Consulting Collusive Experts[1]

Dilip Mookherjee[2], Alberto Motta[3] and Masatoshi Tsumagari[4]

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Abstract

We study the problem of a Principal seeking advice from an expert with better information about an agent's cost, on how to design an incentive contract for the agent. The expert has a prior relationship with the agent, facilitating (weak) ex ante collusion which coordinates their participation and reporting decisions to generate mutual benefits. Delegating contracting with the agent to the expert is never profitable, while consulting the expert is frequently valuable in designing the agent's contract. Changes in bargaining power within the coalition have no effect, while altruism of the expert towards the agent makes the Principal worse off.

KEYWORDS: mechanism design, collusion, delegation, expert

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[2]Department of Economics, Boston University, 270 Bay State Road Boston MA 02215; dilipm@bu.edu

[3]School of Economics, University of New South Wales, Australian School of Business, Sydney 2052, Australia; motta@unsw.edu.au

[4]Corresponding author: Department of Economics, Keio University, 2-15-45 Mita Minato-ku Tokyo 108-8345, Japan; tsuma@econ.keio.ac.jp

1 Introduction

Consider a Principal (P) seeking to procure a service from a contractor, worker or utility firm that is privately informed about its cost. In order to limit the supplier's information rent, P seeks the advice of an expert, referee or monitor (M) endowed with information concerning technology and supply conditions in the relevant industry. The expert has a prior relationship with the agent (A), which creates the potential for ex ante collusion: not only can M and A coordinate their reports to P, but also coordinate their decision whether to participate in the mechanism. Prior literature on hierarchical mechanism design has focused mainly on contexts of interim collusion, restricted to coordination of reporting decisions, after coalition partners have independently agreed to participate (Faure-Grimaud, Laffont and Martimort (FLM hereafter, 2003), Celik (2009)). Interim collusion seems more appropriate in auditing or supervision contexts where P assigns an auditor with no prior relationship with the agent, and the auditor and agent come into contact with one another after agreeing to participate. When M and A know each other prior to contracting with P, they can coordinate their participation decisions, thereby enlarging the scope of collusion. This raises a number of new questions. Under what conditions can P still benefit from the expertise of M? Is it optimal for P to contract with M alone, and delegate contracting with the agent to M? How is the Principal's welfare affected by varying bargaining power or altruistic preferences between M and A?

The existence of prior connections between experts and agents is common in many real world contexts such as the relationship between credit rating agencies and firms raising capital, regulators and private utility companies, company Directors and CEOs, managers and workers, or prime contractors and subcontractors. However, its consequences for the design of hierarchical contracts have not received much attention. By contrast, implications of ex ante collusion in the context of auction design (where bidders collude on participation and bids) have been studied by Che and Kim (2009) and Pavlov (2008). Besides incorporating collusion in participation, our model is similar to existing literature on collusion in hierarchies. M's signal of A's cost is partially informative. Both M and A observe the realization of this signal, resulting in one-sided asymmetric information within the coalition, which represents the sole friction in collusion. If they fail to agree on a side contract they play non-cooperatively thereafter (referred to as *weak* collusion); hence M and A enter into a deviating side contract only if it results in an interim Pareto improving allocation for the coalition. We consider a continuum type space for A's cost, and show how classical Myersonian mechanism design methods based on 'virtual' types can be extended (using techniques in Jullien (2000)) to incorporate ex ante collusion constraints, while allowing for general information structures for M. This is in contrast to previous analyses which have focused on discrete (two or three) type cases and specific information structures.

Our first main result is that collusion in participation has important implications for the optimality of delegating authority to M (where P contracts with M alone and lets M subcontract with A). Some authors have shown that delegation can be optimal in the presence of collusion in specific settings, e.g., with moral hazard for a range of parameter values by Baliga and Sjostrom (1998), or adverse selection with interim collusion and a specific information structure by FLM. Celik (2009) on the other hand shows that the FLM result does not hold under alternative information structures. By contrast in the ex ante setting we show that delegation is *never* optimal, irrespective of the information structure. Indeed, delegation is inferior to P not consulting the expert at all. Intuitively, ex ante collusion results in an additional constraint on the delegation design problem, akin to a limited liability constraint. This prevents P from being able to extract M's collusion rents upfront at the participation stage, resulting in double marginalization of rents (DMR). Contracting directly with A in the absence of M would lower P's procurement costs in all states.[5]

To explore the nature of optimal mechanisms with ex ante collusion, we start by considering the simple case of an indivisible good, with a continuum type space and two possible signals observed by M resulting in posterior beliefs with full support. This setting permits a detailed analysis of optimal contracts under ex ante collusion (EAC), when and how they differ from the interim collusion (INC) context. When P's valuation of the good is low, optimal contracts in the two contexts coincide and incentives are low-powered (in the sense of lower variability of aggregate payments to M and A with the quantity delivered to P). For higher valuations, optimal contracts diverge: the INC setting is characterized by high powered contracts and delegation, resembling a franchise arrangement where M becomes a residual claimant and pays P a fixed upfront fee. Such contracts are infeasible in EAC: M would refuse to participate to avoid paying the upfront fee in states where A experiences a

[5] A similar result is obtained by Mookherjee and Tsumagari (2004) in a team production setting involving two agents privately informed about their respective costs.

high cost and does not deliver the good at the offered price. In such settings, collusion in participation necessitates centralized contracting, low powered incentives, and low sensitivity of prices with respect to cost information. Moreover, when P's valuation of the good is sufficiently high, consulting M is no longer worthwhile under ex ante collusion, but always remains valuable in interim collusion.

We then consider the context of divisible good procurement, and consider a wide range of information structures. Here we show that consulting M is *always* valuable. This helps explain the widespread reliance on experts, despite prospects for ex ante collusion. This difference from the indivisible good context arises because there is greater scope for M's information to affect P's procurement decisions e.g., regarding quantity of the good to be supplied. The proof is based on showing that small variations can be constructed around the optimal contract where M is not consulted, without giving rise to any collusion or changing M's interim payoff. The variation entails raising the output procured over some range of cost types, and lowering it over another range (corresponding to an arbitrary cost signal state). Differences in beliefs of P and M regarding A's cost ensures the existence of 'mutual gains from trade' from such a variation, enabling P to earn higher profits, while preserving M's participation incentives.

The following additional results concerning optimal mechanisms in the ex ante collusion setting are then provided:

- We find sufficient conditions for ex ante collusion to lower P's profits, compared to the setting without any collusion. This contrasts with results obtained by Che and Kim (2006) or Motta (2009) for interim collusion settings where the second-best welfare can generally be achieved by P.

- Altruism of M towards A always hurts P, implying the need for P to consult experts without any personal connections with A. The result is not a priori obvious, owing to two offsetting effects. Increased altruism aids collusion by lowering frictions within the coalition, but also reduces the severity of the DMR problem by limiting the extent to which M seeks to gain personally from lowering the price offered to A. The former effect outweighs the latter when contracting is centralized, while the opposite is true under delegated contracting.

- Changes in bargaining power between M and A over the side-contract do not matter.

Despite the existence of asymmetric information within the coalition, a modified form of the 'Coase Theorem' continues to hold. This is a consequence of the standard assumption that failure to collude results in noncooperative play in P's mechanism.[6]

The paper is organized as follows. Section 2 discusses relation to existing literature in more detail. Section 3 studies the context of an indivisible good and two cost signals. Section 4 considers a perfectly divisible good and a general information structure for M involving a finite number of possible signals, and presents results concerning value of the expert and collusion costs. Section 5 discusses extensions incorporating alternative allocations of bargaining power within the coalition, and altruistic experts. Finally, Section 6 discusses implications of our results, extensions and shortcomings of our analysis. All proofs are presented in the Appendix.

2 Related Literature

Early literature on mechanism design with collusion (e.g., Tirole (1986), Laffont and Tirole (1993)) focused on contexts of 'hard' information where supervisors could only hide information but could not report untruthfully. The subsequent literature examines the case of 'soft' information where no constraints on allowable reports are imposed. They can be classified by the context (auctions, team production or supervision), the nature of collusion (ex ante or interim, weak or strong collusion)[7], and whether type spaces are discrete or continuous.

A large part of existing literature deals with auctions and team production, where there are multiple privately informed agents and no supervisor. For auctions, Dequiedt (2007) considers strong ex ante collusion with binary agent types and shows that efficient collusion is possible, implying that the second-best cannot be achieved. In contrast, Pavlov (2008) considers a model with continuous types where the second-best can be achieved with weak

[6]Mookherjee and Tsumagari (2017) show this result no longer holds in settings of 'strong' collusion, where side contracts include commitments to threats made by each party concerning strategies they will employ should the other party refuse to participate in the collusion.

[7]Ex ante collusion permits collusion over both reporting and participation decisions, while interim collusion pertains only to collusion over reporting. Weak collusion refers to collusion for mutual gain, where failure to agree to collude is followed by noncooperative play. Under strong collusion, each partner commits to a threat pertaining to how it would play in P's mechanism, should the other partner refuse to collude, thereby permitting extortion as well as mutual gain.

ex ante collusion, and Che and Kim (2009) find the same result with either weak or strong ex ante collusion with continuous types. Team production with binary types is studied by Laffont and Martimort (1997), who show the second best can be achieved with weak interim collusion; this analysis is extended to a public goods context in Laffont and Martimort (2000) to explore the role of correlated types. Baliga and Sjostrom (1998) consider a team setting with two productive agents that collude, involving moral hazard and limited liability rather than adverse selection. They show that delegation to one of the agents is an optimal response to collusion for a wide set of parameter values. Che and Kim (2006) show how second-best allocations can be achieved in a team production context with continuous types under weak interim collusion. Quesada (2004) on the other hand shows strong ex ante collusion is costly in a team production model with binary types. Mookherjee and Tsumagari (2004) show delegation to one of the agents is worse than centralized contracting in the presence of weak ex ante collusion. The logic of this result is similar to that underlying our result that delegation to the expert is worse than not consulting the expert at all. Their paper also considers delegation to an expert who is perfectly informed about the costs of each agent, and show that its value relative to centralized contracting depends on complementarity or substitutability between inputs supplied by different agents. The current paper differs insofar as there is only one agent, and there is asymmetric information within the expert-agent coalition owing to the expert receiving a noisy signal of the agent's cost.

In the context of collusion between a supervisor and agent, existing models explore *interim* collusion only. FLM consider a model with binary types and signals (with full support for conditional distributions), a risk-averse supervisor where collusion is costly, where (unconditional) delegation turns out to be an optimal response to collusion. Celik (2009) considers a model with three types and two signals (where the support of conditional distributions depends on the signal), and risk neutral supervisor and agent, in which unconditional delegation is dominated by no supervision, which in turn is dominated strictly by centralized contracting with supervision. Our results show that Celik's results (which apply to a specific information structure in the context of interim collusion) extends to general information structures under ex ante collusion.

3 Illustration: Procurement of an Indivisible Good

P procures an indivisible good with quantity q either 0 or 1 from A who produces it at cost θ. A is privately informed about the realization of θ. The expert M and A jointly observe the realization of signal $i \in \{L, H\}$ of A's cost. Both A and M have outside option payoffs of 0. $F_i(\theta)$ denotes the distribution of θ conditional on i defined on $[\underline{\theta}, \bar{\theta}]$, which has a density $f_i(\theta)$ which is differentiable and positive on $[\underline{\theta}, \bar{\theta}]$. Hence the support of θ does not vary with the signal, and hazard rates are well-defined and finite-valued throughout the support. $\kappa_i \in (0, 1)$ denotes the probability of signal i, with $\kappa_L + \kappa_H = 1$. P does not observe the signal i, and has a prior $F(\theta) \equiv \kappa_L F_L(\theta) + \kappa_H F_H(\theta)$ with density $f(\theta) \equiv \kappa_L f_L(\theta) + \kappa_H f_H(\theta)$.

Assumption 1 *(i)* $\frac{f_L(\theta)}{f_H(\theta)}$ *is decreasing*

(ii) $H(\theta) \equiv \theta + \frac{F(\theta)}{f(\theta)}$, $h_i(\theta) \equiv \theta + \frac{F_i(\theta)}{f_i(\theta)}$ *and* $l_i(\theta) \equiv \theta + \frac{F_i(\theta)-1}{f_i(\theta)}$ $(i = L, H)$ *are increasing*

(iii) $h_L(\bar{\theta}) > V > \underline{\theta}$

Part (i) represents a monotone likelihood ratio property wherein $i = L$ (resp. $i = H$) is a signal of low (resp. high) cost, while (ii) is a standard assumption ensuring monotonicity of (conditional) virtual costs. These imply $F_H(\theta) < F(\theta) < F_L(\theta)$ and $h_H(\theta) < H(\theta) < h_L(\theta)$ for any $\theta \in (\underline{\theta}, \bar{\theta})$. The second inequality of (iii) ensures gains from trade between P and A; the first one ensures that costless access to M's signal is valuable for P in the absence of collusion. These conditions are satisfied in the following example with a uniform prior $F(\theta) = \theta$ on $[0, 1]$ and linear conditional densities: $F_L(\theta) = 2d\theta - (2d-1)\theta^2$, $F_H(\theta) = 2(1-d)\theta + (2d-1)\theta^2$ on $[0, 1]$, $\kappa_L = \kappa_H = 1/2$, $d \in (1/2, 1)$ and V between 0 and $1 + \frac{1}{2(1-d)}$. We shall illustrate our analysis with numerical computations for this example.

The situation where P has no access to M's signal is referred to as the *No Monitor (NM)* case. Here P offers a non-contingent price p^{NM} to maximize $F(p)[V - p]$, which satisfies $V = H(p^{NM})$ if $V < H(\bar{\theta})$, and equals $\bar{\theta}$ otherwise. Let $\Pi^{NM} \equiv F(p^{NM})[V - p^{NM}]$ denote the resulting expected payoff of P. The *second-best allocation* results when there is no collusion whence P can costlessly access M's signal; here P offers A a price p_i^{SB} which maximizes $(V - p_i)F_i(p_i)$. The ordering of virtual cost functions implied by Assumption 1 ensures a lower price elasticity of supply and thus a lower second-best price in the low cost signal state. However, the supply curve is shifted to the right in the low signal state, so the ordering of resulting supply likelihoods between the two states is ambiguous, which turns out to depend on V:

Lemma 1 *(i)* $p_H^{SB} > p^{NM} > p_L^{SB}$ *if* $V < H(\bar{\theta})$, *and* $p_H^{SB} = p^{NM} = \bar{\theta} > p_L^{SB}$ *otherwise*

(ii) There exist V^* *and* V^{**} *such that* $\underline{\theta} < V^* \leq V^{**} < h_H(\bar{\theta})$, *where* $F_L(p_L^{SB}) > F_H(p_H^{SB})$ *for* $V \in (\underline{\theta}, V^*)$ *and* $F_L(p_L^{SB}) < F_H(p_H^{SB})$ *for* $V \in (V^{**}, h_L(\bar{\theta}))$.

3.1 Delegation to Expert with Ex Ante Collusion

Consider P's option to contract solely with M and delegate the authority to contract with A. With ex ante collusion, M does not commit to responding to P's offer before contracting with A. So after P offers M a contract, the latter offers A a contract. Following A's response, M then responds to P.

Given this timing, standard arguments imply that (following any given contract offer) M can confine attention to offering A a take-it-or-leave-it price p_i in state i for delivering the good to P. And similarly P can confine attention to offering M a two part contract X_0, X_1 where X_q is the payment for delivery of output q. There is no added value to P asking M to submit a report of her signal or the outcome of contracting with A, as conditional on the q delivered M would select whichever message would maximize her payment received.

In order to induce M to deliver the good with positive probability, P must offer $X_1 > \underline{\theta}$. Upon observing signal i, M will then decide what price $p_i \in [\underline{\theta}, X_1]$ to offer A, along with participation decision in P's contract in either of the two events where A does or does not accept M's offer. If A accepts, it is optimal for M to agree to participate in P's contract since the optimal price will satisfy $p_i < X_1$. Let $I \in \{0, 1\}$ denote M's participation decision in the event that A does not accept M's offer. Then M selects p_i and I to maximize $F_i(p_i)(X_1 - p_i) + I[1 - F_i(p_i)]X_0$. It follows that $I = 1$ only if $X_0 \geq 0$. If $X_0 < 0$, M will not accept P's offer in the event that A does not accept M's offer. The same outcome is realized if P sets $X_0 = 0$. Hence without loss of generality, $X_0 \geq 0$, and M always accepts P's offer. The constraint $X_0 \geq 0$ plays a key role in the subsequent analysis. It arises owing to ex ante collusion, whereby M contracts and communicate with A prior to responding to P's offer. In an interim collusion setting this constraint does not arise, and is replaced by interim participation constraints for M, whence X_0 can be negative and yet P's contract could be accepted by M.

