Bypassing Intermediaries via Vertical Integration:  
A Transaction-Cost-Based Theory\textsuperscript{1}  
Dilip Mookherjee\textsuperscript{2}, Alberto Motta\textsuperscript{3} and Masatoshi Tsumagari\textsuperscript{4}  

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Abstract  

We provide a theory of vertical integration motivated to bypass intermediaries and contract directly with workers or customers in order to reduce double marginalization of rents. Non-integration results in inefficiencies that cannot be overcome via sophisticated contract design if supplier costs are nonverifiable, owing either to weak accounting systems or collusion between intermediaries and lower layer agents. Vertical integration generate benefits by lowering these inefficiencies, but incur bureaucratic costs in order to control intrafirm collusion. We discuss predictions of our theory concerning determinants of benefits and costs of integration, compare these with Transaction Cost and Property Rights based theories, and with available empirical evidence.  

KEYWORDS: vertical integration, intermediation, collusion, delegation, double marginalization of rents, direct foreign investment

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\textsuperscript{2}Department of Economics, Boston University, 270 Bay State Road Boston MA 02215; dilipm@bu.edu  
\textsuperscript{3}School of Economics, University of New South Wales, Australian School of Business, Sydney 2052, Australia; motta@unsw.edu.au  
\textsuperscript{4}Department of Economics, Keio University, 2-15-45 Mita Minato-ku Tokyo 108-8345, Japan; tsuma@econ.keio.ac.jp
1 Introduction

A frequently cited motive for vertical integration is to bypass intermediaries and contract directly with workers or customers. A manufacturer may integrate forward into distribution and sell directly to consumers, in order to limit rents earned by wholesalers or retailers. A multinational firm that sources an intermediate good from a different country may want to invest directly in an enterprise in that country in order to contract directly with production workers, as an alternative to outsourcing from a local supplier who hires these workers. Apart from reducing intermediary rents, actions of separately owned intermediaries generate externalities that are sought to be reduced via vertical integration. These include cascading distortions arising from the double marginalization of rents (DMR), which have been highlighted by numerous case studies and empirical analyses in the industrial organization literature.\(^5\)

Such forms of vertical integration can be achieved either via acquisition of firms owned by intermediaries, or by setting up new integrated firms that compete with them. Global supply chains in food processing and retailing sectors are becoming increasingly dominated by large MNCs contracting directly with farmers in developing countries, with higher quality standards and increased vertical coordination within these chains (Dries and Swinnen (2004), Maertens et al. (2011), Michelson et al. (2013), Minten et al. (2009), Rao and Qaim (2011), Reardon et al. (1999)). These have been facilitated by FDI (foreign direct investment) deregulation under globalization, and advances in information technology that have made it easier for MNCs to bypass traditional intermediaries and contract directly with farmers, workers and customers. Globalization policy debates concerning FDI involve assessment of their efficiency and distributive consequences, including spillover effects on welfares of workers and consumers, besides the parties directly involved.

As Joskow (2010) and Bresnahan and Levin (2012) explain, ‘transaction cost’ (TC) theories of vertical integration focus on reducing \textit{ex post} inefficiencies arising from contractual imperfections between vertically related parties subject to high degrees of ‘specificity’.\(^6\) A vast body of empirical IO literature provides support of the main prediction of this app-

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\(^5\) Lafontaine and Slade (2005), Joskow (2010) and Bresnahan and Levin (2012) provide useful overviews of this literature.

\(^6\) The TC approach is based on the work of Klein et al. (1978) and Williamson (1971, 1975, 1985); see Tadelis and Williamson (2012) for a survey.
proach, concerning the positive relationship between vertical integration and specificity. However, as elaborated by Gibbons (2005) this approach is subject to a number of conceptual difficulties owing to lack of appropriate microfoundations. The theory is unclear regarding the precise source of benefits and costs of integration. Regarding the benefit side, more sophisticated contract design (e.g., nonlinear pricing, subsidizing intermediary costs) can overcome the DMR problem. In the MNC example, a two-part tariff wherein the supplier pays a fixed fee to the MNC enables the former to become a residual claimant and thereby internalize the DMR externality. Setting the upfront fee appropriately may however be difficult, as suppliers are likely to be better informed about local production conditions — the very reason they are appointed in the first place. Nevertheless, sophisticated contract design can still achieve constrained efficient allocations under suitable conditions such as revenue/cost verifiability and absence of collusion with their subordinates. Villas-Boas (2007) and Mortimer (2008) use structural estimation methods to provide evidence of departures from linear pricing in vertical relationships between manufacturers and retailers in specific US industries, which succeed in overcoming inefficiencies resulting from DMR.

The other puzzle concerns the costs of integration. If integration is costless, one would expect to only observe vertically integrated enterprises which can (in principle) achieve improved efficiency compared to market relationships with independently owned suppliers after acquiring them. What are the costs of internal resource allocation within hierarchies relative to market relationships, that prevent this from happening — leading to the question posed by Williamson (1985): ‘why isn’t the entire economy organized as a single firm?’ In response to this question, Williamson and many others have described various ‘transaction’ or ‘bureaucracy’ costs of internal organization that many large integrated firms seem to be afflicted by in practice. But it has proven difficult to provide a clear theoretical understanding of the source of these costs.

The purpose of this paper is to develop a theory that addresses these questions. We show how contract enforcement problems of verifiability of costs or revenues of vertical subordinates, or prospects of collusion between subordinates located at different layers, can prevent DMR inefficiencies from being eliminated via sophisticated contract design with separately owned intermediaries. This creates room for potential efficiency benefits to be achieved via vertical integration. The integrated firm is a vertical hierarchy that absorbs

\footnote{See Melumad et al. (1995) for a theoretical demonstration.}
former intermediaries as ‘supervisors’, and contracts directly with workers or customers at bottom layers on the basis of information provided by supervisors. This allows greater control compared to resource allocation under non-integration. At the same time it creates prospects for collusion between supervisors and bottom-layer agents within the integrated firm. The ‘bureaucracy’ costs of internal organization arise as a response to such collusion, building on the approach of Tirole (1986), Laffont and Tirole (1993) and Laffont and Martimort (1997, 2000). To control such collusion, the integrated firm optimally designs a mechanism involving low powered incentives for supervisors, and cross-matching of information reports sent to headquarters by supervisor and bottom-layer agents respectively. The costs of internal organization include the allocational distortions arising from low-powered managerial incentives, as well as the ‘institutional’ cost of writing contracts with a larger set of agents, setting up communication channels and internal dispute settlement procedures which are not needed in outsourcing.

Modeling these ‘institutional’ costs of centralization is challenging. But it is evident that such costs do exist and can be substantial. We simplify the analysis by assuming these take the form of an exogenous fixed cost, which depends on the broader context (country, legal system, communication technology, corruption norms). This fixed cost has to be traded off against the (endogenous) benefit from reducing the severity of the DMR problem. Consequently, integration is sometimes but not always observed – it happens when the benefits are large enough to outweigh the institutional cost. Treating the latter as an institutional parameter, we focus thereafter on studying determinants of integration benefits which are endogenous.

For concreteness we focus on the problem faced by a multinational company (MNC) denoted by P (Principal) located in a Northern country deciding whether to outsource to an intermediary supplier S located in a Southern country, or to acquire the firm owned by this supplier. The supplier contracts with a Southern production worker A to get the item produced. Our theory delivers predictions concerning the likelihood of vertical integration and how it depends on ‘specificity’, firm-level attributes and contextual attributes such as distance between the two countries, governance and contract enforcement institutions in the Southern country. We use the model to infer welfare and distributional impacts of

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8This is a common assumption in the literature on multinational firms in international economics; see Keller and Yeaple (2013) and Yeaple (2013).
integration which incorporate spillover effects on welfare of lower-layer agents (e.g., workers or customers), as well as ‘pass-through’ of shocks to the firm’s external revenues to agents at various layers.

Owing to our focus on vertical integration as a way of bypassing intermediaries in order to reduce \textit{ex post} inefficiency arising from DMR problems, our theory belongs to the TC branch of the literature. It differs from Property Rights (PR) theories based on incomplete contracts, which focus on \textit{ex ante} investment effects of integration.\footnote{For surveys of this literature, see Gibbons (2005), Gibbons and Roberts (2012) or Dessein (2014). Our approach is closer in spirit to recent theories of firm scope of Hart and Holmstrom (2010) that focus on \textit{ex post} inefficiencies arising from \textit{ex post} noncontractibility problems, and also derive implications for internal organization of integrated firms. Our theory differs insofar as asymmetric information and collusion are the source of inefficiency within the integrated firm, instead of \textit{ex post} noncontractibility and conflicting nonpecuniary preferences across stakeholders.} A growing literature on MNCs in international economics is based on the PR approach, including both theoretical analyses and empirical testing (see Antras (2013), Antras and Yeaple (2013) for surveys of this literature). As we elaborate in Section 7, many of the predictions of our model are similar to those of the PR approach that have been empirically verified, while there is insufficient evidence on predictions that differ.

The paper is organized as follows. Section 2 provides a self-contained overview of the basic model and main results. Additional model details are presented in Section 3 and results in Section 4. Section 5 provides extensions of the basic model to study effects of higher bargaining power of S in negotiating an acquisition by P; forward integration (where S buys P’s firm) as an alternative to backward integration, and alternative forms of collusion between S and A. We also examine the differences between vertical integration based on acquisition of S, with ‘greenfield’ enterprises allowed (by deregulation or advances in information technology) to set up to compete directly with S. Section 6 describes welfare implications of integration, using numerical computations of an example with uniformly distributed costs and signals with linear likelihood ratios. Section 7 concludes with a summary of the predictions, followed by a comparison of these with predictions of PR-based theories, and a discussion of available empirical evidence related to these predictions.
2 Overview of Model and Main Results

The model is structured as follows. The status quo situation involves two separate firms, one owned by a Principal (P) which corresponds to the Northern MNC, and a Southern firm owned by a supplier S which employs worker A (referred to as the agent). P owns an asset consisting of a product and access to a world market where it can be sold at unit price $V_P$. The asset owned by S is the right to contract exclusively with agent A located within the Southern country, arising either from monopoly ownership of a productive asset that A needs to work with, or a prior connection with (or knowledge of) the ‘right’ agent A otherwise indistinguishable (by P, or any ‘outsider’) from a sea of other potential Southern workers. Prior to P’s arrival, S and A jointly produce a similar product that can be sold on local Southern markets at price $V_S < V_P$. Owing to the absence of other competitors owning similar assets, the relationship between P and S constitutes a bilateral monopoly. The difference $\sigma \equiv V_P - V_S$ represents the extent of appropriable quasi-rents or *specificity* in the relationship between P and S.

