arms-length relationships. The result is not a priori obvious, owing to two offsetting effects. On the one hand, increased altruism aids collusion by lowering frictions within the coalition. On the other hand, it reduces the severity of the DMR problem by limiting the extent to which S seeks to personally gain from lowering the price offered to A. The former effect outweighs the latter when contracting is centralized, while the opposite is true under decentralized contracting.

- Changes in bargaining power between S and A over the side-contract do not matter. Despite the existence of asymmetric information within the coalition, a modified form of the 'Coase Theorem' continues to hold. This is a consequence of the standard assumption employed regarding the nature of collusion, wherein failure to collude results in noncooperative play in P's mechanism.[8]

- A form of modified delegation is always optimal, in which A does not contract directly with P on the equilibrium path, but has the option to do so off the equilibrium path.

The paper is organized as follows. Section 2 studies the context of an indivisible good and two cost signals. Section 3 considers a perfectly divisible good and a general information structure for S involving a finite number of possible signals, and presents results concerning value of supervision and collusion costs. Section 4 discusses extensions incorporating alternative allocations of bargaining power within the coalition, and altruistic supervisors. Finally, Section 5 discusses implications of our results, extensions and shortcomings of our analysis.

2 Indivisible Good Case

P procures an indivisible good with quantity q either 0 or 1 from A who produces it at cost θ. A is privately informed about the realization of θ. The supervisor S and A jointly observe

[8] Mookherjee and Tsumagari (2017) show this result no longer holds in settings of 'strong' collusion, where side contracts include commitments to threats made by each party concerning strategies they will employ should the other party refuse to participate in the collusion.

the realization of signal $i \in \{L, H\}$ of A's cost. Both A and S have outside option payoffs of 0. $F_i(\theta)$ denotes the distribution of θ conditional on i defined on $[\underline{\theta}, \bar{\theta}]$, which has a density $f_i(\theta)$ which is differentiable and positive on $[\underline{\theta}, \bar{\theta}]$. Hence the support of θ does not vary with the signal, and hazard rates are well-defined and finite-valued throughout the support. $\kappa_i \in (0, 1)$ denotes the probability of signal i, with $\kappa_L + \kappa_H = 1$. P does not observe the signal i, and has a prior $F(\theta) \equiv \kappa_L F_L(\theta) + \kappa_H F_H(\theta)$ with density $f(\theta) \equiv \kappa_L f_L(\theta) + \kappa_H f_H(\theta)$.

Assumption 1 *(i)* $\frac{f_L(\theta)}{f_H(\theta)}$ *is decreasing*

(ii) $H(\theta) \equiv \theta + \frac{F(\theta)}{f(\theta)}$, $h_i(\theta) \equiv \theta + \frac{F_i(\theta)}{f_i(\theta)}$ *and* $l_i(\theta) \equiv \theta + \frac{F_i(\theta)-1}{f_i(\theta)}$ *(*$i = L, H$*) are increasing*

(iii) $h_L(\bar{\theta}) > V > \underline{\theta}$

Part (i) represents a monotone likelihood ratio property wherein $i = L$ (resp. $i = H$) is a signal of low (resp. high) cost, while (ii) is a standard assumption ensuring monotonicity of (conditional) virtual costs. These imply $F_H(\theta) < F(\theta) < F_L(\theta)$ and $h_H(\theta) < H(\theta) < h_L(\theta)$ for any $\theta \in (\underline{\theta}, \bar{\theta})$. The last inequality of Condition (iii) ensures gains from trade between P and A; the second one ensures that costless access to S's signal is valuable for P in the absence of collusion. These conditions are satisfied in the following example with a uniform prior $F(\theta) = \theta$ on $[0, 1]$ and linear conditional densities: $F_L(\theta) = 2d\theta - (2d-1)\theta^2$, $F_H(\theta) = 2(1-d)\theta + (2d-1)\theta^2$ on $[0, 1]$, $\kappa_L = \kappa_H = 1/2$, $d \in (1/2, 1)$ and V between 0 and $1 + \frac{1}{2(1-d)}$. We shall illustrate our analysis with numerical computations for this example.

The situation where P has no access to S's signal is referred to as the *No Supervision (NS)* case. Here P offers a non-contingent price p^{NS} to maximize $F(p)[V - p]$, which satisfies $V = H(p^{NS})$ if $V < H(\bar{\theta})$, and equals $\bar{\theta}$ otherwise. Let $\Pi^{NS} \equiv F(p^{NS})[V - p^{NS}]$ denote the resulting expected payoff of P. The *second-best allocation* results when there is no collusion whence P can costlessly access S's signal; here P offers A a price p_i^{SB} which maximizes $(V - p_i)F_i(p_i)$. The ordering of virtual cost functions implied by Assumption 1 ensures a lower price elasticity of supply and thus a lower second-best price in the low cost signal state. However, the supply curve is shifted to the right in the low signal state, so the ordering of resulting supply likelihoods between the two states is ambiguous, which turns out to depend on V:

Lemma 1 *(i)* $p_H^{SB} > p^{NS} > p_L^{SB}$ *if* $V < H(\bar{\theta})$*, and* $p_H^{SB} = p^{NS} = \bar{\theta} > p_L^{SB}$ *otherwise*

(ii) There exist V^* and V^{**} such that $\underline{\theta} < V^* \leq V^{**} < h_H(\bar{\theta})$, where $F_L(p_L^{SB}) > F_H(p_H^{SB})$ for $V \in (\underline{\theta}, V^*)$ and $F_L(p_L^{SB}) < F_H(p_H^{SB})$ for $V \in (V^{**}, h_L(\bar{\theta}))$.

Proof of Lemma 1: (i) is straightforward. To establish (ii), for any $q \in [0,1]$, define $P_i(q) \in [\underline{\theta}, \bar{\theta}]$ such that $F_i(P_i(q)) = q$ and $C_i(q) \equiv qP_i(q)$. So we may interpret $C_i(q)$ as the 'cost' function in state i. Since $C_i'(F_i(\theta)) = h_i(\theta)$, Assumption 1 (ii) implies $C_i'(q)$ is increasing in q on $[0,1]$. Then $q_i^{SB} \equiv F_i(p_i^{SB})$ satisfies $V = C_i'(q_i^{SB})$ for $V \in (\underline{\theta}, h_i(\bar{\theta}))$. From Assumption 1 (i) and $f_i(\theta) > 0$ on $[\underline{\theta}, \bar{\theta}]$ for $i \in \{L, H\}$, $C_L(q) < C_H(q)$ on $q \in (0,1)$ with $C_L(0) = C_H(0) = 0$ and $C_L(1) = C_H(1) = \bar{\theta}$. Hence there are intervals of small q such that $C_L'(q) < C_H'(q)$ and large q such that $C_L'(q) > C_H'(q)$. This guarantees the existence of V^* and V^{**} with the stated properties. ■

2.1 Delegation to Supervisor with Ex Ante Collusion

Consider P's option to contract solely with S and delegate the authority to contract with A. With ex ante collusion, S does not commit to responding to P's offer before contracting with A. So after P offers S a contract, the latter offers A a contract. Following A's response, S then responds to P.

Given this timing, standard arguments imply that (following any given contract offer) S can confine attention to offering A a take-it-or-leave-it price p_i in state i for delivering the good to P. And similarly P can confine attention to offering S a two part contract X_0, X_1 where X_q is the payment for delivery of output q. There is no added value to P asking S to submit a report of her signal or the outcome of contracting with A, as conditional on the q delivered S would select whichever message would maximize her payment received.

In order to induce S to deliver the good with positive probability, P must offer $X_1 > \underline{\theta}$. Upon observing signal i, S will then decide what price $p_i \in [\underline{\theta}, X_1]$ to offer A, along with participation decision in P's contract in either of the two events where A does or does not accept S's offer. If A accepts, it is optimal for S to agree to participate in P's contract since the optimal price will satisfy $p_i < X_1$. Let $I \in \{0,1\}$ denote S's participation decision in the event that A does not accept S's offer. Then S selects p_i and I to maximize $F_i(p_i)(X_1 - p_i) + I[1 - F_i(p_i)]X_0$. It follows that $I = 1$ only if $X_0 \geq 0$. If $X_0 < 0$, S will not accept P's offer in the event that A does not accept S's offer. The same outcome is realized if P sets $X_0 = 0$. Hence without loss of generality, $X_0 \geq 0$, and S always accepts P's offer. The constraint $X_0 \geq 0$ plays a key role in the subsequent analysis. It arises owing

to ex ante collusion, whereby S contracts and communicate with A prior to responding to P's offer. In an interim collusion setting this constraint does not arise, and is replaced by interim participation constraints for S, whence X_0 can be negative and yet P's contract could be accepted by S.

Let b denote the delivery *bonus* $X_1 - X_0$. The choice of p_i will be made by S to maximize $F_i(p_i)(b-p_i)$. If $b \leq \underline{\theta}$, it is optimal for S to offer A a price below $\underline{\theta}$, whence the good is never delivered to P. Otherwise there is a unique optimal price $p_i(b)$ which satisfies $\underline{\theta} < p_i(b) < b$. Eventually P earns expected payoff $[\kappa_L F_L(p_L(b)) + \kappa_H F_H(p_H(b))](V - b) - X_0$, which is sought to be maximized by choosing $b > \underline{\theta}, X_0 \geq 0$. Now note that any such payoff would be strictly dominated by the option of not appointing S at all where P directly offers A a price of b. This follows since $b < V$ is necessary for P to earn a positive payoff; hence $[\kappa_L F_L(p_L(b)) + \kappa_H F_H(p_H(b))](V - b) - X_0 < F(b)(V - b) \leq \Pi^{NS}$. We thus obtain:

Proposition 1 *With an indivisible good and ex ante collusion, delegation to the supervisor is worse for the principal compared to not appointing a supervisor.*

As we shall later see, delegation could dominate the no-supervisor outcome under interim collusion. This represents a stark contrast between the two forms of collusion. In delegation with ex ante collusion, S earns rents which cannot be taxed away upfront by P at the time of contracting with S, thereby generating a double marginalization of rents (DMR). Under interim collusion, P may be able to extract some of S's interim rents (in the absence of knowledge of A's type) via an upfront fee, thereby limiting the DMR problem.

2.2 Centralized Contracting with Ex Ante Collusion

Under ex ante collusion, therefore, if at all P obtains an advantage from appointing S, she needs to contract simultaneously with both S and A. S and A can negotiate a side-contract (SC, for short) prior to responding to P's offer. Following private communication of a cost message by A to S, the SC coordinates their respective messages (which include participation decisions and cost reports) sent to P, besides a side payment between A and S. As shown later, without loss of generality S has all the bargaining power within the coalition and makes a take-it-or-leave-it SC offer to A. If A refuses it, they play P's mechanism non-cooperatively. It turns out (as explained in the online Appendix) P can confine attention to mechanisms that are collusion-proof, i.e., for which it is optimal for S to not offer any

non-null SC to A, and both S and A agree to participate. We now explain the implied individual and coalition incentive compatibility constraints in the context of an indivisible good. In this setting, abstract message spaces can be dispensed with and the allocation can be represented more simply by a set of prices that satisfy the constraints described below.

First, a contract offer to A reduces to a single take-it-or-leave-it price offer p_i when the cost signal is i. Second, in order to deter collusion, P must offer an aggregate payment to S and A which depends only on whether or not the good is produced. Let $X_0 + b, X_0$ denote the aggregate payments when the good is and is not produced respectively. The two prices p_L, p_H combined with X_0, b characterize an allocation entirely. This is associated with a mechanism where S and A are asked to submit reports of the signal i to P. If the two reports happen to match, A is offered the option to produce and deliver the good directly to P in exchange for price p_i, while S is paid X_0 if the good is not delivered, and $b + X_0 - p_i$ if it is delivered. If the two reports do not match, there is no production and both S and A are required to pay a high penalty to P. The key feature distinguishing centralized contracting from delegation is that in the former P makes a contract offer directly to A which is conditioned on reported signals. This provides an outside option to A which S is constrained to match while offering an SC to A. This is an important strategic tool which enables P to manipulate the outcome of collusion between S and A, and reduce the severity of the DMR problem.

Along the equilibrium path where A and S decide to participate, report i truthfully to P, and do not enter into a deviating SC, A produces the good in state i and receives the payment p_i if and only if θ is smaller than p_i. Without loss of generality, A receives no payment in the event of non-production (since any mechanism paying a positive amount to A in the event of non-production is dominated by one that does not). This generates utility to A of $u_A(\theta, i) = \max\{p_i - \theta, 0\}$. S ends up with $X_0 + b - p_i$ in the event that production takes place, and X_0 otherwise.

The allocation p_L, p_H, X_0, b has to satisfy the following constraints. First, in order to ensure that ex post the coalition does not prefer to reject it, the aggregate payment to S and A must be nonnegative in the event that the good is not delivered:

$$X_0 \geq 0. \tag{1}$$

The reason is that if the good is not delivered, A earns no rent; hence rejection of P's contract by the coalition does not entail any payoff consequence for A. If $X_0 < 0$, S would

then benefit from rejecting P's contract; hence it is Pareto improving for the coalition to do so.[9] This constraint is distinctive to the ex ante collusion setting, where participation decisions in P's contract are made after S and A have negotiated a side contract.

Second, in order to induce S to participate ex ante:

$$F_H(p_H)(b - p_H) + X_0 \geq 0 \tag{2}$$

$$F_L(p_L)(b - p_L) + X_0 \geq 0 \tag{3}$$

Individual participation constraints for A are already incorporated into the supply decision represented by a supply likelihood of $F_i(p_i)$ in state i.

Third, S and A should not be tempted to enter a deviating SC. A deviating SC would involve a different set of prices $\tilde{p}_i$ offered to A (in state i) for delivering the good, combined with a lump-sum payment $\tilde{u}_i$. A would then produce if θ is smaller than $\tilde{p}_i$, and S would earn an expected payoff $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 - \tilde{u}_i$. Type θ of A would accept the deviating SC provided

$$\max\{\tilde{p}_i - \theta, 0\} + \tilde{u}_i \geq \max\{p_i - \theta, 0\} \tag{4}$$

We show in the online Appendix that without loss of generality S can restrict attention to side contracts which are accepted by all types of A. Hence collusion-proofness requires $(\tilde{p}_i = p_i, \tilde{u}_i = 0)$ to maximize $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 - \tilde{u}_i$ subject to (4) for all types $\theta \in [\underline{\theta}, \bar{\theta}]$.

This condition can be broken down as follows. First, if $p_i > \underline{\theta}$, S should not benefit by deviating to a price $\tilde{p}_i < p_i$. This would necessitate offering a lump-sum payment of $\tilde{u}_i = p_i - \tilde{p}_i$ to ensure that all types of A accept the SC, which would then generate S an interim expected payoff of $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 - p_i + \tilde{p}_i$. A necessary and sufficient condition for such a deviation to not be worthwhile is that

$$b \geq p_i - \frac{1 - F_i(p_i)}{f_i(p_i)} \equiv l_i(p_i) \tag{5}$$

since $l_i(p)$ is increasing in p as per the monotone hazard rate assumption 1(ii). Intuitively, offering a lower price than p_i is similar to S selling the good back to A. Condition (5) which

[9] No analogous non-negativity constraint on aggregate payments $X_0 + b$ corresponding to delivery of the good is imposed here, because the decision to reject P's contract could result in a loss of rents for A. S would then have to compensate A for this loss, and the required compensation may be large enough that it may be optimal for S to instead accept P's contract despite $X_0 + b$ being negative. The issue of coalition incentive compatibility is addressed in more detail below.

states that the value (b) of the good to S exceeds its virtual value to A, ensures that such a sale is not worthwhile.