Let b denote the delivery *bonus* $X_1 - X_0$. The choice of p_i will be made by M to maximize $F_i(p_i)(b - p_i)$. If $b \leq \underline{\theta}$, it is optimal for M to offer A a price below $\underline{\theta}$, whence the good is never delivered to P. Otherwise there is a unique optimal price $p_i(b)$ which satisfies $\underline{\theta} < p_i(b) < b$. Eventually P earns expected payoff $[\kappa_L F_L(p_L(b)) + \kappa_H F_H(p_H(b))](V - b) - X_0$, which is sought to be maximized by choosing $b > \underline{\theta}, X_0 \geq 0$. Now note that any such payoff would be strictly dominated by the option of not consulting M at all where P directly offers A a price of b. This follows since $b < V$ is necessary for P to earn a positive payoff; hence $[\kappa_L F_L(p_L(b)) + \kappa_H F_H(p_H(b))](V - b) - X_0 < F(b)(V - b) \leq \Pi^{NM}$. We thus obtain:

Proposition 1 *With an indivisible good and ex ante collusion, delegation to the expert is worse for the principal compared to not consulting the expert at all.*

As we shall later see, delegation could dominate the no-monitor outcome under interim collusion. This represents a stark contrast between the two forms of collusion. In delegation with ex ante collusion, M earns rents which cannot be taxed away upfront by P at the time of contracting with M, thereby generating a double marginalization of rents (DMR). Under interim collusion, P may be able to extract some of M's interim rents (in the absence of knowledge of A's type) via an upfront fee, thereby limiting the DMR problem.

3.2 Centralized Contracting with Ex Ante Collusion

Under ex ante collusion, therefore, if at all P obtains an advantage from consulting M, she needs to contract simultaneously with both M and A. M and A can negotiate a side-contract (SC, for short) prior to responding to P's offer. Following private communication of a cost message by A to M, the SC coordinates their respective messages (which include participation decisions and cost reports) sent to P, besides a side payment between A and M. As shown later, without loss of generality M has all the bargaining power within the coalition and makes a take-it-or-leave-it SC offer to A. If A refuses it, they play P's mechanism non-cooperatively. It turns out (as explained in the online Appendix) P can confine attention to mechanisms that are collusion-proof, i.e., for which it is optimal for M to not offer any non-null SC to A, and both M and A agree to participate. We now explain the implied individual and coalition incentive compatibility constraints in the context of an indivisible good.

First, a contract offer to A reduces to a single take-it-or-leave-it price offer p_i when the cost signal is i. Second, in order to deter collusion, P must offer an aggregate payment to M and A which depends only on whether or not the good is produced. Let $X_0 + b, X_0$ denote the aggregate payments when the good is and is not produced respectively. The two prices p_L, p_H combined with X_0, b characterize an allocation entirely. This is associated with a mechanism where M and A are asked to submit reports of the signal i to P. If the two reports happen to match, A is offered the option to produce and deliver the good directly to P in exchange for price p_i, while M is paid X_0 if the good is not delivered, and $b + X_0 - p_i$ if it is delivered. If the two reports do not match, there is no production and both M and A are required to pay a high penalty to P. The key feature distinguishing centralized contracting from delegation is that in the former P makes a contract offer directly to A which is conditioned on reported signals. This provides an outside option to A which M is constrained to match while offering an SC to A. This is an important strategic tool which enables P to manipulate the outcome of collusion between M and A, and reduce the severity of the DMR problem.

Along the equilibrium path where A and M decide to participate, report i truthfully to P, and do not enter into a deviating SC, A produces the good in state i and receives the payment p_i if and only if θ is smaller than p_i. Without loss of generality, A receives no payment in the event of non-production (since any mechanism paying a positive amount to A in the event of non-production is dominated by one that does not). This generates utility to A of $u_A(\theta, i) = \max\{p_i - \theta, 0\}$. M ends up with $X_0 + b - p_i$ in the event that production takes place, and X_0 otherwise.

The allocation p_L, p_H, X_0, b has to satisfy the following constraints. First, in order to ensure that ex post the coalition does not prefer to reject it, the aggregate payment to M and A must be nonnegative in the event that the good is not delivered:

$$X_0 \geq 0. \tag{1}$$

The reason is that if the good is not delivered, A earns no rent; hence rejection of P's contract by the coalition does not entail any payoff consequence for A. If $X_0 < 0$, M would then benefit from rejecting P's contract; hence it is Pareto improving for the coalition to do so.[8] This constraint is distinctive to the ex ante collusion setting, where participation decisions in P's contract are made after M and A have negotiated a side contract.

[8]No analogous non-negativity constraint on aggregate payments $X_0 + b$ corresponding to delivery of the good is imposed here, because the decision to reject P's contract could result in a loss of rents for A. M would then have to compensate A for this loss, and the required compensation may be large enough that it

Second, in order to induce M to participate ex ante:

$$F_H(p_H)(b - p_H) + X_0 \geq 0 \tag{2}$$

$$F_L(p_L)(b - p_L) + X_0 \geq 0 \tag{3}$$

Individual participation constraints for A are already incorporated into the supply decision represented by a supply likelihood of $F_i(p_i)$ in state i.

Third, M and A should not be tempted to enter a deviating SC. A deviating SC would involve a different set of prices $\tilde{p}_i$ offered to A (in state i) for delivering the good, combined with a lump-sum payment $\tilde{u}_i$. A would then produce if θ is smaller than $\tilde{p}_i$, and M would earn an expected payoff $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 - \tilde{u}_i$. Type θ of A would accept the deviating SC provided

$$\max\{\tilde{p}_i - \theta, 0\} + \tilde{u}_i \geq \max\{p_i - \theta, 0\} \tag{4}$$

We show in the online Appendix that without loss of generality M can restrict attention to side contracts which are accepted by all types of A. Hence collusion-proofness requires $(\tilde{p}_i, \tilde{u}_i) = (p_i, 0)$ to maximize $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 - \tilde{u}_i$ subject to (4) for all types $\theta \in [\underline{\theta}, \bar{\theta}]$.

This condition can be broken down as follows. First, if $p_i > \underline{\theta}$, M should not benefit by deviating to a price $\tilde{p}_i < p_i$. This would necessitate offering a lump-sum payment of $\tilde{u}_i = p_i - \tilde{p}_i$ to ensure that all types of A accept the SC, which would then generate M an interim expected payoff of $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 - p_i + \tilde{p}_i$. A necessary and sufficient condition for such a deviation to not be worthwhile is that

$$b \geq p_i - \frac{1 - F_i(p_i)}{f_i(p_i)} \equiv l_i(p_i) \tag{5}$$

since $l_i(p)$ is increasing in p as per the monotone hazard rate assumption 1(ii). Intuitively, offering a lower price than p_i is similar to M selling the good back to A. Condition (5) which states that the value (b) of the good to M exceeds its virtual value to A, ensures that such a sale is not worthwhile.

Similarly, if $p_i < \bar{\theta}$, M should not want to offer A a higher price $\tilde{p}_i$. Unlike the case of a lower offer price, such a variation cannot be accompanied by a negative lump sum payment $\tilde{u}_i$ to A, owing to the need for A's ex post participation constraint to be satisfied in non-delivery states. Offering $\tilde{p}_i > p_i$ will then generate an interim payoff of $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0$.

may be optimal for M to instead accept P's contract despite $X_0 + b$ being negative. The issue of coalition incentive compatibility is addressed in more detail below.

For M to not want to deviate to a higher price, it must be the case that

$$b \leq p_i + \frac{F_i(p_i)}{f_i(p_i)} = h_i(p_i) \tag{6}$$

This condition can be interpreted simply as the value of delivery (b) to M being lower than the virtual cost of A delivering it.

(5, 6) can be combined into the single collusion-proofness condition

$$\max\{\hat{l}_L(p_L), \hat{l}_H(p_H)\} \leq b \leq \min\{\hat{h}_L(p_L), \hat{h}_H(p_H)\}. \tag{7}$$

where $\hat{l}_i(p_i)$ denotes $l_i(p_i)$ if $p_i > \underline{\theta}$ and $-\infty$ otherwise, and $\hat{h}_i(p_i)$ denotes $h_i(p_i)$ if $p_i < \bar{\theta}$ and ∞ otherwise (since the corresponding state i constraint is relevant only when p_i differs from $\underline{\theta}, \bar{\theta}$ respectively). This condition is referred to as coalition incentive constraint henceforth.

As implied by arguments in the online Appendix, these conditions are necessary and sufficient for the allocation (p_L, p_H, b, X_0) to be the outcome of a Perfect Bayesian Equilibrium (PBE) of the ex ante collusion contracting game, which is interim-Pareto-undominated for the coalition by any other PBE. Hence, an optimal allocation must maximize

$$[\kappa_H F_H(p_H) + \kappa_L F_L(p_L)](V - b) - X_0 \tag{8}$$

subject to (1, 2, 3, 7). We refer to these constraints as characterizing ex ante collusion (EAC) feasibility.

It is convenient to restate P's profit as

$$U(p_L, p_H) - R(b, X_0; p_L, p_H) \tag{9}$$

where $U(p_L, p_H) \equiv \kappa_H F_H(p_H)(V - p_H) + \kappa_L F_L(p_L)(V - p_L)$ is the expression for expected profit in the second-best setting, from which M's rent $R(b, X_0; p_L, p_H) \equiv \kappa_H F_H(p_H)(b - p_H) + \kappa_L F_L(p_L)(b - p_L) + X_0$ has to be subtracted in the presence of collusion. Note also that given b, p_L, p_H it is optimal to set $X_0 = \max\{0, \max_i\{F_i(p_i)(p_i - b)\}\}$. With this convention we can henceforth represent an EAC allocation by the triple (p_L, p_H, b).

We start the analysis by making some simple but key observations regarding properties of any EAC-feasible allocation in which M is valuable (i.e, where the resulting profit exceeds the maximum profit attainable in NM).

Lemma 2 *In any EAC-feasible allocation in which M is valuable:*

(i) $b < p_i$ *for some* i *and* $X_0 > 0$

(ii) $p_L < p_H$

(iii) $F_L(p_L) > F_H(p_H)$.

Part (i) states that relevant EAC allocations must involve *low-powered incentives* for M in at least one state i, in the sense that ex post M is worse off in state i if the good is delivered than when it is not. This is the very opposite of delegation, where M earns a nonnegative margin on any transaction in every state. In ex ante collusion, the base pay X_0 must be positive in order to compensate for the 'loss' incurred by M when the good is delivered in state i (so as to ensure that M wants to participate at the interim stage corresponding to state i). Conversely, (i) may be viewed as stating that A is offered higher powered incentives than M in some state; this is a 'countervailing incentive' designed to raise A's outside option in bargaining with M over a side contract, so as to counter the DMR problem.

Part (ii) states that the low cost signal results in a lower price offered to A, just as in the second-best setting. The reason is that when the prices offered to A can vary with the cost signal, P's profit rises only if they result in a lower price being offered following a low cost signal. A variation in the opposite direction would directly result in lower profit, besides possibly entailing some rents paid to M. Part (iii) restricts the extent to which the prices can vary across the two states: the price in the low cost state should not be so low that the resulting supply likelihood becomes smaller in that state. Intuitively, larger variations in prices are not worthwhile because they generate high collusion stakes which raise M's rent excessively.

Lemma 2 indicates the problem of finding an optimal EAC allocation can be broken down into two successive stages. At the first stage, for any given pair of prices p_L, p_H satisfying (ii) and (iii), we find an optimal contract b for M to minimize M's rent subject to the coalition incentive constraint (7), and the requirements that $b < p_H$ and $X_0 = \max_i\{F_i(p_i)(p_i - b)\}$. Then at the second stage, prices p_L, p_H are selected to maximize $U(p_L, p_H) - R^*(p_L, p_H)$ subject to $p_L < p_H, F_L(p_L) > F_H(p_H)$, where $R^*(p_L, p_H)$ denotes the minimized rent of M at the first stage.

The next result describes the solution to the first stage problem, i.e., the optimal bonus for any set of prices satisfying (ii) and (iii). Upon substituting for the optimal base pay

X_0, the expression for M's expected rent reduces to

$$\tilde{R}(b; p_L, p_H) \equiv \kappa_L F_L(p_L)(b-p_L)+\kappa_H F_H(p_H)(b-p_H)-\min\{F_L(p_L)(b-p_L), F_H(p_H)(b-p_H)\}. \tag{10}$$

Clearly $\tilde{R}$ is non-negative and attains a global minimum of zero at $b = \frac{p_L F_L(p_L)-p_H F_H(p_H)}{F_L(p_L)-F_H(p_H)} \equiv B(p_L, p_H) < p_L < p_H$. This turns out to be feasible (snd hence $B(p_L, p_H)$ is optimal) if $B(p_L, p_H) \geq \max\{l_L(p_L), l_H(p_H)\}$, otherwise it is optimal to select the lowest bonus that is feasible, which is $\max\{l_L(p_L), l_H(p_H)\}$.

Lemma 3 *Given p_L, p_H satisfying $p_L < p_H$ and $F_L(p_L) > F_H(p_H)$, the optimal bonus $b(p_L, p_H) = \max\{B(p_L, p_H), l_L(p_L), l_H(p_H)\}$ where $B(p_L, p_H) \equiv \frac{p_L F_L(p_L)-p_H F_H(p_H)}{F_L(p_L)-F_H(p_H)}$.*

Next, we characterize properties of optimal EAC allocations (with p_i^E denoting the corresponding optimal price in state i).

Proposition 2 *With an indivisible good and ex ante collusion:*

(a) There exists $\hat{V}_1 > \underline{\theta}$ such that if $V \in (\underline{\theta}, \hat{V}_1)$ the second-best profit can be achieved;

(b) M is valuable if $V < H(\bar{\theta})$, but not if $V > \hat{V}_2$ for some $\hat{V}_2 \in (H(\bar{\theta}), h_L(\bar{\theta}))$.

(c) $p_H^E \leq p_H^{SB}$

(d) $p_L^E \geq p_L^{SB}$ if $l_L(.)$ is convex.

Part (a) states that the second-best profit can be achieved by P when V is low enough, while (b) says that consulting M is valuable for low values of V but not for sufficiently high values. Parts (c) and (d) describe how prices offered to A deviate from second-best prices. Provided l_L is convex, a condition satisfied in our example with linear conditional density functions and uniform prior, the dispersion between prices in the two states is narrower than in the second-best. The heuristic reason underlying these results is that collusion costs tend to rise with dispersion in prices p_i across the two states. For sufficiently low values of V, the second-best can be implemented, essentially because the dispersion between second-best prices corresponding to the different cost signals is small enough. The value of consulting M tends to decline as V rises, because this raises price dispersion and hence the rents paid to M.

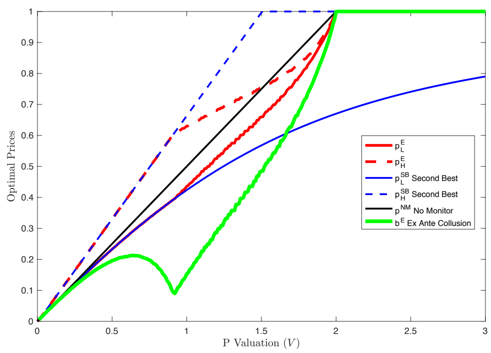

Figure 1: Second-Best, No Monitor and EAC Optimal Prices in Example with $d = 0.99$

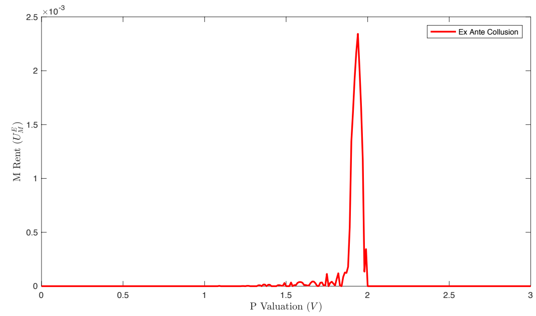

Figure 2: M's payoff in EAC Optimal Allocation in Example with $d = 0.99$

This intuitive argument also helps explain why M is valuable for values of V smaller than $H(\bar{\theta})$. Starting with the optimal NM allocation where an interior price $p^{NM} < \bar{\theta}$ is offered, consulting M enables P to vary the price p_i with the cost signal in the direction of the second-best prices ($p_L^{SB} < p_L < p^{NM} < p_H < p_H^{SB}$). When the variation is slight, the induced stakes of collusion are small enough that M can earn no rents, thereby generating a profit improvement for P. Parts (c) and (d) reinforce this intuition, by showing that the distortion in prices compared with the second-best involves lower dispersion (given convexity of l_L).

These results are illustrated in our numerical example with $d = 0.99$. Figure 1 plots optimal prices offered to A in the second-best (SB), no monitor (NM) and ex ante collusion (E) settings, corresponding to different values of V. It also plots the corresponding EAC-optimal bonus values b^E. Figure 2 plots the corresponding rents earned by M. For low values of V, the second-best is implemented and M earns no rents. Over this range price dispersion rises, as in the second-best. For intermediate values of V, M is valuable despite earning positive rents; over this range price dispersion narrows in contrast to rising dispersion in second-best prices. Eventually the gap between p_L^E and p_H^E is eliminated as V grows further, from which point onwards M ceases to be valuable and earns zero rents. Hence M earns positive rents only for intermediate values of V, as confirmed by Figure 2.

3.3 Contrasting Ex Ante and Interim Collusion

We now describe how (and when) the solution to EAC differs from interim collusion (INC). The formulation of the INC problem differs from the EAC problem in only one respect: the collusive participation constraint $X_0 \geq 0$ does not apply. An INC allocation can also be represented by the triple (b, p_L, p_H), where base pay X_0 is optimally set equal to $\max_i\{F_i(p_i)(p_i - b)\}$ and is permitted to be negative. Part (i) of Lemma 1 then no longer applies, opening up the possibility of providing high powered incentives with a bonus b larger than $\max_i\{p_i\}$ (as in a delegation setting), and then extracting M's rent upfront with a negative base pay. In particular, delegation to M can no longer be ruled out.