To simplify the analysis, we assume that the good to be produced is indivisible.\(^{10}\) Agent A is privately informed about the cost $\theta$ of producing the good. S has special expertise regarding production conditions in the S country, represented by a signal $\eta$ which is partially informative regarding the realization of $\theta$. P does not observe the realization of this signal. Owing to their prior connection, S and A can costlessly communicate and side-contract with one another privately; such communication or transactions are not observed by P.\(^{11}\)

With non-integration (NI) where the two firms are separately owned, P and S negotiate an arms-length contract where P buys the good from S, who in turn contracts with A to produce the good. Throughout we assume P has all the bargaining power *vis-a-vis* S, and S has all the bargaining power *vis-a-vis* A.\(^{12}\) The key contracting friction between P and S

\(^{10}\)Most results extend to the context where the quantity produced is divisible. See Mookherjee et al. (2016) for details.

\(^{11}\)Consistent with the literature on collusion in organizations following Tirole (1986), we assume S and A can enter into side contracts that are costlessly enforced (via a third party or on the basis of other parallel relationships between the two of them). The only friction in side-contracting between S and A is the superior information possessed by A regarding the realization of cost $\theta$. Unlike the Tirole (1986) model, there is no ‘hard information’ that restricts reports that can be submitted, so the potential for collusion is considerably more severe in our setting.

\(^{12}\)In later sections we describe the consequences of alternative distribution of bargaining power.
in NI is that P is unable to verify payments made by S to A. This could either be the result of poor accounting standards in the Southern country (which allow S to costlessly produce ‘fake’ invoices for payments to A), or the result of collusion between S and A (whereby S can enter into side-payments with A that cannot be observed by any third party). This prevents P from entering into cost-sharing contracts with S. Hence non-integration results in DMR, owing to a cascading of information rents along the supply chain. We impose no constraints on contracts or message spaces: and show that as long as P is unable to contract directly with A, sophisticated design of contracts with S will not eliminate the DMR problem.

Controlling procurement cost constitutes the prime motive for vertical integration, in which P acquires the key asset owned by S that enables P to contract directly with A. In the integrated firm, P can hire S as a supervisor or manager, to tap the latter’s expertise in order to design contracts to A. The integrated firm is a hierarchy in which P contracts directly with both S and A. P invites and cross-matches reports from S and A of their respective private information. This gives rise to incentives for S and A to collude, thereby generating one source of (endogenous) transaction cost. The other source of transaction cost is a fixed exogenous cost \( f \) of writing contracts with more players (A as well as S) which are more complicated (in the sense of being based on a larger number of contingencies), and setting up necessary communication channels. In contrast, non-integration involves P offering a single take-it-or-leave-it price to S, to which S responds with a delivery decision. The fixed ‘institutional’ costs of centralization in the integrated enterprise depend on Southern country characteristics (such as legal and communication infrastructure) and distance from the Northern country where P is based (which affect communication costs).

Our first main result is that the gross profit of P in the integrated firm (excluding the fixed set-up cost \( f \)) is strictly higher compared to non-integration, provided the upper bound of the support of the cost distribution is high enough to ensure existence of a DMR problem in the latter. In other words, P is able to reduce the severity of the DMR problem in the integrated firm, despite the problem of collusion. The extent of increase in gross profit is independent of the institutional setup cost \( f \). Hence vertical integration will occur when the set-up cost is smaller than the increase in gross profit achieved; otherwise non-integration will be observed. The ability to contract directly with A and the setting up of a centralized organization enables a reduction in the DMR problem that was not achievable
under non-integration. This is the benefit of integration, which has to be traded off against the setup cost $f$.

The benefit of integration depends on the extent of specificity $\sigma \equiv V_P - V_S$: it approaches zero as specificity approaches zero, and is strictly increasing provided it exceeds some threshold value. The model thus formalizes one of the most important and robust prediction of the transaction cost approach: high specificity renders vertical integration more likely.

The model also generates the following predictions: (i) The integrated firm will be organized as a multi-layer hierarchy, as the benefits of employing S as a supervisor in the integrated firm outweighs the costs of the collusion this may give rise to; (ii) It will not be optimal for $P$ to unconditionally delegate authority to $S$ in contracting with $A$, as this would just reduce the mechanism to that resulting in non-integration;\(^{13}\) (iii) Vertical integration takes the form of $P$ acquiring $S$’s firm rather than vice versa, i.e., backward rather than forward integration ought to result in the production context considered in the model. Conversely, in a marketing setting where the agent is a customer and $S$ a local retailer with private connections to the consumer, integration will take the form of forward integration in which $P$ sells directly to consumers, hiring $S$ as a marketing agent or salesperson. This contrasts with the PR theory prediction that the owner will tend to be the party with the more severe incentive problem. (iv) Vertical integration is more likely to arise if the South country has superior communication and legal infrastructure, the fixed setup costs of FDI in the South country are lower (e.g. when the distance between the two countries is smaller). (v) Integration is more likely in industries with higher value products, and for more Northern firms that are more productive. As discussed in Section 7, many of these predictions are supported by empirical evidence, while evidence on others are currently lacking.

Finally our model yields interesting implications for distributional and welfare impacts of vertical integration. Owing to the difficulty in obtaining explicit analytical solutions, we numerically compute optimal allocations in the vertically integrated firm in an example with

\(^{13}\)However, in Mookherjee et al. (2016), we show in the context of a more general model of mechanism design with collusion that the optimal allocation can also be achieved by a conditional delegation mechanism. In this mechanism, $P$ delegates contracting and communication with $A$ to $S$ on the equilibrium path, but offers $A$ and $S$ an option to trigger a ‘dispute’ which then results in $P$ acquiring control and setting up an arbitration mechanism which decides production and payments following submission of cost reports by $A$ and $S$. 

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uniformly distributed costs. In this example, integration when it occurs results in higher welfare and prices offered to A, in the context involving a bilateral monopoly between P and S. However, this result may not obtain in an extended version of the model where integration could be accompanied by an increase in monopsony power (e.g., where the status quo involves two competing suppliers $S_1, S_2$ in the Southern country, one of which is acquired by P and the remaining supplier is unable to compete with the integrated firm). Returning to the bilateral monopoly context, the aggregate rents of S and A (and therefore aggregate surplus, including P’s welfare) turn out to be higher under integration. For some parameter values involving low specificity S’s rents are unaffected; over this range integration is Pareto improving. The improvement in aggregate efficiency tends to increase in the extent of specificity. For fixed $V_S$, a larger fraction of increases in $V_P$ are ‘passed on’ to A (i.e., A’s welfare increases by more) under integration, implying greater ‘trickle down’ effects of globalization benefits to workers. In Section 7 we discuss empirical evidence concerning effects of FDI that confirm these predictions.

3 Model Details

The status quo corresponds to non-integration (NI) with two separately owned firms P and S. P earns $V_P$ by selling a good to the world market. The good to be supplied is indivisible. S procures the good from a local producer A and can earn $V_S < V_P$ by selling it in the local market. A can supply the good to either S or P but not both, and is privately informed regarding his production cost $\theta$. P and S share a common prior distribution $F(\theta)$ (with a positive, differentiable) density $f(\theta)$ on support $[\theta, \bar{\theta}]$. Not owning a complementary productive asset (which is owned by S) or market reputation, A cannot supply the good to either local or the world market on his own.

From past experience, S has accumulated ‘local’ connections and expertise that P does not possess. This includes an ongoing relationship with A, who is more suited to the production task compared to other local producers in the S country. If P were to try to enter production without S’s help, she would not be able to identify or select A from the pool of potential applicants. Apart from knowledge of the ‘right’ local producer, S has ‘expertise’ represented by access to an informative signal $\eta$ of A’s cost. The realization of this signal is observed by S and A jointly. $\eta$ takes two possible values $\eta_L, \eta_H$. The likelihood of observing
signal $i$ is $a_i(\theta)$, a positive differentiable function on $(\theta, \bar{\theta})$. Let $F_i(\theta) \equiv \frac{1}{\kappa_i} \int_\theta^{\bar{\theta}} a_i(y)f(y)dy$ denote the distribution of $\theta$ conditional on $\eta_i$, where $\kappa_i \equiv \int_\theta^{\bar{\theta}} a_i(y)f(y)dy \in (0,1)$ denotes the probability of $\eta_i$. The density function of $F_i(\theta)$ is denoted by $f_i(\theta)$.

To ensure the problem is interesting we assume $V_P > V_S > \theta$. We also impose standard monotonicity conditions on likelihood ratios and hazard rates:

**Assumption 1**  
(i) $\frac{a_L(\theta)}{a_H(\theta)}$ is decreasing in $\theta$ on $[\theta, \bar{\theta}]$.

(ii) $H(\theta) \equiv \theta + \frac{F_i(\theta)}{f_i(\theta)}$, $h_i(\theta) \equiv \theta + \frac{F_i(\theta)}{f_i(\theta)}$ and $l_i(\theta) \equiv \theta + \frac{F_i(\theta)-1}{f_i(\theta)}$ ($i = L, H$) are increasing in $\theta$.

These can be interpreted in terms of corresponding assumptions regarding supply functions and their elasticity: a low signal corresponds to higher supply (a supply function shifted to the right) and lower price elasticity: $F_H(p) < F(p) < F_L(p)$ and $\sigma_L(p) < \sigma(p) < \sigma_H(p)$ where $F_i(p)$ denotes the likelihood of A supplying the good when offered payment of $p$, and $\sigma_i(p)$ the elasticity $\frac{pF_i(p)}{F_i(p)}$ conditional on signal $\eta_i$, while $F(p), \sigma(p)$ represent corresponding supply and supply elasticity functions. A specific example is a uniform prior ($F(\theta) = \theta$ on $[0,1]$) and linear likelihood ratio function $a_L(\theta) = 1 - \theta$ for $\theta \in [0,1]$ and $a_H(\theta) = \theta$ for $\theta \in [0,1]$. See Figure 1.
In the absence of P, S delivers the good to the local market after procuring from A. Following $\eta = \eta_i$, S offers a take-it-or-leave-it price $p_i(V_S)$ to A which maximizes $F_i(p_i)(V_S - p_i)$, and earns an expected payoff $u_i^S \equiv F_i(p_i(V_S))(V_S - p_i(V_S))$.

