MECHANISM DESIGN WITH LIMITED COMMUNICATION: Implications for Decentralization[1]

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This Version: June 18 2009

Abstract

We develop a theory of mechanism design in a principal-multiagent setting with private information, where communication involves costly delay. The need to make production decisions within a time deadline prevents agents from communicating their entire private information to the principal, rendering revelation mechanisms infeasible. Feasible communication protocols allow only finite number of possible messages sent in a finite number of stages. An extension of the 'Revenue Equivalence Theorem' is obtained, and used to show that an optimal production allocation can be computed by maximizing virtual profits of the Principal subject to communication constraints alone. In this setting delegation of production decisions to agents strictly dominates centralized production decisions, and decentralized communication protocols dominate centralized ones. The value of decentralizing contracting decisions depends on the ability of the principal to verify messages exchanged between agents.

KEYWORDS: communication, mechanism design, decentralization, incentives, principal-agent, organizations

1 Introduction

1.1 Motivation

This paper develops a theory of mechanism design with limitations on the capacity of agents and Principal to communicate, which prevent implementation of 'complete' contracts or revelation mechanisms. We show the theory generates interesting implications for optimal allocation of control rights over different components

[1]We thank Ingela Alger, Richard Arnott, Hideshi Itoh, Ulf Lilienfeld, Bart Lipman, Michael Manove, Preston McAfee, Alberto Motta, Andy Newman, Roy Radner, Stefan Reichelstein, Ilya Segal, Hideo Suehiro and seminar participants at Boston College, Brown, Northwestern, Hitotsubashi, Keio, Kobe, Nagoya, Padua and Stanford GSB for useful comments.

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of the mechanism — contracting, communication and production decisions. The main point of departure from standard mechanism design theory is to incorporate constraints on the ability of agents and principal to communicate with one another. As is well known, the existing literature on mechanism design focuses almost entirely on revelation mechanisms. In such mechanisms agents simultaneously send a report of their entire private information to the Principal, instead of communicating directly with one another. Agents are not delegated authority over (verifiable) production decisions: they await instructions from the Principal on what to do (see, e.g., Myerson (1982)). It is not possible to derive a theory of allocation of control rights in that setting as a completely centralized mechanism can always replicate the performance of any other mechanism. Most organizations involve substantial decentralization of decision-making and direct, interactive communication among agents. The aim of this paper is to explore circumstances where this can be explained by limits on communication abilities on otherwise strategic and rational organization members.

Limitations on communication can be justified by the fact that reading and writing messages are time-consuming tasks, and decisions have to be made under some time deadlines. The clearest example of this arises when agents have technical expertise not shared by the principal. It is not possible for a doctor to explain her full understanding of the medical condition of a patient to members of her family who are not doctors themselves, largely because the latter would not understand the vocabulary used by a doctor to describe medical conditions. Another example is the richness of information related to specialization of agents in certain regions or areas — a diplomat assigned to a certain country, a sales manager assigned to a particular territory, a real estate agent working in a neighborhood, all of whom may not be able to communicate detailed knowledge of their assigned jurisdictions to a central government or corporate headquarters.

Advances in communication technology enhance organizational performance principally because they improve the capacities of agents to communicate information to others, thereby improving planning and coordination of decisions across agents with diverse information. Successive technologies such as the telephone, telegraph, email and video-conferencing allow agents to exchange larger volumes of information at faster speeds. Yet they still impose limits on the extent of information that can be communicated speedily enough to be incorporated in timely decisions. Face-to-face meetings are still valued over video-conferencing for this reason. Corporate executives located at the same office spend large amounts of their time in meetings, the main function of which is to enable them to exchange information and coordinate decisions. Even in that context, the need to make decisions in real time combined with limits on communicational abilities prevents full revelation of information.

In most organizations, therefore, even after the exchange of information agents will still know more about their respective environments than the Principal or any central coordination device. It may then make sense to delegate some decisions to agents. Such ideas can be traced back to Hayek (1945) as well as the 'message space' literature (Hurwicz (1960, 1972), Mount and Reiter (1974), Segal (2006)).

Most of the preceding references as well as the theory of teams developed by Marschak and Radner (1972), abstract from incentive problems by assuming that all agents in the organization share the same goals as the Principal. In such a context, communication limitations typically justify decentralized decision-making, as they permit production decisions to be based on better information. In the presence of incentive considerations where agents pursue self-interested goals, this is no longer obvious: decentralization may encourage agents to pursue their own agendas at the same time that it brings to bear better information on decisions. This creates a key trade-off between the potential 'control loss' or 'abuse of power' against the 'flexibility' advantages of delegation.

This paper explores this trade-off in the context of a standard team production model with two agents with independent private costs. Each agent produces a one-dimensional real-valued input at a real-valued unit cost which it is privately informed about. The inputs of the two agents have to be coordinated to jointly produce a benefit for the Principal. Agents are provided transfers by the Principal to provide them with suitable incentives to report their costs and produce inputs. We pose the problem of design of a mechanism by the Principal to maximize her expected net benefit, subject to communicational, incentive and participation constraints. In particular, agents will have at most a finite number of communicational strategies available, which will be insufficient to communicate the realization of their real-valued cost parameter. This restriction is in turn derived by a time deadline within which production decisions need to be taken, and time taken by agents to read and write messages to one another.[3]

[3]Implicit in this is the notion that time delays in decision making are costly. We do not model these delay costs explicitly. Such an explicit model is unnecessary because we compare different mechanisms for any given level of time delay, and obtain rankings that do not depend on the given extent of delay.

1.2 Relation to Previous Literature

The representation of communicational constraints poses a significant challenge: how should communicational complexity be measured, and how can this be incorporated in mechanism design theory? The classical message-space literature (Hurwicz (1960, 1972), Mount and Reiter (1974)) measured communicational complexity by the dimensionality of message space sizes in iterative communication protocols, and posed the question of the minimal dimensionality required to implement given resource allocations. They abstracted from incentive considerations. More recent literature uses different measures of communicational complexity (e.g., which incorporate both message space size and number of rounds of communication), and asks how the minimal communicational complexity required to implement given allocations increases in the presence of incentive problems (Reichelstein and Reiter (1988), Segal (2006), Fadel and Segal (2007), van Zandt (2006)).