It is easy to check that in INC, part (ii) of Lemma 2 continues to apply (for the same reason), so $p_L < p_H$ is still necessary. However part (iii) need not apply: the likelihood of supply could be higher in the high cost state. The reason is that under interim collusion part (i) of Lemma 2 no longer holds — incentives could be high-powered ($b > p_H$). Part (iii) is then modified as follows (upon using a similar argument as in Lemma 2): an INC allocation where M is valuable must either (i) be low-powered (in the sense that $b < p_H$) and satisfy $F_L(p_L) > F_H(p_H), X_0 > 0$, or (ii) high-powered ($b > p_H$) and satisfy $F_H(p_H) > F_L(p_L), X_0 < 0$. It is evident that (i) is EAC feasible, while (ii) is not. We therefore obtain:

Lemma 4 *The optimal INC allocation differs from the optimal EAC allocation only if the former involves high powered incentives ($b > p_H > p_L$) and $F_H(p_H) > F_L(p_L)$.*

So we now focus on allocations with high-powered incentives where $b > p_H > p_L$ and $F_H(p_H) > F_L(p_L)$. The optimal bonus in ex ante collusion now differs from Lemma 3 as follows.

Lemma 5 *Given p_L, p_H satisfying $p_L < p_H$ and $F_L(p_L) > F_H(p_H)$, the optimal bonus in interim collusion is $b(p_L, p_H) = \min\{B(p_L, p_H), \hat{h}_L(p_L), \hat{h}_H(p_H)\}$. M is valuable only if $b > V$.*

The relevant range of bonuses and their effect on M's rent are thus reversed in interim collusion, compared to the EAC setting: the relevant range of b is $(p_H, \min_i\{\hat{h}_i(p_i)\}]$, over which M's rent is decreasing in b. Whenever M earns positive rents in INC, it is optimal for P to make incentives as high-powered as possible, and set the bonus to the maximum level $\min_i\{\hat{h}_i(p_i)\}$ consistent with the coalition incentive constraint. Moreover, the bonus needs to exceed V in order for M to be valuable.

We are now in a position to characterize some features of INC optimal allocations which are EAC-infeasible.

Lemma 6 *(i) An INC optimal allocation which is not EAC feasible can be implemented via delegation to M.*

(ii) Second-best profits cannot be achieved by an INC optimal allocation which is not EAC feasible.

(iii) There exists $\tilde{V} \leq H(\bar{\theta})$ such that for all $V \in (\tilde{V}, h_L(\bar{\theta}))$ M is valuable in the INC optimal allocation.

Result (i) follows from observing that P's profits are decreasing in each price p_i in INC. Raising prices paid to A raises the likelihood of the good being delivered, which lowers P's profit largely as a consequence of paying a bonus exceeding what the good is worth to P (as shown in the previous Lemma). Hence if $h_i(p_i)$ exceeds b, it is profitable to lower p_i slightly while leaving the bonus b unchanged, as this would preserve feasibility of the allocation. This implies that the price offered to A is exactly what would have been chosen in each state by M under delegation. And under delegation M would earn a higher profit in the low cost state compared with the high cost state, owing to A's 'supply curve' being shifted to the right in the former relative to the latter. It is then impossible for P to fully extract M's rents in the low cost state, as M has to be willing to accept the contract in both states. Hence second-best profits cannot be achieved. Part (iii) shows that unlike the ex ante collusion setting, M remains valuable in interim collusion for all large V between $H(\bar{\theta})$ and $h_L(\bar{\theta})$. Intuitively this is because in the absence of collusion in participation and the associated DMR problem, delegation helps P control the stakes of collusion better.

Combining the various results above, we obtain the following Proposition which contrasts optimal solutions in the ex ante and interim collusion settings.[9] The solution to ex ante collusion involves low powered incentives, and in particular can never be achieved by

[9]The result comparing INC optimal prices with second-best prices in (iii) obtains from observing that prices corresponding to delegation with a bonus of V equal second-best prices, and the optimal bonus must exceed V.

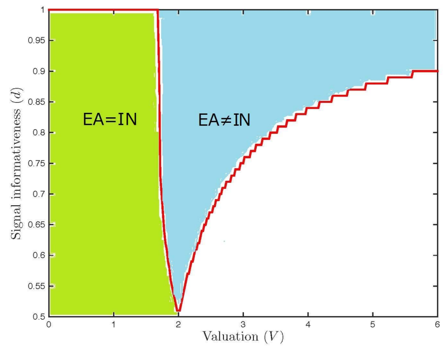

Figure 3: When Interim and Ex Ante Collusion Solutions Differ

delegation. Interim collusion involves a different allocation for large values of V, which is implemented via high-powered incentives (a delivery bonus that exceeds the value of the good to P, combined with delegation). Recall that we consider the range of possible values of V between $\underline{\theta}$ and $h_L(\bar{\theta})$.

Proposition 3 *(i) For sufficiently small values of V, EAC and INC optimal allocations coincide. For sufficiently large V, they are different.*

(ii) M is valuable in INC for all $V > H(\bar{\theta})$, whereas M is not valuable in EAC for sufficiently large V.

(iii) Whenever the INC optimal allocation differs from the EAC optimal allocation, it can be implemented via delegation to M, with prices $p_i^I \geq p_i^{SB}$ for $i = L, H$ and a bonus $b^I > V$ (with (p_i^I, b^I) corresponding to the INC optimal allocation).

In the context of our numerical example, Figure 3 shows different regions of the two dimensional parameter space (V, d) where the INC optimal and EAC optimal solutions do and do not coincide. The unshaded subregion on the extreme right is excluded by our restriction that $V < h_L(\bar{\theta})$. In the subregion on the left (marked "EA=IN") involving relatively low values of V, the EAC and INC solutions coincide. In the middle subregion (marked "EA $\neq$ IN") they diverge. Figure 4 plots the pattern of optimal prices in the INC optimal solution, corresponding to different values of V (with d set equal to 0.99). For intermediate values of V where the second-best is not attained and the two solutions

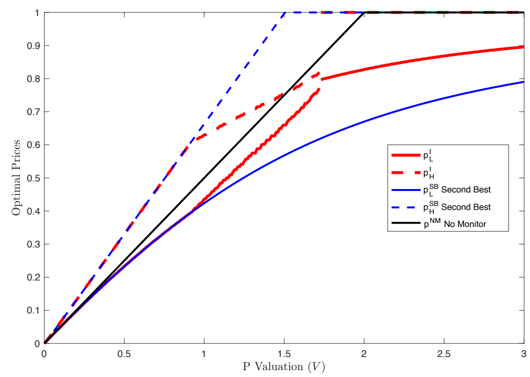

Figure 4: Optimal Prices with Interim Collusion

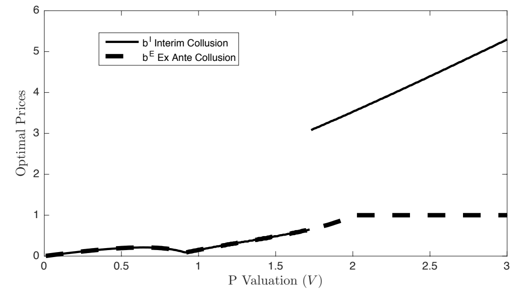

Figure 5: Optimal Bonus in Ex Ante and Interim Collusion

coincide, the price offered in the high cost signal state is smaller than the corresponding second-best price. As V rises further, the INC solution diverges from the EAC, causing a discontinuous switch in the pricing pattern: the price offered in the high cost signal state jumps up to the second-best price, resulting in locally increasing price dispersion. Figure 5 plots the optimal bonus against alternative values of V (with d set equal to 0.99). Over the range where the EAC and INC solutions coincide, incentives are low-powered (the bonus is smaller than V). At the threshold where they just begin to diverge, the INC optimal bonus jumps discontinuously upwards while the EAC bonus continues to remain below V.

Interim collusion is thus characterized by a discontinuous change in contracting strategy as V crosses the threshold, from a 'bureaucracy' (low-powered incentives, centralized contracting and low sensitivity of supplier price to cost information of the expert), to a 'market-like' contract resembling a franchise arrangement (high powered incentives, delegation, revenues earned primarily through franchise fees, and higher sensitivity of price to cost information). The market-based strategy is infeasible in the presence of ex ante collusion, since franchisees can then collude with their suppliers to avoid paying the upfront franchise fee when suppliers cannot deliver owing to high cost realizations.

4 Divisible Good Procurement

4.1 Environment

A delivers an output q to P at a personal cost of θq. Output is perfectly divisible: the range of feasible outputs is $\Re_+$. P's return from q is $V(q)$ where $V(q)$ is twice continuously differentiable, increasing and strictly concave satisfying $\lim_{q\to 0} V^{'}(q) = +\infty$, $\lim_{q\to +\infty} V^{'}(q) = 0$ and $V(0) = 0$. These conditions imply that $q^*(\theta) \equiv \arg\max_q V(q) - \theta q$ is continuously differentiable, positive on $\theta \in [0, \infty)$ and strictly decreasing.

A is privately informed about the realization of θ; P and M share a common prior $F(\theta)$ over θ on the interval $\Theta \equiv [\underline{\theta}, \bar{\theta}] \subset \Re_+$. F has a density function $f(\theta)$ which is continuously differentiable and everywhere positive on its support. The 'virtual cost' $H(\theta) \equiv \theta + \frac{F(\theta)}{f(\theta)}$ is strictly increasing in θ; this assumption simplifies the analysis but is inessential to the results.

M plays no role in production, and costlessly acquires an informative signal η about θ. The underlying assumption is that the relevant knowledge concerning A's cost realization has already been acquired by M prior to contracting. The set of possible realizations of η is Π, a finite set with $\#\Pi \geq 2$. The finiteness of this set is assumed for technical convenience, and is relatively inessential as long as M's information regarding θ is not perfect. It is common knowledge that the realization of η is observed by both M and A. $a(\eta \mid \theta) \in [0, 1]$ denotes the likelihood function of η conditional on θ, which is common knowledge among all agents. $a(\eta \mid \theta)$ is continuously differentiable and positive on $\Theta(\eta)$, where $\Theta(\eta)$ denotes the set of values of θ for which signal η can arise with positive probability. We assume $\Theta(\eta)$ is an interval, for every $\eta \in \Pi$. Define $\underline{\theta}(\eta) \equiv \inf \Theta(\eta)$ and $\bar{\theta}(\eta) \equiv \sup \Theta(\eta)$. We assume that for any $\eta \in \Pi$, $a(\eta \mid \theta)$ is not a constant function on Θ, and there are some subsets of θ with positive measure such that $a(\eta \mid \theta) \neq a(\eta^{'} \mid \theta)$ for any $\eta, \eta^{'} \in \Pi$. In this sense each possible signal realization conveys information about the agent's cost. The information conveyed is partial, since Π is finite. This formulation includes both cases of full support and partition information structures.

The conditional density function and the conditional distribution function are respectively denoted by $f(\theta \mid \eta) \equiv f(\theta)a(\eta \mid \theta)/p(\eta)$ (where $p(\eta) \equiv \int_{\underline{\theta}(\eta)}^{\bar{\theta}(\eta)} f(\tilde{\theta})a(\eta \mid \tilde{\theta})d\tilde{\theta}$) and $F(\theta \mid \eta) \equiv \int_{\underline{\theta}(\eta)}^{\theta} f(\tilde{\theta} \mid \eta)d\tilde{\theta}$. The 'virtual' cost conditional on the signal η is $h(\theta \mid \eta) \equiv \theta + \frac{F(\theta|\eta)}{f(\theta|\eta)}$. We do not impose any monotonicity assumption for $h(\theta \mid \eta)$. Let $\hat{h}(\theta \mid \eta)$ be constructed from $h(\theta \mid \eta)$ and $F(\theta \mid \eta)$ by the ironing procedure introduced by Myerson (1981) (see the online Appendix for details regarding this procedure).

All players are risk neutral. P's objective is to maximize the expected value of $V(q)$, less expected payment to A and M, represented by X_A and X_M respectively. M's objective is to maximize expected transfers $X_M - t$ where t is a transfer from M to A. A seeks to maximize expected transfers received, less expected production costs, $X_A + t - \theta q$. Both A and M have outside options equal to 0.

In this environment, a feasible (deterministic) allocation is represented by $(u_A, u_M, q) = \{(u_A(\theta,\eta), u_M(\theta,\eta), q(\theta,\eta)) \in \Re^2 \times \Re_+ \mid (\theta,\eta) \in K\}$ where $K \equiv \{(\theta,\eta) \mid \eta \in \Pi, \theta \in \Theta(\eta)\}$, u_M, u_A denotes M and A's payoff respectively, and q represents the production level. P's payoff equals $u_P = V(q) - u_M - u_A - \theta q$. These payoffs relate to transfers and productions as follows: $u_A \equiv X_A + t - \theta q; u_M \equiv X_M - t; u_P \equiv V(q) - X_M - X_A$.

In the absence of collusion where P costlessly learns the realization of η, it is well known (e.g., adapting arguments of Baron and Myerson (1982)) that the resulting optimal or *second-best allocation* $(u_A^{SB}, u_M^{SB}, q^{SB})$ is as follows:

$$u_A^{SB}(\theta,\eta) = \int_{\theta}^{\bar{\theta}(\eta)} q^{SB}(y,\eta)dy,$$

$$E[u_M^{SB}(\theta,\eta) \mid \eta] = 0$$

and

$$q^{SB}(\theta,\eta) \equiv q^*(\hat{h}(\theta \mid \eta)) = \arg\max_q [V(q) - \hat{h}(\theta \mid \eta)q].$$

4.2 The Ex Ante Collusion Game

Owing to risk-neutrality of all parties, concavity of V and linearity of A's payoff in q, it is easy to check that P can restrict attention to a deterministic grand contract:

$$GC = (X_A(m_A, m_M), X_M(m_A, m_M), q(m_A, m_M); \mathcal{M}_A, \mathcal{M}_M)$$

where $\mathcal{M}_A$ (resp. $\mathcal{M}_M$) is a message set for A (resp. M).[10] This mechanism assigns a deterministic allocation, i.e. transfers X_M, X_A and output q, for any message $(m_A, m_M) \in \mathcal{M}_A \times \mathcal{M}_M$. $\mathcal{M}_A$ includes A's exit option $e_A \in \mathcal{M}_A$, with the property that $m_A = e_A$ implies

[10]Randomized contracts are optimal in Ortner and Chassang (2017) owing to their assumption that the contract offered to M by P is not observed by A. In our context, contracts are observed by both M and A, so there are no benefits of randomization.

$X_A = q = 0$ for any $m_M \in \mathcal{M}_M$. Similarly $\mathcal{M}_M$ includes M's exit option $e_M \in \mathcal{M}_M$, where $m_M = e_M$ implies $X_M = 0$ for any $m_A \in \mathcal{M}_A$.

The timing of events is as follows.

(C1) A observes θ and η, M observes η.

(C2) P offers a grand contract GC.

(C3) M and A play the side contract game described in more detail below.

As in existing literature, we assume the side-contract is costlessly enforceable. Moreover we assume M can make a take-it-or-leave-it offer of a side-contract. This assumption turns out to be inessential: Section 5.1 explains how the same results obtain with side contracts offered by an uninformed third party that assigns arbitrary welfare weights to M and A.

Conditional on any $\eta \in \Pi$ which is jointly observed by M and A, (C3) consists of the following three stages.

(i) M offers a side-contract SC which determines for any $\tilde{\theta} \in \Theta(\eta)$ to be privately reported by A to M, a probability distribution over joint messages $(m_A, m_M) \in \mathcal{M}_A \times \mathcal{M}_M$, and a side payment from M to A.[11] Formally, it is a pair of functions $\{\tilde{m}(\tilde{\theta}, \eta), t(\tilde{\theta}, \eta)\}$ where $\tilde{m}(\theta, \eta) : \Theta(\eta) \times \{\eta\} \longrightarrow \Delta(\mathcal{M}_A \times \mathcal{M}_M)$, the set of probability measures over $\mathcal{M}_A \times \mathcal{M}_M$, and $t : \Theta(\eta) \times \{\eta\} \longrightarrow \Re$. The case where M does not offer a side contract is represented by a null side-contract (NSC) with zero side payments ($t(\theta, \eta) \equiv 0$), and (deterministic) messages $(m_A(\theta, \eta), m_M(\eta))$ which is a noncooperative Bayesian equilibrium of the grand contract relative to the prior beliefs. We abuse terminology slightly and refer to the situation where no side contract is offered as one where NSC is offered.

(ii) A either accepts or rejects the SC offered, and the game continues as follows.

(iii) If A accepts the offered SC, he sends a private report $\theta' \in \Theta(\eta)$ to M, following which the SC is executed.[12] If A rejects SC, M updates his beliefs which is restricted

[11] The option of randomizing over possible messages is useful for technical reasons. Owing to quasilinearity of payoffs, there is no need to randomize over side transfers.