When P enters, there are two different ways for P to procure the good from the S country:

- **Non-Integration (NI):** The two firms are separately owned, and P procures the good by contracting with S, who becomes a middleman between P and A. In the North-South context, this corresponds to outsourcing.

- **Backward Integration (BI):** P acquires S’s firm, which consists of knowledge of A’s identity and assets that A works with. S gives up the option to procure from A and deliver to the local market, enabling P to procure directly from A. In the North-South context, this corresponds to foreign direct investment (FDI) by P via acquisition of a local firm.

In a later section, we shall also consider a third alternative: forward integration (FI) where S acquires P’s firm, procures from A and supplies to the world market. Figure 2 illustrates contract structures in NI, BI and FI.
3.1 Non-Integration

NI features a sequence of bilateral contracts: first P offers a contract to S, then S offers a contract to A. It is not possible for P to observe payments made by S to A, hence P cannot condition the price offer to S on the latter’s cost.\(^\text{14}\) Cost observability is ruled out either owing to absence of suitable accounting standards in the S country, or collusion between S and A that enables them to manipulate accounting costs via false invoices. P is unable to ensure that S responds to her offer at the interim stage before communicating with A. Hence both contracts are subject to ex post participation constraints.

Under such conditions, P is unable to screen S’s private information regarding cost conditions by conditioning trades on messages sent by S: without loss of generality, P offers a take-it-or-leave-it price \(b\) to S for delivering the good. The reason is that S can wait to obtain a cost report from A before responding to P’s offer, removing the possibility of screening S’s information on the basis of differences in S’s beliefs prior to communicating with A. Conditional on the production level, S will submit whatever message will correspond to the maximal aggregate payment P makes to S and A combined, and then divide this up with A in a suitable manner using a side contract. Hence P can do no better than to base the aggregate payment to S and A on the production level, and delegate authority entirely to S.\(^\text{15}\)

Since the good is indivisible, the ‘outsourcing’ contract between P and S consists of two payments, corresponding to non-delivery \((X_0)\) and delivery \((X_0 + b\) respectively). Payment \(X_0\) in the event of non-delivery must be non-negative, owing to ex post participation constraints.\(^\text{16}\) This prevents P from using a two-part tariff, where S’s information rents can be taxed away at the interim contracting stage. Moreover, payment in the event of delivery \(X_0 + b\) cannot be smaller than \(V_s\), what S can earn by selling instead to the local market. The contract \((X_0, b)\) generates an expected profit of \(\kappa_L F_L(p_L(b)) + \kappa_H F_H(p_H(b))(V_P - b) - X_0\), which is maximized by choosing \(X_0, b\) subject to \(X_0 \geq 0\) and \(b + X_0 \geq V_S\), where \(p_i(b) \equiv \arg \max_{p_i \in [\theta, \bar{\theta}]} F_i(p_i)(b - p_i)\) for \(i = L, H\). Clearly the optimal \(X_0\) is zero. Let \(b^{NI}\) denote the optimal price offer and \(\Pi^{NI}\) be the associated expected profit earned by P.

\(^{14}\)Melumad, Mookherjee and Reichelstein (1995) show verifiability of supplier cost is necessary for sequential bilateral contracting to achieve constrained optimal allocations.

\(^{15}\)See Baliga and Sjostrom (1998) for a similar argument in the context of a model of collusion with moral hazard.

\(^{16}\)A negative transfer in any state would result in S rejecting the contract offer in that state.
The corresponding price offered by S to A is \( p_{ni}^i \equiv p_i(b_{ni}) \).

The solution to NI features double marginalization of rents. S earns rents in contracting with P owing to private information regarding his own procurement cost. At the same time, A also earns rents in contracting with S. S’s monopsony power in contracting with A features the standard trade-off between extracting A’s rents and lowering the probability of A’s supply. In setting a price offer for A, S ignores P’s loss of rents when A fails to supply the item, and ends up offering too low a price to A. Alternatively, the supply curve facing P includes S’s information rents, which lies above the supply curve facing S. See Figure 3 for an illustration of the outcomes in state \( \eta_i \).

3.2 Backward Integration

We now describe allocations that can be achieved in an integrated firm owned by P. Here P can contract directly with A, as well as with S to elicit the latter’s information regarding the cost signal \( \eta_i \). This information can be used by P to lower A’s rents. Knowing that the cost signal is low would motivate P to offer A a lower price, as A’s supply elasticity is lower in this state. A would then have an incentive to bribe S to report a high cost signal instead. Collusion limits the usefulness of P’s effort to elicit S’s information, as S and A can
communicate privately with one another and enter into hidden side-contracts to ‘game’ the mechanism designed by P. We will later show that it is typically optimal for P to contract with S to elicit the latter’s information in the integrated firm. Hence we need to consider the implications of P contracting with both S and A in the integrated firm.

Collusion between S and A takes the ex ante form, where they negotiate a side-contract prior to responding to P’s offer of a mechanism.\textsuperscript{17} We assume, in the tradition of Tirole (1986), that contracts are costlessly enforceable by some third party. Following private communication of a cost message by A to S, the side-contract coordinates their respective messages (which include participation decisions and cost reports) sent to P, besides a side payment between A and S. Unlike Tirole (1986), information is ‘soft’, i.e., message spaces are unrestricted, with both S and A able to send ‘false’ messages. S offers a side contract to A, which A accepts or refuses.

In the event of A refusing this side-contract, they play P’s mechanism noncooperatively. Unlike NI, A has the option of receiving a contract directly from P, which can now be conditioned on reports sent by S regarding the realization of $\eta$. As elaborated in Mookherjee et al. (2016), integration allows P to manipulate the outside options of A in bargaining over a side contract, reducing the severity of the DMR problem. Raising A’s outside option forces S to offer a higher price to A for delivering the good, thereby alleviating the underproduction in NI. It can be shown that P can equivalently delegate contracting with A to S in a conditional manner – where S communicates and contracts privately with A, but A has an option to trigger an ‘appeals’ or ‘dispute settlement’ mechanism in which they submit reports independently to P who then makes all relevant production and payment decisions. Bargaining over the side-contract in the ‘shadow’ of such an internal quasi-legal mechanism allows P to reduce the severity of the DMR problem.

The specific setting considered in this paper (i.e., an indivisible good being procured and two-point cost signals received by S) allows considerable simplification of the analysis of optimal mechanisms in BI. It can be shown that P loses nothing by confining attention to mechanisms that are collusion-proof, which leave no room for S and A to enter into a non-null side contract (see Mookherjee et al. (2016)). Hence P can confine attention to

\textsuperscript{17}This is in contrast to interim collusion where S is required to communicate his participation decision to P before communicating with A, as in the analyses of Faure-Grimaud et al. (2003) or Celik (2009). The implications of this contrast are elaborated in detail in Mookherjee et al. (2016).
mechanisms satisfying a set of individual and coalition incentive compatibility constraints.

In order to describe the mechanism design problem in BI, it is necessary to be explicit about the exact sequence of events by which P negotiates the acquisition of S’s firm (depicted in Figure 4):

(BI-i) P’s offer provides a comprehensive description of the proposed mechanism within the integrated firm to S. We refer to this as the Grand Contract (GC), which is communicated privately to S (so is not observed by A). It specifies message spaces \( M_A, M_S \) for A and S respectively, production decision \( q(m_A, m_S) \) and transfers \( X_A(m_A, m_S), X_S(m_A, m_S) \) conditioned on submitted messages. A’s message space includes an exit option which is followed by absence of production and transfers to A. S has the opportunity to reject P’s offer both before and after communicating with A, so an additional exit option for S does not need to be included in \( M_S \).

(BI-ii) S decides either to (a) reject P’s acquisition offer at this point, prior to communicating with A, or (b) disclose the offer to A and propose a side-contract to A defining how they can jointly collude.\(^{18}\) The side-contract (SC) specifies a private report of the true cost \( \theta \) from A to S, followed by joint messages \( m(\theta, \eta) \in M_A \times M_S \cup \{Exit\} \) they respectively send to P, a private side-payment \( t(\theta, \eta) \) from S to A, and production supplied to the local market \( q(\theta, \eta) \in \{0, 1\} \) for \( (\theta, \eta) \) in the event that they decide to reject P’s mechanism \( (m(\theta, \eta) = Exit) \).

(BI-iii) If at the previous stage S rejected P’s offer without disclosing to A, S contracts

\(^{18}\)There is ‘hard information’ about the offer, implying that if S discloses the offer, there is no scope for lying about any of its details.
with A to deliver the product to the local market. If S disclosed the offer and proposed a side contract SC, A responds by rejecting or accepting SC. If it is accepted, the SC is implemented and the game ends.

(BI-iv) If A rejects SC, S and A proceed noncooperatively thereafter. S decides whether or not to accept the BI offer. If he does, S and A participate in the BI mechanism noncooperatively. If S rejects the BI offer, the game ends with no production or payments between S and A.

The solution concept employed is Weak Perfect Bayesian Equilibrium (WPBE) which is Pareto-undominated for the \{S, A\} coalition, i.e., for any \(\eta\), there does not exist any other WPBE which improves S’s payoff, without making any type of A worse off.\(^{19}\)

We now characterize properties of allocations that can be achieved as outcomes of WPBE satisfying this criterion. To simplify the exposition we focus on equilibria in which BI is accepted by S in both states \(\eta_L, \eta_H\). We show below that such equilibria exist, and allow P to earn higher operating profit compared with NI. Proposition 2 below shows that such equilibria also generate higher profit for P than any equilibrium in which BI is accepted by S in only one of the two states.

Since the good is indivisible and the mechanism has to be collusion-proof, abstract message spaces can be dispensed with and the allocation can be represented more simply by a set of prices that satisfy a set of constraints described below.