An alternative approach is to fix a limited communication protocol and ask what is the constrained optimal allocation mechanism. The Marschak-Radner theory of teams can be viewed as belonging to this approach, in the absence of incentive considerations. Some authors have attempted to extend this to contexts with incentive problems (Green and Laffont (1986, 1987), Melumad, Mookherjee and Reichelstein (MMR) (1992, 1997), Laffont and Martimort (1998)).[4] This literature, however, typically imposes severe restrictions on communication protocols. For instance, MMR assume only one round of communication, and message sets of given (finite) size. They also compare two specific mechanisms: one where contracting, communication and production decisions are all centralized, with another where they are all decentralized.

This paper extends the MMR approach to accommodate a wider range of both communication protocols and mechanisms. We allow an arbitrary number of communication rounds and message space sizes at each round, and arbitrary communication network structures (i.e., who communicates with whom). We do not restrict the class of mechanisms either. 'Mixed' mechanisms are allowed, where different components of the mechanism: contracting, communication and production decisions can be independently centralized or decentralized. This enlarges the relevant class of mechanisms considerably, with varying notions of 'decentralization'.

For instance, the owner of a firm can contract with all workers, and make production decisions personally, but may allow agents to directly communicate with one

[4] Deneckere and Severinov (2003) develop a theory of mechanism design where truthful messages can be sent costlessly, while non-truthful messages may entail some cost.

another. Or the owner may additionally delegate production decisions to workers on the assembly line or shop floor. Going one step further, the owner could contract with a single designated agent or 'manager', delegating to this manager the authority to contract with the other workers, and make (or further delegate) production and payment decisions. MMR restricted attention to the two polar mechanisms where all three components — contracting, communication and decision-making — were centralized or decentralized. In this paper we consider all intermediate notions of decentralization as well.

It is also important to emphasize that earlier approaches such as MMR (1992) imposed *ad hoc* restrictions on the size of message spaces, which were not derived from an explicit theory of limited communication.[5] This paper grounds the theory in an explicit model of communication. This ensures a more careful treatment of the comparative performance of centralized and decentralized contracting. The source of the superior 'flexibility' of decentralized contracting in MMR (1992) was the ability of an intermediate 'manager' to condition subcontracts offered to subordinates on information held privately by the manager at the time of contracting. At the same time the 'manager' was assumed unable to communicate a similar amount of information to the principal. Proposition 1 shows that this asymmetry cannot arise in a model of limited communication, provided the principal can verify *ex post* messages exchanged by agents.[6] These issues and other differences between our approach and MMR (1992) are explained in further detail in Section 3.

1.3 Our Model and Results

Our model is set up to focus exclusively on the implications of time limitations on communication at the *interim* stage — after agents have received their private information, and before production decisions are made. In particular, we abstract from limitations of rationality of agents, or on the complexity of contracts: we assume there are no time limitations at the *ex ante* stage when contracts are designed, offered and read. We also abstract from limitations on *ex post* verifiability of performance and messages on which transfers are conditioned. Under centralized contracting, agents cannot collude by entering into unobserved side contracts or private communication. Proposition 1 shows that under these assumptions,

[5] MMR (1997) was based instead on a restriction on the complexity of contracts, measured by the number of contingencies incorporated — rather than limitations on communication.

[6] However, it can be justified if there are restrictions on *ex post* verifiability of messages. We provide an example of this in Section 9.

decentralized contracting cannot outperform centralized contracting.

We thereafter explore how the mechanism design problem can be posed within the class of centralized contracts. Propositions 3 and 4 show how it can be simplified in a way analogous to methods used for auction design or related problems with quasilinear utilities in the absence of communicational constraints. A version of the 'Revenue Equivalence' theorem is obtained, enabling optimal transfers to be computed as a function of production allocations, so that the problem can be cast in terms of choice of the latter alone. Under the standard assumption of monotone hazard rates for the cost distributions, the problem reduces to selection of a production allocation rule to maximize 'virtual' expected profit of the Principal, subject only to communicational feasibility restrictions. This implies a convenient separation of the incentive and communicational aspects of the design: the former are entirely incorporated in the objective function (in the form of incentive rents of the agents which cause actual costs to be inflated to virtual costs). Improvements in communication technology relax the constraints on the problem, leaving the objective function unchanged. Communicational algorithms to maximize virtual profit can thus be devised without worrying about incentive compatibility of the chosen communication strategies.

This characterization leads to our main result in Section 7. Delegating the production decision for each agent to that agent itself is strictly superior to any system where they are set production targets by others, provided the production function satisfies Inada conditions. This is a consequence of the characterization of optimal production allocations. Owing to the finiteness of the communication protocol, agents are better informed about their own environments at the end of the communication stage. Moreover, the interests of agents and the principal are perfectly aligned by the choice of the incentive mechanism: there is no problem of 'loss of control' associated with delegation. Allocating control rights to agents over their own production levels then allows a more flexible production allocation, without attendant problems of control loss.

Another implication concerns the design of communication protocols, discussed in Section 8. Owing to the alignment of incentives, greater 'flexibility' of decision-making and information flows across agents is desirable. Communication protocols should thus be designed to maximize the flow of information across agents. This will necessitate direct iterative communication between agents.

An implication of these results for organization design is the importance of "mixed" mechanisms in the presence of communication constraints. They indicate the value of mechanisms that combine centralized contracting with decentralized decision-making and communication networks. Such mechanisms have been emphasized as distinctive features of 'Japanese' firms (see, e.g., Aoki (1990)).

Section 9 then discusses the implications of limited ability of the Principal to verify messages exchanged between agents. We provide an example where decentralized contracting strictly dominates centralized contracting. Finally, Section 10 concludes by describing questions left for future research.

2 Model

There is a Principal (P) and two agents 1 and 2. Agent $i = 1, 2$ produces a one-dimensional nonnegative real valued input q_i at cost $\theta_i q_i$, where θ_i is a real-valued parameter distributed over an interval $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ according to a positive, continuously differentiable density function f_i and associated c.d.f. F_i. The distribution satisfies the standard monotone hazard condition that $\frac{F_i(\theta_i)}{f_i(\theta_i)}$ is nondecreasing, implying that the 'virtual cost' $v_i(\theta_i) \equiv \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ is strictly increasing. θ_1 and θ_2 are independently distributed, and these distributions F_1, F_2 are common knowledge among the three players.

The inputs of the two agents combine to produce a gross return $V(q_1, q_2)$ for P, whose net payoff is $V - t_1 - t_2$, where t_i denotes a transfer from P to i. The payoff of i is $t_i - \theta_i q_i$. All are risk-neutral and have autarkic payoffs of 0. We shall assume that V is twice continuously differentiable and strictly concave. We shall also assume that the production function is non-separable: $V_{12} \neq 0$ for all q_1, q_2. This creates a need to coordinate production assignments between the agents.