[12] Standard arguments show that the restriction to direct revelation mechanisms for the side contract entails no loss of generality.

to be prior beliefs if NSC was offered in stage (i) above.[13] A and M then play a noncooperative Bayesian equilibrium of the grand contract relative to the beliefs.

4.3 Suboptimality of Delegated Contracting

First consider the special case of *Delegation to M (DM)* where P delegates authority to M over contracting with A. Here the GC designed by P involves a null contract for A: the latter submits no report to P directly, and receives no production instructions or payments from P. P contracts only with M, requiring the latter to send a message m_M to P which determines the output $q(m_M)$ and aggregate payment $X(m_M)$ to the (M,A) coalition. Following receipt of this offer, M designs a side contract for A which selects an output $Q(m_A)$ and payment $X_A(m_A)$ to the latter as a function of a message m_A sent by A to M, provided A accepts the side contract. After receiving A's message (and conditional on A agreeing to participate), M submits a participation decision and message m_M to P. In contrast to the interim collusion setting, M can postpone submission of the participation decision *after* receiving a report from A.

Our first main result is that delegation is *never* optimal in ex ante collusion, as it is strictly dominated by the case where M is not consulted at all, which we refer to as *No Monitor (NM)*.

Proposition 4 *Delegation to M generates lower expected profit for the Principal compared to the optimal NM mechanism with no monitor.*

The FLM result concerning optimality of delegation in an interim collusion setting with two cost types therefore does not extend to ex ante collusion.[14] The underlying argument

[13]This ensures that it is immaterial whether or not NSC was accepted or rejected, since in either case they play the grand contract non-cooperatively with prior beliefs.

[14]It can be shown, however, that the optimal allocation under ex ante collusion can be achieved by a modified form of delegation, where P communicates and transacts only with M on the equilibrium path. In this arrangement, M is 'normally' expected to contract on behalf of the coalition $\{M, A\}$ with P, sending a joint participation decision and report of the state (θ, η) to P after having entered into a side contract with A. However A has the option of circumventing this 'normal' procedure and asking P to activate a grand contract in which A and M will send independent reports and participation decisions to P. The presence of this option ensures that A has sufficient bargaining power within the coalition; it does not have to be 'actually' used on the equilibrium path.

(extending Proposition 1 in Mookherjee and Tsumagari (2004) for a setting with two agents and no supervisor or intermediary) is simple and very general (e.g., it can be shown to extend to a discrete type setting also). P contracts for delivery of the good with M, so the problem reduces to contracting with a single agent M. In order to deliver the good to P, M needs to procure it in turn from A. The cost that M expects to incur equals A's virtual cost function $h(\theta|\eta)$ corresponding to the signal observed by M. This is unambiguously higher than the delivery cost θ of A if P were to contract directly with A. This is the well-known problem of double marginalization of rents (DMR), arising due to exercise of monopsony power by M in side-contracting with A. Unlike the context of interim collusion, M can postpone her own participation decision *after* receiving A's report. This effectively translates into a kind of 'limited liability' constraint for M, which prevents P from taxing away upfront the rents earned by M.[15]

Given this result, we hereafter focus on centralized contracting, where P offers a non-null contract to both M and A in GC.

4.4 Centralized Contracting and Ex Ante Collusion Proofness

We now introduce the notion of ex ante collusion proofness in the context of centralized contracts. A more detailed justification for this solution concept is provided in the online Appendix.

Informally, an allocation is ex ante collusion proof (EACP) if M cannot benefit from offering a non-null side contract when the Principal selects a grand contract based on the associated direct revelation mechanism (i.e., when A and M make consistent reports about the state, the allocation corresponding to that state is chosen). Equivalently, the null side contract is optimal for M, when the outside option of A corresponds to the latter's payoff in the resulting allocation.

Before proceeding to the formal definition, note that a *deterministic allocation* can be represented by payoff functions $(u_A(\theta,\eta), u_M(\theta,\eta))$ of the true state (θ,η) combined with the output function $q(\theta,\eta)$, as these determine the Principal's payoff function $u_P(\theta,\eta) \equiv$

[15] While it is relatively easy to show that DM cannot dominate NM, the proof establishes the stronger result that DM is **strictly** dominated by NM. The proof of strict domination is also straightforward in the case that $h(\theta|\eta)$ is continuous and nondecreasing in θ over a common support $[\underline{\theta}, \bar{\theta}]$ for every η. In that case an argument based on Proposition 1 in Mookherjee and Tsumagari (2004) can be applied. In the general case there are a number of additional technical complications, but we show that the result still goes through.

$V(q(\theta,\eta)) - u_M(\theta,\eta) - u_A(\theta,\eta) - \theta q(\theta,\eta)$, and the aggregate net transfers of M (equals $u_M(\theta,\eta)$) and A (equals $u_A(\theta,\eta) + \theta q(\theta,\eta)$). For technical convenience we consider randomized allocations, though it will turn out they will never actually need to be used on the equilibrium path.[16] In a randomized allocation, $(u_A(\theta,\eta), u_M(\theta,\eta), q(\theta,\eta))$ denotes the expected payoffs of A, M and the expected output, conditional on the state (θ,η).

We now introduce notation for 'coalitional' contracts and incentives as follows; this will be useful in representing constraints imposed by ex ante collusion proofness. For (conditional expected) allocation $(u_A(\theta,\eta), u_M(\theta,\eta), q(\theta,\eta))$, define functions $(\hat{X}(m), \hat{q}(m))$ on domain $m \in \hat{\mathcal{M}} \equiv K \cup \{e\}$ (where $K \equiv \{(\theta,\eta) \mid \theta \in \Theta(\eta), \eta \in \Pi\}$) as follows:

$$(\hat{X}(\theta,\eta), \hat{q}(\theta,\eta)) = (u_A(\theta,\eta) + \theta q(\theta,\eta) + u_M(\theta,\eta), q(\theta,\eta))$$

where the message e represents a coordinated coalitional decision for both M and A to exit from P's mechanism, while the message (θ,η) represents a coordinated decision for M and A to agree to participate and send the common report (θ,η) to P. The key constraint distinguishing ex ante from interim collusion is:

$$(\hat{X}(e), \hat{q}(e)) = (0, 0).$$

Let $(\hat{X}(\theta,\eta), \hat{q}(\theta,\eta))$ denote corresponding expected values of the sum of payments $X_M + X_A$ made by the principal, and the output delivered, in state (θ,η). Also, let $\Delta(\hat{\mathcal{M}})$ denote the set of the probability measures on $\hat{\mathcal{M}}$, and use $\tilde{m} \in \Delta(\hat{\mathcal{M}})$ to denote a randomized message submitted by the coalition to P. With a slight abuse of notation, we shall denote the corresponding conditional expected allocation by $(\hat{X}(\tilde{m}), \hat{q}(\tilde{m}))$, which is defined on $\Delta(\hat{\mathcal{M}})$. $\tilde{m} = (\theta,\eta)$ or e will be used to denote the probability measure concentrated at (θ,η) or e respectively.

M's choice of an optimal (randomized) side-contract can be formally posed as follows. Given a grand contract and a noncooperative equilibrium recommended by P, let the corresponding conditional expected allocation as defined above be denoted by $(u_A(\theta,\eta), u_M(\theta,\eta), q(\theta,\eta))$ and $(\hat{X}(\tilde{m}), \hat{q}(\tilde{m}))$. For any $\eta \in \Pi$, the associated side-contracting problem $P(\eta)$ is to select $(\tilde{m}(\theta \mid \eta), \tilde{u}_A(\theta,\eta))$ to maximize M's expected payoff

$$E[\hat{X}(\tilde{m}(\theta \mid \eta)) - \theta \hat{q}(\tilde{m}(\theta \mid \eta)) - \tilde{u}_A(\theta,\eta) \mid \eta]$$

[16] This owes to the assumption that A's payoff is linear in the output produced.

subject to $\tilde{m}(\theta \mid \eta) \in \Delta(\hat{\mathcal{M}})$,

$$\tilde{u}_A(\theta, \eta) \geq \tilde{u}_A(\theta', \eta) + (\theta' - \theta)\hat{q}(\tilde{m}(\theta' \mid \eta))$$

for any $\theta, \theta' \in \Theta(\eta)$, and

$$\tilde{u}_A(\theta, \eta) \geq u_A(\theta, \eta)$$

for all $\theta \in \Theta(\eta)$. The first constraint states truthful revelation of the agent's true cost to M is consistent with the agent's incentives, and the second constraint requires A to attain a payoff at least as large as what he would expect to attain by playing the grand contract noncooperatively.

Definition 1 *The (conditional expected) allocation* $(u_A(\theta, \eta), u_M(\theta, \eta), q(\theta, \eta)) : K \rightarrow \Re^2 \times \Re_+$ *is ex ante collusion proof (EACP) if for every* $\eta \in \Pi$*:* $(\tilde{m}(\theta \mid \eta), \tilde{u}_A(\theta, \eta)) = ((\theta, \eta), u_A(\theta, \eta))$ *solves problem* $P(\eta)$*.*

4.5 Characterization of EACP Allocations

We now characterize EACP allocations. This requires us to define a family of 'modified' virtual cost functions, representing the effective cost incurred by the coalition in delivering a unit of output to P, following selection of an optimal side-contract.

Definition 2 *For any* $\eta \in \Pi$*,* $Y(\eta)$ *is a collection of* **coalition shadow cost (CSC)** *functions* $\pi(\cdot \mid \eta) : \Theta(\eta) \rightarrow \Re$ *which satisfy the following property. For any function in this collection, there exists a real-valued function* $\Lambda(\theta|\eta)$ *which is non-decreasing in* $\theta \in \Theta(\eta)$ *with* $\Lambda(\underline{\theta}(\eta) \mid \eta) = 0$ *and* $\Lambda(\bar{\theta}(\eta) \mid \eta) = 1$*, such that*

$$\pi(\theta|\eta) \equiv \theta + \frac{F(\theta \mid \eta) - \Lambda(\theta \mid \eta)}{f(\theta \mid \eta)} \tag{11}$$

Equation (11) modifies the usual expression for virtual cost $h(\theta|\eta) \equiv \theta + \frac{F(\theta|\eta)}{f(\theta|\eta)}$ by subtracting from it the non-negative term $\frac{\Lambda(\theta|\eta)}{f(\theta|\eta)}$. In order to overcome the DMR problem in delegation, in the centralized regime P contracts with both M and A, thereby providing A an outside option (of $u_A(\theta, \eta)$) that effectively raises his bargaining power *vis-a-vis* M while negotiating the side contract. Meeting a larger outside option for A effectively induces M to deliver a higher output to P: this is what paying a higher rent to A necessitates. The extent of DMR is then curbed: the shadow cost for the coalition in delivering a unit of output to

P is lowered. This lowering of the virtual cost is represented by the subtraction of the term $\frac{\Lambda(\theta|\eta)}{f(\theta|\eta)}$ from what it would have been $(h(\theta|\eta))$ under delegation. In the analogous context of contracting with a single agent with type dependent outside options (Jullien (2000)), $\Lambda(\theta \mid \eta)$ represents the shadow value of a uniform reduction in A's outside option for all types below θ. Clearly, the $\Lambda(\theta \mid \eta)$ function must be non-decreasing.

However, $\pi(\theta|\eta)$ is not the correct expression for the shadow cost of output for the coalition, if it is non-monotone in θ. In that case, it has to be replaced by its 'ironed' version, using the distribution function $F(\theta|\eta)$. Let the corresponding ironed version of $\pi(\theta|\eta)$ be denoted by $z(\theta|\eta)$: we call this a *coalition virtual cost function.*

Definition 3 *For any $\eta \in \Pi$, the set of* **coalition virtual cost (CVC)** *functions is the set*

$$Z(\eta) \equiv \{z(\cdot \mid \eta) \quad \textit{is the ironed version of some} \quad \pi(\cdot \mid \eta) \in Y(\eta)\}$$

of functions obtained by applying the ironing procedure to the set $Y(\eta)$ of CSC functions.[17] *Denote by $\Theta(\pi(\cdot \mid \eta), \eta)$ the corresponding pooling region of θ where $\pi(\cdot|\eta)$ is flattened by the ironing procedure.*

As the next result shows, every EACP allocation satisfies coalition participation and incentive constraints corresponding to some coalition virtual cost function z. Combined with an individual incentive compatibility constraint for A, and a constraint that output must be constant over regions where the ironing procedure flattens the underlying CSC function, these coalition constraints characterize EACP allocations.[18] The proof of this Proposition is provided in the online Appendix, as it extends well known methods from Jullien (2000).

Proposition 5 *The allocation (u_A, u_M, q) is EACP if and only if the following conditions hold for every η. There exists a CVC function $z(\cdot|\eta) \in Z(\eta)$ such that*

(i) For every $(\theta, \eta), (\theta^{'}, \eta^{'}) \in K \equiv \{(\theta, \eta) \mid \theta \in \Theta(\eta), \eta \in \Pi\}$,

$$X(\theta, \eta) - z(\theta \mid \eta)q(\theta, \eta) \geq X(\theta^{'}, \eta^{'}) - z(\theta \mid \eta)q(\theta^{'}, \eta^{'})$$

[17] The ironing procedure ensures these functions are continuous and non-decreasing. For further details, see the online Appendix.

[18] See Mookherjee and Tsumagari (2004), Celik (2008) and Pavlov (2008) for similar characterizations of collusion proof mechanisms.

$$X(\theta, \eta) - z(\theta \mid \eta)q(\theta, \eta) \geq 0$$

where $X(\theta, \eta) \equiv u_A(\theta, \eta) + u_M(\theta, \eta) + \theta q(\theta, \eta)$.

(ii) For any $\theta, \theta' \in \Theta(\eta)$, $u_A(\theta, \eta) \geq u_A(\theta', \eta) + (\theta' - \theta)q(\theta', \eta)$.

(iii) $q(\theta, \eta)$ *is constant on any interval of* θ *which is a subset of the corresponding pooling region of the CVC function* z.

Define an allocation to be EAC feasible if it is EACP and satisfies interim participation constraints for M and A: $E[u_M(\theta, \eta)|\eta] \geq 0$ for all $\eta \in \Pi$, $u_A(\theta, \eta) \geq 0$ for all $(\theta, \eta) \in K$. Finally, P's problem is to select among EAC feasible allocations to maximize her expected profit $\Pi \equiv E[V(q(\theta, \eta)) - u_M(\theta, \eta) - u_A(\theta, \eta) - \theta q(\theta, \eta)]$, where expectation is taken with respect to P's prior beliefs. Condition (i) represents the coalition incentive and participation constraints corresponding to contracting with a single agent with a unit cost of z. Condition (ii) is the individual incentive compatibility constraint for A. Condition (iii) states that the output must be constant over every interval in the pooling region. In comparing with the analysis of the indivisible good case, the choice of the output function $q(\theta, \eta)$ and agent's payoff $u_A(\theta, \eta)$ corresponds to the prices offered to A, while the payment function $X(\theta, \eta)$ corresponds to the choice of base pay X_0 and bonus b for M. It can be checked that requirement (i) above that the payment function $X(\theta, \eta)$ be incentive compatible for the coalition with a unit cost function $z(\cdot|\eta) \in Z(\eta)$ corresponds to the coalition incentive constraint (7) in the indivisible good case.

4.6 Results

Proposition 6 *With a divisible good and ex ante collusion, consulting M is always valuable.*

This result contrasts with the indivisible good case, where consulting M was sometimes not valuable. Intuitively, M is more valuable in the divisible good setting, owing to the additional scope for varying the quantity procured based on M's information. The proof starts with the optimal no-monitor contract, and constructs a small variation in the output function $q(., \eta^{**})$ for a cost signal state η^{**} satisfying a regularity condition.[19] The divisibility of the good implies the existence of such a state. The variation in output schedule

[19]The regularity condition requires that $[F(\theta|\eta^{**})/f(\theta|\eta^{**})]/[F(\theta)/f(\theta)]$ is increasing over some interval of θ with positive measure conditional on η^{**}. The proof shows that the informativeness of M's signal implies that such a state always exists.

corresponding to this state is constructed as follows. Starting with the optimal allocation in NM (which corresponds to the special case $\Lambda(\theta \mid \eta)$ is chosen equal to $F(\theta \mid \eta)$, ensuring that the CSC and CVC functions both reduce to the identity function ($\pi(\theta|\eta) = z(\theta|\eta) = \theta$)), P can construct a small variation $\tilde{z}$ in the CVC function in state η^{**}, raising it above θ for some interval Θ_H and lowering it for some other interval Θ_L. The corresponding quantity procured $q(\theta, \eta^{**})$ is set equal to $q^{NM}(\tilde{z}(\theta|\eta^{**}))$, the quantity procured in NM when the agent reported a cost of $\tilde{z}(\theta|\eta^{**})$. This corresponds to raising the quantity procured from the coalition over Θ_L and lowering it over Θ_H. Payments to the coalition are set analogously at $X^{NM}(\tilde{z}(\theta|\eta^{**}))$, what the agent would have been paid in NM following such a cost report. The agent is offered the associated rent: $u_A(\theta, \eta^{**}) = \int_\theta^{\bar{\theta}} q^{NM}(\tilde{z}(y|\eta^{**}))dy$.