First, a contract offer to A in the BI mechanism reduces to a single take-it-or-leave-it price offer \(p_i\) made to A when the cost signal is \(\eta_i\). Second, in order to deter collusion, P must offer an aggregate payment to S and A which depends only on whether or not the good is produced. Let \(X_0 + b, X_0\) denote the aggregate payments when the good is and is not produced respectively. The two prices \(p_L, p_H\) combined with \(X_0, b\) characterize a BI allocation entirely. This is associated with a mechanism where S and A are asked to submit reports (\(\hat{\eta}_S, \hat{\eta}_A\)) of the signal \(\eta\) to P. If the two reports happen to match (\(\hat{\eta}_S = \hat{\eta}_A = \eta_i\)), A is offered the option to produce and deliver the good directly to P in exchange for price \(p_i\), while S is paid \(X_0\) if the good is not delivered, and \(b + X_0 - p_i\) if it is delivered. If the two reports do not match, there is no production and S and A are required to pay a high penalty to P. The key feature distinguishing BI from NI allocations is that in the former P

\(^{19}\)This refinement is essential to capture the prospect of collusion between S and A, as explained in Mookherjee et al. (2016).
makes a contract offer directly to A which is conditioned on reported signals. This provides an outside option to A which S is constrained to match while offering a side contract to A. This is an important strategic tool that enables P to manipulate the outcome of collusion between S and A, and reduce the severity of the DMR problem.

Along the equilibrium path where A and S decide to participate, report $\eta_i$ truthfully to P, and do not enter into a deviating side-contract, A produces the good in state $\eta_i$ and receives the payment $p_i$ if and only if $\theta_i$ is smaller than $p_i$. Without loss of generality, A receives no payment in the event of non-production.\footnote{It can be checked that any mechanism paying a positive amount to A in the event of non-production is dominated by one that does not.} This generates utility to A of $u_A(\theta, \eta_i) = \max\{p_i - \theta, 0\}$. S ends up with $X_0 + b - p_i$ in the event that production takes place, and $X_0$ otherwise.

The BI allocation $p_L, p_H, X_0, b$ has to satisfy the following feasibility constraints. First, in order to ensure that \textit{ex post} the coalition does not prefer to reject it or supply to the local market instead:

\begin{align*}
b + X_0 & \geq V_S \\
X_0 & \geq 0.
\end{align*}

Second, in order to induce S to accept the acquisition offer \textit{ex ante}, S’s interim expected utility cannot fall below what he could earn by rejecting it at stage (BI-ii):

\begin{align*}
F_H(p_H)(b - p_H) + X_0 & \geq u_S^H \\
F_L(p_L)(b - p_L) + X_0 & \geq u_S^L.
\end{align*}

Third, S and A should not be tempted to enter a deviating SC. A deviating SC would involve a different set of prices $\tilde{p}_i$ offered to A (in state $\eta_i$) for delivering the good, combined with a lump-sum payment $\tilde{u}_i$. A would then produce if $\theta$ is smaller than $\tilde{p}_i$, and S would earn an expected payoff $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 - \tilde{u}_i$. A would accept the deviating SC provided

\begin{equation*}
\max\{\tilde{p}_i - \theta, 0\} + \tilde{u}_i \geq \max\{p_i - \theta, 0\}
\end{equation*}

Hence collusion-proofness requires ($\tilde{p}_i = p_i, \tilde{u}_i = 0$) to maximize $F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 - \tilde{u}_i$ subject to (5).

This condition can be broken down as follows. First, if $p_i > \theta$, S should not benefit by deviating to a price $\tilde{p}_i < p_i$. This would necessitate offering a lumpsum payment of
\( \tilde{u}_i = p_i - \tilde{p}_i \) to ensure that A accepts the SC, which would then generate S an interim expected payoff of \( F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 - p_i + \tilde{p}_i \). This is equivalent to requiring that
\[
b \geq p_i - \frac{1 - F_i(p_i)}{f_i(p_i)} \equiv l_i(p_i)
\] (6)
since \( l_i(p) \) is increasing in \( p \) as per the monotone hazard rate assumption 1(ii). Intuitively, offering a lower price than \( p_i \) is similar to S selling the good back to A. Condition (6) which states that the value of the good to S (\( b \)) exceeds its virtual value to A ensures that such a sale is not worthwhile.

Similarly, if \( p_i < \bar{\theta} \), S should not want to offer A a higher price \( \tilde{p}_i \). Unlike the case of a lower offer price, such a variation cannot be accompanied by a negative lump sum payment \( \tilde{u}_i \) to A, owing to the need for A’s ex post participation constraint to be satisfied in non-delivery states. Offering \( \tilde{p}_i > p_i \) will then generate an interim payoff of \( F_i(\tilde{p}_i)(b - \tilde{p}_i) + X_0 \). For S to not want to deviate to a higher price, it must be the case that
\[
b \leq p_i + \frac{F_i(p_i)}{f_i(p_i)} = h_i(p_i)
\] (7)
given the monotone hazard rate assumption. This condition can be interpreted simply as the value of delivery (\( b \)) to S being lower than the virtual cost of A of delivering it.

(6, 7) can be combined into the single collusion-proofness condition
\[
\max\{\hat{l}_L(p_L), \hat{l}_H(p_H)\} \leq b \leq \min\{\hat{h}_L(p_L), \hat{h}_H(p_H)\}.
\] (8)
where \( \hat{h}_i(p) \) denotes \( h_i(p) \) for \( p \neq \bar{\theta} \) and \( \infty \) otherwise, and likewise \( \hat{l}_i(p) \) denotes \( l_i(p) \) for \( p \neq \theta \) and \(-\infty \) otherwise.

The preceding arguments explain the necessity of conditions (1, 2, 3, 4, 8) for an allocation \((p_L, p_H, b, X_0)\) to be feasible in BI. They are also sufficient: in the Appendix we show that a coalition-Pareto-undominated WPBE can be constructed which results in this allocation.

**Lemma 1** A BI allocation \((p_L, p_H, b, X_0)\) is feasible, i.e., incentive compatible and collusion-proof, if and only if it satisfies conditions (1, 2, 3, 4, 8).

Finally, an optimal BI allocation must maximize
\[
[\kappa_H F_H(p_H) + \kappa_L F_L(p_L)](V_P - b) - X_0
\] (9)
subject to (1, 2, 3, 4, 8). We shall denote the solution by \((p_H^{BI}, p_L^{BI}, b^{BI}, X_0^{BI})\), and the accompanying expected profit of \(P\) by \(\Pi^{BI}\). This excludes the institutional fixed cost \(f\) that the BI mechanism entails. This fixed cost corresponds to additional communication channels in BI: \(P\) needs to offer and sign contracts with \(A\) directly, and elicit reports of the cost signal from \(S\) and \(A\). Price offers are conditioned on the reported costs. In contrast, in NI, \(P\) only needed to contract with \(S\); no cost reports needed to be submitted by \(S\), as \(P\) offered a single take-it-or-leave-it price \(b\). We shall hereafter refer to \(\Pi^{BI}\) as the operating profit of \(P\) in BI, which excludes the fixed cost \(f\), so that the net profit equals \(\Pi^{BI} - f\). This needs to be compared with \(\Pi^{NI}\) when \(P\) decides whether or not to acquire \(S\)'s firm. An acquisition will occur only if BI earns a higher operating profit by enough to cover the fixed cost \(f\): \(\Pi^{BI} - \Pi^{NI} > f\).

It is evident that at least one of either (1), (3) and (4) must be binding in the optimal allocation.\(^{21}\) It is also evident that \(P\)'s maximal profit \(\Pi^{BI}\) approaches zero as the extent of specificity \(V_P - V_S\) approaches zero.\(^{22}\) Hence it is necessary that there be a non-negligible degree of specificity for BI to be chosen rather than NI.

## 4 Results

We first compare \(P\)'s operating profits in NI and BI, excluding the institutional fixed cost \(f\) associated with BI. Note first that \(P\) can always attain in BI at least the profits achieved in NI, since the latter is equivalent to unconditionally delegating authority to \(S\) to contract with \(A\) within BI (i.e., where \(P\) does not offer a contract to \(A\), so \(A\) has no outside option in bargaining with \(S\) over the side contract).\(^{23}\) The question is whether \(P\) can achieve strictly higher profit in BI by enough to overcome its additional institutional cost to be worthwhile.

This cannot happen when \(V_S\) is large enough relative to the upper bound \(\bar{\theta}\) (specifically, if \(V_S \geq h_L(\bar{\theta})\)) that \(P\) always procures the good in NI, by offering a price large enough to guarantee that the good is delivered \((p_L^{NI} = p_H^{NI} = \bar{\theta})\).\(^{24}\) In that case NI involves no

\(^{21}\)Otherwise \(X_0 = 0\) and \(b = \max\{\hat{l}_L(p_L), \hat{l}_H(p_H)\}\). Then \(b < p_i\) for each \(i\), and \(S\)'s participation constraint will be violated.

\(^{22}\)(1) implies aggregate payments \(b + X_0\) to the coalition in the event of the good being delivered approaches what \(P\) can sell the good for, so \(P\)'s profit in this event approaches zero. And (2) ensures that \(P\) cannot make any profit if the good is not delivered.

\(^{23}\)Specifically, the optimal NI allocation corresponds to a BI allocation with \(p_i = p_L^{NI}, X_0 = 0, b = b^{NI}\).

\(^{24}\)Recall that \(b^{NI} \geq V_S\) is necessary to satisfy \(S\)'s participation constraint in NI. Hence \(V_S \geq h_L(\bar{\theta})\) implies
underproduction and hence is not subject to any DMR problem: there cannot be any scope for achieving higher operating profit by acquiring S’s firm.

Our first main result is that in all other cases, BI does attain a higher operating profit.

**Proposition 1** \( \Pi_{BI} > \Pi_{NI} \) if and only if \( h_L(\bar{\theta}) > V_S \).

Proposition 1 implies that whenever NI involves a price below the maximum cost \( \bar{\theta} \) and is thereby potentially subject to a DMR problem, BI will be preferred if \( f \) is small enough, and NI will be preferred otherwise. The reasoning underlying this result is quite simple, illustrated in Figure 5. Suppose that the price offered to A in NI in state L is smaller than \( \bar{\theta} \), so there is scope for raising the price further in this state. Let P select \( p_L = p'_L \) in BI which is slightly higher than \( p_{NI}^L \), while leaving the price in state H and S’s delivery bonus \( b \) unchanged \( (p_H = p_{NI}^H, b = b_{NI}) \). This raises the probability of the good being delivered, resulting in a first-order increase \( [F_L(p'_L) - F_L(p_{NI}^L)](V_P - b_{NI}) \) in P’s expected profit. On the other hand, S’s payoff in L falls since \( p_{NI}^L \) had been optimally chosen by S in NI given the delivery bonus \( b_{NI} \) which remains unchanged. To compensate S for this, P needs to offer a positive lump-sum payment \( X_0 = F_L(p_{NI}^L)(b_{NI} - p_{NI}^L) - F_L(p'_L)(b_{NI} - p'_L) \). But S’s \( b_{NI} \geq h_L(\bar{\theta}) \). Then \( p_{NI}^L = \bar{\theta} \).
loss is second-order, so the cost of this compensation is smaller than the gain in P’s profit owing to the higher probability of delivery. As the resulting allocation is feasible in BI, i.e., satisfies conditions (1, 2, 3, 4, 8), it follows that P earns a higher operating profit in BI. Contracting directly with A allows the DMR problem to be reduced, as P offers a higher price to A which S is forced to match in BI.