Similarly, if $p_i < \bar{\theta}$, S should not want to offer A a higher price $\tilde{p}_i$. Unlike the case of a lower offer price, such a variation cannot be accompanied by a negative lump sum payment $\tilde{u}_i$ to A, owing to the need for A's ex post participation constraint to be satisfied in non-delivery states. Offering $\tilde{p}_i > p_i$ will then generate an interim payoff of $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0$. For S to not want to deviate to a higher price, it must be the case that

$$b \leq p_i + \frac{F_i(p_i)}{f_i(p_i)} = h_i(p_i) \tag{6}$$

This condition can be interpreted simply as the value of delivery (b) to S being lower than the virtual cost of A delivering it.

(5, 6) can be combined into the single collusion-proofness condition

$$\max\{\hat{l}_L(p_L), \hat{l}_H(p_H)\} \leq b \leq \min\{\hat{h}_L(p_L), \hat{h}_H(p_H)\}. \tag{7}$$

where $\hat{l}_i(p_i)$ denotes $l_i(p_i)$ if $p_i > \underline{\theta}$ and $-\infty$ otherwise, and $\hat{h}_i(p_i)$ denotes $h_i(p_i)$ if $p_i < \bar{\theta}$ and ∞ otherwise (since the corresponding state i constraint is relevant only when p_i differs from $\underline{\theta}, \bar{\theta}$ respectively).

As implied by arguments in the online Appendix, these conditions are necessary and sufficient for the allocation (p_L, p_H, b, X_0) to be the outcome of a Perfect Bayesian Equilibrium (PBE) of the ex ante collusion contracting game, which is interim-Pareto-undominated for the coalition by any other PBE. Hence, an optimal allocation must maximize

$$[\kappa_H F_H(p_H) + \kappa_L F_L(p_L)](V - b) - X_0 \tag{8}$$

subject to (1, 2, 3, 7). We refer to these constraints as characterizing ex ante collusion (EAC) feasibility.

It is convenient to restate P's profit as

$$U(p_L, p_H) - R(b, X_0; p_L, p_H) \tag{9}$$

where $U(p_L, p_H) \equiv \kappa_H F_H(p_H)(V - p_H) + \kappa_L F_L(p_L)(V - p_L)$ is the expression for expected profit in the second-best setting, from which S's rent $R(b, X_0; p_L, p_H) \equiv \kappa_H F_H(p_H)(b - p_H) + \kappa_L F_L(p_L)(b - p_L) + X_0$ has to be subtracted in the presence of collusion. Note also that given b, p_L, p_H it is optimal to set $X_0 = \max\{0, \max_i\{F_i(p_i)(p_i - b)\}\}$. With this convention we can henceforth represent an EAC allocation by the triple (p_L, p_H, b).

We start the analysis by making some simple but key observations regarding properties of any EAC-feasible allocation in which S is valuable (i.e, where the resulting profit exceeds the maximum profit attainable in NS).

Lemma 2 *In any EAC-feasible allocation in which S is valuable:*

(i) $b < p_i$ *for some* i *and* $X_0 > 0$

(ii) $p_L < p_H$

(iii) $F_L(p_L) > F_H(p_H)$.

Proof of Lemma 2: (i) If $b \geq p_i, i = L, H$, the optimal $X_0 = 0$. P's profit (8) then equals $[\kappa_H F_H(p_H) + \kappa_L F_L(p_L)](V - b)$, which is non-negative only if $V - b \geq 0$. This implies that P's profit is (weakly) dominated by the allocation $\tilde{p}_H = \tilde{p}_L = b$, which in turn is weakly dominated by what P could earn in NS. (ii) The interim participation constraints imply that S will attain a nonnegative rent. Hence P's profit is bounded above by $U(p_L, p_H)$. If $p_L \geq p_H$, the value of $U(p_L, p_H)$ is smaller than the maximum value of $U(\tilde{p}_L, \tilde{p}_H)$ subject to the constraint that $\tilde{p}_L \geq \tilde{p}_H$. The constraint must bind, since the unconstrained solution is represented by second-best prices which violate the constraint. Hence the maximum value of the constrained problem is realized at $\tilde{p}_H = \tilde{p}_L = p^{NS}$. The expected profit of P would then be dominated by the NS allocation where P offers p^{NS} to A in both states. (iii) Parts (i) and (ii) imply that in order to dominate the best NS allocation, an EAC feasible allocation must satisfy $p_H - b > \max\{0, p_L - b\}$. So if (iii) did not hold, $F_H(p_H)(p_H - b) \geq F_L(p_L)(p_L - b)$, and optimal $X_0 = F_H(p_H)(p_H - b)$. Then as $F_H(p_H) \geq F_L(p_L)$ implies $\kappa_H F_L(p_L) + \kappa_H F_H(p_H) \leq F_H(p_H)$, and $V \geq b$ to ensure that P earns non-negative profit, it follows that P's profit equal $[\kappa_L F_L(p_L) + \kappa_H F_H(p_H)](V - b) - F_H(p_H)(p_H - b) \leq F_H(p_H)(V - p_H) \leq F(p_H)(V - p_H) \leq \Pi^{NS}$, a contradiction. ■

Part (i) states that relevant EAC allocations must involve *low-powered incentives* for S in at least one state i, in the sense that ex post S is worse off in state i if the good is delivered than when it is not. This is the very opposite of delegation, where S earns a nonnegative margin on any transaction in every state. In ex ante collusion, the base pay X_0 must be positive in order to compensate for the 'loss' incurred by S when the good is delivered in state i (so as to ensure that S wants to participate at the interim stage corresponding to state i). Conversely, (i) may be viewed as stating that A is offered higher

powered incentives than S in some state; this is a 'countervailing incentive' designed to raise A's outside option in bargaining with S over a side contract, so as to counter the DMR problem.

Part (ii) states that the low cost signal results in a lower price offered to A, just as in the second-best setting. The reason is that when the prices offered to A can vary with the cost signal, P's profit rises only if they result in a lower price being offered following a low cost signal. A variation in the opposite direction would directly result in lower profit, besides possibly entailing some rents paid to S in addition. Part (iii) restricts the extent to which the prices can vary across the two states: the price in the low cost state should not be so low that the resulting supply likelihood becomes smaller in that state. Contrasting this with part (ii) of Lemma 1, we see that ex ante collusion entails a distortion relative to the second-best where (if V is large enough) the price offered in state H is so much higher than in state L that the supply likelihood is larger in the former state. The intuitive explanation of this distortion is that large variations in prices offered between the two cost states generate high collusion stakes which raise S's rent so much as not to be worthwhile for P.

Lemma 2 indicates the problem of finding an optimal EAC allocation can be broken down into two successive stages. At the first stage, for any given pair of prices p_L, p_H satisfying (ii) and (iii), we find an optimal contract b for S to minimize S's rent subject to the coalition incentive constraint (7), and the requirements that $b < p_H$ and $X_0 = \max_i\{F_i(p_i)(p_i - b)\}$. Then at the second stage, prices p_L, p_H are selected to maximize $U(p_L, p_H) - R^*(p_L, p_H)$ subject to $p_L < p_H, F_L(p_L) > F_H(p_H)$, where $R^*(p_L, p_H)$ denotes the minimized rent of S at the first stage.

Lemma 3 *Given p_L, p_H satisfying $p_L < p_H, F_L(p_L) > F_H(p_H)$, the optimal bonus $b(p_L, p_H) = \max\{B(p_L, p_H), l_L(p_L), l_H(p_H)\}$ where $B(p_L, p_H) \equiv \frac{p_L F_L(p_L) - p_H F_H(p_H)}{F_L(p_H) - F_H(p_H)}$.*

The proof of this result as well as all subsequent results are provided in the Appendix. This Lemma characterizes the solution to the first stage problem, which reduces to the problem of selecting the bonus to minimize S's rent. At the bonus $B(p_L, p_H)$, S's rent becomes zero. Hence this is the optimal bonus as long as it satisfies the coalition incentive constraint (7), which reduces to the condition that $B(p_L, p_H)$ exceeds $\max_i\{l_i(p_i)\}$.[10] When

[10] Since $B(p_L, p_H) < p_L$, Lemma 2 implies that the only relevant part of constraint (7) is $b \geq \max_i\{\hat{l}_i(p_i)\}$.

this condition is not met, attention has to be restricted to a range of bonuses where S earns a positive rent (i.e., earns different expected profit $F_i(p_i)(b - p_i) + X_0$ between the two states). Over this range, the rent is rising in b, because S's profit $F_L(p_L)(b - p_L)$ in the low cost signal state rises faster in b than P can extract by lowering X_0 which equals $-F_H(p_H)(b - p_H)$. Hence the optimal bonus minimizes the the bonus consistent with the incentive constraint (7), implying it should be set at $\max_i\{l_i(p_i)\}$.

We are now in a position to characterize properties of optimal EAC allocations (with p_i^E denoting the corresponding optimal price in state i).

Proposition 2 *With an indivisible good and ex ante collusion:*

(a) There exists $\hat{V}_1 > \underline{\theta}$ such that if $V \in (\underline{\theta}, \hat{V}_1)$ the second-best profit can be achieved;

(b) S is valuable if $V < H(\bar{\theta})$, but not if $V > \hat{V}_2$ for some $\hat{V}_2 \in (H(\bar{\theta}), h_L(\bar{\theta}))$.

(c) $p_H^E \leq p_H^{SB}$

(d) $p_L^E \geq p_L^{SB}$ if $l_L(.)$ is convex.

Part (a) states that the second-best profit can be achieved by P when V is low enough, while (b) says that appointing S is valuable for low values of V but not for sufficiently high values. Parts (c) and (d) describe how prices offered to A deviate from second-best prices. Provided l_L is convex, a condition satisfied in our example with linear conditional density functions and uniform prior, the dispersion between prices in the two states is narrower than in the second-best. The heuristic reason underlying these results is that collusion costs tend to rise with dispersion in prices p_i across the two states. For instance, small enough dispersion is associated with zero collusion costs, as the 'ideal' bonus $B(p_L, p_H)$ approaches p_H as p_L approaches p_H from below; since $p_H > l_H(p_H)$ this bonus is coalitionally incentive compatible (i.e., satisfies (7)). For sufficiently low values of V, the second-best can be implemented, essentially because the dispersion between second-best prices corresponding to the different cost signals is small enough. The value of appointing S tends to decline as V rises, because this tends to raise price dispersion which generate growing rents for S.

This intuitive argument also helps explain why S is valuable for values of V smaller than $H(\bar{\theta})$. Starting with the optimal NS allocation where an interior price $p^{NS} < \bar{\theta}$ is

And the imposed restriction on prices implies they are interior, hence $\hat{l}_i(p_i) = l_i(p_i)$.

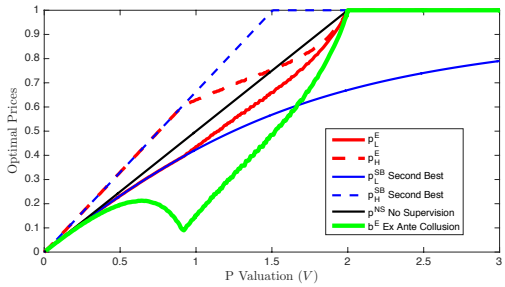

Figure 1: Second-Best, No Supervision and EAC Optimal Prices in Example with $d = 0.99$

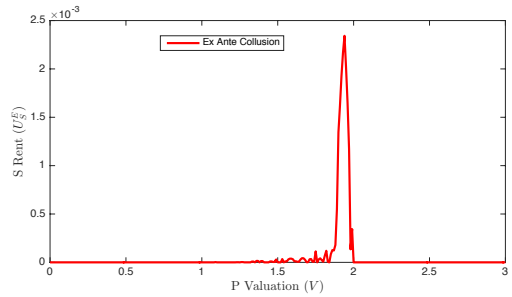

Figure 2: S's payoff in EAC Optimal Allocation in Example with $d = 0.99$

offered, appointing S enables P to vary the price p_i with the cost signal in the direction of the second-best prices ($p_L^{SB} < p_L < p^{NS} < p_H < p_H^{SB}$). When the variation is slight, the induced stakes of collusion are small enough that S can earn no rents, thereby generating a profit improvement for P. Parts (c) and (d) reinforce this intuition, by showing that the distortion in prices compared with the second-best involves lower dispersion (given convexity of l_L).

These results are illustrated in our numerical example with $d = 0.99$. Figure 1 plots optimal prices offered to A in the second-best (SB), no supervisor (NS) and ex ante collusion (E) settings, corresponding to different values of V. It also plots the corresponding EAC-optimal bonus values b^E. Figure 2 plots the corresponding rents earned by S. For low values of V, the second-best is implemented and S earns no rents. Over this range price dispersion rises, as in the second-best. For intermediate values of V, S is valuable despite earning positive rents; over this range price dispersion narrows in contrast to rising dispersion in second-best prices. Eventually the gap between p_L^E and p_H^E is eliminated as V grows further, from which point onwards S ceases to be valuable and earns zero rents. Hence S earns positive rents only for intermediate values of V, as confirmed by Figure 2.

2.3 Contrasting Ex Ante and Interim Collusion

We now describe how (and when) the solution to EAC differs from interim collusion (IC). The formulation of the IC problem differs from the EAC problem in only one respect: the collusive participation constraint $X_0 \geq 0$ does not apply. An IC allocation can also be represented by the triple (b, p_L, p_H), where base pay X_0 is optimally set equal to $\max_i\{F_i(p_i)(p_i - b)\}$ and is permitted to be negative. Part (i) of Lemma 1 then no longer applies, opening up the possibility of providing high powered incentives with a bonus b larger than $\max_i\{p_i\}$ (as in a delegation setting), and then extracting S's rent upfront with a negative base pay. In particular, delegation to S can no longer be ruled out.

It is easy to check that in IC, part (ii) of Lemma 2 continues to apply (for the same reason), so $p_L < p_H$ is still necessary. However part (iii) need not apply: the likelihood of supply could be higher in the high cost state. The reason is that under interim collusion part (i) of Lemma 2 no longer holds — incentives could be high-powered ($b \geq p_H$). Part (iii) is then modified as follows (upon using a similar argument as in Lemma 2): an IC allocation where S is valuable must either (i) be low-powered (in the sense that $b \leq p_H$) and satisfy $F_L(p_L) > F_H(p_H), X_0 \geq 0$, or (ii) high-powered ($b > p_H$) and satisfy $F_L(p_L) < F_H(p_H), X_0 < 0$. It is evident that (i) is EAC feasible, while (ii) is not. We therefore obtain:

Lemma 4 *The optimal IC allocation differs from the optimal EAC allocation only if the former involves high powered incentives ($b > p_H > p_L$) and $F_H(p_H) > F_L(p_L)$.*

So we now focus on allocations with high-powered incentives where $p_L < p_H < b$ and $F_H(p_H) > F_L(p_L)$. The optimal bonus in ex ante collusion now differs from Lemma 3 as follows.