By construction, this allocation satisfies the agent's incentive and participation constraints, as well as the coalition incentive constraint. Proposition 5 ensures such an allocation is EAC feasible, i.e., M's interim participation constraint is satisfied. The variation over Θ_L lowers rents earned by M, and over Θ_H raises them. Since M does not earn any rents to start with (i.e, in NM), it is necessary to construct the variation such that M's expected rents in state η^{**} do not go down. The rate at which M's rents vary locally in state θ with the quantity procured equals $\frac{F(\theta|\eta^{**})}{f(\theta|\eta^{**})}$.[20] Intuitively this is the saving that can be pocketed by M when procuring one less unit of the good from A. Maintaining M's expected rent therefore implies a marginal rate of substitution between output variations over Θ_L and Θ_H that equals the ratio of the (average) conditional inverse hazard rates $\frac{F(\theta|\eta^{**})}{f(\theta|\eta^{**})}$ over these two intervals respectively. On the other hand, P's benefit from a small expansion in output delivered in state θ equals $V'(q^{NM}(\theta)) - \theta$, where $q^{NM}(\theta)$ denotes the optimal allocation in NM.[21] This allocation satisfies $V'(q^{NM}(\theta)) = H(\theta) \equiv \theta + \frac{F(\theta)}{f(\theta)}$, the virtual cost of procurement without conditioning on information regarding η. Hence P's marginal benefit from output expansion in state θ equals the unconditional inverse hazard rate $\frac{F(\theta)}{f(\theta)}$. This implies that P's marginal rate of substitution between output variations over Θ_L and Θ_H equals the ratio of the (average) unconditional inverse hazard rates $\frac{F(\theta)}{f(\theta)}$ over these two intervals. The informativeness of M's signals implies that P's marginal rate of substitution differs from M's in state η^{**} over Θ_L, Θ_H. Hence there exist variations of the type described

[20] M's interim rent in state η^{**} equals the expected value conditional on η^{**} of $X^{NM}(\tilde{z}(\theta|\eta^{**})) - u_A(\tilde{z}(\theta|\eta^{**})) - \theta q^{NM}(\tilde{z}(\theta|\eta^{**}))$, i.e., equals $E[\{\tilde{z}(\theta|\eta^{**}) - h(\theta|\eta^{**})\}q^{NM}(\tilde{z}(\theta|\eta^{**})) - \int_{\tilde{z}(\theta|\eta^{**})}^{\bar{\theta}} q^{NM}(z)dz|\eta^{**}]$.

[21] This follows from the fact that $\frac{\partial X^{NM}(z)}{\partial z} = zq^{NM'}(z)$, implying that the marginal increase in payment evaluated at $z = \theta$ equals θ times the marginal output change.

above which raise P's expected payoff, while preserving the expected payoff of M.

The next result provides sufficient conditions for ex ante collusion to be costly, thereby providing a contrast with interim collusion in team production (Che-Kim (2006)) or supervision (Motta (2009)) settings.

Proposition 7 *With a divisible good and ex ante collusion, the second-best payoff cannot be attained if :*

(i) The support of θ does not vary with the signal: $\Theta(\eta) = \Theta$ for any $\eta \in \Pi$;

(ii) There exists $\eta^* \in \Pi$ such that $f(\theta|\eta^*)$ and $\frac{f(\theta|\eta^*)}{f(\theta|\eta)}$ are both strictly decreasing in θ for any $\eta \neq \eta^*$;

(iii) $\underline{\theta} f(\underline{\theta} \mid \eta^*) > 1$;

(iv) $V'''(q) \leq \frac{(V''(q))^2}{V'(q)}$ for any $q \in Q^{SB} \equiv \{\tilde{q} | \tilde{q} = q^{SB}(\theta, \eta)$ for some $(\theta, \eta) \in K\}$.

Condition (ii) includes a weaker version of the monotone likelihood ratio property: there is a signal realization η^* which is 'better' news about θ than any other realization, in the sense of shifting weight in favor of low realizations of θ. It additionally requires that the conditional density $f(\theta|\eta^*)$ is strictly decreasing in θ, i.e., higher realizations of θ are less likely than low realizations when $\eta = \eta^*$. (ii) is satisfied for instance when θ has a uniform prior and there are just two possible signal values satisfying the standard monotone likelihood ratio property. Condition (iii) says that costs are high in the sense that the support of the cost distribution is shifted sufficiently to the right. Finally (iv) is a condition on the benefit function, which is satisfied if V is exponential ($V = V_0[1 - \exp(-rq)]; V_0 > 0, r > 0$).[22]

The proof develops necessary conditions for the second best to be EAC feasible given the distributional properties (i) and (ii). If the outputs are second-best, they must be a monotone decreasing function of the (ironed) virtual cost $\hat{h}(\theta \mid \eta)$ in the second-best setting. If they also satisfy the coalition incentive constraints, they must be monotone in CVC $z(\theta \mid \eta)$. These conditions imply the existence of a monotone transformation from

[22]This benefit function does not satisfy the Inada conditions assumed in the model. However, the only purpose of imposing the Inada conditions was to ensure that optimal allocations would always involve strictly positive quantities procured in all states. This will be the case here if V_0 is large enough.

$\hat{h}$ to z, and enable M's ex post rent to be expressed as a function of $\hat{h}$ alone. Condition (iv) is used to show that this rent function is strictly convex; combined with (i) and (ii) this implies that the expected rents of M must be strictly higher (hence strictly positive) in state η^* than any other state. Then M must earn positive rents in state η^*, which ensures the second best cannot be achieved.

5 Extensions

5.1 Side Contracts Designed by a Third Party, and Alternative Allocations of Bargaining Power

We now explain how the preceding results extend when the side contract is designed not by M, but instead by a third-party that manages the coalition and assigns arbitrary welfare weights to the payoffs of M and A respectively. Such a formulation has been used by a number of authors to model collusion, such as Laffont and Martimort (1997, 2000), Dequiedt (2007) and Celik and Peters (2011). An advantage of this approach is that it enables us to examine effects of varying the allocation of bargaining power between colluding partners.

Our results extend to such a setting, under the following formulation of side contracts designed by a third party. We assume the third-party's objective is to maximize a weighted sum of M and A's interim payoffs. In the subgame (C3) following choice of a grand contract by P, the third party designs the side contract after learning the realization of η.[23] Both M and A have the option to reject the side contract; if either of them does, they play the grand contract noncooperatively. Otherwise the side contract mechanism is executed.

The notion of EACP allocations is extended as follows. Letting $\alpha \in [0, 1]$ denote the welfare weight assigned by the third-party to A's payoff, the side contract design problem reduces to selecting randomized message $\tilde{m}(\theta \mid \eta)$ and A's payoff $\tilde{u}_A(\theta, \eta)$ to (using the same notation for the formulation $P(\eta)$ of side contracts in Section 4.4):

$$\max E[(1-\alpha)\{\hat{X}(\tilde{m}(\theta \mid \eta)) - \theta\hat{q}(\tilde{m}(\theta \mid \eta)) - \tilde{u}_A(\theta, \eta)\} + \alpha\tilde{u}_A(\theta, \eta) \mid \eta]$$

subject to $\tilde{m}(\theta \mid \eta) \in \Delta(\hat{\mathcal{M}})$,

$$\tilde{u}_A(\theta, \eta) \geq u_A(\theta, \eta)$$

[23] This assumption can be dropped without affecting the results, since it can be shown the third-party can use cross-reporting of η by M and A to learn its true value.

for all $\theta \in \Theta(\eta)$,

$$\tilde{u}_A(\theta, \eta) \geq \tilde{u}_A(\theta^{'}, \eta) + (\theta^{'} - \theta)\hat{q}(\tilde{m}(\theta^{'} \mid \eta))$$

for any $\theta, \theta^{'} \in \Theta(\eta)$, and

$$E[\hat{X}(\tilde{m}(\theta \mid \eta)) - \theta\hat{q}(\tilde{m}(\theta \mid \eta)) - \tilde{u}_A(\theta, \eta) \mid \eta] \geq E[u_M(\theta, \eta) \mid \eta].$$

Besides modifying the objective function, this formulation adds a participation constraint for M. We refer to this as problem $TP(\eta; \alpha)$. The definition of EACP can be extended to EACP(α) by requiring the null side contract to be optimal in $TP(\eta; \alpha)$ for every η. Further details concerning the justification of this solution concept is provided in the online Appendix.

We now show that the set of EACP(α) allocations is independent of α. This implies that all our preceding results extend to side contracts designed by a third party.[24]

Proposition 8 *The set of EACP(α) allocations is independent of* $\alpha \in [0, 1]$.

Despite the existence of asymmetric information within the coalition, the Coase Theorem applies. The reasoning is straightforward. The EACP criterion amounts to the absence of incentive compatible deviations that are Pareto improving for the coalition: this property does not vary with the precise welfare weights. Consider any $\alpha \in (0, 1)$. A given allocation is EACP(α) if and only if there is no other allocation attainable by some non-null side contract which satisfies the incentive constraint for A, and which Pareto-dominates it (for A and M) with at least one of them strictly better off. The same characterization applies to any interior $\alpha' \in (0, 1)$, implying that the set of EACP(α) allocations is independent of $\alpha \in (0, 1)$. The transferability of utility can then be used to show that the set of EACP allocations for interior welfare weights are also the same at the boundary.

5.2 Altruistic Expert

Now consider a different variant, where M offers a side-contract to A, but M is altruistic towards A rather than just concerned with his own income. Suppose M's payoff is $u_M = X_M + t + \alpha[X_A - t - \theta q]$, where $\alpha \in [0, 1]$ is the weight M places on A's payoff. A's payoff function remains the same as in the previous section: $u_A = X_A - t - \theta q$.

[24] FLM provide an analogous result for the case of interim collusion.

Our analysis extends as follows. The expression for coalition shadow cost is now modified to

$$\pi_\alpha(\theta|\eta) \equiv \theta + (1-\alpha)\frac{F(\theta \mid \eta) - \Lambda(\theta \mid \eta)}{f(\theta \mid \eta)}$$

instead of $\pi(\theta|\eta)$ in Definition 2. When P delegates to M, the corresponding expression for the cost of procuring one unit from M is modified from $h(\theta \mid \eta)$ to $h_\alpha(\theta \mid \eta) = \theta + (1-\alpha)\frac{F(\theta|\eta)}{f(\theta|\eta)}$. As long as $\alpha < 1$, this is strictly higher than θ, so delegation will still continue to result in a lower profit than NM. The proof that M is valuable under centralized contracting also goes through *in toto*.

It is interesting to examine the effect of changes in the degree of altruism on P's payoffs. When P delegates to M, an increase in α lowers M's shadow cost of output $h_\alpha(\theta \mid \eta)$, which benefits P. This is intuitive: the DMR problem becomes less acute with a more altruistic expert. Note that with perfect altruism $\alpha = 1$, and the DMR problem disappears: delegation then becomes equivalent to NM.

On the other hand, an increase in altruism cannot benefit P in centralized contracting. The set of EACP allocations can be shown to be non-increasing in α. Take any EACP allocation corresponding to α: the following argument shows that it is a EACP allocation corresponding to any $\alpha' < \alpha$. Let $z(\theta \mid \eta)$ be the CVC function that is associated with the allocation at α, i.e., it is the ironed version of $\pi_\alpha(\theta|\eta)$ corresponding to some function $\Lambda_\alpha(\cdot|\eta)$ satisfying the stipulated requirements in the definition of CSC functions on $[\underline{\theta}(\eta), \bar{\theta}(\eta)]$. We can then select

$$\Lambda_{\alpha'}(\theta \mid \eta) = \frac{\alpha - \alpha'}{1-\alpha'}F(\theta \mid \eta) + \frac{1-\alpha}{1-\alpha'}\Lambda_\alpha(\theta \mid \eta)$$

when the altruism parameter is α', which satisfies the stipulated requirements since $\alpha > \alpha'$. This ensures that the same CSC and CVC function is available when the altruism parameter is α', since by construction $\pi_\alpha(\theta|\eta) = \pi_{\alpha'}(\theta|\eta)$. Hence the allocation satisfies the sufficient condition for EACP when the altruism parameter is α'.

Finally, if $\alpha = 1$, the CSC function π_α coincides with the identity function θ, the cost of the agent in NM. We thus obtain

Proposition 9 *In centralized contracting, P's optimal payoff is non-increasing in α. When P delegates to M, P's optimal payoff is increasing in α. When $\alpha = 1$, P's optimal payoffs in delegation, centralized contracting coincide and equal that in NM, so M is not valuable.*

6 Concluding Comments

Our results have interesting implications for hierarchical contract design. They provide a rationale for the widespread practice of consulting third party experts in the design of incentive contracts, even when they have 'prior connections' with the agent that could facilitate ex ante collusion. In such circumstances, unconditional delegation is suboptimal; the mechanism must allow the agent to communicate directly with the Principal. The Principal could appoint the expert as a 'manager', i.e., contract only with the expert and delegate subcontracting with the agent on the equilibrium path, but the agent must be provided the option to bypass the manager and register an 'appeal' with the Principal, prompting the latter to intervene directly. The existence of such off-equilibrium-path options is essential for the Principal to be able to control the prospect of ex ante collusion sufficiently to make it profitable to consult the expert. Within firms, it explains the role of worker rights to appeal the evaluations reported by their managers to higher level managers or an ombudsman appointed for this purpose, thereby formalizing Williamson's (1975) claim that such dispute settlement procedures constitute an advantage of hierarchies over market relationships.[25]

The result concerning effects of altruism of M towards A implies that the Principal ought to appoint 'external' experts rather than 'insiders' likely to be altruistic towards the agent. In the context of corporate governance, for instance, this is an argument in favor of appointing 'outsiders' rather than 'insiders' to a company's Board of Directors.[26] In the context of regulation, it confirms the normal intuition in favor of preventing any direct conflict of interest for the regulator (e.g., who should not have a financial stake in the agent's fortunes, nor have any social or personal connections with the agent).

Extensions of the model to bilateral asymmetric information within the coalition (e.g., if A does not observe M's signal) and to discrete type spaces are examined respectively in Tsumagari (2016a,b). Mookherjee and Tsumagari (2017) show that the allocation of bargaining power between M and A does matter in the case of 'strong' collusion, e.g., where the side contract includes commitments regarding subsequent actions by one partner

[25]It also relates to Hirschman's (1970) depiction of the value of 'voice' within organizations over and above exit options.

[26]See Harris and Raviv (2008) for a model based on limited commitment by P where this result may not hold in some settings.

if the other refuses it.

Our analysis is subject to a number of shortcomings. We excluded the possibility of other coalitions that may co-exist with the M-A coalition, a topic studied by Ortner and Chassang (2017). We also ignored the enforcement of side contracts within the coalition; modeling self-enforcing collusion via a relational contract in a side game between colluding parties seems to be an interesting extension that could be pursued in future research.

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Appendix: Proofs

Proof of Lemma 1: (i) is straightforward. To establish (ii), for any $q \in [0,1]$, define $P_i(q) \in [\underline{\theta}, \bar{\theta}]$ such that $F_i(P_i(q)) = q$ and $C_i(q) \equiv qP_i(q)$. So we may interpret $C_i(q)$ as the 'cost' function in state i. Since $C_i^{'}(F_i(\theta)) = h_i(\theta)$, Assumption 1 (ii) implies $C_i^{'}(q)$ is increasing in q on $[0,1]$. Then $q_i^{SB} \equiv F_i(p_i^{SB})$ satisfies $V = C_i^{'}(q_i^{SB})$ for $V \in (\underline{\theta}, h_i(\bar{\theta}))$. From Assumption 1 (i) and $f_i(\theta) > 0$ on $[\underline{\theta}, \bar{\theta}]$ for $i \in \{L, H\}$, $C_L(q) < C_H(q)$ on $q \in (0,1)$ with $C_L(0) = C_H(0) = 0$ and $C_L(1) = C_H(1) = \bar{\theta}$. Hence there are intervals of small q such that $C_L^{'}(q) < C_H^{'}(q)$ and large q such that $C_L^{'}(q) > C_H^{'}(q)$. This guarantees the existence of V^* and V^{**} with the stated properties. ■

Proof of Lemma 2: (i) If $b \geq p_i, i = L, H$, the optimal $X_0 = 0$. P's profit (8) then equals $[\kappa_H F_H(p_H) + \kappa_L F_L(p_L)](V - b)$, which is non-negative only if $V - b \geq 0$. This implies that P's profit is (weakly) dominated by the allocation $\tilde{p}_H = \tilde{p}_L = b$, which in turn is weakly dominated by what P could earn in NM. (ii) The interim participation constraints imply that M will attain a nonnegative rent. Hence P's profit is bounded above by $U(p_L, p_H)$. If $p_L \geq p_H$, the value of $U(p_L, p_H)$ is smaller than the maximum value of $U(\tilde{p}_L, \tilde{p}_H)$ subject to the constraint that $\tilde{p}_L \geq \tilde{p}_H$. The constraint must bind, since the unconstrained solution is represented by second-best prices which violate the constraint. Hence the maximum value of the constrained problem is realized at $\tilde{p}_H = \tilde{p}_L = p^{NM}$. The expected profit of P would then be dominated by the NM allocation where P offers p^{NM} to A in both states. (iii) Parts (i) and (ii) imply that in order to dominate the best NM allocation, an EAC feasible allocation must satisfy $p_H - b > \max\{0, p_L - b\}$. So if (iii) did not hold, $F_H(p_H)(p_H - b) \geq F_L(p_L)(p_L - b)$, and optimal $X_0 = F_H(p_H)(p_H - b)$. Then as $F_H(p_H) \geq F_L(p_L)$ implies $\kappa_H F_L(p_L) + \kappa_H F_H(p_H) \leq F_H(p_H)$, and $V \geq b$ to ensure that P earns non-negative profit, it follows that P's profit equal $[\kappa_L F_L(p_L) + \kappa_H F_H(p_H)](V - b) - F_H(p_H)(p_H - b) \leq F_H(p_H)(V - p_H) \leq F(p_H)(V - p_H) \leq \Pi^{NM}$, a contradiction. ■