It is worth noting that auction design (with private values) represents a special case of this model, if no further assumptions are imposed. For instance suppose the function V takes the following form: it equals $\bar{V}$ if $q_1+q_2 \geq 1$ and 0 otherwise. For $\bar{V}$ sufficiently large, the desired sum q_1+q_2 will equal 1 irrespective of type realizations of the agents. Owing to the linearity of costs, the problem then reduces to selection of one of the two agents to supply 1 unit of the good. Most of the results in the paper apply to this setting. However, Proposition 4 concerning strict superiority of decentralized production decisions additionally requires the assumption that V satisfies Inada conditions, which ensures that each agent produces a strictly positive amount in all states.

3 The MMR Approach

In the classical setting (e.g., Myerson (1982)) without restrictions on communication or contracting, attention can be focused on a class of revelation mechanisms in which contracting, communication and decision-making are centralized. P designs a mechanism in which contracts are offered to both agents at the *ex ante* or *interim* stage. Agents respond to these at the *interim* stage: after observing the realization of θ_i agent i decides whether or not to participate. Conditional on participating, this agent sends a message $\tilde{\theta}_i \in \Theta_i$ to P. The two agents operate independently and simultaneously at this stage. Following this, production and transfers are determined: $q_i(\tilde{\theta}_1, \tilde{\theta}_2), t_i(\tilde{\theta}_1, \tilde{\theta}_2); i = 1, 2$ as per the contract offered by P.

An alternative to such a centralized revelation mechanism is the following decentralized mechanism, in which P contracts only with agent 1 (called the manager), and delegates subcontracting and production decisions to the manager. An example of this is the following 'profit-center' arrangement, where P evaluates the performance of the manager by measuring the output $V(q_1, q_2)$ delivered, and the transfer payment t_2 authorized by the manager for agent 2 (called the subordinate). This payment is made by P to the subordinate, and can be thought of as the 'cost' incurred by the profit center. P does not contract directly with the subordinate, nor monitor communication or subcontracting between the manager or the subordinate.

Specifically, the profit center delegation mechanism is the following. At the *exante* stage P offers a contract to the manager, which specifies a transfer $t_1(V, t_2; \tilde{\theta}_1)$ based on the output and cost of the center, and a message $\tilde{\theta}_1$ sent by the manager to P at the *interim* stage. At the *interim* stage, the manager observes the realization of θ_1, and decides whether or not to participate. Conditional on participating, the manager submits a message $\tilde{\theta}_1$ to P, and designs a subcontract for the subordinate. This subcontract consists of decisions concerning production q_1, q_2 and transfer t_2 to the subordinate, as a function of a cost reports $\hat{\theta}_1, \tilde{\theta}_2$ to be exchanged between the manager and the subordinate (conditional on the latter agreeing to participate). The subcontract will of course depend on the private information of the manager and the report already communicated to P. So the subcontract actually offered can be denoted by $t_2(\hat{\theta}_1, \tilde{\theta}_2|\theta_1, \tilde{\theta}_1), \{q_i(\hat{\theta}_1, \tilde{\theta}_2|\theta_1, \tilde{\theta}_1)\}_{i=1,2}$. Following this, the subordinate observes the realization of θ_2 and decides whether or not to participate in the subcontract.[7] Conditional on participating, the manager and subordinates

[7] It may seem that the need for the manager to submit a cost report $\hat{\theta}_1$ to the subordinate is

exchange cost reports $\hat{\theta}_1, \tilde{\theta}_2$. Production and transfers are subsequently determined as per the signed contracts.

MMR (1992) showed that absent any restrictions on communication or contracting, there exists a profit-center mechanism which can achieve the second-best expected profit. In this setting, the centralized and decentralized mechanisms achieve equivalent performance. They then went on to compare the performance of the two mechanisms under a common finite restriction on the size of the message spaces that could be employed by any agent.

Specifically, agent i is restricted to selecting reports from a message set M_i containing $k_i < \infty$ elements. The content of these messages (i.e., the specific vocabulary) does not matter; only the richness of the vocabulary (i.e., the size k_i) matters. MMR (1992) considered the following version of the centralized mechanism. P offers contracts specifying production and transfers to each agent as a function of reports sent by the agents: $q_i(m_1, m_2), t_1(m_1, m_2); i = 1, 2$ where $m_i \in M_i$. The rest is as in a revelation mechanism: at the *interim* stage each agent i independently makes participation decisions and communicates a report m_i to P conditional on agreeing to participate.

This was compared with the corresponding version of the profit-center mechanism, where the same message space restrictions applied. The contract of P with the manager is now replaced by a transfer rule $t_1(V, t_2; m_1)$, and the subcontract by $t_2(\hat{m}_1, m_2|\theta_1, m_1)$, $\{q_i(\hat{m}_1, m_2|\theta_1, m_1)\}_{i=1,2}$. Here the finite messages $m_1, \hat{m}_1, m_2$ substitute for the type reports $\tilde{\theta}_1, \hat{\theta}_1, \tilde{\theta}_2$ respectively in the setting with unlimited communication. However, no restriction was imposed on how offered subcontracts could vary with the true type θ_1 of the manager. This played an important role in the 'flexibility' advantage of the profit-center arrangement. Under centralization, the contract offered to agent 2 by P could depend only on a coarse report m_1 submitted by agent 1 regarding the realization of θ_1. It could not match the way that the subcontract could be fine-tuned to information possessed by the manager regarding θ_1.

In a context where the limitation on message spaces arises from limited ability of agents to communicate with one another, this formulation of the profit-center arrangement exaggerates its flexibility. In this formulation the subcontract must be designed by the manager at the *interim* stage, after learning the realization of

redundant, as the subcontract offered and its dependence on the realization of θ_1 obviates the need for any further communication by the manager to the subordinate. This does turn out to be true, but we suppress those details here.

θ_1. For the subordinate to be able to react to the offered subcontract, this agent must be able to read all its details. This could effectively require the subordinate agent to be able to read a real-valued message sent by the manager. If communication at the *interim* stage is restricted owing to time limitations — e.g., if only a finite number of bits of information can be exchanged — this will be infeasible. Subcontracts then have to be restricted further to belong to a finite set of possible subcontracts, to allow managers and subordinates to negotiate them in finite time at the *interim* stage. This corresponds to a restriction on the 'flexibility' of delegated subcontracting. In particular, the manager will be restricted at the *interim* stage to selecting one of various subcontracts from a pre-specified finite menu. As we shall show in the next Section, a centralized contract can be chosen to mimic such a subcontracting arrangement, provided the principal can *ex post* verify messages exchanged between agents.