**Corollary 1**

(i) If \( h_L(\bar{\theta}) > V_S \), and given specificity \( V_P - V_S \), P prefers BI to NI if fixed cost \( f \) of BI is sufficiently small.

(ii) Higher specificity enlarges the range of fixed costs for which P prefers BI, over a range of high levels of specificity (i.e., when \( V_S \) is small relative to \( V_P \)).

(iii) Given any \( f \), if specificity is sufficiently low, P prefers NI to BI.

(i) is evident, while (ii) follows from the following argument. For fixed \( V_P \) consider the implications of varying the degree of specificity, i.e., letting \( V_S \) vary over the range \([0, V_P]\). When specificity is high (i.e., \( V_S \) is low), the solution to NI is locally independent of \( V_S \) as S’s participation constraint is not binding. On the other hand, some participation constraint is always binding in BI, and a fall in \( V_S \) relaxes these constraints, so P’s profit in BI increases as a result. Hence integration becomes more attractive with higher specificity. Over low ranges of specificity, optimal profits in both NI and BI are decreasing in \( V_S \), and it is difficult to compare the rates at which they respectively fall. See however the numerical examples in Section 6 where benefits of integration are everywhere increasing in specificity. Finally, result (iii) follows from the fact that P’s profits approach zero under either NI and BI when specificity approaches zero.

The next Proposition provides a rationale for focusing on equilibria where BI results in both states \( \eta_L, \eta_H \). It shows that such equilibria generate higher profits for P compared with those in which BI results in only one state \( \eta_i \), while in the other state \( \eta_j, j \neq i \) S refuses to sell the firm to P, with either NI resulting in that state, or S does not sell to P at all and sells to the local market instead. The argument is essentially similar to that used in Proposition 1 above: reductions in DMR resulting from integration generate benefits to P in each and every state separately, though the argument is complicated by the feature that the feasibility constraints pertain jointly to both states.

\(^{25}\)Condition (8) holds since \( b^{NI} = h_L(p_{L}^{NI}) > l_L(p_{L}^{NI}) \), so \( p_{L}' \) slightly higher than \( p_{L}^{NI} \) implies \( h_L(p_{L}') > b^{BI} > l_L(p_{L}') \). It is evident that the other conditions also hold.
**Proposition 2**  
(a) Any equilibrium in which BI results in only one state $\eta_i$, while in the other state $\eta_j, j \neq i$ there is no trade between P and S (i.e., S supplies to the local market) generates less profit for P than an equilibrium in which BI results in both states.

(b) Suppose $V_P < h_H(\bar{\theta})$. Then any equilibrium in which BI results in one state, and NI in the other, generates less profit for P than an equilibrium in which BI results in both states.

### 4.1 Value of Hiring Supervisor in Integrated Firm

In BI, the potential advantage of hiring S is that signal reported by S helps P reduce A’s rents. On the other hand, S will earn some rents owing to collusion, which cannot be taxed away upfront by P (owing to the *ex ante* nature of the collusion resulting from S’s prior connection with A). Is it then beneficial for P to hire S as a supervisor? Above we restricted attention to a particular form of BI in which P contracted with S to provide a cost signal report which is used by P to contract with A. We now compare the profits from P hiring and not hiring S.

What exactly does it mean to not hire S as a supervisor in BI? By this we mean an arrangement in which S sells his firm to P in exchange for a lumpsum amount $X_0$ in both states $\eta_L, \eta_H$. S does not send any report of the signal $\eta$, and the payment to S does not depend on the output produced by A (i.e., $b = 0$). After acquiring S’s firm, P then contracts with A on the basis of his prior beliefs over $\theta$. Later we consider other more complicated alternatives, e.g., where S sells the firm only in one state but not the other.

Let the mechanism where P acquires S’s firm but does not hire S as a supervisor be denoted by NS. In this mechanism, P will directly offer A a price $p$ (which does not depend on $\eta$), and offer S a lumpsum $X_0$ for acquiring the firm. These will be selected to maximize

$$\max F(p)(V_P - p) - X_0$$

subject to

$$X_0 \geq \max \{u_{iH}^S, u_{iL}^S\}$$

$$X_0 + p \geq V_S.$$
The first constraint is required to ensure S is willing to sell the firm in both states \( \eta_L, \eta_H \). Since \( u^S_L \geq u^S_H \) by Assumption 1(i), it reduces to \( X_0 \geq u^S_L \). The second constraint prevents coalitional exit from the grand contract. Let \( (p^{NS}, X_0^{NS}) \) denote the solution to this problem, and \( \Pi^{NS} \) be the associated profit.

**Proposition 3** Assume that \( H(\bar{\theta}) > V_P > V_S > 0 \). Then \( \Pi^{BI} > \Pi^{NS} \).

The reasoning is as follows. Without S, it is optimal for P to offer an interior price \( p^{NS} < \bar{\theta} \) to A, since \( H(\bar{\theta}) > V_P \). The acquisition price \( X_0 \) paid to S is at least \( u^S_S \) which is strictly greater than \( u^S_S \), since \( V_S > 0 \). Hence S’s participation constraint is slack in state \( H \). If P hires S, P can raise \( p_H \) slightly above \( p^{NS} \), while selecting \( b = p_L = p^{NS} \) and leaving \( X_0 \) unchanged. Owing to positive slack in S’s participation constraint in state \( H \), this allocation is feasible in BI, and generates higher profit for P.

The result continues to hold when NS involves a sale of S’s firm in only one state, but not the other. Here P can learn the state from observing whether S accepts the BI offer. The case where S sells the firm in one state \( \eta_i \) but is not hired as a supervisor is a special case of an allocation where BI results only in state \( \eta_i \) in which the payment to S is independent of what A produces \( (b = 0) \). Proposition 2 shows P can earn higher profit from an allocation where BI results with S being hired in both states.

One qualification however ought to be noted: the preceding argument is based on the assumption that NS and BI both involve the same institutional fixed cost. This pertains to contexts where contracting costs with A constitute the dominant source of the fixed cost. If communication costs are more important, NS could involve a substantially lower fixed cost, as P only contracts with A and does not have to set up a cross-reporting or dispute settlement mechanism. In that case it is possible that P may prefer not to hire S as a supervisor in BI.

5 Variations and Extensions

5.1 Improved Contract Enforcement Institutions

What are the consequences of better contract enforcement institutions? The answer depends on the precise nature of the improvement. If the key problem with NI is poor accounting standards in the Southern country, there will be an improvement if S’s payments
to A can be verified by P. In that case, sophisticated cost-based contracts can overcome the DMR problem, leaving no scope for integration to increase P’s profits.

**Proposition 4** Suppose that \( V_P < h_L(\theta) \). If P can verify side-payments between A and S, second-best profits (\( \equiv \Sigma_i \kappa_i |F_i(p_i(V_P))(V_P - p_i(V_P)) - u_i^S| \)) can be achieved in NI.

The argument (provided in the Appendix) is that with verifiable costs P can effectively mandate what price \( p_i \) S can pay A for delivering the output following a cost report of \( \eta_i \) made by S to P. Corresponding payments \( X_i, b_i \) from P to S in the event of output being not delivered and delivered can also be stipulated in the NI contract. The only room for S to behave strategically is to misrepresent the true cost signal to P. This turns out to not be a problem: under the same condition as in Proposition 1, P has enough instruments to induce S to report truthfully while implementing the second-best allocation in NI. Note in particular that with cost verifiability, collusion between S and A has no bite in NI.

Suppose on the other hand that accounting standards are poor (resulting in nonverifiability of costs of separately owned firms), and improved contract enforcement consists of reduced prospects for collusion between S and A. For instance, side contracts can no longer be enforced or involve considerable enforcement costs that generate deadweight losses in side-contracts. Then NI continues to be plagued by DMR, while BI achieves higher profits owing to lower collusion costs. If collusion between S and A takes the *interim* rather than *ex ante* form, P can elicit their respective information at the participation stage by offering each a menu of contracts (Motta (2009)) and achieve second-best profits in BI. In this case, improved contract enforcement institutions make vertical integration more likely.

### 5.2 Varying Bargaining Power between P and S

So far we assumed P has all the bargaining power in negotiating the acquisition with S. What happens if S also has some bargaining power? Suppose, for instance, that after P has decided to try to acquire S’s firm and has incurred the institutional cost \( f \) for setting up, a third-party assigning welfare weight \( \alpha \in [0, 1] \) to S designs the grand contract instead of P.\(^{26}\) If P decides to go the outsourcing route instead, the NI contract is designed by the same third-party with the same welfare weight \( \alpha \) assigned to S.

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\(^{26}\)A participation constraint for P has to be added, to ensure that P earns non-negative expected profit.
If $\alpha \geq \frac{1}{2}$, S is assigned greater bargaining power than P. In this case, the optimal mechanisms in both NI and BI award zero rent to P, whence the DMR problem disappears and both NI and BI can attain second-best allocations. Hence shifting bargaining power in favor of S makes BI less likely. This is the consequence of the assumption that in the bargaining between P and S, there is one-sided asymmetric information; whence raising the bargaining power of the uninformed party reduces the inefficiency underlying DMR. If asymmetric information were bilateral (e.g., if P were privately informed regarding the realization of $V_P$), the result would depend on the allocation of bargaining power vis-a-vis private information.

5.3 Forward Integration

Here we consider an alternative form of integration, where S acquires P’s firm and thus the right to sell the product in the world market at price $V_P$. Call this FI. The game corresponding to P’s offering FI instead of BI is as follows.