Proof of Lemma 3: To start with, note that the restrictions $p_L < p_H$ and $F_L(p_L) > F_H(p_H)$ imply that the prices are interior: $\underline{\theta} < p_i < \bar{\theta}, i = H, L$. Hence the coalition incentive constraint (7) simplifies to $\max_i\{l_i(p_i)\} \leq b \leq \min_i\{h_i(p_i)\}$. Next, note that upon substituting for the optimal base pay X_0, the expression for M's expected rent reduces to

$$\tilde{R}(b; p_L, p_H) \equiv \kappa_L F_L(p_L)(b - p_L) + \kappa_H F_H(p_H)(b - p_H) - \min\{F_L(p_L)(b - p_L), F_H(p_H)(b - p_H)\}. \tag{12}$$

Clearly $\tilde{R}$ is non-negative and attains a global minimum of zero at $b = B(p_L, p_H) < p_L < p_H$. If $B(p_L, p_H) \geq \max\{l_L(p_L), l_H(p_H)\}$, it is feasible to select $b = B(p_L, p_H)$ as the coalition incentive constraint (7) is satisfied (given that $p_i \leq h_i(p_i), i = H, L$), as well as the constraint that $b < p_H$. Hence in this case the optimal bonus equals $B(p_L, p_H)$. If $B(p_L, p_H) < \max\{l_L(p_L), l_H(p_H)\}$, then observe that over the range $b \geq B(p_L, p_H)$, $(b - p_L)F_L(p_L) \geq (b - p_H)F_H(p_H)$, implying that $X_0 = F_H(p_H)(b - p_H)$, or

$$\tilde{R} = \kappa_L[\{F_L(p_L) - F_H(p_H)\}b - p_L F_L(p_L) + p_H F_H(p_H)]. \tag{13}$$

Hence $\tilde{R}$ is strictly increasing in b over the range $b \geq B(p_L, p_H)$, and the optimal bonus in this case equals $\max\{l_L(p_L), l_H(p_H)\}$. ■

Having solved for the optimal bonus corresponding to a given set of prices, we can now turn to the problem of selecting these prices optimally.

Proof of Proposition 2: (a) By Lemma 1, $F_L(p_L^{SB}) > F_H(p_H^{SB})$ for V close to $\underline{\theta}$. As V approaches $\underline{\theta}$, p_i^{SB} approaches $\underline{\theta}$ for both $i = H, L$, and $B(p_L^{SB}, p_H^{SB})$ approaches $\underline{\theta} > \max_i\{l_i(\underline{\theta})\}$, implying $b(p_L^{SB}, p_H^{SB}) = B(p_L^{SB}, p_H^{SB})$ for V sufficiently close to $\underline{\theta}$. So $(p_L, p_H, b) = (p_L^{SB}, p_H^{SB}, B(p_L^{SB}, p_H^{SB}))$ is EAC feasible, implying the second-best profit can be achieved for V close to $\underline{\theta}$.

(b) $V < H(\bar{\theta})$ implies $p^{NM} < \bar{\theta}$. For any such V, we can find p_L, p_H sufficiently close to p^{NM} satisfying $p_L^{SB} \leq p_L < p^{NM} < p_H \leq p_H^{SB}, F_L(p_L) > F_H(p_H)$ and $\max_i\{l_i(p_i)\} < B(p_L, p_H)$ (since $B(p, p) = p > l_i(p), i = L, H$ for any $p < \bar{\theta}$). The allocation $(p_L, p_H, B(p_L, p_H))$ is then EAC feasible, in which M earns zero rent, and P earns a profit of $U(p_L, p_H) > U(p^{NM}, p^{NM}) = \Pi^{NM}$.

Next we show that M is not valuable at $V = \hat{V} \equiv \kappa_L h_L(\bar{\theta}) + \kappa_H h_H(\bar{\theta}) < h_L(\bar{\theta})$. Suppose otherwise, whence $F_L(p_L^E) > F_H(p_H^E)$ by Lemma 2. Note that $\hat{V} = \bar{\theta} + [\kappa_L \frac{1}{f_L(\bar{\theta})} + \kappa_H \frac{1}{f_H(\bar{\theta})}] > \bar{\theta} + \frac{1}{\kappa_L f_L(\bar{\theta}) + \kappa_H f_H(\bar{\theta})} = H(\bar{\theta})$. Hence $\Pi^{NM}(\hat{V}) = \hat{V} - \bar{\theta} = \kappa_L(h_L(\bar{\theta}) - \bar{\theta}) + \kappa_H(h_H(\bar{\theta}) - \bar{\theta})$. Now $\bar{\theta}$ is the second-best price when V equals $h_i(\bar{\theta})$ in state i, implying $h_i(\bar{\theta}) - \bar{\theta} \geq F_i(p_i^E)(h_i(\bar{\theta}) - p_i^E)$. Hence $\Pi^{NM}(\hat{V}) \geq \kappa_L F_L(p_L^E)(h_L(\bar{\theta}) - p_L^E) + \kappa_H F_H(p_H^E)(h_H(\bar{\theta}) - p_H^E) \geq \kappa_L F_L(p_L^E)(\hat{V} - p_L^E) + \kappa_H F_H(p_H^E)(\hat{V} - p_H^E) = U(p_L^E, p_H^E)$, where the second inequality follows from the definition of $\hat{V}$ and $F_L(p_L^E) > F_H(p_H^E)$. Since P's expected profit in EAC is bounded above by $U(p_L^E, p_H^E)$, we obtain a contradiction. Hence it is optimal to offer $p_i = \bar{\theta}$ for both i at $\hat{V}$. By a standard revealed preference argument, these prices are also optimal at any higher V. Hence M is not valuable at any $V > \hat{V}$.

(c) We first show that M's rent is locally non-decreasing in p_H at (p_L^E, p_H^E). If $B(p_L^E, p_H^E) > \max_i\{l_i(p_i^E)\}$, M earns zero rent which is unaffected by small variations in p_H. So suppose $B(p_L^E, p_H^E) \leq \max_i\{l_i(p_i^E)\}$ in which case $b^E = \max_i\{l_i(p_i^E)\}$ and $R^*(p_L^E, p_H^E) = \kappa_L[\{F_L(p_L^E) - F_H(p_H^E)\}\max_i\{l_i(p_i^E)\} + F_H(p_H^E)p_H^E - F_L(p_L^E)p_L^E] = \kappa_L \max_i \rho_i(p_H^E, p_L^E)$ where $\rho_i(p_H, p_L) \equiv \{F_L(p_L) - F_H(p_H)\}l_i(p_i) + F_H(p_H)p_H - F_L(p_L)p_L$. Now ρ_L is locally nondecreasing in p_H at (p_L^E, p_H^E) because $F_H(p_H)[p_H - l_L(p_L)]$ is increasing in p_H at (p_L^E, p_H^E) (the latter in turn follows from Lemma 2 and (7) which together imply $p_H^E > b^E = \max_i\{l_i(p_i^E)\} \geq l_L(p_L^E)$). And ρ_H is nondecreasing in p_H over the range of p_H which satisfies $F_L(p_L) > F_H(p_H)$ since $l_H'(p_H)[F_L(p_L) - F_H(p_H)] + f_H(p_H)[h_H(p_H) - l_H(p_H)] \geq 0$.

It now follows that if $p_H^E > p_H^{SB}$, a slight lowering of p_H will have a positive first order effect on $U(p_L, p_H)$, without raising M's rent. Hence $p_H^E \leq p_H^{SB}$.

(d) We show that M's rent is locally non-increasing in p_L at (p_L^E, p_H^E) if $l_L(p_L)$ is convex. When $B(p_L^E, p_H^E) > \max_i\{l_i(p_i^E)\}$, M's rent is zero which does not vary locally with p_L. So suppose $B(p_L^E, p_H^E) \leq \max_i\{l_i(p_i^E)\}$ implying that $R^*(p_L^E, p_H^E) = \kappa_L \max_i \rho_i(p_H^E, p_L^E)$. Now $F_L(p_L)[l_H(p_H^E) - p_L]$ is locally non-increasing in p_L at p_L^E, since its partial derivative with respect to p_L at p_L^E equals $f_L(p_L^E)[l_H(p_H^E) - h_L(p_L^E)]$, which is non-positive as (7) implies $l_H(p_H^E) \leq b^E \leq h_L(p_L^E)$. Hence ρ_H is locally nonincreasing in p_L at (p_L^E, p_H^E). The result therefore holds when $\rho_L(p_L^E, p_H^E) < \rho_H(p_L^E, p_H^E)$.

Next consider the case where $\rho_H(p_L^E, p_H^E) \leq \rho_L(p_L^E, p_H^E) = \frac{R^*(p_L^E, p_H^E)}{\kappa_L}$. Since $\frac{\partial \rho_L}{\partial p_L} = l_L'(p_L)[F_L(p_L) - F_H(p_H)] - 1$, the convexity of $l_L(p_L)$ implies the conxevity of ρ_L in p_L over the range of p_L which satisfies $F_L(p_L) > F_H(p_H)$ for any fixed value of p_H. Now as p_L approaches p_H^E, $\rho_L(p_L, p_H^E)$ approaches $[F_L(p_H^E) - F_H(p_H^E)][l_L(p_H^E) - p_H^E] < 0$. Since $\rho_L(p_L^E, p_H^E) \geq 0$, there must exist $\tilde{p}_L \in [p_L^E, p_H^E)$ where $\rho_L(\tilde{p}_L, p_H^E) = 0$ and ρ_L is locally decreasing in p_L. The convexity of $\rho_L(p_L, p_H^E)$ in p_L then implies that $\rho_L(p_L, p_H^E)$ is also locally decreasing in p_L at every p_L which satisfies $p_L \leq \tilde{p}_L$ and $F_L(p_L) > F_H(p_H^E)$. Since $p_L^E \leq \tilde{p}_L$, it follows that ρ_L is locally decreasing in p_L at (p_L^E, p_H^E).

It now follows that if $p_L^E < p_L^{SB}$, a slight increase in p_L will have a positive first-order effect on $U(p_L, p_H)$, without raising M's rent. Hence $p_L^E \geq p_L^{SB}$. ■

Proof of Lemma 5: Given any pair of prices satisfying $p_L < p_H$ and $F_H(p_H) > F_L(p_L)$, the optimal bonus must minimize M's rent subject to $b > p_H$ and the coalition incentive constraint (7). M earns zero rent at the bonus $B(p_L, p_H) = \frac{p_H F_H(p_H) - p_L F_L(p_L)}{F_H(p_H) - F_L(p_L)}$ which is now larger than p_H. Since the choice of b is restricted to the range $b > p_H$ where

$b > \max_i\{\hat{l}_i(p_i)\}$ is automatically satisfied, the bonus $B(p_L, p_H)$ is optimal if $B(p_L, p_H) \leq \min_i\{\hat{h}_i(p_i)\}$. Otherwise, $B(p_L, p_H) > \min_i\{\hat{h}_i(p_i)\}$ and the choice of b is restricted to the range $(p_H, \min_i\{\hat{h}_i(p_i)\}]$. Over this range $b < B(p_L, p_H)$ which implies $F_H(p_H)(b - p_H) < F_L(p_L)(b - p_L)$ and therefore $X_0 = -F_H(p_H)(b - p_H)$. The expression for M's rent is then modified to $\tilde{R}(b; p_L, p_H) = \kappa_L F_L(p_L)(b - p_L) + \kappa_H F_H(p_H)(b - p_H) - F_H(p_H)(b - p_H) = \kappa_L[\{F_L(p_L) - F_H(p_H)\}b + F_H(p_H)p_H - F_L(p_L)p_L]$, which is now *decreasing* in b.

To see that $b > V$ is necessary for M to be valuable, note that since the function $\tilde{R}(b; p_L, p_H)$ is decreasing in b, if $b \leq V$ then $p_H < V$, implying that P's profit is bounded above by $U(p_L, p_H) - \kappa_L[\{F_L(p_L) - F_H(p_H)\}V + F_H(p_H)p_H - F_L(p_L)p_L] = F_H(p_H)(V - p_H) \leq F(p_H)(V - p_H)$, the profit attained in NM upon choosing the price of p_H in both states. ■

Proof of Lemma 6: (i) As explained in the text, an INC optimal allocation which is infeasible in EAC must involve $p_L < p_H, F_H(p_H) > F_L(p_L)$ and in which M is valuable (since any allocation in NM is feasible in EAC). P attains profit $\Pi = [\kappa_H F_H(p_H) + \kappa_L F_L(p_L)](V - b) + F_H(p_H)(b - p_H) = \kappa_L F_L(p_L)(V - b) + F_H(p_H)[\kappa_H V + \kappa_L b - p_H]$. By Lemma 5, it is necessary that $b > V$. To show that this can be attained via INC with delegation, we need to show that if $p_i < \bar{\theta}$ then $b = h_i(p_i)$, while if $p_i = \bar{\theta}$ then $b \geq h_i(\bar{\theta})$.

Suppose first that $p_i < \bar{\theta}$ for either i. Then $\hat{h}_i(p_i) = h_i(p_i) \geq b$. If $i = L$ and $h_L(p_L) > b$, note that Π is strictly decreasing in p_L, so profit can be raised by lowering p_L slightly. Similarly, if $i = H$ and $b < h_H(p_H)$, we have $\kappa_H V + \kappa_L b < h_H(p_H)$, implying $F_H(p_H)[\kappa_H V + \kappa_L b - p_H]$ is locally strictly decreasing in p_H, and profit can be raised by lowering p_H slightly.

Next, suppose $p_i = \bar{\theta}$. If $b < h_i(\bar{\theta})$, the same argument as above applies: profit can be raised by lowering p_i slightly. Hence it must be the case that $b \geq h_i(\bar{\theta})$.

(ii) From (i), an INC optimal allocation which is EAC infeasible satisfies $p_i = p_i(b)$ which maximizes $F_i(p)(b - p)$ with respect to choice of $p \in [\underline{\theta}, \bar{\theta}]$. Since $F_L(p) > F_H(p)$ for all $p \in (\underline{\theta}, \bar{\theta})$, it must be true that $F_L(p_L)(b - p_L) \geq F_H(p_H)(b - p_H)$, with strict inequality if $b < h_L(\bar{\theta})$. Hence $b < h_L(\bar{\theta})$ implies M earns positive rent in state L (as $X_0 = -F_H(p_H)(b - p_H)$), and second-best profits cannot be achieved. And if $b \geq h_L(\bar{\theta})$, it must be the case that $p_L = p_H = \bar{\theta}$, in which case the INC optimal allocation can be attained in NM and therefore also in EAC.

(iii) Consider any $V \geq H(\bar{\theta})$, whence $\Pi^{NM} = V - \bar{\theta}$. The optimal INC profit is bounded below by what can be achieved via delegation in the interim collusion setting. If the bonus is b, the resulting prices will be $p_i(b)$, base pay will be set equal to $-F_H(p_H(b))(b - p_H(b))$ (using the argument in (ii) above), so the resulting profit will be $\Pi^{IND}(b;V) \equiv [\kappa_L F_L(p_L(b)) + \kappa_H F_H(p_H(b))][V - b] + F_H(p_H(b))[b - p_H(b)]$. The derivative of Π^{IND} with respect to b evaluated at $b = V$ then equals $\kappa_L[F_H(p_H(V)) - F_L(p_L(V))]$. Now observe that by definition of the $p_i(b)$ function, $p_i(V) = p_i^{SB}$. So $V \geq H(\bar{\theta})$ implies $p_H^{SB} = p^{NM} = \bar{\theta}$, so $p_H(V) = \bar{\theta}$. On the other hand, $p_L(V) = p_L^{SB} < \bar{\theta}$ since $V < h_L(\bar{\theta})$, so $F_L(p_L(V)) < 1 = F_H(p_H(V))$. It follows that Π^{IND} is strictly increasing in b when evaluated at $b = V$. Since $\Pi^{IND}(V;V) = V - \bar{\theta} = \Pi^{NM}$, it follows that M adds value in the INC optimal allocation. ■

Proof of Proposition 4:

At the first step, note that the optimal side contract problem for M in DM involves an outside option for A which is identically zero. This reduces to a standard problem of contracting with a single agent with adverse selection and an outside option of zero, where M has a prior distribution $F(\theta|\eta)$ over the agent's cost θ in state η. The expected procurement cost incurred by M is then $\hat{h}(\theta|\eta)$.