The other limitation of the MMR (1992) approach is that it ignores the possibility of other mechanisms with intermediate 'degrees' of decentralization. Even if we accept the MMR formulation of the superior flexibility of the profit-center arrangement, there may be other allocations of control rights that also permit similar flexibility. For instance, P could retain authority over contracting, but delegate production decisions to the agents.

To explain this more concretely, consider the following mechanism with centralized contracting and communication, and decentralized decisions. P contracts with both agents, who are required to submit cost reports m_1, m_2 to P subsequent to agreeing to participate. The mechanism allows each agent to choose his own production: i decides q_i. The contract offered by P specifies a transfer $t_i(q_1, q_2; m_1, m_2)$ to agent i, which depends on the production decisions and cost reports chosen by the two agents respectively. Here q_i can depend on real-valued private information θ_i available to i regarding his own cost realization, which permits production decisions to be fine-tuned to the latter. This flexibility is not possible under the purely centralized arrangement studied by MMR (1992). Yet it does not go all the way to the other extreme of the profit-center arrangement, where contracting and communication with subordinates are decentralized along with production decisions.

It is also possible to consider mechanisms with centralized contracting and decision-making, but with decentralized communication. P could design a communication protocol where agents communicate with one another and P. Iterative communication mechanisms are an example. P could design a multi-stage communication game with n rounds of communication: at stage $k = 1, \ldots, n$ agent i may be assigned finite message spaces M_{ijk}, M_{iPk} with agent j and P respectively. Agent i will send $m_{ijk} \in M_{ijk}$ to j and $m_{iPk} \in M_{iPk}$ to P at round k. Between rounds k and $k+1$ each agent will read the messages received at round k, and write messages to be sent at $k+1$. The latter can be conditioned on the history of messages exchanged by i until round k. The rest of the mechanism could be centralized, with P making production and transfer decisions at the end of round n based on all messages received by P by then.

Interactive communication between agents permits greater flexibility in production decisions than simultaneous one-round communication, even subject to the same aggregate time limits for communication. Consider, for instance, the case where each agent i has a message space consisting of four elements – i.e., consisting of two binary bits of information $(m_1^i, m_2^i) \in \{0,1\}^2$.[8] Suppose that it takes one unit of time for an agent to communicate one bit, and there is a total of two units of time available at the *interim* stage for communication prior to making production decisions. The purely centralized mechanism considered by MMR (1992) corresponds to the simultaneous communication version, with a single round of communication: agent i independently communicates (m_1^i, m_2^i) to P. It is shown in their paper that this mechanism reduces to selection of a rectangular partition of the type spaces of the agents, of the sort depicted in Figure 1. P will select an interval partition of Θ_i containing four subintervals, with all types of i in the same subinterval pooling on a common message. There are then 16 possible combination of messages received by P from the two agents, each corresponding to a sub-rectangle of the type space.

An alternative protocol would allow two rounds of communication, with agents directly allowed to communicate with one another as well as with P. At stage $k = 1, 2$, agent i sends a binary message $m_k^i \in \{0,1\}$ to agent j as well as P. This allows agent j to condition the message m_2^j sent to P at the second round on the message m_1^i received from i at round 1. This allows the partition of the type space shown in Figure 2. It also contains 16 rectangles but allows sub-interval boundaries for each agent that depend on the realization of types of the other agent.

While some interactive dependence can be achieved by multi-stage centralized protocols, the need to route all information flows through P will typically restrict the flow of information across agents. For instance, suppose in our example above that it takes each agent and P one unit of time to read one bit of information, while it takes no time for agents to write information. Then a centralized protocol cannot

[8]In this example, we also abstract from the time it would take the principal to communicate production targets to the agents. We suppose the time required for communication of targets is fixed, and focus on how the agents report their information to the principal.