The grand contract offered by P consists of a ‘price’ $Q$ at which P is willing to sell her firm to S. It is easy to check that there is no value from basing this on a message submitted by S, for the same reason that there is no value from basing the outsourcing price in NI on messages sent by S (i.e., that S can respond to P’s offer after consulting A). If S decides to buy, thereafter S operates the integrated firm FI, hiring A to produce the product which is sold abroad at price $V_P$. S then ends up earning a net price of $V_P - Q$ for selling abroad, after subtracting the cost of purchasing the firm from P. This is equivalent to the NI alternative we have already considered where P offers an outsourcing price of $b = V_P - Q$. Hence if $\Pi^{BI} - f > \Pi^{NI}$, P will prefer to acquire S rather than sell his own firm to S.

If we altered the model to reverse roles of P and S, e.g., where P has ‘prior connections’ with customers on the world market and is privately informed about the latters’ valuations, while S could not collude with A, then our preceding analysis would be reversed with FI replacing BI. In that case we would observe FI rather than BI. In the more general case with two-sided ‘prior connections’, the choice between FI and BI becomes a non-trivial question. The two extreme cases considered above suggest that ownership is more likely to rest with the party with the ‘less severe’ incentive problem. This prediction contrasts with those of PR-based theories of ownership.
5.4 Local Outsiders versus Insiders

An alternative to BI would be for P to set up a new firm in the Southern country with the help of another expert (S') in the Southern country who does not have prior connections with workers or industry expertise in the status quo. Upon investing \( s' \), this expert can locate worker A, acquire the complementary assets that A needs to be able to produce the good that P needs, and gain access to a cost signal \( \eta' \) which may be less informative than \( \eta \). A high enough cost \( s' \) would have deterred this expert from entering the industry to compete with the incumbent S, prior to the arrival of P.

Once P arrives, she can consider setting up a firm with the help of S', engaging the latter to ‘find’ A and inform P regarding the realization of \( \eta' \). Call this option GI. GI is not subject to the problem of ex ante collusion, owing to the absence of a prior connection between S’ and A. P can contract with S’ and get the latter to commit to paying an upfront fee, before S’ starts searching for A and cost information. Once S’ finds A, the two can subsequently collude over joint reports to P. However, P can tax away collusion rents of S’ (net of the entry cost \( s' \) incurred by S’) with an upfront fee. Hence P can replicate the operating profits that can be achieved in GI by letting S’ set up a firm (denoted by G) and outsourcing to G. This option would dominate setting up GI which would incur an additional institutional cost of \( f \). One can also interpret G as a form of forward integration where P sells his firm to S’.

Hence optimal organizational forms associated with local ‘insiders’ and ‘outsiders’ can be strikingly different. There can also be a choice between the two alternatives. If \( s' \) is large and \( \eta' \) is less informative than \( \eta \), BI with S will be a superior option for P. Conversely if \( s' \) is small and \( \eta' \) is as informative as \( \eta \), dealing with the outsider S’ will be better as the problem of collusion is less severe.

5.5 Direct Entry via Greenfield Ventures, based on IT Advances

Yet another alternative is for P to enter without tying up with any local entrepreneur. Advances in information technology such as the internet may enable P to directly identify and make price offers to A, and acquire local cost signal \( \eta \) available previously only to local insiders. Then P can enter with a greenfield firm, that competes directly with the incumbent S. In other words, the local intermediary S can be directly bypassed, without requiring S’s consent.
Assume that subsequent to such entry, P and S compete in Bertrand fashion to employ A. As P can sell the product at a higher price \( V_P \) compared to what S can earn, the latter will be unable to compete with P. Define \( p^+_i \equiv \arg \max_{p_i \in \theta, \bar{\theta}} F_i(p_i)(V_P - p_i) \). Then if \( V_S < p^+_i \), \( p^*_P = p^+_i \) and P’s payoff is \( F_i(p^+_i)(V_P - p^+_i) \) and if \( V_S > p^+_i \), \( p^*_P = V_S \) and P’s payoff is \( F_i(V_S)(V_P - V_S) \). In either case, S receives zero payoff. The greenfield venture will be superior to the option of acquiring S’s firm: P no longer needs S’s special access to local workers and local information, and can dispense with the cost \( f \) of setting up a hierarchy that employs S as an intermediary. If the technology is effective and relatively low cost, it will render BI involving acquisitions of local incumbents less likely.

Enhanced IT platforms may however entail some cost, and may not succeed in locating agent A with certainty. In such cases, P may not actually enter with a greenfield venture, but use the threat of entering with such a venture to reduce S’s outside options in bargaining over an acquisition of S’s firm. In that event, BI may become more likely.

6 Incentive and Welfare Implications of Integration

In this section we address questions pertaining to production, incentive and welfare implications of vertical integration, using non-integration as a benchmark. Owing to the complexity of the mechanism design problem within BI, we are unable to derive analytical results concerning these questions. However, optimal BI and NI allocations can be numerically computed, in specific examples. Here we consider the case where \( V_P = 1 \), \( \theta \) is uniformly distributed on \([0,1]\), signal probabilities \( \kappa_H = \kappa_L = 1/2 \) and the distributions of \( \theta \) conditional on signal realizations are given by \( F_L(\theta) = 2\theta - \theta^2 \), \( F_H(\theta) = \theta^2 \) (which correspond to linear likelihood functions \( a_L(\theta) = 1 - \theta \) for \( \theta \in [0,1] \) and \( a_H(\theta) = \theta \) for \( \theta \in [0,1] \) of the signal conditional on \( \theta \)).

Figure 6 plots operating profits of P under BI and NI respectively, as \( V_S \) (and hence degree of specificity) is varied over the range \([0,V_P] = [0,1]\). It shows \( \Pi^{BI} \) and \( \Pi^{BI} - \Pi^{NI} \) are both decreasing in \( V_S \), while \( \Pi^{NI} \) is decreasing over a range of high \( V_S \) where S’s participation constraint binds and is constant for lower values of \( V_S \) where it does not.

The likelihood of procurement in either regime depends on the prices offered to A. We expect that BI will feature higher prices owing to a reduction in DMR. This is confirmed in Figure 7.
Figure 6: NI vs BI

Figure 7: Optimal Prices in NI and BI

Figure 8: Incentive Schemes

Figure 9: A and S’s payoffs
It is often argued that hierarchies feature low-powered incentives compared to market relationships. The comparison of prices offered to A indicates that the hierarchy offers higher incentives to production level workers at the bottom of the organization. However, at the divisional level the power of incentives is measured by the sum of the incremental payments to S and A associated with higher production, which is given by the bonus $b$. Figure 8 shows that BI features a lower bonus than NI for high levels of specificity, and the same bonus for low specificity. At the same time BI features a positive base payment $X_0$ when the division produces nothing, at low levels of specificity. BI therefore involves a reallocation of incentive payments between A and S: increasing them for bottom layer members while lowering them for managers at intermediate layers, with the latter effect dominating the former.

Consider next the welfare impacts of BI. Figure 9 plots expected payoffs of A and S respectively. It is evident that production workers welfare increases, owing to the higher prices (i.e., efficiency wages) offered to them. For high specificity (low $V_S$) S is worse off under BI, while for higher specificity S’s payoff is unaffected (owing to a binding participation constraint over this range). Hence BI redistributes welfare from S to A when specificity is high. Figure 10 shows a higher impact on welfare in the Southern country, the sum of A and S’s payoffs. As P is better off with BI whenever it occurs, this is reinforced when we consider world welfare, the the sum of P, S and A’s payoffs. The black line in Figure 11 plots world welfare, corresponding to fixed cost set at $f = 0.05$. BI occurs only when specificity is large: $V_S$ is smaller than 0.55. The integration decision involves an externality: P makes the decision based on consequences of P’s own profit, disregarding the benefits accruing to the South country. Hence there is a discontinuous downward drop in world welfare at $V_S = 0.55$: as $V_S$ rises slightly above the threshold, P decides not to integrate. Over a range of values of $V_S$ slightly above 0.55, there is too ‘little’ integration owing to this externality. However, for $V_S$ close enough to $V_P = 1$ non-integration is welfare optimal and this externality ceases to be relevant.

Figure 12 examines ‘pass-through’ of increases in $V_P$ to A and S’s payoffs, by fixing $V_S = 0.2$ and varying $V_P$ over the range [0.2, 1]. We see higher pass-through to A and lower pass-through to S in BI. Hence a larger fraction of benefits of increases in export prices are passed on to workers, and less to intermediaries under integration.

These results concerning benign effects of FDI on worker welfare are however sensitive
Figure 10: Implication for Southern Welfare

Figure 11: Implication for Global Welfare with $f = 0.05$

Figure 12: Trickle Down Effects

Figure 13: Effect of Uncertainty
to our assumption concerning market concentration. So far we have considered a bilateral monopoly between P and S; in such a context BI replaces the monopsony of the local employer S (in contracting with A) by that of the foreign employer P. Concerns about possible adverse impacts of FDI on worker welfare are often based on the possibility that it may increase employer market power. The following example provides support for such concerns. Suppose there are two identical entrepreneurs \( S_1 \) and \( S_2 \) in the Southern country, instead of just one. Both \( S_1 \) and \( S_2 \) know A and own the assets that A needs to produce with. Prior to P’s arrival, they compete to employ A, and have access to the same local market where either can sell the good at price \( V_S \). To simplify, assume that neither obtains any cost signal. With Bertrand competition in the labor market, \( S_1 \) and \( S_2 \) earn zero profit and offer A a price of \( V_S \), prior to the arrival of P.

P earns \( V_P \) by selling the good on the world market. In NI, P can procure the good from either \( S_1 \) or \( S_2 \). If P offers them both a price \( b(\geq V_S) \), subsequent competition between \( S_1 \) and \( S_2 \) induces them to both offer A a price equal to \( b \). This eliminates intermediary rents and hence DMR in NI. P selects \( b \) which maximizes \( F(b)(V_P - b) \) subject to \( b \geq V_S \). Let \( p^+ \) denote the unconstrained maximizer of \( F(p)(V_P - p) \). Then it is optimal for P to offer \( b^* = \max\{p^+, V_S\} \). The payoff of A is \( \max\{b^* - \theta, 0\} \) in state \( \theta \), while P’s expected payoff is \( F(b^*)(V_P - b^*) \).