Given this, P's contract with M in DM is effectively a contracting problem for P with a single supplier whose unit supply cost is $\hat{h}(\theta|\eta)$. P's prior over this supplier's cost is given by distribution function $G(h) \equiv \Pr((\theta,\eta) \mid \hat{h}(\theta \mid \eta) \leq h)$ for $h \geq \min_\eta \hat{h}(\underline{\theta}(\eta) \mid \eta) = \underline{\theta}$ and $G(h) = 0$ for $h < \underline{\theta}$. Let $G(h \mid \eta)$ denote the cumulative distribution function of $h = \hat{h}(\theta \mid \eta)$ conditional on η: $G(h \mid \eta) \equiv \Pr(\theta \mid \hat{h}(\theta \mid \eta) \leq h, \eta)$ for $h \geq \hat{h}(\underline{\theta}(\eta) \mid \eta)(= \underline{\theta}(\eta))$ and $G(h \mid \eta) = 0$ for $h < \underline{\theta}(\eta)$. Then $G(h) = \Sigma_{\eta \in \Pi} p(\eta) G(h \mid \eta)$. Since $\hat{h}(\theta \mid \eta)$ is continuous and nondecreasing on $\Theta(\eta)$, $G(h \mid \eta)$ is strictly increasing in h on $[\underline{\theta}(\eta), \hat{h}(\bar{\theta}(\eta) \mid \eta)]$. However, $G(h \mid \eta)$ may fail to be left-continuous.

Hence P's problem in DM reduces to max $E_h[V(q(h)) - X(h)]$ subject to $X(h) - hq(h) \geq X(h^{'}) - hq(h^{'})$ for any $h, h^{'} \in [\underline{\theta}, \bar{h}]$ and $X(h) - hq(h) \geq 0$ for any $h \in [\underline{\theta}, \bar{h}]$ where the distribution function of h is $G(h)$ and $\bar{h} \equiv \max_{\eta \in \Pi} \hat{h}(\bar{\theta}(\eta) \mid \eta)$. The corresponding problem in NM is max $E_\theta[V(q(\theta)) - X(\theta)]$ subject to $X(\theta) - \theta q(\theta) \geq X(\theta^{'}) - \theta q(\theta^{'})$ for any $\theta, \theta^{'} \in \Theta$ and $X(\theta) - \theta q(\theta) \geq 0$ for any $\theta \in \Theta$. The two problems differ only in the underlying cost distributions of P: $G(h)$ in the case of DM and $F(\theta)$ in the case of NM. Since $\theta < \hat{h}(\theta \mid \eta)$

for $\theta > \underline{\theta}(\eta)$,

$$G(h \mid \eta) \equiv \Pr(\theta \mid \hat{h}(\theta \mid \eta) \leq h, \eta) < \Pr(\theta \mid \theta \leq h, \eta) = F(h \mid \eta)$$

for $h \in (\underline{\theta}(\eta), \hat{h}(\bar{\theta}(\eta) \mid \eta))$, implying $G(h) = \Sigma_{\eta \in \Pi} p(\eta) G(h \mid \eta) < \Sigma_{\eta \in \Pi} p(\eta) F(h \mid \eta) = F(h)$ for any $h \in (\underline{\theta}, \bar{h})$. Therefore the distribution of h in DM (strictly) dominates that of θ in NM in the first order stochastic sense. $\bar{h} > \bar{\theta}$, since $\hat{h}(\bar{\theta}(\eta) \mid \eta) > \bar{\theta}(\eta)$ for any η.

It remains to show that this implies that P must earn a lower profit in DM. We prove the following general statement. Consider two contracting problems with a single supplier which differ only in regard to the cost distributions G_1 and G_2, where $G_1(h) < G_2(h)$ for any $h \in (\underline{h}, \bar{h})$ and $G_2(h) = 1$ on $h \in [\bar{\theta}, \bar{h}]$. Standard arguments imply the problem can be reduced to selecting $q(h)$ to maximize the expected value of $V(q(h)) - hq(h) - \int_h^{\bar{h}_i} q(y)dy$ (where $\bar{h}_1 \equiv \bar{h}$ and $\bar{h}_2 \equiv \bar{\theta}$) with respect to distribution G_i, subject to the constraint that $q(.)$ is nonincreasing. Let the maximized profit of P with distribution G be denoted $W(G)$. We will show $W(G_1) < W(G_2)$.

Let $q_1(h)$ denote the optimal solution of the problem based on $G_1(h)$. If $q_1(h)$ is constant on $(\underline{h}, \bar{h})$ with $q_1(h) = q > 0$, $W(G_1) = V(q) - \bar{h}q$. It is feasible for P to select this output schedule when the cost distribution is G_2, generating expected profit $V(q) - \bar{\theta}q$. Then $W(G_2) \geq V(q) - \bar{\theta}q > W(G_1)$ since $\bar{h} > \bar{\theta}$. We henceforth focus on the case where $q_1(h)$ is not constant.

(i) First we show that $V'(q_1(h)) < h$ does not hold for any set of values of h with positive measure. Suppose otherwise that there exists some interval over which $V'(q_1(h)) < h$. Then we can replace the portion of $q_1(h)$ with $V'(q_1(h)) < h$ by $q^*(h)$ with $V'(q^*(h)) = h$, without violating the constraint that $q(h)$ is non-increasing. It raises the value of the objective function, since $V(q_1(h)) - hq_1(h) < V(q^*(h)) - hq^*(h)$ for h where $q_1(h)$ is replaced by $q^*(h)$, and $\int_h^{\bar{h}} q(y)dy$ decreases with this replacement. This is a contradiction.

(ii) Define

$$\Phi(h) \equiv V(q_1(h)) - hq_1(h) - \int_h^{\bar{h}} q_1(y)dy.$$

We claim that $\Phi(h)$ is left-continuous. First we show that our attention can be restricted to the case that $q_1(h)$ is left-continuous. Otherwise, there exists $h' \in (\underline{h}, \bar{h})$ such that $q_1(h'-) > q_1(h')$. Now consider $\tilde{q}_1(h)$ (which is left-continuous at h') such that $\tilde{q}_1(h') = q_1(h'-)$ and $\tilde{q}_1(h) = q_1(h)$ for any $h \neq h'$. Defining $\tilde{\Phi}(h) \equiv V(\tilde{q}_1(h)) - h\tilde{q}_1(h) - \int_h^{\bar{h}} \tilde{q}_1(y)dy$, observe that $\tilde{\Phi}(h) = \Phi(h)$ for $h \neq h^{'}$ and $\tilde{\Phi}(h) > \Phi(h)$ when $h = h^{'}$. Then

$$
\begin{aligned}
\int_{[\underline{h},\bar{h}]} \tilde{\Phi}(h)dG(h) &= \int_{[\underline{h},\bar{h}]\setminus h^{'}} \tilde{\Phi}(h)dG(h) + \tilde{\Phi}(h^{'})[G(h^{'}+) - G(h^{'}-)] \\
\geq \int_{[\underline{h},\bar{h}]\setminus h^{'}} \tilde{\Phi}(h)dG(h) &+ \Phi(h^{'})[G(h^{'}+) - G(h^{'}-)] = \int_{[\underline{h},\bar{h}]} \Phi(h)dG(h)
\end{aligned}
$$

with strict inequality if $G(h)$ is discontinuous at $h = h^{'}$. When $q_1(h)$ is left-continuous, $\Phi(h)$ is also so.

(iii) We claim that $\Phi(h)$ is non-increasing in h and is not constant on $(\underline{h}, \bar{h})$. To show the former, note that for any h, we have

$$
\begin{aligned}
& \lim_{\epsilon\to 0+} \frac{\Phi(h+\epsilon) - \Phi(h)}{\epsilon} \\
= \; & \lim_{\epsilon\to 0+} (1/\epsilon)[V(q_1(h+\epsilon)) - (h+\epsilon)q_1(h+\epsilon) - \int_{h+\epsilon}^{\bar{h}} q_1(y)dy - [V(q_1(h)) - hq_1(h) - \int_{h}^{\bar{h}} q_1(y)dy]] \\
= \; & [V^{'}(\hat{q}(h)) - h] \lim_{\epsilon\to 0+} \frac{q_1(h+\epsilon) - q_1(h)}{\epsilon} - q_1(h+) + \lim_{\epsilon\to 0+} (1/\epsilon) \int_{h}^{h+\epsilon} q_1(y)dy \\
= \; & [V^{'}(\hat{q}(h)) - h] \lim_{\epsilon\to 0+} \frac{q_1(h+\epsilon) - q_1(h)}{\epsilon}
\end{aligned}
$$

for some $\hat{q}(h) \in [q_1(h+), q_1(h)]$. This is non-positive since $V^{'}(\hat{q}(h)) \geq V^{'}(q_1(h)) \geq h$ and $\lim_{\epsilon\to 0+} \frac{q_1(h+\epsilon)-q_1(h)}{\epsilon} \leq 0$. Because of left-continuity of $\Phi(h)$, it implies that $\Phi(h)$ is non-increasing in h. Next to show $\Phi(h)$ is not constant, suppose otherwise. Then $\Phi(h) = \Phi(\bar{h}-) = V(q_1(\bar{h}-)) - \bar{h}q_1(\bar{h}-)$ which must be equal to $W(G_1)$. It means that $W(G_1)$ is attainable with constant output schedule ($q_1(h) = q_1(\bar{h}-)$ for any $h \in (\underline{h}, \bar{h})$). We obtain a contradiction, since we supposed that $q_1(h)$ is not constant.

(iv) Now consider the contracting problem with cost distribution $G_2(h)$. Since $q_1(h)$ is non-increasing in h, it is feasible for P to select this output schedule when the cost distribution is G_2. Hence $W(G_2) \geq \int_{\underline{h}}^{\bar{h}} \Phi(h)dG_2(h)$. Therefore if $\int_{\underline{h}}^{\bar{h}} \Phi(h)dG_2(h) > \int_{\underline{h}}^{\bar{h}} \Phi(h)dG_1(h) = W(G_1)$, it follows that $W(G_2) > W(G_1)$. Since $G_i(h)$ $(i = 1, 2)$ is right-continuous and $\Phi(h)$ is left-continuous, we can integrate by parts:

$$\int_{\underline{h}}^{\bar{h}} \Phi(h)dG_i(h) + \int_{\underline{h}}^{\bar{h}} G_i(h)d\Phi(h) = \Phi(\bar{h}).$$

Hence

$$\int_{\underline{h}}^{\bar{h}} \Phi(h)dG_2(h) - \int_{\underline{h}}^{\bar{h}} \Phi(h)dG_1(h) = \int_{\underline{h}}^{\bar{h}} [G_1(h) - G_2(h)]d\Phi(h).$$

By (iii) and $G_2(h) > G_1(h)$ for $h \in (\underline{h}, \bar{h})$, this is positive. ■

Proof of Proposition 6:

Step 1:

Define

$$A(\theta \mid \eta) \equiv \frac{F(\theta \mid \eta)}{f(\theta \mid \eta)} \Big/ \frac{F(\theta)}{f(\theta)} \equiv \frac{\int_{\underline{\theta}(\eta)}^{\theta} f(y)a(\eta|y)dy}{a(\eta|\theta)F(\theta)}.$$

We show that there exist $\eta^{**} \in \Pi$ and a closed interval $I = [\theta^{'}, \theta^{''}]$ with positive measure (conditional on η^{**}) such that $A(\theta \mid \eta^{**})$ is increasing in θ over I.

Evidently this holds for η such that $\underline{\theta} < \underline{\theta}(\eta)$, since $A(\underline{\theta}(\eta) \mid \eta)$ is continuous, $\lim_{\theta \to \underline{\theta}(\eta)} A(\theta \mid \eta) = 0$ and $A(\theta \mid \eta) > 0$ for $\theta > \underline{\theta}$. Suppose otherwise; then $\underline{\theta}(\eta) = \underline{\theta}$ for all η. Using l'Hôpital's rule, $\lim_{\theta \to \underline{\theta}} A(\theta \mid \eta) = 1$. If $A(\theta \mid \eta)$ is non-increasing in θ for all η, $A(\theta \mid \eta) \leq 1$ or equivalently $\int_{\underline{\theta}}^{\theta} f(y)a(\eta|y)dy \leq a(\eta|\theta)F(\theta)$ for all $(\theta, \eta) \in K$. Since

$$\Sigma_\eta \int_{\underline{\theta}}^{\theta} f(y)a(\eta|y)dy = \Sigma_\eta a(\eta|\theta)F(\theta) = F(\theta)$$

for all θ, $A(\theta \mid \eta) = 1$ for all $(\theta, \eta) \in K$. Then $h(\theta \mid \eta) = H(\theta)$ for any $(\theta, \eta) \in K$. This is a contradiction, since η is informative about θ.

For η^{**} and I, we choose $\lambda > 0$, closed intervals $\Theta_L = [\underline{\theta}^L, \bar{\theta}^L] \subset I$ and $\Theta_H = [\underline{\theta}^H, \bar{\theta}^H] \subset I$ with $\bar{\theta}^L < \underline{\theta}^H$ such that

$$\frac{F(\theta)}{f(\theta)} \Big/ \frac{F(\theta \mid \eta^{**})}{f(\theta \mid \eta^{**})} < \lambda < \frac{F(\tilde{\theta})}{f(\tilde{\theta})} \Big/ \frac{F(\tilde{\theta} \mid \eta^{**})}{f(\tilde{\theta} \mid \eta^{**})} \ \text{ for } \tilde{\theta} \in \Theta_L,\ \theta \in \Theta_H.$$

These conditions are equivalent to

$$H(\theta) - (1-\lambda)\theta - \lambda h(\theta \mid \eta^{**}) > 0 \ \text{ for } \theta \in \Theta_L$$

and

$$H(\theta) - (1-\lambda)\theta - \lambda h(\theta \mid \eta^{**}) < 0 \ \text{ for } \theta \in \Theta_H.$$

Step 2: Construction of $z(\cdot \mid \eta)$

Now let us construct $z(\cdot \mid \eta)$ which satisfies the following conditions.

(A) For $\eta \neq \eta^{**}$, $z(\theta \mid \eta) = \theta$ for any $\theta \in \Theta(\eta)$.

(B) For η^{**}, $z(\cdot \mid \eta^{**}) \in Z(\eta^{**})$ which satisfies

(i) $z(\theta \mid \eta^{**}) = \theta$ for any $\theta \notin \Theta_H \cup \Theta_L$

(ii) For $\theta \in \Theta_L$, $z(\theta \mid \eta^{**})$ satisfies (a) $z(\theta \mid \eta^{**}) \leq \theta$ with strict inequality for some subinterval of Θ_L of positive measure, and (b) $H(z) - (1-\lambda)z - \lambda h(\theta \mid \eta^{**}) > 0$ for any $z \in [z(\theta \mid \eta^{**}), \theta]$.

(iii) For $\theta \in \Theta_H$, $z(\theta \mid \eta^{**})$ satisfies (a) $z(\theta \mid \eta^{**}) \geq \theta$ with strict inequality for some some subinterval of Θ_H of positive measure, (b) $z(\theta \mid \eta^{**}) < h(\theta \mid \eta^{**})$ and (c) $H(z) - (1-\lambda)z - \lambda h(\theta \mid \eta^{**}) < 0$ for any $z \in [\theta, z(\theta \mid \eta^{**})]$.

(iv) $E[(z(\theta \mid \eta^{**}) - h(\theta \mid \eta^{**}))q^{NM}(z(\theta \mid \eta^{**})) + \int_{z(\theta|\eta^{**})}^{\bar{\theta}(\eta^{**})} q^{NM}(z)dz \mid \eta^{**}] = 0.$

It is shown in the online Appendix that there exists $z(\cdot \mid \eta^{**}) \in Z(\eta^{**})$ which satisfies (B(i)-(iv)).

Step 3

For $z(\cdot \mid \eta)$ constructed in Step 2, consider the following allocation (u_A, u_M, q):

$$q(\theta, \eta) = q^{NM}(z(\theta \mid \eta))$$

$$u_A(\theta, \eta) = \int_{\theta}^{\bar{\theta}} q^{NM}(z(y \mid \eta))dy$$

$$u_M(\theta, \eta) = X^{NM}(z(\theta \mid \eta)) - \theta q^{NM}(z(\theta \mid \eta)) - \int_{\theta}^{\bar{\theta}(\eta)} q^{NM}(z(y \mid \eta))dy - \int_{\bar{\theta}(\eta)}^{\bar{\theta}} q^{NM}(y)dy.$$

where

$$X^{NM}(z(\theta \mid \eta)) \equiv z(\theta \mid \eta)q^{NM}(z(\theta \mid \eta)) + \int_{z(\theta|\eta)}^{\bar{\theta}} q^{NM}(z)dz.$$

In the online Appendix it is shown that (u_A, u_M, q) is a EAC feasible allocation.