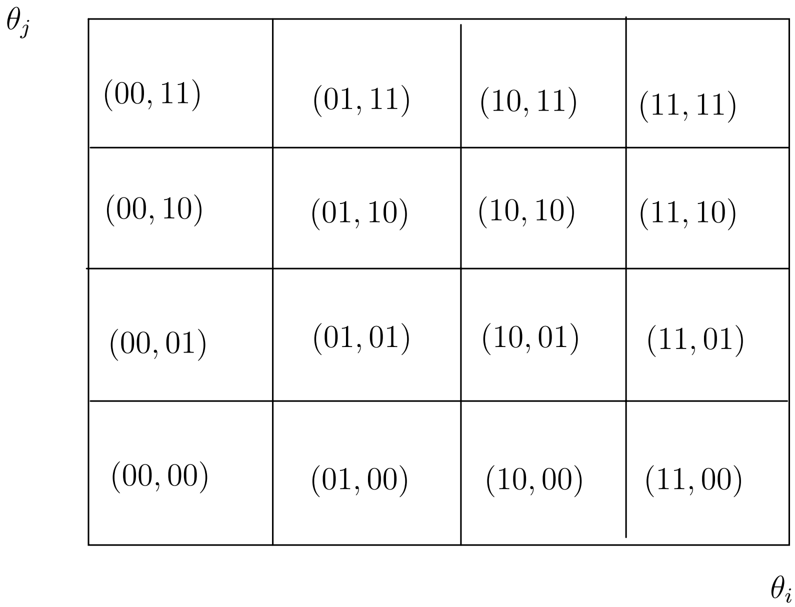


Figure 1: One Stage Simultaneous Communication with $k_1, k_2 = 4$

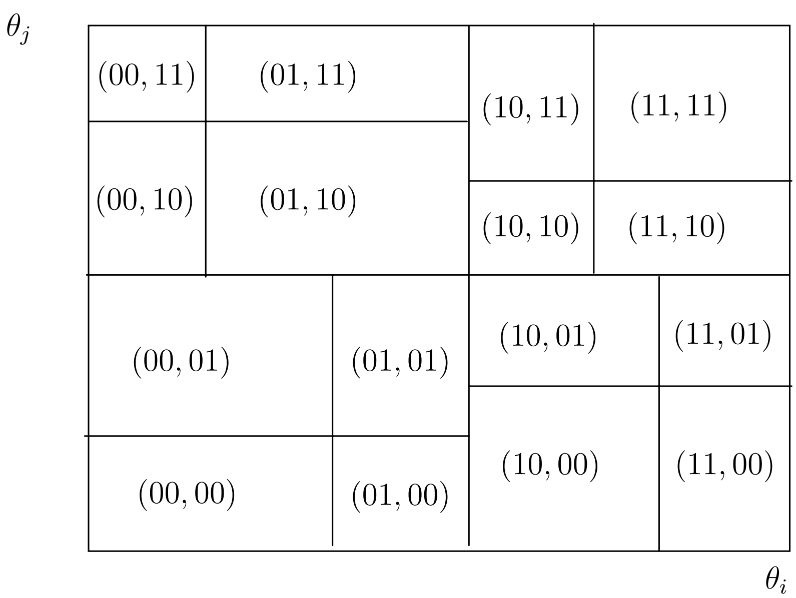


Figure 2: One Stage Sequential Communication with $k_1, k_2 = 4$

allow any information to be exchanged between the two agents within a deadline of two units of time. It will take P one unit of time to read a binary message sent by agent 1, say, which P can subsequently send to agent 2. But it will take another unit of time for 2 to read this message, leaving no further time for 2 to 'respond' to this with a supplementary message to P. With direct communication between the agents, agent 1 can send the same message to agent 2 at the same time as P. Then both agent 2 and P can read this message during the first unit of time. Agent 2's subsequent message to P can then be conditioned on the message received from 1.

More generally, when each agent can communicate an arbitrary finite number of bits of information within a given time limit, there is a trade-off between the number of stages of interactive communication, and the number of bits exchanged at any given stage. Agents can make long unilateral speeches or write long and detailed memos. Or they can engage in dialogues or mail exchanges with one another, employing briefer sentences or responses. The construction of optimal communicational protocols subject to restrictions on communicational complexity (e.g., the total number of bits communicated) in general is a difficult problem. In this paper we shall allow a rich range of such multi-stage protocols. The choice of protocol is obviously an important component of the overall mechanism. However, we will attempt to obtain insights concerning optimal allocation of control rights without addressing the problem of the optimal communication protocol directly. Later in Section 8 we shall compare centralized and decentralized protocols.

4 Contracting and Communication

Our formulation is based on a number of assumptions described in the Introduction: there are no constraints on the complexity of contracts as these are offered and read at an *ex ante* stage not characterized by time limits. Nor are there any constraints on the ability of P to verify *ex post* messages exchanged between agents for the purpose of deciding on transfers, after production decisions have been made. In centralized contracting, the agents cannot side-contract or engage in any private communication. The only constraint is one the time taken at the *interim* stage for the agents to communicate their private information for the purposes of coordinating production decisions. For the sake of expositional ease, we restrict attention to a particular class of communication protocols below. However, the theory applies more generally to less structured protocols which permit each agent a finite set of communication strategies.

4.1 Centralized Contracting

4.1.1 Timing

In centralized contracting, P selects a *communication protocol* (explained further below) at $t = -1$, and offers contracts to both agents. There is enough time between $t = -1$ and $t = 0$ for agents to read the offered contracts. We abstract from the time needed to read or negotiate contracts at the *ex ante* stage and focus entirely on communication delays arising at the *interim* stage. P commits to the communication protocol and the contracts. The agents do not commit to their participation decisions until after they observe their private information.

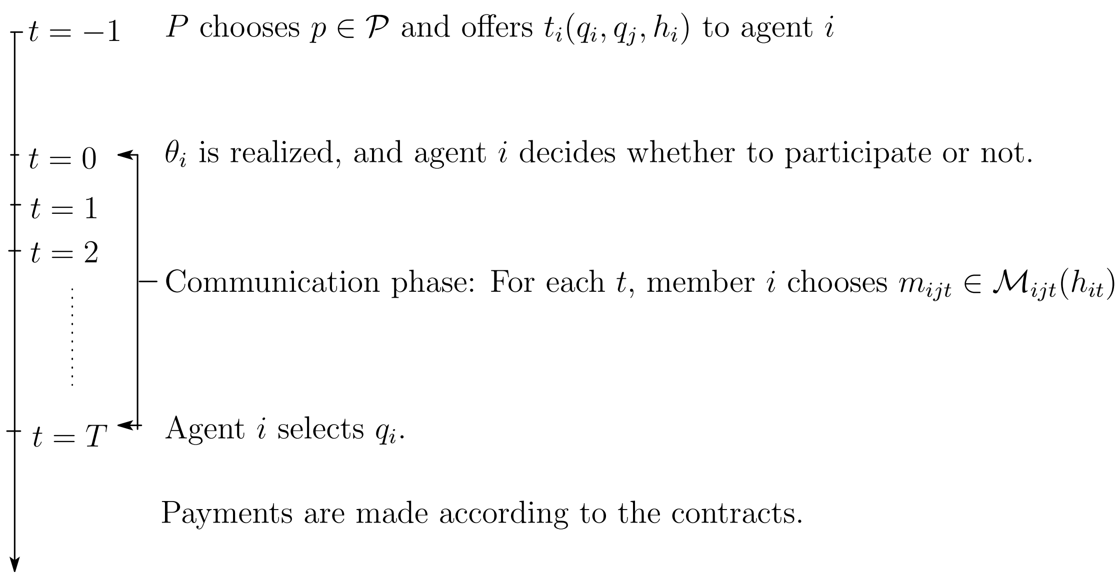
At $t = 0$, each agent i privately observes the realization of θ_i, and independently decides whether to participate or opt out of the mechanism. If either agent opts out the game ends; otherwise they enter the planning or communication phase which lasts until $t = T$. We treat the deadline T as given; it can be chosen subsequently by the organization designer to trade off the cost of delayed production with the benefit of added communication. Our results do not depend on the specific deadline chosen.

At $t = T$, agent i selects production level q_i. This does not necessarily mean that i 'decides' q_i in any meaningful sense. As discussed further below, someone else may set a target for q_i — this could be a message sent to i by the target-setter during the communication phase — and the incentive scheme for i may effectively force i to meet this target. Messages exchanged during the communication phase thus include 'instructions' or 'targets' that might be set when production decisions are not decentralized to the concerned agents.

Finally, after production decisions have been made, P verifies messages exchanged and production levels achieved. Payments are made based on these, according to the contracts signed at the *ex ante* stage.

The timeline for centralized contracting is depicted below.