Lemma 5 *Given p_L, p_H satisfying $p_L < p_H, F_L(p_L) > F_H(p_H)$, the optimal bonus in interim collusion is $b(p_L, p_H) = \min\{B(p_L, p_H), \hat{h}_L(p_L), \hat{h}_H(p_H)\}$. S is valuable only if $b > V$.*

The relevant range of bonuses and their effect on S's rent are reversed in interim collusion, compared to the EAC setting: the relevant range of b is $(p_H, \min_i\{\hat{h}_i(p_i)\}]$, over which S's rent is decreasing in b. Hence whenever S earns positive rents in IC, it is optimal for P to make incentives as high-powered as possible, and set the bonus to the maximum level

$\min_i\{\hat{h}_i(p_i)\}$ consistent with the coalition incentive constraint. Moreover, the bonus needs to exceed V in order for S to be valuable.

We are now in a position to characterize some features of IC optimal allocations which are EAC-infeasible.

Lemma 6 *(i) An IC optimal allocation which is not EAC feasible can be implemented via delegation to S.*

(ii) Second-best profits cannot be achieved by an IC optimal allocation which is not EAC feasible.

(iii) There exists $\tilde{V} \leq H(\bar{\theta})$ such that for all $V \in (\tilde{V}, h_L(\bar{\theta}))$ S is valuable in the IC optimal allocation.

Result (i) follows from observing that P's profits are decreasing in each price p_i in IC. Raising prices paid to A raises the likelihood of the good being delivered, which lowers P's profit largely as a consequence of paying a bonus exceeding what the good is worth to P (as shown in the previous Lemma). Hence if p_i is interior and $h_i(p_i)$ exceeds b, it is profitable to lower p_i slightly while leaving the bonus b unchanged, as this would preserve feasibility of the allocation. This implies that the price offered to A is exactly what would have been chosen in each state by S under delegation. And under delegation S would earn a higher profit in the low cost state compared with the high cost state, owing to A's 'supply curve' being shifted to the right in the former relative to the latter. It is then impossible for P to fully extract S's rents in the low cost state, as S has to be willing to accept the contract in both states. Hence second-best profits cannot be achieved. Part (iii) shows that unlike the ex ante collusion setting, S remains valuable in interim collusion for all large V between $H(\bar{\theta})$ and $h_L(\bar{\theta})$. Intuitively this is because in the absence of collusion in participation and the associated DMR problem, delegation helps P control the stakes of collusion better.

This leads us to our main result contrasting optimal solutions in the ex ante and interim collusion settings. Recall that we consider the range of possible values of V between $\underline{\theta}$ and $h_L(\bar{\theta})$.

Proposition 3 *(i) For sufficiently small values of V, EAC and IC optimal allocations coincide. For sufficiently large V, they are different.*

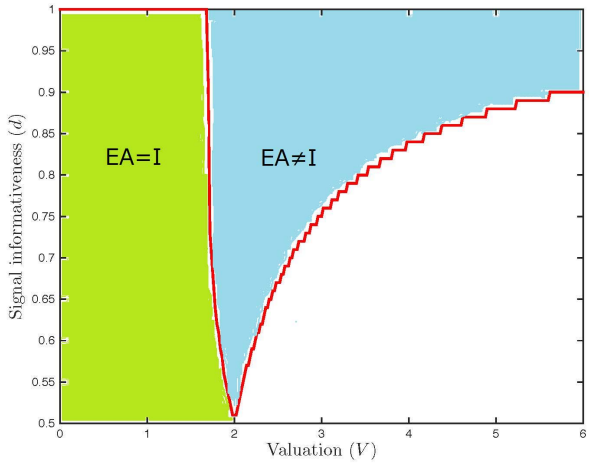

Figure 3: When Interim and Ex Ante Collusion Solutions Differ

(ii) S is valuable in IC for all $V > H(\bar{\theta})$, whereas S is not valuable in EAC for sufficiently large V.

(iii) Whenever the IC optimal allocation differs from the EAC optimal allocation, it can be implemented via delegation to S, with prices $p_i \geq p_i^{SB}$ for $i = L, H$ and a bonus $b > V$.

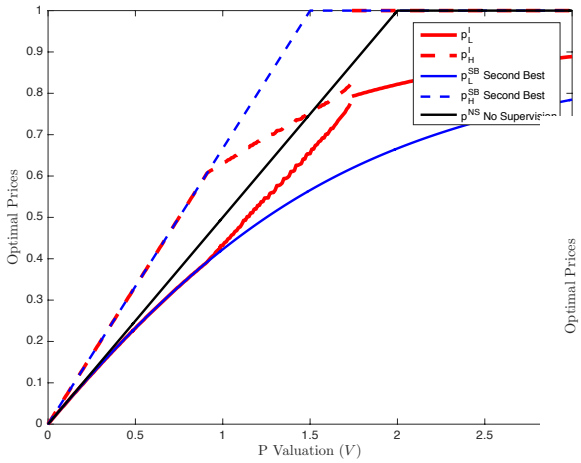

Figure 4: Optimal Prices with Interim Collusion

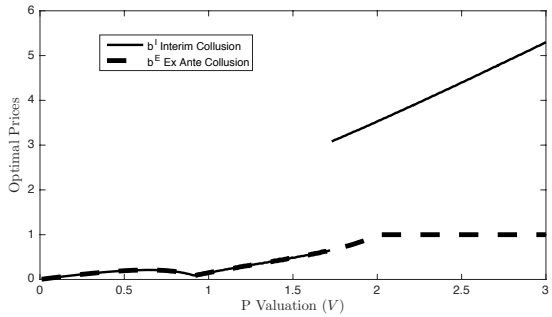

Figure 5: Optimal Bonus in Ex Ante and Interim Collusion

These follow upon combining various Lemmas and Propositions above.[11] The solution

[11] The result comparing IC optimal prices with second-best prices in (iii) obtains from observing that prices corresponding to delegation with a bonus of V equal second-best prices, and the optimal bonus must exceed V.

to ex ante collusion involves low powered incentives, and in particular can never be achieved by delegation. Interim collusion involves a different allocation for large values of V, which is implemented via high-powered incentives (a delivery bonus that exceeds the value of the good to P, combined with delegation).

In the context of our numerical example, Figure 3 shows different regions of the two dimensional parameter space (V, d) where the IC optimal and EAC optimal solutions do and do not coincide. The unshaded subregion on the extreme right is excluded by our restriction that $V < h_L(\bar{\theta})$. In the subregion on the left (marked "EA=I") involving relatively low values of V, the EAC and IC solutions coincide. In the middle subregion (marked "EA $\neq$ I") they diverge. Figure 4 plots the pattern of optimal prices in the IC optimal solution, corresponding to different values of V (with d set equal to 0.99). For intermediate values of V where the second-best is not attained and the two solutions coincide, the price offered in the high cost signal state is smaller than the corresponding second-best price. As V rises further, the IC solution diverges from the EAC, causing a discontinuous switch in the pricing pattern: the price offered in the high cost signal state jumps up to the second-best price, resulting in locally increasing price dispersion. Figure 5 plots the optimal bonus against alternative values of V. Over the range where the EAC and IC solutions coincide, incentives are low-powered (the bonus is smaller than V). At the threshold where they just begin to diverge, the IC optimal bonus jumps discontinuously upwards while the EAC bonus continues to remain below V.

Interim collusion is thus characterized by a discontinuous change in organizational strategy as V crosses the threshold, from a 'bureaucracy' (low-powered incentives, centralized contracting and low sensitivity of supplier price to cost information of supervisor), to a 'market-like' contract resembling a franchise arrangement (high powered incentives, delegation, revenues earned primarily through franchise fees, and higher sensitivity of price to cost information revenues). The market-based strategy however cannot be sustained in the presence of ex ante collusion, since franchisees can then collude with their suppliers to avoid paying the upfront franchise fee when suppliers cannot deliver owing to high cost realizations.

3 Divisible Good Procurement

3.1 Environment

A delivers an output q to P at a personal cost of θq. Output is perfectly divisible: the range of feasible outputs is $\Re_+$. P's return from q is $V(q)$ where $V(q)$ is twice continuously differentiable, increasing and strictly concave satisfying $\lim_{q\to 0} V^{'}(q) = +\infty$, $\lim_{q\to +\infty} V^{'}(q) = 0$ and $V(0) = 0$. These conditions imply that $q^*(\theta) \equiv \arg_q \max V(q) - \theta q$ is continuously differentiable, positive on $\theta \in [0, \infty)$ and strictly decreasing.

A is privately informed about the realization of θ; P and S share a common prior $F(\theta)$ over θ on the interval $\Theta \equiv [\underline{\theta}, \bar{\theta}] \subset \Re_+$. F has a density function $f(\theta)$ which is continuously differentiable and everywhere positive on its support. The 'virtual cost' $H(\theta) \equiv \theta + \frac{F(\theta)}{f(\theta)}$ is strictly increasing in θ; this assumption simplifies the analysis but is inessential to the results.

S plays no role in production, and costlessly acquires an informative signal η about θ.[12] The set of possible realizations of η is Π, a finite set with $\#\Pi \geq 2$. The finiteness of this set is assumed for technical convenience, and is relatively inessential as long as S's information regarding θ is not perfect. It is common knowledge that the realization of η is observed by both S and A. $a(\eta \mid \theta) \in [0, 1]$ denotes the likelihood function of η conditional on θ, which is common knowledge among all agents. $a(\eta \mid \theta)$ is continuously differentiable and positive on $\Theta(\eta)$, where $\Theta(\eta)$ denotes the set of values of θ for which signal η can arise with positive probability. We assume $\Theta(\eta)$ is an interval, for every $\eta \in \Pi$. Define $\underline{\theta}(\eta) \equiv \inf \Theta(\eta)$ and $\bar{\theta}(\eta) \equiv \sup \Theta(\eta)$. We assume that for any $\eta \in \Pi$, $a(\eta \mid \theta)$ is not a constant function on Θ, and there are some subsets of θ with positive measure such that $a(\eta \mid \theta) \neq a(\eta^{'} \mid \theta)$ for any $\eta, \eta^{'} \in \Pi$. In this sense each possible signal realization conveys information about the agent's cost. The information conveyed is partial, since Π is finite. This formulation includes both cases of full support and partition information structures.

The conditional density function and the conditional distribution function are respectively denoted by $f(\theta \mid \eta) \equiv f(\theta)a(\eta \mid \theta)/p(\eta)$ (where $p(\eta) \equiv \int_{\underline{\theta}(\eta)}^{\bar{\theta}(\eta)} f(\tilde{\theta})a(\eta \mid \tilde{\theta})d\tilde{\theta}$) and $F(\theta \mid \eta) \equiv \int_{\underline{\theta}(\eta)}^{\theta} f(\tilde{\theta} \mid \eta)d\tilde{\theta}$. The 'virtual' cost conditional on the signal η is $h(\theta \mid \eta) \equiv \theta + \frac{F(\theta|\eta)}{f(\theta|\eta)}$.

[12] If signal acquisition involves a fixed cost, P will need to reimburse S for this cost. Hence it will have to be subtracted from P's payoff when S is appointed. The extension of our results to this context is straightforward.

We do not impose any monotonicity assumption for $h(\theta \mid \eta)$. Let $\hat{h}(\theta \mid \eta)$ be constructed from $h(\theta \mid \eta)$ and $F(\theta \mid \eta)$ by the ironing procedure introduced by Myerson (1981) (see the online Appendix for details regarding this procedure).

All players are risk neutral. P's objective is to maximize the expected value of $V(q)$, less expected payment to A and S, represented by X_A and X_S respectively. S's objective is to maximize expected transfers $X_S - t$ where t is a transfer from S to A. A seeks to maximize expected transfers received, less expected production costs, $X_A + t - \theta q$. Both A and S have outside options equal to 0.

In this environment, a feasible (deterministic) allocation is represented by $(u_A, u_S, q) = \{(u_A(\theta,\eta), u_S(\theta,\eta), q(\theta,\eta)) \in \Re^2 \times \Re_+ \mid (\theta,\eta) \in K\}$ where $K \equiv \{(\theta,\eta) \mid \eta \in \Pi, \theta \in \Theta(\eta)\}$, u_S, u_A denotes S and A's payoff respectively, and q represents the production level. P's payoff equals $u_P = V(q) - u_S - u_A - \theta q$. These payoffs relate to transfers and productions as follows: $u_A \equiv X_A + t - \theta q; u_S \equiv X_S - t; u_P \equiv V(q) - X_S - X_A$.

In the absence of collusion where P costlessly learns the realization of η, it is well known (e.g., adapting arguments of Baron and Myerson (1982)) that the resulting optimal or *second-best allocation* $(u_A^{SB}, u_S^{SB}, q^{SB})$ is as follows:

$$u_A^{SB}(\theta,\eta) = \int_{\theta}^{\bar{\theta}(\eta)} q^{SB}(y,\eta)dy,$$

$$E[u_S^{SB}(\theta,\eta) \mid \eta] = 0$$

and

$$q^{SB}(\theta,\eta) \equiv q^*(\hat{h}(\theta \mid \eta)) = \arg\max_q [V(q) - \hat{h}(\theta \mid \eta)q]$$

where $\hat{h}(\theta \mid \eta)$ is constructed from $h(\theta \mid \eta)$ and $F(\theta \mid \eta)$ by the ironing procedure.

3.2 The Ex Ante Collusion Game

Owing to risk-neutrality of all parties, concavity of V and linearity of A's payoff in q, it is easy to check that P can restrict attention to a deterministic grand contract:

$$GC = (X_A(m_A, m_S), X_S(m_A, m_S), q(m_A, m_S); M_A, M_S)$$

where M_A (resp. M_S) is a message set for A (resp. S).[13] This mechanism assigns a deterministic allocation, i.e. transfers X_S, X_A and output q, for any message $(m_A, m_S) \in M_A \times M_S$. M_A includes A's exit option $e_A \in M_A$, with the property that $m_A = e_A$ implies $X_A = q = 0$ for any $m_S \in M_S$. Similarly M_S includes S's exit option $e_S \in M_S$, where $m_S = e_S$ implies $X_S = 0$ for any $m_A \in M_A$. The set of all possible deterministic grand contracts is denoted by $\mathcal{GC}$.

The timing of events is as follows.

(C1) A observes θ and η, S observes η.

(C2) P offers a grand contract $GC \in \mathcal{GC}$.

(C3) S and A play the side contract game described in more detail below.

As in existing literature, we assume the side-contract is costlessly enforceable. Moreover we assume S can make a take-it-or-leave-it offer of a side-contract. This assumption turns out to be inessential: Section 4.1 explains how the same results obtain with side contracts offered by an uninformed third party that assigns arbitrary welfare weights to the supervisor and agent.

Conditional on any $\eta \in \Pi$ which is jointly observed by S and A, (C3) consists of the following three stages.

(i) S offers a side-contract SC which determines for any $\tilde{\theta} \in \Theta(\eta)$ to be privately reported by A to S, a probability distribution over joint messages $(m_A, m_S) \in M_A \times M_S$, and a side payment from S to A.[14] Formally, it is a pair of functions $\{\tilde{m}(\tilde{\theta}, \eta), t(\tilde{\theta}, \eta)\}$ where $\tilde{m}(\theta, \eta) : \Theta(\eta) \times \{\eta\} \longrightarrow \Delta(M_A \times M_S)$, the set of probability measures over $M_A \times M_S$, and $t : \Theta(\eta) \times \{\eta\} \longrightarrow \Re$. The case where S does not offer a side contract is represented by a null side-contract (NSC) with zero side payments ($t(\theta, \eta) \equiv 0$), and (deterministic) messages $(m_A(\theta, \eta); m_S(\eta))$. We abuse terminology slightly and refer to the situation where no side contract is offered as one where NSC is offered.

(ii) A either accepts or rejects the SC offered, and the game continues as follows.