Now consider the case where P offers BI to both \( S_1 \) and \( S_2 \) for a small payment of \( \epsilon > 0 \). If accepted by \( S_i \) (\( i = 1, 2 \)), \( S_i \) promises to disclose A’s identity to P and not compete with P in offering a contract to A. If both \( S_1 \) and \( S_2 \) accept this offer, P selects one of them \( S_i \) randomly, and thereafter contracts directly with A, offering \( p^+ \) which maximizes \( F(p)(V_P - p) \), while \( S_i \) receives \( \epsilon \). P’s payoff is then \( F(p^+)(V_P - p^+) - 2\epsilon \). If both reject P’s offer, \( S_1 \) and \( S_2 \) receive zero payoff as in the status quo. If \( S_i \) accepts, while \( S_j \) does not, competition between P and \( S_j \) results in \( S_j \) earning zero while \( S_i \) earns \( \epsilon \). Therefore accepting P’s offer is a dominant strategy for both \( S_1 \) and \( S_2 \). Since \( \epsilon \) can be made arbitrarily small, P can earn the right to contract directly with A via BI at negligible cost in this manner.

It is evident that P will be better off with BI than NI when \( V_S > p^+ \) and \( F(p^+)(V_P - p^+) - f > F(V_S)(V_P - V_S) \). In contrast to our results above, A is worse off whenever BI is selected, receiving a lower price \( (p^+ \text{ instead of } V_S) \). Here vertical integration eliminates competition between \( S_1 \) and \( S_2 \) which ends up hurting A. Non-integration which involves
zero rents earned by intermediaries, is replaced by integration where P earns rents at the expense of A. This suggests that integration raises worker (and Southern) welfare, as long as it does not raise employer market power.

Finally, many empirical studies of FDI have shown that it is more likely to happen in industries with more R&D intensive products involving higher export values as well as greater uncertainty in production costs. Such products would involve higher values of $V_P$ and $V_S$, as well as greater uncertainty of cost $\theta$. The effect of scaling up $V_P$ and $V_S$ uniformly will tend to make BI more likely; in our model this is equivalent to scaling down the institutional cost $f$ for fixed $V_P, V_S$. In other words if we consider a given pair of North and South countries, with two different industries varying in scale of $V_P, V_S$, and a given institutional cost $f$ of setting up an integrated firm in the South country in either industry, BI would be more likely to occur in the high value industry. The effect of higher cost uncertainty is more complicated, as shown by the following numerical example. Suppose cost $\theta$ is uniformly distributed on $[k, 1-k]$ for $k \in (0, 1/2)$. Increasing $k$ then corresponds to a mean-preserving reduction in cost uncertainty. Figure 13 shows the effect of varying $k$ on $\Pi^{BI} - \Pi^{NI}$ with $V_P = 1$ and $V_S = 0$. The effect on relative profitability of $BI$ is non-monotonic. Reducing cost uncertainty reduces information rents of A and S. Since P tends to procure more often in BI, falling costs owing to lower information rents tends to generate a higher impact on profits in BI. On the other hand, as uncertainty vanishes so do rents, and both BI and NI profits converge to the same level. These two effects operate in different directions, and we see the latter effect dominates over a range of low levels of uncertainty. Hence our model is consistent with increased uncertainty raising the likelihood of BI, but the result could also go the other way.

7 Summary of Predictions and Related Literature

Our model yields the following predictions: vertical integration is more likely to be observed when (a) specificity is high; (b) fixed costs of setting up an integrated firm in the Southern country are low, owing to fewer regulations, superior communication and information technology, a more efficient legal system, and low distance between the two countries; (c) in industries and products which involve high value and more specialized, and firms which are
more productive.\textsuperscript{27} (d) The effects of improved contract enforcement institutions depend on the precise source of improvement: improved accounting standards \textit{per se} lower the value of integration, while lower collusion prospects raise the value of integration.

Other predictions pertain to the nature of integrated firms, and their welfare effects. (e) Intermediaries whose firms are acquired will be appointed to management positions in the integrated firm. Delegation of authority to such managers is limited, in order to ensure better treatment of workers compared with non-integration. (f) Worker welfare, wages and productivity will be higher in integrated firms. (g) Intermediaries will be worse off, if specificity is high enough. In such instances they will lobby Southern country governments to prevent FDI deregulation, though South country welfare will tend to be higher with FDIs. (h) Integrated firms will pass on a larger share of increased firm revenues to workers. (i) Backward rather than forward integration occurs when the Southern country supplier rather than the Northerm firm is subject to incentive problems.

The empirical literature on multinational firms provides evidence consistent with predictions (a), (b) and (c), which are also predictions made by PR-based theories. Many studies have confirmed that the share of intra-firm trade in total trade is positively correlated with capital intensity, R&D intensity and skill intensity both across industries and across firms.\textsuperscript{28} More productive firms are more likely to engage in FDI rather than outsourcing (Tomiura (2007)). Greater distance (both physical and cultural) between countries makes FDI less likely (Gorodnichenko et al. (2015)), while enhanced information and communication technology raise intra-firm trade shares (Chen and Kamal (2016), Cristea (2015)).

Regarding prediction (d), no study that we are aware of distinguishes between effects of improved accounting standards and reduced collusion. While some studies (e.g., Corcos et al. (2013)) show FDI positively correlated with governance and contract enforcement institutions in the host country, other studies show conflicting results (e.g., Bernard et al. (2010) find that increased governance quality raises the probability that foreign affiliates are present, it also lowers intrafirm trade shares conditional on existence of a foreign affiliate).

Standard PR-based theories do not make any particular predictions analogous to (e)\textendash{(h)} concerning internal organization of integrated firms, spillover welfare effects or pass-through of firm revenues to workers or customers. A number of empirical papers provide evidence

\textsuperscript{27}These correspond to higher values of $V_P$, $V_S$ and cost $\theta$, relative to the fixed cost $f$ which tends to be country-specific rather than industry or firm-specific.

consistent with our predictions. Neiman (2010) and Hellerstein and Villas-Boas (2010) show in specific US industries that integrated firms pass on effects of exchange rate or other external shocks at a significantly rate to customers; they explain this result by lower incidence of DMR. Conyon et al. (1999) show that acquisitions by foreign firms raised worker wages significantly while those acquired by domestic owners lowered wages, after controlling for firm, industry and year dummies in a sample of 600 British firms. Similar wage effects of FDI are reported by Lipsey (2004). Studies of FDI effects on farming sector in various African, Asian and East European countries generally show positive effects on farmers and small suppliers (Dries and Swinnen (2004), Minten et al. (2009), Maertens et al. (2011), Rao and Qaim (2011) and Michelson et al. (2013)). Atkin et al. (2016) provide evidence of large and significant welfare gains for Mexican consumers owing to arrival of retail FDI, most of which are accounted by reduced prices in new foreign supermarkets. At the same time traditional retailers experience exits and reductions in store profits.

Finally, we are not aware of any evidence pertaining to prediction (i), which differentiates our theory from the PR-approach.

References


Appendix: Proofs

Proof of Lemma 1:

For an arbitrary allocation which satisfies (1, 2, 3, 4, 8), construct the following grand contract. If A and S report the same \( \eta = \eta_i \) to P, S receives \( b + X_0 - p_i \) and A receives \( p_i \) when A delivers the good, while S receives \( X_0 \) and A receives 0 when A does not deliver. If they submit different reports, they are punished with large negative transfers.

Given this grand contract, there exists a WPBE in which S discloses P’s BI offer to A and offers a null side-contract to A (i.e. involving truthful reporting and zero side transfers). Along the equilibrium path, S and A play P’s mechanism noncooperatively, decide to participate in the mechanism and report \( \eta_i \) truthfully. A produces the good if and only if \( \theta \leq p_i \). If S offers a non-null SC, without loss of generality attention can be confined to SC’s which A always accepts and behaves in an incentive compatible fashion. The stated conditions ensure that there is a WPBE where S offers a null side contract, and there does not exist any alternative WPBE which is interim Pareto dominated for the coalition.

Proof of Proposition 1: If \( p_L^{NI} < \tilde{\theta} \), the argument described in the text shows that \( \Pi^{BI} > \Pi^{NI} \). So suppose that \( p_L^{NI} = \tilde{\theta} \). Since \( p_L^{NI} \leq p_H^{NI} \) (owing to \( h_L(\theta) > h_H(\theta) \) on \( (\theta, \tilde{\theta}] \) by Assumption 1(i)), we have \( p_L^{NI} = p_H^{NI} = \tilde{\theta} \). This implies that \( b^{NI} \geq h_L(\tilde{\theta}) = h_H(\tilde{\theta}) \).

First consider the case that \( h_L(\tilde{\theta}) > V_S \). Then P would never want to raise \( b^{NI} \) above \( h_L(\tilde{\theta}) \) as this is an upper bound to the cost incurred by S in ensuring that the good is delivered. Hence we have \( b^{NI} = h_L(\tilde{\theta}) \) and P attains a profit of \( \Pi^{NI} = V_P - h_L(\tilde{\theta}) \). On the
other hand, P can select the following allocation in BI: \( (p_L, p_H, b, X_0) = (\bar{\theta}, \bar{h}, h_L(\bar{\theta}) - \epsilon, 0) \). For sufficiently small \( \epsilon > 0 \), this satisfies all constraints of the problem in BI and P earns a profit of \( V_P - b^NI + \epsilon \), which is higher than \( \Pi^{NI} \).

Next consider the case that \( h_L(\bar{\theta}) \leq V_S \). Then \( b^NI = V_S \) and \( \Pi^{NI} = V_P - V_S \). On the other hand, P’s payoff in BI cannot exceed \( V_P - V_S \), since \( [\kappa_H F_H(p_H) + \kappa_L F_L(p_L)](V_P - b) - X_0 \leq [\kappa_H F_H(p_H) + \kappa_L F_L(p_L)](V_P - b - X_0) \leq V_P - V_S \). This completes the proof.

**Proof of Proposition 2:**

(a) Here we show that any allocation achieved with BI only for state \( \eta_i \), with no trade in the other state \( \eta_j \) generates strictly lower payoff than \( \Pi^BI \). Since S earns at least \( u_i^S \) in \( \eta_i \), an upper bound to P’s payoff is

\[
\kappa_i[F_i(p_i(V_P))(V_P - p_i(V_P)) - u_i^S] \tag{10}
\]

where \( p_i(V_P) \equiv \arg \max_{p_i \in [0,1]} F_i(p_i)(V_P - p_i) \).