Centralized Contract



4.1.2 Communication Protocol

A communication protocol is a dynamic process of exchange of messages among agents and the principal. We shall subsequently refer to the two agents $1, 2$ and the principal P as members of the communication network. The protocol is represented by a set of dates $t = 0, 1, \ldots, T$ at which messages are exchanged, with the time interval between successive dates taken up by reading messages recently received, and writing new messages to be sent at the next date. The protocol specifies a message space $\mathcal{M}_{ijt}(h_{i,t-1})$ for messages that can be sent by each member i to every other member j on the network at any date t, following a history of messages $h_{i,t-1}$ exchanged by i until $t - 1$.

Message histories are generated recursively as follows. For member i with history $h_{i,t-1}$ until date $t - 1$, h_{it} is generated by adding the messages sent and received by i to and from other members at t. We assume absence of communication noise or errors, so messages sent are received without fail or distortion.

The specification of protocols does not require all agents to simultaneously communicate with one another. Alternating senders and receivers at different dates can be accommodated by assigning null message spaces to receivers. It can incorporate different network structures: if i is not allowed to communicate with j at t, then

$\mathcal{M}_{ijt}(h_{i,t-1})$ is empty.

A *communication plan* c_i for i specifies, for any given date t and message history h_{it}, and any other member j, a message $c_{ij}(h_{it})$ selected from $\mathcal{M}_{ijt}(h_{it})$ to be sent to j at t. Throughout, we restrict attention to pure strategies. Let $\mathcal{C}_i$ denote the set of feasible communication plans for i in a given protocol.

The message space of each agent is finite at every date, as reading and writing messages are time-consuming tasks. Hence the set of possible communication plans $\mathcal{C}_i$ for every member i is finite.

A feasible communication protocol p is represented by a finite set of communication plans $\mathcal{C} \equiv (\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_P)$, and a history $h_{it}(c)$ of messages exchanged by i until any date t as a function of a tuple of communication plans $c \equiv (c_1, c_2, c_P) \in \mathcal{C}$. The 'technology' of reading and writing messages is incorporated in the specification of the set of feasible communication protocols relative to any given deadline T. Let $\mathcal{P}$ denote the set of feasible protocols.

Within a given protocol, execution of the communication plans of the various members generates a history of messages h_{iT} exchanged until the end of the communication phase, which we shall simply denote by h_i. And H_{it} will denote $\{\tilde{h}_{it} | \exists c \in \mathcal{C} : \tilde{h}_{it} = h_{it}(c)\}$, the set of all date t histories that could be generated by the protocol.

It is assumed that reading and writing messages do not generate any private costs for members. Hence there are no moral hazard problems associated with communication *per se*, nor is there the possibility of unobserved communication among agents.

Communication plans chosen by each agent $i = 1, 2$ will be a function of their type θ_i. We shall refer to this as i's *communication strategy* $c_i(\theta_i) \in \mathcal{C}_i$.

Since P has no private information, c_P the communication plan of P is conditioned only on messages received from the agents. We assume P can commit to this communication plan, which is specified along with the communication protocol as part of the mechanism.

4.1.3 Contracts and Production Decisions

As explained in the Introduction, we assume that the *ex post* stage P costlessly observes inputs q_i supplied by each agent, in addition to the message histories; these are also verifiable by third-party enforcers. Hence payments can be conditioned on these.

A centralized contract for i is represented by a transfer rule $t_i(q_i, q_j, h_i)$. It can be checked that no benefit can be derived by conditioning the payment to i on message histories of other agents (i.e., h_j). Moreover, contracts conditioning on message histories of other agents may raise 'privacy' concerns.

At the deadline T each agent i will select a production level q_i. This can depend on its type θ_i and message history h_i. The restrictions on communication force production decisions to depend on 'coarse' messages about the state of the other agent: the production of agent i must depend on θ_j only through h_i. Specifically, agent i's production strategy is a function $\hat{q}_i(\theta_i, h_i(c_i(\theta_i), c_j(\theta_j), c_P))$. They can be fine-tuned to information about i's own cost θ_i, which constitutes the potential 'flexibility' advantage of delegating production decisions.

Formally, we shall say that the *production decision* q_i *is centralized* if it is measurable with respect to h_P, and *partially decentralized* if it is measurable with respect to $h_j, j \neq i$ but not with respect to h_P. In these cases, production decisions concerning q_i can be thought of as being made by P or j respectively.

In contrast, the production decision q_i is said to be *completely decentralized* if it is not the case that q_i is measurable with respect to h_P or $h_j, j \neq i$. The production decision of i cannot then be predicted by P or j at $t = T$ based on the messages they have sent or read. In other words it is not possible that agent i is assigned a production target by someone else, combined with an incentive scheme that forces i to abide by the target. Instead, i will make the production decision personally, a choice that will be influenced, though not completely determined, by the messages sent by others that enter as arguments of the incentive scheme.

4.2 Decentralized Contracting

4.2.1 Timing

In decentralized contracting, P contracts with the manager (agent i), who subsequently contracts with the employee (agent j). The communication network is also hierarchical: the employee communicates with the manager, and the manager with P. Contracts are offered at $t = -1$: P offers a contract for i and selects an 'upper-level' communication protocol between herself and the manager. The manager then offers a subcontract to j, and selects the 'lower level' communication protocol.[9]

The rest is as under centralized contracting. At $t = 0$ agents i, j observe the realization of θ_i, θ_j respectively, and each independently decides whether or not to opt out. If neither opts out, they enter the communication phase from $t = 0$ until $t = T$. At T agents i, j decide q_i, q_j respectively, and payments are thereafter made as stipulated in the contracts. The timeline for decentralized contracting is depicted below.

Decentralized Contract

$t = -1$ — P chooses $p_1 \in \mathcal{P}_D^1$ and offers $t_i(q_i, q_j, h_{Pi})$ to agent i
Agent i chooses $p_2 \in \mathcal{P}^2(p_1)$ and offers $t_j(q_i, q_j, h_{ij})$ to agent j.

$t = 0$ — θ_k is realized, and agent k decides whether to participate or not.

$t = 1$

$t = 2$

⋮

Communication phase: For each t, member k chooses $m_{klt} \in \mathcal{M}_{klt}(h_{kt})$

$$kl \notin \{Pj, jP\}$$

$t = T$ — Agent k selects q_k.

Payments are made according to the contracts.