[13] Randomized contracts are optimal in Ortner and Chassang (2017) owing to their assumption that the contract offered to S by P is not observed by A. In our context, contracts are observed by both S and A, so there are no benefits of randomization.

[14] The option of randomizing over possible messages is useful for technical reasons. Owing to quasilinearity of payoffs, there is no need to randomize over side transfers.

(iii) If A accepts the offered SC, he sends a private report $\theta' \in \Theta(\eta)$ to S, following which the SC is executed.[15] If A rejects SC, S updates his beliefs to $b(SC;\eta)$ which is restricted to be $b_{\emptyset}(\eta)$ if NSC was offered in stage (i) above.[16] A and S then play a noncooperative Bayesian equilibrium c of the grand contract relative to beliefs $b(SC;\eta)$.

The key feature of ex ante collusion is that the side contract allows S and A to coordinate their participation decisions after A sends S the private report θ', since the messages (m_S, m_A) sent to P include their respective participation decisions. In interim collusion, the timing of moves is different: S and A have to select their respective participation decisions noncooperatively after $C2$. Provided both agree to participate, S and A subsequently negotiate a side contract which coordinates their subsequent reports (which do not include participation decisions).

3.3 Suboptimality of Delegated Supervision

Before proceeding further, we consider the special case of *Delegated Supervision (DS)* where P delegates authority to S over contracting with A. Here the GC designed by P involves a null contract for A: the latter submits no report to P directly, and receives no production instructions or payments from P. P contracts only with S, requiring the latter to send a message m_S to P which determines the output $q(m_S)$ and aggregate payment $X(m_S)$ to the (S,A) coalition. Following receipt of this offer, S designs a side contract for A which selects an output $Q(m_A)$ and payment $X_A(m_A)$ to the latter as a function of a message m_A sent by A to S, provided A accepts the side contract. After receiving A's message (and conditional on A agreeing to participate), S submits a participation decision and message m_S to P. In contrast to the interim collusion setting, S can postpone submission of the participation decision *after* receiving a report from A.

Our first main result is that delegation is *never* optimal in ex ante collusion, as it is strictly dominated by the case where S is not appointed at all, which we refer to as *No Supervision (NS)*.

[15] Standard arguments show that the restriction to direct revelation mechanisms for the side contract entails no loss of generality.

[16] This ensures that it is immaterial whether or not NSC was accepted or rejected, since in either case they play the grand contract non-cooperatively with prior beliefs.

Proposition 4 *Delegated Supervision is worse for the Principal compared to No Supervision.*

The FLM result concerning optimality of delegation in an interim collusion setting with two cost types therefore does not extend to the setting of our model with ex ante collusion, risk neutrality and continuous types. The underlying argument is simple and very general (it can be shown to extend to a discrete type setting also). P contracts for delivery of the good with S, so the problem reduces to contracting with a single agent S. In order to deliver the good to P, S needs to procure it in turn from A. The cost that S expects to incur equals A's virtual cost function $h(\theta|\eta)$ corresponding to the signal observed by S. This is unambiguously higher than the delivery cost θ of A if P were to contract directly with A, in the absence of any supervision. This is the well-known problem of double marginalization of rents (DMR), arising due to exercise of monopsony power by S in side-contracting with A. Unlike the context of interim collusion, S can postpone her own participation decision *after* receiving A's report. This effectively translates into a kind of 'limited liability' constraint for A, which prevents P from taxing away upfront the rents earned by S.[17]

Given this result, we hereafter focus on centralized contracting with supervision, where P offers a non-null contract to both S and A in GC.

3.4 Centralized Contracting and Ex Ante Collusion Proofness

We now introduce the notion of ex ante collusion proofness in the context of centralized contracts. A justification for this solution concept is provided in the online Appendix.

Informally, an allocation is ex ante collusion proof (EACP) if the supervisor cannot benefit from offering a non-null side contract when the Principal selects a grand contract based on the associated direct revelation mechanism (i.e., when agent and supervisor make consistent reports about the state, the allocation corresponding to that state is chosen). In other words, null side contract is optimal for S, when the outside option of A corresponds to the latter's payoff resulting from the allocation.

[17] While it is relatively easy to show that DS cannot dominate NS, the proof establishes the stronger result that DS is **strictly** dominated by NS. The proof of strict domination is also straightforward in the case that $h(\theta|\eta)$ is continuous and nondecreasing in θ over a common support $[\underline{\theta}, \bar{\theta}]$ for every η. In that case an argument based on Proposition 1 in Mookherjee and Tsumagari (2004) can be applied. In the general case there are a number of additional technical complications, but we show that the result still goes through.

Before proceeding to the formal definition, note that a *deterministic allocation* can be represented by payoff functions $(u_A(\theta,\eta), u_S(\theta,\eta))$ of the true state (θ,η) combined with the output function $q(\theta,\eta)$, as these determine the Principal's payoff function $u_P(\theta,\eta) \equiv V(q(\theta,\eta)) - u_S(\theta,\eta) - u_A(\theta,\eta) - \theta q(\theta,\eta)$, and the aggregate net transfers of S (equals $u_S(\theta,\eta)$) and A (equals $u_A(\theta,\eta) + \theta q(\theta,\eta)$). For technical convenience we consider randomized allocations, though it will turn out they will never actually need to be used on the equilibrium path.[18] In a randomized allocation, $(u_A(\theta,\eta), u_S(\theta,\eta), q(\theta,\eta))$ denotes the expected payoffs of A, S and the expected output, conditional on the state (θ,η).

We now introduce notation for 'coalitional' contracts and incentives as follows; this will prove to be useful in representing constraints imposed by ex ante collusion proofness. For (conditional expected) allocation $(u_A(\theta,\eta), u_S(\theta,\eta), q(\theta,\eta))$, define functions $(\hat{X}(m), \hat{q}(m))$ on domain $m \in \hat{M} \equiv K \cup \{e\}$ (where $K \equiv \{(\theta,\eta) \mid \theta \in \Theta(\eta), \eta \in \Pi\}$) as follows:

$$(\hat{X}(\theta,\eta), \hat{q}(\theta,\eta)) = (u_A(\theta,\eta) + \theta q(\theta,\eta) + u_S(\theta,\eta), q(\theta,\eta))$$

where the message e represents a coordinated coalitional decision for both S and A to exit from P's mechanism, while the message (θ,η) represents a coordinated decision for S and A to agree to participate and send the common report (θ,η) to P. The key constraint distinguishing ex ante from interim collusion is:

$$(\hat{X}(e), \hat{q}(e)) = (0,0).$$

Let $(\hat{X}(\theta,\eta), \hat{q}(\theta,\eta))$ denote corresponding expected values of the sum of payments $X_S + X_A$ made by the principal, and the output delivered, in state (θ,η). Also, let $\Delta(\hat{M})$ denote the set of the probability measures on $\hat{M}$, and use $\tilde{m} \in \Delta(\hat{M})$ to denote a randomized message submitted by the coalition to P. With a slight abuse of notation, we shall denote the corresponding conditional expected allocation by $(\hat{X}(\tilde{m}), \hat{q}(\tilde{m}))$, which is defined on $\Delta(\hat{M})$. $\tilde{m} = (\theta,\eta)$ or e will be used to denote the probability measure concentrated at (θ,η) or e respectively.

S's choice of an optimal (randomized) side-contract can be formally posed as follows. Given a grand contract and a noncooperative equilibrium recommended by P, let the corresponding conditional expected allocation as defined above be denoted by $(u_A(\theta,\eta), u_S(\theta,\eta), q(\theta,\eta))$ and $(\hat{X}(\tilde{m}), \hat{q}(\tilde{m}))$. For any $\eta \in \Pi$, the associated side-contracting problem $P(\eta)$ is to select

[18] This owes to the assumption that A's payoff is linear in the output produced.

$(\tilde{m}(\theta \mid \eta), \tilde{u}_A(\theta, \eta))$ to maximize S's expected payoff

$$E[\hat{X}(\tilde{m}(\theta \mid \eta)) - \theta\hat{q}(\tilde{m}(\theta \mid \eta)) - \tilde{u}_A(\theta, \eta) \mid \eta]$$

subject to $\tilde{m}(\theta \mid \eta) \in \Delta(\hat{M})$,

$$\tilde{u}_A(\theta, \eta) \geq \tilde{u}_A(\theta^{'}, \eta) + (\theta^{'} - \theta)\hat{q}(\tilde{m}(\theta^{'} \mid \eta))$$

for any $\theta, \theta^{'} \in \Theta(\eta)$, and

$$\tilde{u}_A(\theta, \eta) \geq u_A(\theta, \eta)$$

for all $\theta \in \Theta(\eta)$. The first constraint states truthful revelation of the agent's true cost to S is consistent with the agent's incentives, and the second constraint requires A to attain a payoff at least as large as what he would expect to attain by playing the grand contract noncooperatively.

Definition 1 *The (conditional expected) allocation* $(u_A(\theta, \eta), u_S(\theta, \eta), q(\theta, \eta)) : K \to \Re^2 \times \Re_+$ *is ex ante collusion proof (EACP) if for every* $\eta \in \Pi$*:* $(\tilde{m}(\theta \mid \eta), \tilde{u}_A(\theta, \eta)) = ((\theta, \eta), u_A(\theta, \eta))$ *solves problem* $P(\eta)$*.*

3.5 Characterization of EACP Allocations

We now characterize EACP allocations. This requires us to define a family of 'modified' virtual cost functions, representing the effective cost incurred by the coalition in delivering a unit of output to P, following selection of an optimal side-contract.

Definition 2 *For any* $\eta \in \Pi$*,* $Y(\eta)$ *is a collection of* **coalition shadow cost (CSC)** *functions* $\pi(\cdot \mid \eta) : \Theta(\eta) \to \Re$ *which satisfy the following property. For any function in this collection, there exists a real-valued function* $\Lambda(\theta|\eta)$ *which is non-decreasing in* $\theta \in \Theta(\eta)$ *with* $\Lambda(\underline{\theta}(\eta) \mid \eta) = 0$ *and* $\Lambda(\bar{\theta}(\eta) \mid \eta) = 1$*, such that*

$$\pi(\theta|\eta) \equiv \theta + \frac{F(\theta \mid \eta) - \Lambda(\theta \mid \eta)}{f(\theta \mid \eta)} \tag{10}$$

Equation (10) modifies the usual expression for virtual cost $h(\theta|\eta) \equiv \theta + \frac{F(\theta|\eta)}{f(\theta|\eta)}$ by subtracting from it the non-negative term $\frac{\Lambda(\theta|\eta)}{f(\theta|\eta)}$. In order to overcome the DMR problem in Delegated Supervision, in the centralized regime P contracts with both S and A, thereby providing A an outside option (of $u_A(\theta, \eta)$) that effectively raises his bargaining power *vis-a-vis* S while negotiating the side contract. Meeting a larger outside option for A effectively

induces S to deliver a higher output to P: this is what paying a higher rent to A necessitates. The extent of DMR is then curbed: the shadow cost for the coalition in delivering a unit of output to P is lowered. This lowering of the virtual cost is represented by the subtraction of the term $\frac{\Lambda(\theta|\eta)}{f(\theta|\eta)}$ from what it would have been ($h(\theta|\eta)$) under Delegated Supervision. In the analogous context of contracting with a single agent with type dependent outside options (Jullien (2000)), $\Lambda(\theta \mid \eta)$ represents the shadow value of a uniform reduction in A's outside option for all types below θ. Clearly, the $\Lambda(\theta \mid \eta)$ function must be non-decreasing.

However, $\pi(\theta|\eta)$ is not the correct expression for the shadow cost of output for the coalition, if it is non-monotone in θ. In that case, it has to be replaced by its 'ironed' version, using the distribution function $F(\theta|\eta)$. Let the corresponding ironed version of $\pi(\theta|\eta)$ be denoted by $z(\theta|\eta)$: we call this a *coalition virtual cost function.*

Definition 3 *For any $\eta \in \Pi$, the set of* **coalition virtual cost (CVC)** *functions is the set*

$$Z(\eta) \equiv \{z(\cdot \mid \eta) \quad \textit{is the ironed version of some} \quad \pi(\cdot \mid \eta) \in Y(\eta)\}$$

of functions obtained by applying the ironing procedure to the set $Y(\eta)$ of CSC functions.[19] *Denote by $\Theta(\pi(\cdot \mid \eta), \eta)$ the corresponding pooling region of θ where $\pi(\cdot|\eta)$ is flattened by the ironing procedure.*

As the next result shows, every EACP allocation satisfies coalition participation and incentive constraints corresponding to some coalition virtual cost function z. Combined with an individual incentive compatibility constraint for A, and a constraint that output must be constant over regions where the ironing procedure flattens the underlying CSC function, these coalition constraints characterize EACP allocations.[20] The proof of this Proposition is provided in the online Appendix, as it borrows well known methods from Jullien (2000).

Proposition 5 *The allocation (u_A, u_S, q) is EACP if and only if the following conditions hold for every η. There exists a CVC function $z(\cdot|\eta) \in Z(\eta)$ such that*

[19] The ironing procedure ensures these functions are continuous and non-decreasing. For further details, see the online Appendix.

[20] See Mookherjee and Tsumagari (2004), Celik (2008) and Pavlov (2008) for similar characterizations of collusion proof mechanisms.

(i) For every $(\theta,\eta),(\theta',\eta') \in K \equiv \{(\theta,\eta) \mid \theta \in \Theta(\eta), \eta \in \Pi\}$,

$$X(\theta,\eta) - z(\theta \mid \eta)q(\theta,\eta) \geq X(\theta',\eta') - z(\theta \mid \eta)q(\theta',\eta')$$

$$X(\theta,\eta) - z(\theta \mid \eta)q(\theta,\eta) \geq 0$$

where $X(\theta,\eta) \equiv u_A(\theta,\eta) + u_S(\theta,\eta) + \theta q(\theta,\eta)$.

(ii) For any $\theta,\theta' \in \Theta(\eta)$, $u_A(\theta,\eta) \geq u_A(\theta',\eta) + (\theta' - \theta)q(\theta',\eta)$.

(iii) $q(\theta,\eta)$ *is constant on any interval of* θ *which is a subset of the corresponding pooling region of the CVC function* z.

Define an allocation to be EAC feasible if it is EACP and satisfies interim participation constraints for S and A: $E[u_S(\theta,\eta)|\eta] \geq 0$ for all $\eta \in \Pi$, $u_A(\theta,\eta) \geq 0$ for all $(\theta,\eta) \in K$. Finally, P's problem is to select among EAC feasible allocations to maximize her expected profit $\Pi \equiv E[V(q(\theta,\eta)) - u_S(\theta,\eta) - u_A(\theta,\eta) - \theta q(\theta,\eta)]$, where expectation is taken with respect to P's prior beliefs. Condition (i) represents the coalition incentive and participation constraints corresponding to contracting with a single agent with a unit cost of z. Condition (ii) is the individual incentive compatibility constraint for A. Condition (iii) states that the output must be constant over every interval in the pooling region. In comparing with the analysis of the indivisible good case, the choice of the output function $q(\theta,\eta)$ and agent's payoff $u_A(\theta,\eta)$ corresponds to the prices offered to A, while the payment function $X(\theta,\eta)$ corresponds to the choice of base pay X_0 and bonus b for S. It can be checked that requirement (i) above that the payment function $X(\theta,\eta)$ be incentive compatible for the coalition with a unit cost function $z(\cdot|\eta) \in Z(\eta)$ corresponds to the coalition incentive constraint (7) in the indivisible good case.