We show that \( \Pi^BI \) is strictly greater than (10). Without loss of generality, we can restrict attention to situations where (10) is positive, or equivalently

\[
V_P > p_i(V_P) + \frac{u_i^S}{F_i(p_i(V_P))}.
\]

We consider two cases: (Case 1) \( p_i(V_P) + \frac{u_i^S}{F_i(p_i(V_P))} \geq p_j(V_P) + \frac{u_j^S}{F_j(p_j(V_P))} \) and (Case 2) \( p_i(V_P) + \frac{u_i^S}{F_i(p_i(V_P))} < p_j(V_P) + \frac{u_j^S}{F_j(p_j(V_P))} \).

**Case 1**

Consider allocation \( (p_i, p_j, b, X_0) = (p_i(V_P), p_j(V_P), p_i(V_P) + \frac{u_i^S}{F_i(p_i(V_P))}, 0) \). P’s payoff in this allocation is

\[
\kappa_i[F_i(p_i(V_P))(V_P - p_i(V_P)) - u_i^S] + \kappa_j F_j(p_j(V_P))(V_P - b),
\]

---

\(^{29}\)P can design the BI mechanism which exactly achieves (10) in an equilibrium. Consider the BI mechanism as follows. In state \( i \), S receives \( b - p_i + X_0 \) (or \( X_0 \)) for the delivery (or non-delivery) of the good, while A does \( p_i \) (or none) for the delivery (or non-delivery). In state \( j \) (\( j \neq i \)), S receives \( b - p_j + X_0 - u_{A_j} \) (or \( X_0 - u_{A_j} \)) and A does \( p_j + u_{A_j} \) (or \( u_{A_j} \)) for the delivery (or non-delivery). In the non-cooperative play of the mechanism, the truthful telling of each state (or \( j \)) is ensured by the cross checking scheme. P can select \( u_{A_j} \) such that S prefers to reject BI offer only in \( j \). It is easy to find \( (b, X_0, p_i, p_j) \) which achieves (10), satisfying all conditions.
which is strictly greater than (10) since \( V_P > b \). We need to check that this allocation satisfies all conditions in Lemma 1. Since \((2, 3, 4)\) are obviously satisfied from the construction. Since \( u_i^S \geq F_i(p_i(V_P))(V_S - p_i(V_P)) \) implies \( b + X_0 = b \geq V_S \) or (1). The selection of \( b \) implies \( b \geq \max\{p_i(V_P), p_j(V_P)\} \). If \( p_i(V_P) < \tilde{\theta} \), \( h_i(p_i(V_P)) = V_P > p_i(V_P) + \frac{u_i^S}{F_i(p_i(V_P))} = b \). Similarly if \( p_j(V_P) < \tilde{\theta} \), \( h_j(p_j(V_P)) = V_P > b \). This argument guarantees (8).

**Case 2**

By the definition of \( u_j^S, u_i^S = F_i(p_i(V_S))(V_S - p_i(V_S)) \geq F_i(p_i(V_P))(V_S - p_i(V_P)) \), implying

\[
p_j(V_P) + \frac{u_j^S}{F_j(p_j(V_P))} > p_i(V_P) + \frac{u_i^S}{F_i(p_i(V_P))} \geq V_S = p_j(V_S) + \frac{u_j^S}{F_j(p_j(V_S))}.
\]

Since \( p_j(V_S) < p_j(V_P) \), there exists \( \hat{p}_j \in [p_j(V_S), p_j(V_P)) \) such that

\[
\hat{p}_j + \frac{u_j^S}{F_j(\hat{p}_j)} = p_i(V_P) + \frac{u_i^S}{F_i(p_i(V_P))}
\]

and

\[
d[\hat{p}_j + \frac{u_j^S}{F_j(\hat{p}_j)}]/dp_j |_{p_j=\hat{p}_j} = 1 - \frac{u_j^S f_j(\hat{p}_j)}{F_j(\hat{p}_j)^2} \geq 0.
\]

Obviously \( \hat{p}_j > \tilde{\theta} \). The latter condition can be rewritten as

\[
h_i(\hat{p}_j) \geq \hat{p}_j + \frac{u_j^S}{F_j(\hat{p}_j)} = b.
\]

Consider allocation \((p_i, p_j, b, X_0) = (p_i(V_P), \hat{p}_j, p_i(V_P) + \frac{u_j^S}{F_i(p_i(V_P))}, 0)\). P’s payoff in this allocation is

\[
\kappa_i[F_i(p_i(V_P))(V_P - p_i(V_P)) - u_i^S] + \kappa_jF_j(\hat{p}_j)(V_P - b),
\]

which is strictly greater than (10) since \( V_P > b \) and \( \hat{p}_j > \tilde{\theta} \). (2, 3, 4) are obviously satisfied. The same argument as (Case 1) can apply to show (1), \( b \geq \max\{\hat{l}_i(p_i(V_P)), \hat{l}_j(\hat{p}_j)\} \) and \( h_i(p_i(V_P)) > b \) for \( p_i(V_P) < \tilde{\theta} \). We also have already checked that \( h_i(\hat{p}_j) \geq \hat{p}_j + \frac{u_j^S}{F_j(\hat{p}_j)} = b \), guaranteeing (8). This completes the proof of (a).

We now prove (b). Suppose that the optimal payoff is achieved with NI for signal state \( i \) \((i = L, H)\) and BI for signal state \( j \) \((j \neq i)\). Then optimal allocation \((b^*, X_0^*, p_1^*, p_2^*)\) satisfies \( h_i(p_1^*) = b^* \) and \( p_i^* < \tilde{\theta} \), since \( b^* < V_P < h_H(\tilde{\theta}) < h_L(\tilde{\theta}) \). Now consider a small rise of \( p_i \) from \( p_i^* \) to \( p_i^{**} \) such that \( p_i^{**} = p_i^* + \epsilon < \tilde{\theta} \) with \( \epsilon > 0 \). \( X_0 \) is also raised from \( X_0^* \) to

\[
X_0^{**} = F_i(p_1^*) (b^* - p_i^*) + X_0^* - F_i(p_1^{**}) (b^* - p_i^{**}).
\]
Notice that $X_0^* - X_0^*$ is greater than $X_0^*$ since $p_i^*$ maximizes $F_i(p_i)(b^* - p_i)$. Now consider allocation $(b^*, X_0^*, p_i^*, p_j^*)$. It is evident that P’s payoff in this allocation is greater than that in the original one for sufficiently small $\epsilon$. We can also check that this allocation satisfies all conditions in Lemma 1 for sufficiently small $\epsilon$. This completes the proof of Proposition 2.

Proof of Proposition 3: Let $p^*$ be the maximizer of $F(p)(V_P - p)$. Then $p_L(V_S) < p^* < \tilde{\theta}$ from our conditions. First it is shown that $p^{NS} < \tilde{\theta}$ (or interior solution) under $H(\tilde{\theta}) > V_P > V_S > \tilde{\theta}$. We can consider two cases: $u^S_L \geq V_S - p^*$ and $u^S_L < V_S - p^*$:

(i) If $u^S_L \geq V_S - p^*$ (which occurs with small $V_S$), $X_0^* + p \geq V_S$ is not binding. Then the solution is $(p^{NS}, X_0^{NS}) = (p^*, u^S_L)$. It also implies $p^{NS} < \tilde{\theta}$.

(ii) If $u^S_L < V_S - p^*$, the second constraint is binding in the solution or $X_0^{NS} + p^{NS} = V_S$. Then $X_0^{NS} = V_S - p^{NS} \geq u^S_L$. Since $V_S - \tilde{\theta} < u^S_L$ with $p_L(V_S) < p^* < \tilde{\theta}$, $p^{NS} < \tilde{\theta}$.

Next let us consider allocation $(p_L, p_H, b, X_0) = (p^{NS}, p^{NS}, p^{NS}, X_0^{NI})$ as a starting point. It is evident that this satisfies all constraints of BI problem and generates $\Pi^{NS}$ to P. Now we consider a small variation from this allocation to

$$(p'_L, p'_H, b'_0, X'_0) = (p^{NS}, p^{NS} + \epsilon, p^{NS}, X_0^{NI}).$$

Since $u^S_L > u^S_H$ for $V_S > 0$, this satisfies

$$F_H(p'_H)(b'_0 - p'_H) + X'_0 = -\epsilon F_H(p^{NS} + \epsilon) + X'_0 \geq -\epsilon F_H(p^{NS} + \epsilon) + u^S_L \geq u^S_H$$

for sufficiently small $\epsilon > 0$. It means that this allocation also satisfies all constraints of the problem in BI, and P’s payoff is greater than $\Pi^{NS}$.

Proof of Proposition 4: Suppose that P offers $(b_i, X_i, p_i)$ to S in NI. For S’s report of $\eta = \eta_i$, this specifies payments to S $(b_i + X_i$ and $X_i)$ for the delivery and the non-delivery, and also price $p_i$ paid from S to A. Without loss of generality, our attention is restricted to a mechanism which induces the S’s participation and truthful telling of $\eta$, which satisfies the following conditions:

$$F_i(p_i)(b_i - p_i) + X_i \geq u^S_i$$

and

$$F_i(p_i)(b_i - p_i) + X_i \geq F_i(p_j)(b_j - p_j) + X_j.$$
We check that the second-best allocation is achievable in this mechanism. The second-best allocation requires \( p_i = p_i^{SB} \equiv p_i(VP) \) and \( F_i(p_i^{SB})(b_i - p_i^{SB}) + X_i = u_i^S \) to be satisfied for \( i = L, H \). There conditions are equivalent to \((b_L, b_H)\) which satisfies

\[
\begin{align*}
  u_L^S &\geq u_H^S + [F_L(p_H^{SB}) - F_H(p_H^{SB})](b_H - p_H^{SB}) \\
  u_H^S &\geq u_L^S + [F_H(p_L^{SB}) - F_L(p_L^{SB})](b_L - p_L^{SB}).
\end{align*}
\]

Our assumption \((VP < h_L(\tilde{\theta}))\) implies \( p_L^{SB} < \tilde{\theta} \) and \( F_H(p_L^{SB}) < F_L(p_L^{SB}) \). These conditions are satisfied at \( b_H = p_H^{SB} \) and

\[
b_L \geq p_L^{SB} + \frac{u_L^S - u_H^S}{F_L(p_L^{SB}) - F_H(p_L^{SB})}.
\]