4.2.2 Communication Protocol

The set of communication protocols consistent with decentralized contracting do not allow any direct communication between the employee j and the owner P. Each of them communicates only with the manager i. Moreover, P does not monitor

[9] The two networks are linked by the participation of the manager: messages sent by the manager on one network may be based on messages received in previous stages on the other network. For instance, the manager may receive a cost report from the employee, combine this with her own cost information to produce a summary cost report to P. Following this P may set an output target, with the manager subsequently allocating production responsibility between herself and the employee.

communication between the manager and the employee, neither does the employee monitor communication between the manager and P.[10]

Let $\mathcal{P}_D^l$ denote the set of communication protocols at level $l = 1, 2$ of the hierarchy. P selects a protocol $p_1 \in \mathcal{P}_D^1$ at the upper layer $l = 1$. This determines the communicational responsibilities of the manager *vis-a-vis* P, and constrains the protocols that the manager can choose from for the bottom layer $l = 2$.[11] Given p_1, the subset of protocols that i can choose for the lower level network is a subset of $\mathcal{P}_D^2$, represented by a correspondence $\mathcal{P}^2(p_1) : \mathcal{P}_D^1 \rightrightarrows \mathcal{P}_D^2$. Hence $\mathcal{P}_D$, the set of communication protocols feasible in the decentralized contracting regime, is represented by $\mathcal{P}_D^1$ the set of protocols for the upper tier, along with the correspondence $\mathcal{P}^2(.)$.

Note that $\mathcal{P}_D \subset \mathcal{P}_C$, i.e., any protocol feasible under decentralized contracting is also feasible under centralized contracting, while the converse is not true. In centralized contracting, P has the option of selecting the same hierarchical communication protocol as in decentralized contracting. However, decentralized contracting must necessarily involve a hierarchical protocol, whereas centralized contracting is not so constrained.

Since $\mathcal{P}_D \subset \mathcal{P}_C$, we do not need any fresh notation for communication protocols in decentralized contracting, apart from noting that they form a strict subset of communication protocols in centralized contracting.

4.2.3 Contracts and Production Decisions

In decentralized contracting P can contract with agent i based on the level of aggregate benefit V delivered, and messages exchanged between P and i. In addition, P may be able to monitor the inputs supplied by each agent separately, as in centralized contracting. P may also be able to monitor the payments made by i to j in the subcontract (e.g., if i is the manager of a division of a firm owned by P, and j is an employee of that division). In what follows we consider the scenario most favorable to decentralized contracting, where both production assignments and transfers between i and j are contractible. We shall show below

[10] We use the term 'monitor' as a shorthand for 'observe' and/or 'ex post verify'.

[11] For instance, it should allow the manager enough time to be able to execute his communicational responsibilities on both networks. If P wants the manager to report on some cost estimate for delivering V, for which prior communication with j is necessary, the protocol in the lower level network should ensure that the manager communicates with j before the time to report to P arrives.

that any allocations attainable under decentralized contracting will be attainable under centralized contracting as well; *a fortiori* the same will be true if contracting options under decentralization are more restricted.

The contract between P and the manager is therefore represented by the transfer rule $t_i = t_i(q_1, q_2, t_j, h_{Pi})$, where h_{Pi} denotes (h_P, h_i^U) the history of messages exchanged between P and i on the upper level network. Here h_P denotes the messages sent and received by P, and h_i^U denotes messages sent (or received) by i to (or from) P.

The subcontracts offered by the manager to the employee specifies transfers t_j as a function of q_1, q_2 and messages exchanged on the lower level network $h_{12} \equiv (h_i^L, h_j^L)$. Here h_i^L and h_j^L denotes messages sent or received by i and j respectively among one another.

Production decisions q_i, q_j are made at $t = T$ by i, j respectively, based on their personal information at that point. The manager decides $\hat{q}_i(\theta_i, h_i)$ where $h_i \equiv (h_i^U, h_i^L)$, and the employee decides $\hat{q}_j(\theta_j, h_j)$ where $h_j = h_j^L$. Production decisions may be centralized or decentralized, as in the centralized contracting regime. The same formal definitions of (completely, partially) decentralized and centralized production decisions apply here as in the centralized contracting regime.[12]

5 Allocations Attainable under Centralized and Decentralized Contracting

Given a particular set of contracts offered by P in centralized contracting and a given communication protocol $p \in \mathcal{P}_C$ (including communication strategy $\tilde{c}_P$), we have a well-defined Bayesian game of incomplete information.

An *allocation attainable under centralized contracting* is a state-contingent production and transfer rule $q_a(\theta_1, \theta_2), t_a(\theta_1, \theta_2), a = 1, 2$ such that there exists a communication protocol $p \in \mathcal{P}_C$, centralized contracts $\tilde{t}_a(q_1, q_2, h_a), a = 1, 2$, and an asso-

[12]In our formulation, subcontracts and communication protocol for the lower level network are designed by the manager at the *ex ante* stage. If they were designed instead at the *interim* stage, employees would need time to read the subcontract offered, which would cut into the time available for coordinating production plans. In that case, the set of possible subcontracts offered and accepted at the interim stage will belong to a finite set. Our formulation is equivalent to this: one can think of the subcontract offered *ex ante* as a finite menu of subcontracts offered at the interim stage, with subsequent communication between the agents between $t = 0$ and T serving to select one from the menu.

ciated tuple of communication and production strategies $\tilde{c}(.) \equiv \{\tilde{c}_i(\theta_i), \tilde{c}_j(\theta_j), \tilde{c}_P\}$ and $\tilde{q}(.) \equiv \{\tilde{q}_i(\theta_i, h_i), \tilde{q}_j(\theta_j, h_j)\}$ which constitutes a Perfect Bayesian Equilibrium (PBE) of the corresponding subgame, such that for any state $\theta \equiv (\theta_1, \theta_2)$ and any $a = 1, 2$:

$$q_a(\theta) = \tilde{q}_a(\theta_a, h_a(\tilde{c}(\theta))) \tag{1}$$

$$t_a(\theta) = \tilde{t}_a(q_1(\theta), q_2(\theta), h_a(\tilde{c}(\theta))) \tag{2}$$

Under decentralized contracting, a different Bayesian game is induced by choice of a contract for the manager and 'upper layer' communication protocol p_1 by P. Agent i, the manager, must select contracts t_j for agent j, a communication protocol $p_2 \in \mathcal{P}^2(p_1)$, and a communication strategy $c_i(\theta_i)$ to be executed by i during the communication phase. Production decisions and the strategies of agent j are similar to what they are under centralized contracting.