3.6 Results

Proposition 6 *With a divisible good and ex ante collusion, hiring S is always valuable.*

The generality of this result contrasts with the indivisible good case, where hiring S was sometimes not valuable. Intuitively, S is more valuable in the divisible good setting, owing to the additional scope for varying the quantity procured based on S's information. The proof starts with the optimal no-supervision contract, and constructs a small variation in

the output function $q(., \eta^{**})$ for a cost signal state η^{**} satisfying a regularity condition.[21] The divisibility of the good permits such an intra-state variation to be constructed, whereas in the indivisible good case output functions (i.e, prices offered to A) must be perturbed in at least two different states.

Note that the optimal allocation in NS corresponds to the special case $\Lambda(\theta \mid \eta)$ is chosen equal to $F(\theta \mid \eta)$, ensuring that the CSC and CVC functions both reduce to the identity function ($\pi(\theta|\eta) = z(\theta|\eta) = \theta$)). P can construct a small variation $\tilde{z}$ in the CVC function in state η^{**}, raising it above θ for some interval Θ_H and lowering it for some other interval Θ_L. The corresponding quantity procured $q(\theta, \eta^{**})$ is set equal to $q^{NS}(\tilde{z}(\theta|\eta^{**}))$, the quantity procured in NS when the agent reported a cost of $\tilde{z}(\theta|\eta^{**})$. This corresponds to raising the quantity procured from the coalition over Θ_L and lowering it over Θ_H. Payments to the coalition are set analogously at $X^{NS}(\tilde{z}(\theta|\eta^{**}))$, what the agent would have been paid in NS following such a cost report. The agent is offered the associated rent: $u_A(\theta, \eta^{**}) = \int_{\theta}^{\bar{\theta}} q^{NS}(\tilde{z}(y|\eta^{**}))dy$. By construction, this allocation satisfies the agent's incentive and participation constraints, as well as the coalition incentive constraint.

Proposition 5 ensures such an allocation is EAC feasible, i.e., S's interim participation constraint is satisfied. The variation over Θ_L lowers rents earned by S, and over Θ_H raises them. Since S does not earn any rents to start with (i.e, in NS), it is necessary to construct the variation such that S's expected rents in state η^{**} do not go down. The rate at which S's rents vary locally in state θ with the quantity procured equals $\frac{F(\theta|\eta^{**})}{f(\theta|\eta^{**})}$.[22] Intuitively this is the saving that can be pocketed by S when procuring one less unit of the good from A. Maintaining S's expected rent therefore implies a marginal rate of substitution between output variations over Θ_L and Θ_H that equals the ratio of the (average) conditional inverse hazard rates $\frac{F(\theta|\eta^{**})}{f(\theta|\eta^{**})}$ over these two intervals respectively. On the other hand, P's benefit from a small expansion in output delivered in state θ equals $V'(q^{NS}(\theta)) - \theta$, where $q^{NS}(\theta)$ denotes the optimal allocation in NS.[23] This allocation satisfies $V'(q^{NS}(\theta)) = H(\theta) \equiv$

[21] The regularity condition requires that $[F(\theta|\eta^{**})/f(\theta|\eta^{**})]/[F(\theta)/f(\theta)]$ is increasing over some interval of θ with positive measure conditional on η^{**}. The proof shows that the informativeness of S's signal implies that such a state always exists.

[22] S's interim rent in state η^{**} equals the expected value conditional on η^{**} of $X^{NS}(\tilde{z}(\theta|\eta^{**})) - u_A(\tilde{z}(\theta|\eta^{**})) - \theta q^{NS}(\tilde{z}(\theta|\eta^{**}))$, i.e., equals $E[\{\tilde{z}(\theta|\eta^{**}) - h(\theta|\eta^{**})\}q^{NS}(\tilde{z}(\theta|\eta^{**})) - \int_{\tilde{z}(\theta|\eta^{**})}^{\bar{\theta}} q^{NS}(z)dz|\eta^{**}]$.

[23] This follows from the fact that $\frac{\partial X^{NS}(z)}{\partial z} = zq^{NS'}(z)$, implying that the marginal increase in payment evaluated at $z = \theta$ equals θ times the marginal output change.

$\theta + \frac{F(\theta)}{f(\theta)}$, the virtual cost of procurement without conditioning on information regarding η. Hence P's marginal benefit from output expansion in state θ equals the unconditional inverse hazard rate $\frac{F(\theta)}{f(\theta)}$. This implies that P's marginal rate of substitution between output variations over Θ_L and Θ_H equals the ratio of the (average) unconditional inverse hazard rates $\frac{F(\theta)}{f(\theta)}$ over these two intervals. The informativeness of S's signals implies that P's marginal rate of substitution differs from S's in state η^{**} over Θ_L, Θ_H. Hence there exist variations of the type described above which raise P's expected payoff, while preserving the expected payoff of S.

The next result provides sufficient conditions for ex ante collusion to be costly, thereby providing a contrast with interim collusion in team production (Che-Kim (2006)) or supervision (Motta (2009)) settings.

Proposition 7 *With a divisible good and ex ante collusion, the second-best payoff cannot be attained if :*

(i) The support of θ does not vary with the signal: $\Theta(\eta) = \Theta$ for any $\eta \in \Pi$;

(ii) There exists $\eta^* \in \Pi$ such that $f(\theta|\eta^*)$ and $\frac{f(\theta|\eta^*)}{f(\theta|\eta)}$ are both strictly decreasing in θ for any $\eta \neq \eta^*$;

(iii) $\underline{\theta} f(\underline{\theta} \mid \eta^*) > 1$;

(iv) $V'''(q) \leq \frac{(V''(q))^2}{V'(q)}$ for any $q \in Q^{SB} \equiv \{\tilde{q} | \tilde{q} = q^{SB}(\theta, \eta)$ for some $(\theta, \eta) \in K\}$.

Condition (ii) includes a weaker version of the monotone likelihood ratio property: there is a signal realization η^* which is 'better' news about θ than any other realization, in the sense of shifting weight in favor of low realizations of θ. It additionally requires that the conditional density $f(\theta|\eta^*)$ is strictly decreasing in θ, i.e., higher realizations of θ are less likely than low realizations when $\eta = \eta^*$. (ii) is satisfied for instance when θ has a uniform prior and there are just two possible signal values satisfying the standard monotone likelihood ratio property. Condition (iii) says that costs are high in the sense that the support of the cost distribution is shifted sufficiently to the right. Finally (iv) is a condition on the benefit function, which is satisfied if V is exponential ($V = V_0[1 - \exp(-rq)]; V_0, r > 0$).[24]

[24]This benefit function does not satisfy the Inada conditions assumed in the model. However, the only purpose of imposing the Inada conditions was to ensure that optimal allocations would always involve strictly

The proof develops necessary conditions for the second best to be EAC feasible given the distributional properties (i) and (ii). If the outputs are second-best, they must be a monotone decreasing function of the (ironed) virtual cost $\hat{h}(\theta \mid \eta)$ in the second-best setting. If they also satisfy the coalition incentive constraints, they must be monotone in CVC $z(\theta \mid \eta)$. These conditions imply the existence of a monotone transformation from $\hat{h}$ to z, and enable S's ex post rent to be expressed as a function of $\hat{h}$ alone. Condition (iv) is used to show that this rent function is strictly convex; combined with (i) and (ii) this implies that the expected rents of S must be strictly higher (hence strictly positive) in state η^* than any other state. Then S must earn positive rents in state η^*, which ensures the second best cannot be achieved.

4 Extensions

4.1 Side Contracts Designed by a Third Party, and Alternative Allocations of Bargaining Power

We now explain how the preceding results extend when the side contract is designed not by S, but instead by a third-party that manages the coalition and assigns arbitrary welfare weights to the payoffs of S and A respectively. Such a formulation has been used by a number of authors to model collusion, such as Laffont and Martimort (1997, 2000), Dequiedt (2007) and Celik and Peters (2011). An advantage of this approach is that it enables us to examine effects of varying the allocation of bargaining power between colluding partners.

Our results extend to such a setting, under the following formulation of side contracts designed by a third party. We assume the third-party's objective is to maximize a weighted sum of S and A's interim payoffs. In the subgame (C3) following choice of a grand contract by P, the third party designs the side contract after learning the realization of η.[25] Both S and A have the option to reject the side contract; if either of them does, they play the grand contract noncooperatively. Otherwise the side contract mechanism is executed.

The notion of EACP allocations is extended as follows. Letting $\alpha \in [0, 1]$ denote the welfare weight assigned by the third-party to A's payoff, the side contract design problem

positive quantities procured in all states. This will be the case here if V_0 is large enough.

[25] This assumption can be dropped without affecting the results, since it can be shown the third-party can use cross-reporting of η by S and A to learn its true value.

reduces to selecting randomized message $\tilde{m}(\theta \mid \eta)$ and A's payoff $\tilde{u}_A(\theta, \eta)$ to (using the same notation for the formulation $P(\eta)$ of side contracts in Section 3.4):

$$\max E[(1-\alpha)\{\hat{X}(\tilde{m}(\theta \mid \eta)) - \theta\hat{q}(\tilde{m}(\theta \mid \eta)) - \tilde{u}_A(\theta, \eta)\} + \alpha\tilde{u}_A(\theta, \eta) \mid \eta]$$

subject to $\tilde{m}(\theta \mid \eta) \in \Delta(\hat{M})$,

$$\tilde{u}_A(\theta, \eta) \geq u_A(\theta, \eta)$$

$$\tilde{u}_A(\theta, \eta) \geq \tilde{u}_A(\theta', \eta) + (\theta' - \theta)\hat{q}(\tilde{m}(\theta' \mid \eta))$$

$$E[\hat{X}(\tilde{m}(\theta \mid \eta)) - \theta\hat{q}(\tilde{m}(\theta \mid \eta)) - \tilde{u}_A(\theta, \eta) \mid \eta] \geq E[u_S(\theta, \eta) \mid \eta].$$

Besides modifying the objective function, this formulation adds a participation constraint for S. We refer to this as problem $TP(\eta; \alpha)$. The definition of EACP can be extended to EACP(α) by requiring the null side contract to be optimal in $TP(\eta; \alpha)$ for every η. Further details concerning the justification of this solution concept is provided in the online Appendix.

We now show that the set of EACP(α) allocations is independent of α. This implies that all our preceding results extend to side contracts designed by a third party.[26]

Proposition 8 *The set of EACP(α) allocations is independent of $\alpha \in [0, 1]$.*

Despite the existence of asymmetric information within the coalition, the result of the Coase Theorem applies here. The reasoning is straightforward, so the formal proof is relegated to the online Appendix. The EACP criterion amounts to the absence of incentive compatible deviations that are Pareto improving for the coalition: this property does not vary with the precise welfare weights. Consider any $\alpha \in (0, 1)$. A given allocation is EACP(α) if and only if there is no other allocation attainable by some non-null side contract which satisfies the incentive constraint for A, and which Pareto-dominates it (for A and S) with at least one of them strictly better off. The same characterization applies to any interior $\alpha' \in (0, 1)$, implying that the set of EACP(α) allocations is independent of $\alpha \in (0, 1)$. The transferability of utility can then be used to show that the set of EACP allocations for interior welfare weights are also the same at the boundary.

[26] FLM provide an analogous result for the case of interim collusion.

4.2 Altruistic Supervisor

Now consider a different variant, where S offers a side-contract to A, but S is altruistic towards A rather than just concerned with his own income. Suppose S's payoff is $u_S = X_S + t + \alpha[X_A - t - \theta q]$, where $\alpha \in [0, 1]$ is the weight S places on A's payoff. A's payoff function remains the same as in the previous section: $u_A = X_A - t - \theta q$.

Our analysis extends as follows. The expression for coalition shadow cost is now modified to

$$\pi_\alpha(\theta|\eta) \equiv \theta + (1-\alpha)\frac{F(\theta \mid \eta) - \Lambda(\theta \mid \eta)}{f(\theta \mid \eta)}$$

instead of $\pi(\theta|\eta)$ in Definition 2. When P delegates to S, the corresponding expression for the cost of procuring one unit from S is modified from $h(\theta \mid \eta)$ to $h_\alpha(\theta \mid \eta) = \theta + (1-\alpha)\frac{F(\theta|\eta)}{f(\theta|\eta)}$. As long as $\alpha < 1$, this is strictly higher than θ, so delegation will still continue to result in a lower profit than NS. The proof that S is valuable under centralized contracting also goes through *in toto*.

It is interesting to examine the effect of changes in the degree of altruism on P's payoffs. When P delegates to S, an increase in α lowers S's shadow cost of output $h_\alpha(\theta \mid \eta)$, which benefits P. This is intuitive: the DMR problem becomes less acute with a more altruistic supervisor. Note that with perfect altruism $\alpha = 1$, and the DMR problem disappears: delegation then becomes equivalent to NS.

On the other hand, an increase in altruism cannot benefit P in centralized contracting. The set of EACP allocations can be shown to be non-increasing in α. Take any EACP allocation corresponding to α: the following argument shows that it is a EACP allocation corresponding to any $\alpha^{'} < \alpha$. Let $z(\theta \mid \eta)$ be the CVC function that is associated with the allocation at α, i.e., it is the ironed version of $\pi_\alpha(\theta|\eta)$ corresponding to some function $\Lambda_\alpha(\cdot|\eta)$ satisfying the stipulated requirements in the definition of CSC functions on $[\underline{\theta}(\eta), \bar{\theta}(\eta)]$. We can then select

$$\Lambda_{\alpha'}(\theta \mid \eta) = \frac{\alpha - \alpha^{'}}{1 - \alpha^{'}} F(\theta \mid \eta) + \frac{1-\alpha}{1-\alpha^{'}} \Lambda_\alpha(\theta \mid \eta)$$

when the altruism parameter is $\alpha^{'}$, which satisfies the stipulated requirements since $\alpha > \alpha^{'}$. This ensures that the same CSC and CVC function is available when the altruism parameter is $\alpha^{'}$, since by construction $\pi_\alpha(\theta|\eta) = \pi_{\alpha'}(\theta|\eta)$. Hence the allocation satisfies the sufficient condition for EACP when the altruism parameter is $\alpha^{'}$.

Finally, if $\alpha = 1$, the CSC function π_α coincides with the identity function θ, the cost

of the agent in NS. We thus obtain

Proposition 9 *In centralized contracting, P's optimal payoff is non-increasing in α. When P delegates to S, P's optimal payoff is increasing in α. When $\alpha = 1$, P's optimal payoffs in delegation, centralized contracting coincide and equal that in NS, so S is not valuable.*

4.3 Optimality of Conditional Delegation

The online Appendix shows that the optimal allocation under ex ante collusion can be achieved by a modified form of delegation, where P communicates and transacts only with S on the equilibrium path. In this arrangement, S is 'normally' expected to contract on behalf of the coalition $\{S, A\}$ with P, sending a joint participation decision and report of the state (θ, η) to P after having entered into a side contract with A. However A has the option of circumventing this 'normal' procedure and asking P to activate a grand contract in which A and S will send independent reports and participation decisions to P. The presence of this option ensures that A has sufficient bargaining power within the coalition; it does not have to be 'actually' used on the equilibrium path.