Now we show that this allocation generates a higher payoff to P than the optimal allocation in NM. Define $\Phi_P(z)$ and $\Phi_M(z, \theta)$ as

$$\Phi_P(z) \equiv V(q^{NM}(z)) - zq^{NM}(z) - \int_z^{\bar{\theta}} q^{NM}(\tilde{z})d\tilde{z}$$

and

$$\Phi_M(z, \theta) \equiv (z - h(\theta \mid \eta^{**}))q^{NM}(z) + \int_z^{\bar{\theta}(\eta^{**})} q^{NM}(\tilde{z})d\tilde{z}.$$

P's resulting expected payoff conditional on η^{**} is $E[\Phi_P(z(\theta \mid \eta^{**})) \mid \eta^{**}]$. P's expected payoff conditional on η^{**} in the optimal allocation in NM is $E[\Phi_P(\theta) \mid \eta^{**}]$. By the definition of $\Phi_M(z,\theta)$ and $E[u_M(\theta,\eta^{**}) \mid \eta^{**}] = 0$, $E[\Phi_M(z(\theta \mid \eta^{**}),\theta) \mid \eta^{**}] = E[\Phi_M(\theta,\theta) \mid \eta^{**}] = 0$. Then the difference between two payoffs is

$$\begin{aligned}
& E[\Phi_P(z(\theta \mid \eta^{**})) \mid \eta^{**}] - E[\Phi_P(\theta) \mid \eta^{**}] \\
= \; & E[\Phi_P(z(\theta \mid \eta^{**})) + \lambda\Phi_M(z(\theta \mid \eta^{**}),\theta) \mid \eta^{**}] - E[\Phi_P(\theta) + \lambda\Phi_M(\theta,\theta) \mid \eta^{**}] \\
= \; & E[\int_\theta^{z(\theta|\eta^{**})} \{\Phi_P^{'}(z) + \lambda\partial\Phi_M(z,\theta)/\partial z\} dz \mid \eta^{**}] \\
= \; & E[\int_\theta^{z(\theta|\eta^{**})} [V^{'}(q^{NM}(z)) - \{(1-\lambda)z + \lambda h(\theta \mid \eta^{**})\}] q^{NM'}(z) dz \mid \eta^{**}] \\
= \; & E[\int_\theta^{z(\theta|\eta^{**})} [H(z) - \{(1-\lambda)z + \lambda h(\theta \mid \eta^{**})\}] q^{NM'}(z) dz \mid \eta^{**}].
\end{aligned}$$

The last equality follows from $V^{'}(q^{NM}(z)) = H(z)$. From the construction of $z(\theta \mid \eta^{**})$ in Step 2 and $q^{NM'}(z) < 0$, this is positive. We have thus found an EAC feasible allocation which generates a higher payoff to P compared to the optimal allocation in NM. ■

Proof of Proposition 7:

Conditions (i) and (ii) imply $h(\theta \mid \eta)$ satisfies the following properties:

- $h(\theta \mid \eta^*) = \hat{h}(\theta \mid \eta^*)$ is strictly increasing and continuously differentiable in θ
- $\hat{h}(\theta \mid \eta^*) > \hat{h}(\theta \mid \eta)$ for $\theta \in (\underline{\theta}, \bar{\theta}]$ and $\hat{h}(\underline{\theta} \mid \eta^*) = \hat{h}(\underline{\theta} \mid \eta) = \underline{\theta}$ for any $\eta \neq \eta^*$
- Define $G(h \mid \eta) \equiv \int_{\{\theta|\hat{h}(\theta|\eta)\leq h\}} f(\theta \mid \eta) d\theta$. Then $G(h \mid \eta^*)$ is a mean-preserving spread of $G(h \mid \eta)$ for any $\eta \neq \eta^*$

The first one is evident, since $f(\theta \mid \eta^*)$ is decreasing in θ and $h(\theta \mid \eta^*)$ is increasing in θ. Then $q^*(\hat{h}(\theta \mid \eta^*))$ is also continuously differentiable and strictly decreasing in θ. By the second property, the range of $\hat{h}$ conditional on η^* (which is denoted by H) includes the range of $\hat{h}$ conditional on η. The proof of the second and third properties are provided in the online Appendix.

Suppose the result is false, and the second best allocation $(u_A^{SB}, u_M^{SB}, q^{SB})$ is EAC feasible. Then Proposition 5 implies the existence of $\pi(\cdot \mid \eta) \in Y(\eta)$ such that for any $(\theta,\eta), (\theta^{'},\eta^{'}) \in K$,

$$q^{SB}(\theta,\eta) = q^*(\hat{h}(\theta \mid \eta))$$

$$X^{SB}(\theta,\eta) - z(\theta \mid \eta) q^{SB}(\theta,\eta) \geq 0$$

$$X^{SB}(\theta,\eta) - z(\theta \mid \eta) q^{SB}(\theta,\eta) \geq X^{SB}(\theta^{'},\eta^{'}) - z(\theta \mid \eta) q^{SB}(\theta^{'},\eta^{'})$$

where $z(\cdot \mid \eta)$ is an ironing transformation of $\pi(\cdot \mid \eta)$ based on $F(\theta \mid \eta)$, and

$$X^{SB}(\theta,\eta) \equiv u_A^{SB}(\theta,\eta) + u_M^{SB}(\theta,\eta) + \theta q^{SB}(\theta,\eta).$$

Step 1: Existence of $(\tilde{X}(h), \phi(h))$.

Consider $(\theta,\eta),(\theta',\eta')$ such that $\hat{h}(\theta \mid \eta) = \hat{h}(\theta' \mid \eta')$. Then $q^{SB}(\theta,\eta) = q^{SB}(\theta',\eta')$. The coalitional incentive constraint implies $X^{SB}(\theta,\eta) = X^{SB}(\theta',\eta')$, since otherwise the coalition would misrepresent a state with higher payment in the other state where the same output is produced. It guarantees the existence of $\tilde{X}(h)$ such that $X^{SB}(\theta,\eta) = \tilde{X}(\hat{h}(\theta \mid \eta))$ for any (θ,η).

Next suppose that $\hat{h}(\theta'' \mid \eta'') = \hat{h}(\theta' \mid \eta')$ and $z(\theta'' \mid \eta'') > z(\theta' \mid \eta')$ for some $(\theta',\eta'),(\theta'',\eta'')$. The ironing procedure ensures $z(\theta \mid \eta)$ and $\hat{h}(\theta \mid \eta)$ are continuous and non-decreasing for θ on Θ. Since $\hat{h}(\underline{\theta} \mid \eta) = \underline{\theta} < \bar{\theta} < \hat{h}(\bar{\theta} \mid \eta)$, $\hat{h}(\theta \mid \eta)$ is not constant on Θ. Then by adjusting θ' and θ'', we can find $(\tilde{\theta}',\tilde{\theta}'')$ such that $\hat{h}(\tilde{\theta}'' \mid \eta'') < \hat{h}(\tilde{\theta}' \mid \eta')$ and $z(\tilde{\theta}'' \mid \eta'') > z(\tilde{\theta}' \mid \eta')$. We obtain a contradiction, since the coalition incentive constraint implies that whenever $z(\tilde{\theta}'' \mid \eta'') > z(\tilde{\theta}' \mid \eta')$, $q^{SB}(\tilde{\theta}'',\eta'') \leq q^{SB}(\tilde{\theta}',\eta')$ or equivalently $\hat{h}(\tilde{\theta}'' \mid \eta'') \geq \hat{h}(\tilde{\theta}' \mid \eta')$. Hence there exists $\phi(h)$ which satisfies $z(\theta \mid \eta) = \phi(\hat{h}(\theta \mid \eta))$ for any (θ,η). Since $\hat{h}(\theta \mid \eta^*)$ and $z(\theta \mid \eta^*)$ are continuous and non-decreasing for θ, $\phi(h)$ is continuous and non-decreasing on H.

The coalitional incentive constraint implies that for any $h \in H$, h maximizes $\tilde{X}(h') - \phi(h)q^*(h')$ subject to $h' \in H$. By the continuity of $\phi(h)$ and the differentiability of $q^*(h)$, we obtain the differentiability of $\tilde{X}(h)$ and the first order condition $\tilde{X}'(h) = \phi(h)q^{*'}(h)$.

Step 2: Properties of $\phi(h)$

Here we show that (a) $\phi(h) \geq 0$ on H and (b) $h - \phi(h)$ is non-negative and increasing in h.

Since $q^{SB}(\theta,\eta^*) = q^*(\hat{h}(\theta \mid \eta^*))$ is strictly decreasing in θ, the pooling region $\Theta(\pi(\cdot \mid \eta^*),\eta^*)$ must be empty. Hence it must be the case that

$$z(\theta \mid \eta^*) = \phi(\hat{h}(\theta \mid \eta^*)) = \theta + \frac{F(\theta \mid \eta^*) - \Lambda(\theta \mid \eta^*)}{f(\theta \mid \eta^*)}.$$

Since $\phi(\hat{h}(\theta \mid \eta^*))$ is non-decreasing in θ and $\Lambda(\theta \mid \eta^*) \leq 1$,

$$\phi(\hat{h}(\theta \mid \eta^*)) \geq \phi(\underline{\theta}) \geq \underline{\theta} - 1/f(\underline{\theta} \mid \eta^*) > 0$$

by property (iii), which implies (a). The above equality can be rewritten as

$$\frac{\Lambda(\theta \mid \eta^*)}{f(\theta \mid \eta^*)} = \hat{h}(\theta \mid \eta^*) - \phi(\hat{h}(\theta \mid \eta^*)).$$

The LHS is non-negative and increasing in θ, since $f(\theta \mid \eta^*)$ is decreasing in θ and $\Lambda(\theta \mid \eta^*)$ is non-negative and non-decreasing in θ. It implies (b).

Step 3: M earns positive rent.

Define $L(h) \equiv \tilde{X}(h) - hq^*(h)$. M's interim payoff is

$$E[X^{SB}(\theta, \eta) - h(\theta \mid \eta)q^{SB}(\theta, \eta) \mid \eta] = E[L(\hat{h}(\theta \mid \eta)) \mid \eta],$$

utilizing a property of the ironing transformation. If the second best allocation is EAC feasible, $E[L(\hat{h}(\theta \mid \eta)) \mid \eta] = 0$ holds for any η. The first derivative of $L(h)$ is

$$L^{'}(h) = (\phi(h) - h)q^{*'}(h) - q^*(h).$$

Since $q^*(h)$ is continuously differentiable and $\phi(h)$ is continuous and almost everywhere differentiable, $L^{'}(h)$ is continuous and also differentiable almost everywhere and

$$L^{''}(h) = (\phi^{'}(h) - 1)q^{*'}(h) + (\phi(h) - h)q^{*''}(h) - q^{*'}(h).$$

By using $V^{'}(q^*(h)) = h$, we can show that $V^{'''}(q) \leq 0$ implies $q^{*''}(h) \leq 0$, and $0 < V^{'''}(q) \leq \frac{(V^{''}(q))^2}{V^{'}(q)}$ implies $q^{*''}(h) > 0$ and $hq^{*''}(h) + q^{*'}(h) < 0$. By $\phi^{'}(h) - 1 < 0$ and $0 \leq \phi(h) \leq h$, it follows that $L^{''}(h) > 0$.

By the strict convexity of L and the mean-preserving spread property of $G(h \mid \eta^*)$,

$$E[L(\hat{h}(\theta \mid \eta^*)) \mid \eta^*] = \int L(h)dG(h \mid \eta^*) > \int L(h)dG(h \mid \eta) = E[L(\hat{h}(\theta \mid \eta)) \mid \eta] \geq 0$$

for any $\eta \neq \eta^*$. Therefore M must earn a positive rent in state η^*. This is a contradiction. ■

Proof of Proposition 8:

We show that the set of EACP(α) is independent of $\alpha \in [0, 1]$. Suppose otherwise that $(u_A(\theta, \eta), u_M(\theta, \eta), q(\theta, \eta))$ is a EACP(α) allocation, but not a EACP($\alpha^{'}$) ($\alpha \neq \alpha^{'}$) allocation. It implies that for some η, $(\tilde{m}(\theta \mid \eta), \tilde{u}_A(\theta, \eta)) = ((\theta, \eta), u_A(\theta, \eta))$ is not the solution of $TP(\eta;\alpha^{'})$ defined for $(u_A(\theta,\eta),u_M(\theta,\eta),q(\theta,\eta))$. If $(\tilde{m}^*(\theta \mid \eta),u_A^*(\theta,\eta))(\neq ((\theta,\eta),u_A(\theta,\eta)))$ is a solution of $TP(\eta;\alpha^{'})$, it satisfies all constraints of $TP(\eta;\alpha^{'})$ and realizes a higher payoff to the third party than in the choice of $(\tilde{m}(\theta \mid \eta),\tilde{u}_A(\theta,\eta)) = ((\theta,\eta),u_A(\theta,\eta))$:

$$
\begin{aligned}
& E[(1-\alpha^{'})[\hat{X}(\tilde{m}^*(\theta \mid \eta)) - \theta\hat{q}(\tilde{m}^*(\theta \mid \eta)) - u_A^*(\theta,\eta)] + \alpha^{'}u_A^*(\theta,\eta) \mid \eta] \\
> \quad & E[(1-\alpha^{'})[\hat{X}(\theta,\eta) - \theta\hat{q}(\theta,\eta) - u_A(\theta,\eta)] + \alpha^{'}u_A(\theta,\eta) \mid \eta].
\end{aligned}
$$

It also satisfies A and M's participation constraints:

$$u_A^*(\theta,\eta) \geq u_A(\theta,\eta)$$

and

$$E[\hat{X}(\tilde{m}^*(\theta \mid \eta)) - \theta\hat{q}(\tilde{m}^*(\theta \mid \eta)) - u_A^*(\theta,\eta) \mid \eta] \geq E[u_M(\theta,\eta) \mid \eta].$$

On the other hand, since $(\tilde{m}(\theta \mid \eta),\tilde{u}_A(\theta,\eta)) = ((\theta,\eta),u_A(\theta,\eta))$ solves $TP(\eta;\alpha)$,

$$
\begin{aligned}
& E[(1-\alpha)[\hat{X}(\theta,\eta) - \theta\hat{q}(\theta,\eta) - u_A(\theta,\eta)] + \alpha u_A(\theta,\eta) \mid \eta] \\
\geq \quad & E[(1-\alpha)[\hat{X}(\tilde{m}^*(\theta \mid \eta)) - \theta\hat{q}(m^*(\theta \mid \eta)) - u_A^*(\theta,\eta)] + \alpha u_A^*(\theta,\eta) \mid \eta]
\end{aligned}
$$

Let us consider three cases: (i) $\alpha \in (0,1)$, (ii) $\alpha = 1$ and (iii) $\alpha = 0$.

(i) $\alpha \in (0,1)$

The last three inequalities imply

$$u_A^*(\theta,\eta) = u_A(\theta,\eta)$$

and

$$E[\hat{X}(\tilde{m}^*(\theta \mid \eta)) - \theta\hat{q}(m^*(\theta \mid \eta)) - u_A^*(\theta,\eta) \mid \eta] = E[u_M(\theta,\eta) \mid \eta].$$

But this is not compatible with the first inequality. We obtain a contradiction.

(ii) $\alpha = 1$

With $\alpha = 1$, the above four inequalities imply

$$E[u_A(\theta,\eta) \mid \eta] = E[u_A^*(\theta,\eta) \mid \eta]$$

and

$$E[\hat{X}(\tilde{m}^*(\theta \mid \eta)) - \theta\hat{q}(\tilde{m}^*(\theta \mid \eta)) - u_A^*(\theta,\eta) \mid \eta] > E[u_M(\theta,\eta) \mid \eta].$$

But for sufficiently small $\epsilon > 0$, the choice of

$$(\tilde{m}(\theta \mid \eta), \tilde{u}_A(\theta, \eta)) = (\tilde{m}^*(\theta, \eta), u_A^*(\theta, \eta) + \epsilon)$$

(instead of $((\theta, \eta), u_A(\theta, \eta)))$ in $TP(\eta; \alpha = 1)$ generates a higher value of the objection function without violating any constraint. We obtain a contradiction.

(iii) $\alpha = 0$

With $\alpha = 0$, the four inequalities imply

$$E[u_M(\theta, \eta) \mid \eta] = E[\hat{X}(\tilde{m}^*(\theta \mid \eta)) - \theta \hat{q}(\tilde{m}^*(\theta \mid \eta)) - u_A^*(\theta, \eta) \mid \eta]$$

and

$$E[u_A^*(\theta, \eta) \mid \eta] > E[u_A(\theta, \eta) \mid \eta].$$

Since $u_A^*(\theta, \eta) \geq u_A(\theta, \eta)$ for any θ, there is a subset of θ with the positive measure such that $u_A^*(\theta, \eta) > u_A(\theta, \eta)$. Consider a modified problem of $TP(\eta; \alpha = 0)$ such that the constraint $\tilde{u}_A(\theta, \eta) \geq u_A(\theta, \eta)$ is replaced by $\tilde{u}_A(\theta, \eta) \geq u_A^*(\theta, \eta)$ in $TP(\eta; \alpha = 0)$. Since the optimal solution $(\tilde{m}(\theta \mid \eta), \tilde{u}_A(\theta, \eta)) = ((\theta, \eta), u_A(\theta, \eta))$ in $TP(\eta; \alpha = 0)$ violates the constraint, the maximum value of the objective function in the modified problem would become lower. On the other hand, $(\tilde{m}^*(\theta \mid \eta), u_A^*(\theta, \eta))$ satisfies all the constraints of the modified problem, and brings

$$E[\hat{X}(\tilde{m}^*(\theta \mid \eta)) - \theta \hat{q}(\tilde{m}^*(\theta \mid \eta)) - u_A^*(\theta, \eta) \mid \eta].$$

The argument implies

$$E[u_M(\theta, \eta) \mid \eta] > E[\hat{X}(\tilde{m}^*(\theta \mid \eta)) - \theta \hat{q}(\tilde{m}^*(\theta \mid \eta)) - u_A^*(\theta, \eta) \mid \eta].$$

We obtain a contradiction. ■