5 Concluding Comments

Our results have interesting implications for hierarchical organizational design. They provide a rationale for the widespread prevalence of supervisors within firms and contracting networks, even when they have 'prior connections' with the agent that may give rise to ex ante collusion. In such circumstances, unconditional delegation is suboptimal; the mechanism must allow agents at lower tiers to 'appeal' and trigger direct communications with the Principal. Within firms, it explains the role of worker rights to appeal the evaluations reported by their managers to higher level managers or an ombudsman appointed for this purpose, thereby formalizing Williamson's (1975) claim that such dispute settlement procedures constitute an advantage of hierarchies over market relationships.[27] The result concerning the irrelevance of allocation of bargaining power within the coalition implies that collusion costs are unaffected by alternative mechanisms for matching supervisors and

[27] It also relates to Hirschman's (1970) depiction of the value of 'voice' within organizations over and above exit options.

agents, e.g., whether an agent should be allowed to select an auditor on a competitive market, or whether the Principal should appoint the auditor instead. The result concerning effects of altruism of S towards A implies that the Principal ought to appoint 'outside' self-interested supervisors rather than 'insiders' likely to be altruistic towards the agent. In the context of corporate governance, for instance, this is an argument in favor of appointing 'outsiders' rather than 'insiders' to a company's Board of Directors.[28] In the context of regulation, it confirms the normal intuition in favor of preventing any direct conflict of interest for the supervisor (e.g., who should not have a financial stake in the agent's fortunes, nor have any social or personal connections with the agent).

Extensions of the model to bilateral asymmetric information within the coalition (e.g., if A does not observe S's signal) and to discrete type spaces are examined respectively in Tsumagari (2016a,b). Mookherjee and Tsumagari (2017) show that the allocation of bargaining power between S and A does matter in the case of 'strong' collusion, e.g., where the side contract includes commitments regarding subsequent actions by one partner if the other refuses it.

Our analysis is subject to a number of shortcomings. We excluded the possibility of other coalitions that may co-exist with the S-A coalition, a topic studied by Ortner and Chassang (2017). Also we ignored the enforcement of side contracts within the coalition; modeling self-enforcing collusion via a relational contract in a side game between colluding parties seems to be an interesting extension that could be pursued in future research.

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[28] See Harris and Raviv (2008) for a model based on limited commitment by P where this result may not hold in some settings.

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Appendix: Proofs

Proof of Lemma 3: To start with, note that the restrictions $p_L < p_H$ and $F_L(p_L) > F_H(p_H)$ imply that the prices are interior: $\underline{\theta} < p_i < \bar{\theta}, i = H, L$. Hence the coalition incentive constraint (7) simplifies to $\max_i\{l_i(p_i)\} \leq b \leq h_i(p_i)\}$. Next, note that upon substituting for the optimal base pay X_0, the expression for S's expected rent reduces to

$$\tilde{R}(b; p_L, p_H) \equiv \kappa_L F_L(p_L)(b-p_L)+\kappa_H F_H(p_H)(b-p_H)-\min\{F_L(p_L)(b-p_L), F_H(p_H)(b-p_H)\} \tag{11}$$

Clearly $\tilde{R}$ is non-negative and attains a global minimum of zero at $b = B(p_L, p_H) < p_L < p_H$. If $B(p_L, p_H) \geq \max\{l_L(p_L), l_H(p_H)\}$, it is feasible to select $b = B(p_L, p_H)$ as the coalition incentive constraint (7) is satisfied (given that $p_i \leq h_i(p_i), i = H, L$), as well as the constraint that $b < p_H$. Hence in this case the optimal bonus equals $B(p_L, p_H)$. If $B(p_L, p_H) < \max\{l_L(p_L), l_H(p_H)\}$, then observe that over the range $b \geq B(p_L, p_H)$, $(b - p_L)F_L(p_L) \geq (b - p_H)F_H(p_H)$, implying that $X_0 = F_H(p_H)(b - p_H)$, or

$$\tilde{R} = \kappa_L[\{F_L(p_L) - F_H(p_H)\}b - p_L F_L(p_L) + p_H F_H(p_H)]. \tag{12}$$

Hence $\tilde{R}$ is strictly increasing in b over the range $b \geq B(p_L, p_H)$, and the optimal bonus in this case equals $\max\{l_L(p_L), l_H(p_H)\}$. ■

Proof of Proposition 2: (a) By Lemma 1, $F_L(p_L^{SB}) > F_H(p_H^{SB})$ for V close to $\underline{\theta}$. As V approaches $\underline{\theta}$, p_i^{SB} approaches $\underline{\theta}$ for both $i = H, L$, and $B(p_L^{SB}, p_H^{SB})$ approaches $\underline{\theta} > \max_i\{l_i(\underline{\theta})\}$, implying $b(p_L^{SB}, p_H^{SB}) = B(p_L^{SB}, p_H^{SB})$. So $(p_L, p_H, b) = (p_L^{SB}, p_H^{SB}, B(p_L^{SB}, p_H^{SB}))$ is EAC feasible, implying the second-best profit can be achieved for V close to $\underline{\theta}$.

(b) $V < H(\bar{\theta})$ implies $p^{NS} < \bar{\theta}$. For any such V, we can find p_L, p_H sufficiently close to p^{NS} satisfying $p_L^{SB} \leq p_L < p^{NS} < p_H \leq p_H^{SB}, F_L(p_L) > F_H(p_H)$ and $\max_i\{l_i(p_i)\} < B(p_L, p_H)$ (since $B(p, p) = p > l_i(p), i = L, H$ for any $p < \bar{\theta}$). The allocation $(p_L, p_H, B(p_L, p_H))$ is then EAC feasible, in which S earns zero rent, and P earns a profit of $U(p_L, p_H) > U(p^{NS}, p^{NS}) = \Pi^{NS}$.

Next we show that S is not valuable at $V = \hat{V} \equiv \kappa_L h_L(\bar{\theta}) + \kappa_H h_H(\bar{\theta}) < h_L(\bar{\theta})$. Suppose otherwise, whence $F_L(p_L^E) > F_H(p_H^E)$ by Lemma 2. Note that $\hat{V} = \bar{\theta} + [\kappa_L \frac{1}{f_L(\bar{\theta})} + \kappa_H \frac{1}{f_H(\bar{\theta})}] > \bar{\theta} + \frac{1}{\kappa_L f_L(\bar{\theta}) + \kappa_H f_H(\bar{\theta})} = H(\bar{\theta})$. Hence $\Pi^{NS}(\hat{V}) = \hat{V} - \bar{\theta} = \kappa_L(h_L(\bar{\theta}) - \bar{\theta}) + \kappa_H(h_H(\bar{\theta}) - \bar{\theta})$. Now $\bar{\theta}$ is the second-best price when V equals $h_i(\bar{\theta})$ in state i, implying $h_i(\bar{\theta}) - \bar{\theta} \geq F_i(p_i^E)(h_i(\bar{\theta}) - p_i^E)$. Hence $\Pi^{NS}(\hat{V}) \geq \kappa_L F_L(p_L^E)(h_L(\bar{\theta}) - p_L^E) + \kappa_H F_H(p_H^E)(h_H(\bar{\theta}) - p_H^E) \geq \kappa_L F_L(p_L^E)(\hat{V} -$

$p_L^E) + \kappa_H F_H(p_H^E)(\hat{V} - p_H^E) = U(p_L^E, p_H^E)$, where the second inequality follows from the definition of $\hat{V}$ and $F_L(p_L^E) > F_H(p_H^E)$. Since P's expected profit in EAC is bounded above by $U(p_L^E, p_H^E)$, we obtain a contradiction. Hence it is optimal to offer $p_i = \bar{\theta}$ for both i at $\hat{V}$. By a standard revealed preference argument, these prices are also optimal at any higher V. Hence S is not valuable at any $V > \hat{V}$.

(c) We first show that S's rent is locally non-decreasing in p_H at (p_L^E, p_H^E). If $B(p_L^E, p_H^E) > \max_i\{l_i(p_i^E)\}$, S earns zero rent which is unaffected by small variations in p_H. So suppose $B(p_L^E, p_H^E) \leq \max_i\{l_i(p_i^E)\}$ in which case $b^E = \max_i\{l_i(p_i^E)\}$ and $R^*(p_L^E, p_H^E) = \kappa_L[\{F_L(p_L^E) - F_H(p_H^E)\}\max_i\{l_i(p_i^E)\} + F_H(p_H^E)p_H^E - F_L(p_L^E)p_L^E] = \kappa_L \max_i \rho_i(p_H^E, p_L^E)$ where $\rho_i(p_H, p_L) \equiv \{F_L(p_L) - F_H(p_H)\}l_i(p_i) + F_H(p_H)p_H - F_L(p_L)p_L$. Now ρ_L is locally nondecreasing in p_H at (p_L^E, p_H^E) because $F_H(p_H)[p_H - l_L(p_L)]$ is increasing in p_H at (p_L^E, p_H^E) (the latter in turn follows from Lemma 2 and (7) which together imply $p_H^E > b^E \geq \max_i\{l_i(p_i^E)\} \geq l_L(p_L^E)$). And ρ_H is everywhere locally nondecreasing in p_H since $l'_H(p_H)[F_L(p_L) - F_H(p_H)] + f_H(p_H)[p_H - l_H(p_H)] \geq 0$.

It now follows that if $p_H^E > p_H^{SB}$, a slight lowering of p_H will have a positive first order effect on $U(p_L, p_H)$, without raising S's rent. Hence $p_H^E \leq p_H^{SB}$.

(d) We show that S's rent is locally non-increasing in p_L at (p_L^E, p_H^E) if $l_L(p_L)$ is convex. When $B(p_L^E, p_H^E) > \max_i\{l_i(p_i^E)\}$, S rents are zero which do not vary locally with p_L. So suppose $B(p_L^E, p_H^E) \leq \max_i\{l_i(p_i^E)\}$ implying that $R^*(p_L^E, p_H^E) = \kappa_L \max_i \rho_i(p_H^E, p_L^E)$. Now $F_L(p_L)[l_H(p_H^E) - p_L]$ is locally non-increasing in p_L at p_L^E, since its partial derivative with respect to p_L at p_L^E equals $f_L(p_L^E)[l_H(p_H^E) - h_L(p_L^E)]$, which is non-positive as (7) implies $l_H(p_H^E) \leq b^E \leq h_L(p_L^E)$. Hence ρ_H is locally nonincreasing in p_L at (p_L^E, p_H^E). The result therefore holds when $\rho_L(p_L^E, p_H^E) < \rho_H(p_L^E, p_H^E)$.

Next consider the case where $\rho_H(p_L^E, p_H^E) \leq \rho_L(p_L^E, p_H^E) = \frac{R^*(p_L^E, p_H^E)}{\kappa_L} > 0$. Since $\frac{\partial \rho_L}{\partial p_L} = l'_L(p_L)[F_L(p_L) - F_H(p_H)] - 1$, the convexity of $l_L(p_L)$ implies the conxevity of ρ_L in p_L for any fixed value of p_H. Now as p_L approaches p_H^E, $\rho_L(p_L, p_H^E)$ approaches $[F_L(p_H^E) - F_H(p_H^E)][l_L(p_H^E) - p_H^E] < 0$. Hence there must exist $\tilde{p}_L \in (p_L^E, p_H^E)$ where $\rho_L(\tilde{p}_L, p_H^E) = 0$ and ρ_L is locally decreasing in p_L. The convexity of $\rho_L(p_L, p_H^E)$ in p_L then implies that $\rho_L(p_L, p_H^E)$ is also locally decreasing in p_L at every $p_L < \tilde{p}_L$. Since $p_L^E < \tilde{p}_L$, it follows that ρ_L is decreasing in p_L at (p_L^E, p_H^E).

It now follows that if $p_L^E < p_L^{SB}$, a slight increase in p_L will have a positive first-order effect on $U(p_L, p_H)$, without raising S's rent. Hence $p_L^E \geq p_L^{SB}$. ■

Proof of Lemma 5: Given any pair of prices satisfying $p_L < p_H, F_H(p_H) > F_L(p_L)$, the optimal bonus must minimize S's rent subject to $b > p_H$ and the coalition incentive constraint (7). S earns zero rent at the bonus $B(p_L, p_H) = \frac{p_H F_H(p_H) - p_L F_L(p_L)}{F_H(p_H) - F_L(p_L)}$ which is now larger than p_H. Since the choice of b is restricted to the range $b > p_H$ where $b > \max_i\{\hat{l}_i(p_i)\}$ is automatically satisfied, the bonus $B(p_L, p_H)$ is optimal if $B(p_L, p_H) \leq \min_i\{\hat{h}_i(p_i)\}$. Otherwise, $B(p_L, p_H) > \min_i\{\hat{h}_i(p_i)\}$ and the choice of b is restricted to the range $(p_H, \min_i\{\hat{h}_i(p_i)\}]$. Over this range $b < B(p_L, p_H)$ which implies $F_H(p_H)(b - p_H) < F_L(p_L)(b - p_L)$ and therefore $X_0 = -F_H(p_H)(b - p_H)$. The expression for S's rent is then modified to $\tilde{R}(b; p_L, p_H) = \kappa_L F_L(p_L)(b - p_L) + \kappa_H F_H(p_H)(b - p_H) - F_H(p_H)(b - p_H) = \kappa_L[\{F_L(p_L) - F_H(p_H)\}b + F_H(p_H)p_H - F_L(p_L)p_L]$, which is now *decreasing* in b.

To see that $b > V$ is necessary for S to be valuable, note that since the function $\tilde{R}(b; p_L, p_H)$ is decreasing in b, if $b \leq V$ then $p_H < V$, implying that P's profit is bounded above by $U(p_L, p_H) - \kappa_L[\{F_L(p_L) - F_H(p_H)\}V + F_H(p_H)p_H - F_L(p_L)p_L] = F_H(V - p_H) \leq F(p_H)(V - p_H)$, the profit attained in NS upon choosing the price of p_H in both states. ■

Proof of Lemma 6: (i) As explained in the text, an IC optimal allocation which is infeasible in EAC must involve $p_L < p_H, F_H(p_H) > F_L(p_L)$ and in which S is valuable (since any allocation in NS is feasible in EAC). P attains profit $\Pi = [\kappa_H F_H(p_H) + \kappa_L F_L(p_L)](V - b) + F_H(p_H)(b - p_H) = \kappa_L F_L(p_L)(V - b) + F_H(p_H)[\kappa_H V + \kappa_L b - p_H]$. By Lemma 5, it is necessary that $b > V$, and $b = \min_i\{\hat{h}_i(p_i)\}$. To show that this can be attained via IC with delegation, we need to show that if $p_i < \bar{\theta}$ then $b = h_i(p_i)$, while if $p_i = \bar{\theta}$ then $b \geq h_i(\bar{\theta})$.

Suppose first that $p_i < \bar{\theta}$ for either i. Then $\hat{h}_i(p_i) = h_i(p_i) \geq b$. If $i = L$ and $h_L(p_L) > b$, note that Π is strictly decreasing in p_L, so profit can be raised by lowering p_L slightly. Similarly, if $i = H$ and $b < h_H(p_H)$, we have $\kappa_H V + \kappa_L b < h_H(p_H)$, implying $F_H(p_H)[\kappa_H V + \kappa_L b - p_H]$ is locally strictly decreasing in p_H, and profit can be raised by lowering p_H slightly.

Next, suppose $p_i = \bar{\theta}$. If $b < h_i(\bar{\theta})$, the same argument as above applies: profit can be raised by lowering p_i slightly. Hence it must be the case that $b \geq h_i(\bar{\theta})$.

(ii) From (i), an IC optimal allocation which is EAC infeasible satisfies $p_i = p_i(b)$ which maximizes $F_i(p)(b - p)$ with respect to choice of $p \in [\underline{\theta}, \bar{\theta}]$. Since $F_L(p) > F_H(p)$ for all $p \in (\underline{\theta}, \bar{\theta})$, it must be true that $F_L(p_L)(b - p_L) \geq F_H(p_H)(b - p_H)$, with strict inequality if $b < h_L(\bar{\theta})$. Hence $b < h_L(\bar{\theta})$ implies S earns positive rent in state L (as $X_0 = -F_H(p_H)(b - p_H)$), and second-best profits cannot be achieved. And if $b \geq h_L(\bar{\theta})$, it

must be the case that $p_L = p_H = \bar{\theta}$, in which case the IC optimal allocation can be attained in NS and therefore also in EAC.

(iii) Consider any $V \geq H(\bar{\theta})$, whence $\Pi^{NS} = V - \bar{\theta}$. The optimal IC profit is bounded below by what can be achieved via delegation in the interim collusion setting. If the bonus is b, the resulting prices will be $p_i(b)$, base pay will be set equal to $-F_H(p_H(b))(b - p_H(b))$ (using the argument in (ii) above), so the resulting profit will be $\Pi^{ID}(b; V) \equiv [\kappa_L F_L(p_L(b)) + \kappa_H F_H(p_H(b))][V - b] + F_H(p_H(b))[b - p_H(b)]$. The derivative of Π^{ID} with respect to b evaluated at $b = V$ then equals $\kappa_L[F_H(p_H(V)) - F_L(p_L(V))]$. Now observe that by definition of the $p_i(b)$ function, $p_i(V) = p_i^{SB}$. So $V \geq H(\bar{\theta})$ implies $p_H^{SB} = p^{NS} = \bar{\theta}$, so $p_H(V) = \bar{\theta}$. On the other hand, $p_L(V) = p_L^{SB} < \bar{\theta}$ since $V < h_L(\bar{\theta})$, so $F_L(p_L(V)) < 1 = F_H(p_H(V))$. It follows that Π^{ID} is strictly increasing in b when evaluated at $b = V$. Since $\Pi^{ID}(V; V) = V - \bar{\theta} = \Pi^{NS}$, it follows that S adds value in the IC optimal allocation. ■

Proof of Proposition 4:

At the first step, note that the optimal side contract problem for S in Delegation to S (denoted DS) involves an outside option for A which is identically zero. This reduces to a standard problem of contracting with a single agent with adverse selection and an outside option of zero, where S has a prior distribution $F(\theta|\eta)$ over the agent's cost θ in state η. The expected procurement cost incurred by S is then $\hat{h}(\theta|\eta)$.

Given this, P's contract with S in DS is effectively a contracting problem for P with a single supplier whose unit supply cost is $\hat{h}(\theta|\eta)$. P's prior over this supplier's cost is given by distribution function $G(h) \equiv \Pr((\theta, \eta) \mid \hat{h}(\theta \mid \eta) \leq h)$ for $h \geq \underline{\theta}$ and $G(h) = 0$ for $h < \underline{\theta}$. Let $G(h \mid \eta)$ denote the cumulative distribution function of $h = \hat{h}(\theta \mid \eta)$ conditional on η: $G(h \mid \eta) \equiv \Pr(\theta \mid \hat{h}(\theta \mid \eta) \leq h, \eta)$ for $h \geq \hat{h}(\underline{\theta}(\eta) \mid \eta)(= \underline{\theta}(\eta))$ and $G(h \mid \eta) = 0$ for $h < \underline{\theta}(\eta)$. Then $G(h) = \Sigma_{\eta \in \Pi} p(\eta) G(h \mid \eta)$. Since $\hat{h}(\theta \mid \eta)$ is continuous and nondecreasing on $\Theta(\eta)$, $G(h \mid \eta)$ is strictly increasing in h on $[\underline{\theta}(\eta), \hat{h}(\bar{\theta}(\eta) \mid \eta)]$. However, $G(h \mid \eta)$ may fail to be left-continuous.

Hence P's problem in DS reduces to $\max E_h[V(q(h)) - X(h)]$ subject to $X(h) - hq(h) \geq X(h') - hq(h')$ for any $h, h' \in [\underline{\theta}, \bar{h}]$ and $X(h) - hq(h) \geq 0$ for any $h \in [\underline{\theta}, \bar{h}]$ where the distribution function of h is $G(h)$ and $\bar{h} \equiv \max_{\eta \in \Pi} \hat{h}(\bar{\theta}(\eta) \mid \eta)$. The corresponding problem in NS is $\max E_\theta[V(q(\theta)) - X(\theta)]$ subject to $X(\theta) - \theta q(\theta) \geq X(\theta') - \theta q(\theta')$ for any $\theta, \theta' \in \Theta$ and $X(\theta) - \theta q(\theta) \geq 0$ for any $\theta \in \Theta$. The two problems differ only in the underlying cost distributions of P: $G(h)$ in the case of DS and $F(\theta)$ in the case of NS. Since $\theta < \hat{h}(\theta \mid \eta)$

for $\theta > \underline{\theta}(\eta)$,

$$G(h \mid \eta) \equiv \Pr(\theta \mid \hat{h}(\theta \mid \eta) \leq h, \eta) < \Pr(\theta \mid \theta \leq h, \eta) = F(h \mid \eta)$$

for $h \in (\underline{\theta}(\eta), \hat{h}(\bar{\theta}(\eta) \mid \eta))$, implying $G(h) = \Sigma_{\eta \in \Pi} p(\eta) G(h \mid \eta) < \Sigma_{\eta \in \Pi} p(\eta) F(h \mid \eta) = F(h)$ for any $h \in (\underline{\theta}, \bar{h})$. Therefore the distribution of h in DS (strictly) dominates that of θ in NS in the first order stochastic sense. $\bar{h} > \bar{\theta}$, since $\hat{h}(\bar{\theta}(\eta) \mid \eta) > \bar{\theta}(\eta)$ for any η.

It remains to show that this implies that P must earn a lower profit in DS. We prove the following general statement. Consider two contracting problems with a single supplier which differ only in regard to the cost distributions G_1 and G_2, where $G_1(h) < G_2(h)$ for any $h \in (\underline{h}, \bar{h})$ and $G_2(h) = 1$ on $h \in [\bar{\theta}, \bar{h}]$. Standard arguments imply the problem can be reduced to selecting $q(h)$ to maximize the expected value of $V(q(h)) - hq(h) - \int_h^{\bar{h}_i} q(y)dy$ (where $\bar{h}_1 \equiv \bar{h}$ and $\bar{h}_2 \equiv \bar{\theta}$) with respect to distribution G_i, subject to the constraint that $q(.)$ is nonincreasing. Let the maximized profit of P with distribution G be denoted $W(G)$. We will show $W(G_1) < W(G_2)$.

Let $q_1(h)$ denote the optimal solution of the problem based on $G_1(h)$. If $q_1(h)$ is constant on $(\underline{h}, \bar{h})$ with $q_1(h) = q > 0$, $W(G_1) = V(q) - \bar{h}q$. It is feasible for P to select this output schedule when the cost distribution is G_2, generating expected profit $V(q) - \bar{\theta}q$. Then $W(G_2) \geq V(q) - \bar{\theta}q > W(G_1)$ since $\bar{h} > \bar{\theta}$. We henceforth focus on the case where $q_1(h)$ is not constant.

(i) First we show that $V^{'}(q_1(h)) < h$ does not hold for any set of values of h with positive measure. Suppose otherwise that there exists some interval over which $V^{'}(q_1(h)) < h$. Then we can replace the portion of $q_1(h)$ with $V^{'}(q_1(h)) < h$ by $q^*(h)$ with $V^{'}(q^*(h)) = h$, without violating the constraint that $q(h)$ is non-increasing. It raises the value of the objective function, since $V(q_1(h)) - hq_1(h) < V(q^*(h)) - hq^*(h)$ for h where $q_1(h)$ is replaced by $q^*(h)$, and $\int_h^{\bar{h}} q(y)dy$ decreases with this replacement. This is a contradiction.

(ii) Define

$$\Phi(h) \equiv V(q_1(h)) - hq_1(h) - \int_h^{\bar{h}} q_1(y)dy.$$

We claim that $\Phi(h)$ is left-continuous. First we show that our attention can be restricted to the case that $q_1(h)$ is left-continuous. Otherwise, there exists $h^{'} \in (\underline{h}, \bar{h})$ such that $q_1(h^{'}-) > q_1(h^{'})$. Now consider $\tilde{q}_1(h)$ (which is left-continuous at $h^{'}$) such that $\tilde{q}_1(h^{'}) = q_1(h^{'}-)$ and $\tilde{q}_1(h) = q_1(h)$ for any $h \neq h^{'}$. Defining $\tilde{\Phi}(h) \equiv V(\tilde{q}_1(h)) - h\tilde{q}_1(h) - \int_h^{\bar{h}} \tilde{q}_1(y)dy$,

observe that $\tilde{\Phi}(h) = \Phi(h)$ for $h \neq h'$ and $\tilde{\Phi}(h) > \Phi(h)$ when $h = h'$. Then

$$\begin{aligned}
\int_{[\underline{h},\bar{h}]} \tilde{\Phi}(h)dG(h) &= \int_{[\underline{h},\bar{h}]\setminus h'} \tilde{\Phi}(h)dG(h) + \tilde{\Phi}(h')[G(h'+) - G(h'-)] \\
\geq \int_{[\underline{h},\bar{h}]\setminus h'} \tilde{\Phi}(h)dG(h) &+ \Phi(h')[G(h'+) - G(h'-)] = \int_{[\underline{h},\bar{h}]} \Phi(h)dG(h)
\end{aligned}$$

with strict inequality if $G(h)$ is discontinuous at $h = h'$. When $q_1(h)$ is left-continuous, $\Phi(h)$ is also so.

(iii) We claim that $\Phi(h)$ is non-increasing in h and is not constant on $(\underline{h}, \bar{h})$. To show the former, note that for any h, we have

$$\begin{aligned}
&\lim_{\epsilon\to 0+} \frac{\Phi(h+\epsilon) - \Phi(h)}{\epsilon} \\
= &\lim_{\epsilon\to 0+} (1/\epsilon)[V(q_1(h+\epsilon)) - (h+\epsilon)q_1(h+\epsilon) - \int_{h+\epsilon}^{\bar{h}} q_1(y)dy - [V(q_1(h)) - hq_1(h) - \int_h^{\bar{h}} q_1(y)dy]] \\
= &[V'(\hat{q}(h)) - h] \lim_{\epsilon\to 0+} \frac{q_1(h+\epsilon) - q_1(h)}{\epsilon} - q_1(h+) + \lim_{\epsilon\to 0+} (1/\epsilon) \int_h^{h+\epsilon} q_1(y)dy \\
= &[V'(\hat{q}(h)) - h] \lim_{\epsilon\to 0+} \frac{q_1(h+\epsilon) - q_1(h)}{\epsilon}
\end{aligned}$$

for some $\hat{q}(h) \in [q_1(h+), q_1(h)]$. This is non-positive since $V'(\hat{q}(h)) \geq V'(q_1(h)) \geq h$ and $\lim_{\epsilon\to 0+} \frac{q_1(h+\epsilon)-q_1(h)}{\epsilon} \leq 0$. Because of left-continuity of $\Phi(h)$, it implies that $\Phi(h)$ is non-increasing in h. Next to show $\Phi(h)$ is not constant, suppose otherwise. Then $\Phi(h) = \Phi(\bar{h}-) = V(q_1(\bar{h}-)) - \bar{h}q_1(\bar{h}-)$ which must be equal to $W(G_1)$. It means that $W(G_1)$ is attainable with constant output schedule ($q_1(h) = q_1(\bar{h}-)$ for any $h \in (\underline{h}, \bar{h})$). We obtain a contradiction, since we supposed that $q_1(h)$ is not constant.

(iv) Now consider the contracting problem with cost distribution $G_2(h)$. Since $q_1(h)$ is non-increasing in h, it is feasible for P to select this output schedule when the cost distribution is G_2. Hence $W(G_2) \geq \int_{\underline{h}}^{\bar{h}} \Phi(h)dG_2(h)$. Therefore if $\int_{\underline{h}}^{\bar{h}} \Phi(h)dG_2(h) > \int_{\underline{h}}^{\bar{h}} \Phi(h)dG_1(h) = W(G_1)$, it follows that $W(G_2) > W(G_1)$. Since $G_i(h)$ $(i = 1, 2)$ is right-continuous and $\Phi(h)$ is left-continuous, we can integrate by parts:

$$\int_{\underline{h}}^{\bar{h}} \Phi(h)dG_i(h) + \int_{\underline{h}}^{\bar{h}} G_i(h)d\Phi(h) = \Phi(\bar{h}).$$

Hence

$$\int_{\underline{h}}^{\bar{h}} \Phi(h)dG_2(h) - \int_{\underline{h}}^{\bar{h}} \Phi(h)dG_1(h) = \int_{\underline{h}}^{\bar{h}} [G_1(h) - G_2(h)]d\Phi(h).$$

By (iii) and $G_2(h) > G_1(h)$ for $h \in (\underline{h}, \bar{h})$, this is positive. ■

Proof of Proposition 5: See online Appendix.

Proof of Proposition 6:

Step 1:

Define

$$A(\theta \mid \eta) \equiv \frac{F(\theta \mid \eta)}{f(\theta \mid \eta)} / \frac{F(\theta)}{f(\theta)} \equiv \frac{\int_{\underline{\theta}(\eta)}^{\theta} f(y)a(\eta|y)dy}{a(\eta|\theta)F(\theta)}.$$

We show that there exists $\eta^{**} \in \Pi$ and a closed interval $I = [\theta', \theta'']$ with positive measure (conditional on η^{**}) such that $A(\theta \mid \eta^{**})$ is increasing in θ over I.

Evidently this holds for η such that $\underline{\theta} < \underline{\theta}(\eta)$, since $A(\underline{\theta}(\eta) \mid \eta)$ is continuous, $A(\underline{\theta}(\eta) \mid \eta) = 0$ and $A(\theta \mid \eta) > 0$ for $\theta > \underline{\theta}$. Suppose otherwise; then $\underline{\theta}(\eta) = \underline{\theta}$ for all η. Using l'Hopital's rule, $\lim_{\theta \to \underline{\theta}} A(\theta \mid \eta) = 1$. If $A(\theta \mid \eta)$ is non-increasing in θ for all η, $A(\theta \mid \eta) \leq 1$ or equivalently $\int_{\underline{\theta}}^{\theta} f(y)a(\eta|y)dy \leq a(\eta|\theta)F(\theta)$ for all $(\theta, \eta) \in K$. Since

$$\Sigma_\eta \int_{\underline{\theta}}^{\theta} f(y)a(\eta|y)dy = \Sigma_\eta a(\eta|\theta)F(\theta) = F(\theta)$$

for all θ, $A(\theta \mid \eta) = 1$ for all $(\theta, \eta) \in K$. Then $h(\theta \mid \eta) = H(\theta)$ for any $(\theta, \eta) \in K$. This is a contradiction, since η is informative about θ.

For η^{**} and I, we choose $\lambda > 0$, closed intervals $\Theta_L = [\underline{\theta}^L, \bar{\theta}^L] \subset I$ and $\Theta_H = [\underline{\theta}^H, \bar{\theta}^H] \subset I$ with $\bar{\theta}^L < \underline{\theta}^H$ such that

$$\frac{F(\theta)}{f(\theta)} / \frac{F(\theta \mid \eta^{**})}{f(\theta \mid \eta^{**})} < \lambda < \frac{F(\tilde{\theta})}{f(\tilde{\theta})} / \frac{F(\tilde{\theta} \mid \eta^{**})}{f(\tilde{\theta} \mid \eta^{**})} \text{ for } \tilde{\theta} \in \Theta_L,\ \theta \in \Theta_H.$$

These conditions are equivalent to

$$H(\theta) - (1-\lambda)\theta - \lambda h(\theta \mid \eta^{**}) > 0 \text{ for } \theta \in \Theta_L$$

and

$$H(\theta) - (1-\lambda)\theta - \lambda h(\theta \mid \eta^{**}) < 0 \text{ for } \theta \in \Theta_H.$$

Step 2: Construction of $z(\cdot \mid \eta)$

Now let us construct $z(\cdot \mid \eta)$ which satisfies the following conditions.

(A) For $\eta \neq \eta^{**}$, $z(\theta \mid \eta) = \theta$ for any $\theta \in \Theta(\eta)$.

(B) For η^{**}, $z(\cdot \mid \eta^{**}) \in Z(\eta^{**})$ which satisfies

(i) $z(\theta \mid \eta^{**}) = \theta$ for any $\theta \notin \Theta_H \cup \Theta_L$

(ii) For $\theta \in \Theta_L$, $z(\theta \mid \eta^{**})$ satisfies (a) $z(\theta \mid \eta^{**}) \leq \theta$ with strict inequality for some subinterval of Θ_L of positive measure, and (b) $H(z) - (1-\lambda)z - \lambda h(\theta \mid \eta^{**}) > 0$ for any $z \in [z(\theta \mid \eta^{**}), \theta]$.

(iii) For $\theta \in \Theta_H$, $z(\theta \mid \eta^{**})$ satisfies (a) $z(\theta \mid \eta^{**}) \geq \theta$ with strict inequality for some some subinterval of Θ_H of positive measure, (b) $z(\theta \mid \eta^{**}) < h(\theta \mid \eta^{**})$ and (c) $H(z) - (1-\lambda)z - \lambda h(\theta \mid \eta^{**}) < 0$ for any $z \in [\theta, z(\theta \mid \eta^{**})]$.

(iv) $E[(z(\theta \mid \eta^{**}) - h(\theta \mid \eta^{**}))q^{NS}(z(\theta \mid \eta^{**})) + \int_{z(\theta|\eta^{**})}^{\bar{\theta}(\eta^{**})} q^{NS}(z)dz \mid \eta^{**}] = 0$.

It is shown in the online Appendix that there exists $z(\cdot \mid \eta^{**}) \in Z(\eta^{**})$ which satisfies (B(i)-(iv)).

Step 3

For $z(\cdot \mid \eta)$ constructed in Step 2, consider the following allocation (u_A, u_S, q):

$$q(\theta, \eta) = q^{NS}(z(\theta \mid \eta))$$

$$u_A(\theta, \eta) = \int_{\theta}^{\bar{\theta}} q^{NS}(z(y \mid \eta))dy$$

$$u_S(\theta, \eta) = X^{NS}(z(\theta \mid \eta)) - \theta q^{NS}(z(\theta \mid \eta)) - \int_{\theta}^{\bar{\theta}(\eta)} q^{NS}(z(y \mid \eta))dy - \int_{\bar{\theta}(\eta)}^{\bar{\theta}} q^{NS}(y)dy.$$

where

$$X^{NS}(z(\theta \mid \eta)) \equiv z(\theta \mid \eta)q^{NS}(z(\theta \mid \eta)) + \int_{z(\theta|\eta)}^{\bar{\theta}} q^{NS}(z)dz.$$

In the online Appendix it is shown that (u_A, u_S, q) is a EAC feasible allocation.

Now we show that this allocation generates a higher payoff to P than the optimal allocation in NS. Define $\Phi_P(z)$ and $\Phi_S(z, \theta)$ as

$$\Phi_P(z) \equiv V(q^{NS}(z)) - zq^{NS}(z) - \int_{z}^{\bar{\theta}} q^{NS}(\tilde{z})d\tilde{z}$$

and

$$\Phi_S(z, \theta) \equiv (z - h(\theta \mid \eta^{**}))q^{NS}(z) + \int_{z}^{\bar{\theta}(\eta^{**})} q^{NS}(\tilde{z})d\tilde{z}.$$

P's resulting expected payoff conditional on η^{**} is $E[\Phi_P(z(\theta \mid \eta^{**})) \mid \eta^{**}]$. P's expected payoff conditional on η^{**} in the optimal allocation in NS is $E[\Phi_P(\theta) \mid \eta^{**}]$. By the definition

of $\Phi_S(z,\theta)$ and $E[u_S(\theta,\eta^{**}) \mid \eta^{**}] = 0$, $E[\Phi_S(z(\theta \mid \eta^{**}),\theta) \mid \eta^{**}] = E[\Phi_S(\theta,\theta) \mid \eta^{**}] = 0$. Then the difference between two payoffs is

$$\begin{aligned}
& E[\Phi_P(z(\theta \mid \eta^{**})) \mid \eta^{**}] - E[\Phi_P(\theta) \mid \eta^{**}] \\
= \; & E[\Phi_P(z(\theta \mid \eta^{**})) + \lambda\Phi_S(z(\theta \mid \eta^{**}),\theta) \mid \eta^{**}] - E[\Phi_P(\theta) + \lambda\Phi_S(\theta,\theta) \mid \eta^{**}] \\
= \; & E[\int_{\theta}^{z(\theta|\eta^{**})} \{\Phi_P^{'}(z) + \lambda\partial\Phi_S(z,\theta)/\partial z\}dz \mid \eta^{**}] \\
= \; & E[\int_{\theta}^{z(\theta|\eta^{**})} [V^{'}(q^{NS}(z)) - \{(1-\lambda)z + \lambda h(\theta \mid \eta^{**})\}]q^{NS'}(z)dz \mid \eta^{**}] \\
= \; & E[\int_{\theta}^{z(\theta|\eta^{**})} [H(z) - \{(1-\lambda)z + \lambda h(\theta \mid \eta^{**})\}]q^{NS'}(z)dz \mid \eta^{**}].
\end{aligned}$$

The last equality uses $V^{'}(q^{NS}(z)) = H(z)$. From the construction of $z(\theta \mid \eta^{**})$ in Step 2 and $q^{NS'}(z) < 0$, this is positive. We have thus found an EAC feasible allocation which generates a higher payoff to P compared to the optimal allocation in NS. ■

Proof of Proposition 7:

Conditions (i) and (ii) imply $h(\theta \mid \eta)$ satisfies the following properties:

- $h(\theta \mid \eta^*) = \hat{h}(\theta \mid \eta^*)$ is strictly increasing and continuously differentiable in θ
- $\hat{h}(\theta \mid \eta^*) > \hat{h}(\theta \mid \eta)$ for $\theta \in (\underline{\theta}, \bar{\theta}]$ and $\hat{h}(\underline{\theta} \mid \eta^*) = \hat{h}(\underline{\theta} \mid \eta) = \underline{\theta}$ for any $\eta \neq \eta^*$
- Define $G(h \mid \eta) \equiv \int_{\{\theta|\hat{h}(\theta|\eta)\leq h\}} f(\theta \mid \eta)d\theta$. Then $G(h \mid \eta^*)$ is a mean-preserving spread of $G(h \mid \eta)$ for any $\eta \neq \eta^*$

The first one is evident, since $f(\theta \mid \eta^*)$ is decreasing in θ and $h(\theta \mid \eta^*)$ is increasing in θ. Then $q^*(\hat{h}(\theta \mid \eta^*))$ is also continuously differentiable and strictly decreasing in θ. By the second property, the range of $\hat{h}$ conditional on η^* (which is denoted by H) includes the range of $\hat{h}$ conditional on η. The proof of the second and third properties are provided in the online Appendix.

Suppose the result is false, and the second best allocation $(u_A^{SB}, u_S^{SB}, q^{SB})$ is EAC feasible. Then Proposition 5 implies existence of $\pi(\cdot \mid \eta) \in Y(\eta)$ such that for any $(\theta,\eta), (\theta^{'},\eta^{'})$,

$$q^{SB}(\theta,\eta) = q^*(\hat{h}(\theta \mid \eta))$$

$$X^{SB}(\theta,\eta) - z(\theta \mid \eta)q^{SB}(\theta,\eta) \geq 0$$

$$X^{SB}(\theta,\eta) - z(\theta \mid \eta)q^{SB}(\theta,\eta) \geq X^{SB}(\theta^{'},\eta^{'}) - z(\theta \mid \eta)q^{SB}(\theta^{'},\eta^{'})$$

where $z(\theta \mid \eta) \equiv z(\theta, \pi(\theta \mid \eta), \eta)$, which is an ironing transformation of $\pi(\cdot \mid \eta)$ based on $F(\theta \mid \eta)$, and

$$X^{SB}(\theta, \eta) \equiv u_A^{SB}(\theta, \eta) + u_S^{SB}(\theta, \eta) + \theta q^{SB}(\theta, \eta).$$

Step 1: Existence of $(\tilde{X}(h), \phi(h))$.

Consider $(\theta, \eta), (\theta', \eta')$ such that $\hat{h}(\theta \mid \eta) = \hat{h}(\theta' \mid \eta')$. Then $q^{SB}(\theta, \eta) = q^{SB}(\theta', \eta')$. The coalitional incentive constraint implies $X^{SB}(\theta, \eta) = X^{SB}(\theta', \eta')$, since otherwise the coalition would misrepresent a state with higher payment in the other state state where the same output is produced. It guarantees the existence of $\tilde{X}(h)$ such that $X^{SB}(\theta, \eta) = \hat{X}(\hat{h}(\theta \mid \eta))$ for any (θ, η).

Next suppose that $\hat{h}(\theta'' \mid \eta'') = \hat{h}(\theta' \mid \eta')$ and $z(\theta'' \mid \eta'') > z(\theta' \mid \eta')$ for some $(\theta', \eta'), (\theta'', \eta'')$. The ironing procedure ensures $z(\theta \mid \eta)$ and $\hat{h}(\theta \mid \eta)$ are continuous and non-decreasing for θ on Θ. Since $\hat{h}(\underline{\theta} \mid \eta) = \underline{\theta} < \bar{\theta} < \hat{h}(\bar{\theta} \mid \eta)$, $\hat{h}(\theta \mid \eta)$ is not constant on Θ. Then by adjusting θ' and θ'', we can find $(\tilde{\theta}', \tilde{\theta}'')$ such that $\hat{h}(\tilde{\theta}'' \mid \eta'') < \hat{h}(\tilde{\theta}' \mid \eta')$ and $z(\tilde{\theta}'' \mid \eta'') > z(\tilde{\theta}' \mid \eta')$. We obtain a contradiction, since the coalition incentive constraint implies that whenever $z(\tilde{\theta}'' \mid \eta'') > z(\tilde{\theta}' \mid \eta')$, $q^{SB}(\tilde{\theta}'', \eta'') \leq q^{SB}(\tilde{\theta}', \eta')$ or equivalently $\hat{h}(\tilde{\theta}'' \mid \eta'') \geq \hat{h}(\tilde{\theta}' \mid \eta')$. Hence there exists $\phi(h)$ which satisfies $z(\theta \mid \eta) = \phi(\hat{h}(\theta \mid \eta))$ for any (θ, η). Since $\hat{h}(\theta \mid \eta^*)$ and $z(\theta \mid \eta^*)$ are continuous and non-decreasing for θ, $\phi(h)$ is continuous and non-decreasing on H.

The coalitional incentive constraint implies that for any $h \in H$, h maximizes $\tilde{X}(h') - \phi(h)q^*(h')$ subject to $h' \in H$. By the continuity of $\phi(h)$ and the differentiability of $q^*(h)$, we obtain the differentiability of $\tilde{X}(h)$ and the first order condition $\tilde{X}'(h) = \phi(h)q^{*\prime}(h)$.

Step 2: Properties of $\phi(h)$

Here we show that (a) $\phi(h) \geq 0$ on H and (b) $h - \phi(h)$ is non-negative and increasing in h.

Since $q^{SB}(\theta, \eta^*) = q^*(\hat{h}(\theta \mid \eta^*))$ is strictly decreasing in θ, the pooling region $\Theta(\pi(\cdot \mid \eta^*), \eta^*)$ must be empty. Hence it must be the case that

$$z(\theta \mid \eta^*) = \phi(\hat{h}(\theta \mid \eta^*)) = \theta + \frac{F(\theta \mid \eta^*) - \Lambda(\theta \mid \eta^*)}{f(\theta \mid \eta^*)}.$$

Since $\phi(\hat{h}(\theta \mid \eta^*))$ is non-decreasing in θ and $\Lambda(\theta \mid \eta^*) \leq 1$,

$$\phi(\hat{h}(\theta \mid \eta^*)) \geq \phi(\underline{\theta}) \geq \underline{\theta} - 1/f(\underline{\theta} \mid \eta^*) > 0$$

by property (iii), which implies (a). The above equality can be rewritten as

$$\frac{\Lambda(\theta \mid \eta^*)}{f(\theta \mid \eta^*)} = \hat{h}(\theta \mid \eta^*) - \phi(\hat{h}(\theta \mid \eta^*)).$$

The LHS is non-negative and increasing in θ, since $f(\theta \mid \eta^*)$ is decreasing in θ and $\Lambda(\theta \mid \eta^*)$ is non-negative and non-decreasing in θ. It implies (b).

Step 3: S earns positive rent.

Define $L(h) \equiv \tilde{X}(h) - hq^*(h)$. S's interim payoff is

$$E[X^{SB}(\theta, \eta) - h(\theta \mid \eta)q^{SB}(\theta, \eta) \mid \eta] = E[L(\hat{h}(\theta \mid \eta)) \mid \eta],$$

utilizing a property of the ironing transformation. If the second best allocation is EAC feasible, $E[L(\hat{h}(\theta \mid \eta)) \mid \eta] = 0$ holds for any η. The first derivative of $L(h)$ is

$$L^{'}(h) = (\phi(h) - h)q^{*'}(h) - q^*(h).$$

Since $q^*(h)$ is continuously differentiable and $\phi(h)$ is continuous and almost everywhere differentiable, $L^{'}(h)$ is continuous and also differentiable almost everywhere and

$$L^{''}(h) = (\phi^{'}(h) - 1)q^{*'}(h) + (\phi(h) - h)q^{*''}(h) - q^{*'}(h).$$

By using $V^{'}(q^*(h)) = h$, we can show that $V^{'''}(q) \leq 0$ implies $q^{*''}(h) \leq 0$, and $0 < V^{'''}(q) \leq \frac{(V^{''}(q))^2}{V^{'}(q)}$ implies $q^{*''}(h) > 0$ and $hq^{*''}(h) + q^{*'}(h) < 0$. By $\phi^{'}(h) - 1 < 0$ and $0 \leq \phi(h) \leq h$, it follows that $L^{''}(h) > 0$.

By the strict convexity of L and the mean-preserving spread property of $G(h \mid \eta^*)$,

$$E[L(\hat{h}(\theta \mid \eta^*)) \mid \eta^*] = \int L(h)dG(h \mid \eta^*) > \int L(h)dG(h \mid \eta) = E[L(\hat{h}(\theta \mid \eta)) \mid \eta] \geq 0$$

for any $\eta \neq \eta^*$. Therefore S must earn a positive rent in state η^*. This is a contradiction. ■

Proof of Proposition 8: See online Appendix.

Proof of Proposition 9: Sketched in the text.