An *allocation attainable under decentralized contracting* is a state-contingent production and transfer rule $q_a(\theta), t_a(\theta), a = 1, 2$ such that there exists a contract $\hat{t}_i(q_1, q_2, t_j, h_{Pi})$ and communication protocol $p_1 \in \mathcal{P}_D^1$ (with communication plan $\hat{c}_P$) selected by P for the top tier, and a Perfect Bayesian Equilibrium (PBE) of the associated subcontracting subgame consisting of a subcontract offered by i: $\hat{t}_j(q_1, q_2, h_{12})$, a communication protocol $p_2 \in \mathcal{P}^2(p_1)$, a tuple of communication strategies $\hat{c}(.) \equiv \{\hat{c}_i(\theta_i), \hat{c}_j(\theta_j), \hat{c}_P\}$ and production strategies $\hat{q}(.) \equiv \{\hat{q}_i(\theta_i, h_i), \hat{q}_j(\theta_j, h_j)\}$, such that for any state $\theta \equiv (\theta_i, \theta_j)$:

$$\begin{aligned} q_a(\theta) &= \hat{q}_a(\theta_a, h_a(\hat{c}(\theta))), a = 1, 2 \\ t_i(\theta) &= \hat{t}_i(q_1(\theta), q_2(\theta), t_j(\theta), h_{Pi}(\hat{c}(\theta))) - t_j(\theta) \\ t_j(\theta) &= \hat{t}_j(q_1(\theta), q_2(\theta), h_{12}(\hat{c}(\theta))) \end{aligned}$$

The following result is a trivial consequence of the fact that decentralized contracting involves a restricted set of communication protocols relative to centralized contracting.

Proposition 1 *Any allocation attainable under decentralized contracting can also be attained under centralized contracting.*

Proof. Consider an allocation $q_a(\theta), t_a(\theta), a = i, j$ attained by decentralized contracting with protocols p_1, p_2 at the two layers, transfer rules $\hat{t}_i, \hat{t}_j$, communication and production strategies $\hat{c}, \hat{q}_1, \hat{q}_2$. Recall that the communication protocol

$p \equiv (p_1, p_2)$ is feasible in centralized contracting. Recall also the assumption that P can verify all messages sent and received by all agents in centralized contracting. Therefore h_{12} is verifiable by P in centralized contracting. So P can select the protocol p, and the following contracts:

$$\begin{aligned} \tilde{t}_j(q_1, q_2, h_{12}) &= \hat{t}_j(q_1, q_2, h_{12}) \\ \tilde{t}_i(q_1, q_2, h_{12}, h_{Pi}) &= \hat{t}_i(q_1, q_2, \hat{t}_j(q_1, q_2, h_{12}), h_{Pi}) - \hat{t}_j(q_1, q_2, h_{12}) \end{aligned}$$

Then from $t = 0$ the continuation game involving choice of communication and production strategies by the agents is the same as under decentralized contracting, so the same strategies constitute a PBE of this game. ∎

The argument of Proposition 1 resembles that underlying the Revelation Principle: under identical communication and contracting constraints, centralized contracts can be designed by the Principal to duplicate any mechanism based on decentralized contracts. It confirms the notion that delegation of subcontracting offers no advantages over centralized contracting, once we are careful to incorporate the communication inherent in the act of selecting and offering a subcontract by the 'managing' agent. In particular, this setting does not allow the flexibility that constituted the key advantage of delegation in MMR (1992).

However, the argument utilizes the assumption that P can costlessly verify *ex post* messages exchanged by the agents. The role of this assumption will be discussed in Section 9 below.

6 Characterizing Optimal Allocations in Centralized Contracting

Proposition 1 pertains to allocation of control over contracting, but says nothing about control over production, or the design of communication. Having designed contracts, P needs to decide whether to retain control rights over production as well. For instance, following exchange of messages with the agents, should the Principal set production targets? Or should the Principal let the agents decide what to produce, under the influence of an incentive mechanism where transfers depend on outputs and exchanged messages?

In order to address this, we need to make some progress in characterizing optimal allocations subject to the communication restrictions.

One restriction on output allocations is obvious: the output of any given agent can depend on information about the type of the other agent only in a coarse manner, on the basis of exchanged messages. The finiteness of the message spaces implies that an agent cannot signal its entire private information to others. This is represented in the following notion of communication-feasibility.

Definition 1 *The output allocation* $(q_1(\theta_1, \theta_2), q_2(\theta_1, \theta_2))$ *is communication feasible if and only if there exists* $p \in \mathcal{P}$, $c(\theta) = (c_i(\theta_i), c_j(\theta_j), c_P) \in C$ *and* $\hat{q}_i(\theta_i, h_i)$ *so that for all* (θ_i, θ_j)*:*

$$q_i(\theta_i, \theta_j) = \hat{q}_i(\theta_i, h_i(c(\theta)))$$

Of course, i's production q_i can depend finely on θ_i , provided this decision is (completely) decentralized to i. Whereas if the Principal makes production decisions then both q_1 and q_2 will depend coarsely on θ_1, θ_2. The key question concerns which system is in the Principal's *ex ante* interest.

The first step of the analysis is to note a 'rectangle property' that holds for any feasible communication protocol. The proof of this and subsequent results is provided in the Appendix.

Lemma 1 [Rectangle property]: *Consider any communication protocol* $p \in \mathcal{P}$. *Then for any* $h_{it} \in H_{it}$*:*

$$\{c \in \mathcal{C} \mid h_{it}(c) = h_{it}\}$$

is a rectangle set in the sense that if $h_{it}(c_i, c_{-i}) = h_{it}(c_i^{'}, c_{-i}^{'}) = h_{it}$ *for* $(c_i, c_{-i}) \neq (c_i^{'}, c_{-i}^{'})$*, then*

$$h_{it}(c_i^{'}, c_{-i}) = h_{it}(c_i, c_{-i}^{'}) = h_{it}$$

Lemma 1 has the following implication. Consider any history h_{it} observed by i until t. Then (given knowledge of the communication plan c_P of the Principal), the set of possible configurations of communication plans of the two agents that could have generated h_{it} can be expressed as the (Cartesian) product of $\tilde{\mathcal{C}}_1(h_{it})$ and $\tilde{\mathcal{C}}_2(h_{it})$, where $\tilde{\mathcal{C}}_a(h_{it})$ is a subset of $\mathcal{C}_a$, a=1,2. So defining

$$\Theta_a(h_{it}) \equiv \{\theta_a | c_a(\theta_a) \in \tilde{\mathcal{C}}_a(h_{it})\}$$

it follows that the set of types (θ_1, θ_2) that could have generated the history h_{it} can be expressed as the Cartesian product of subsets $\Theta_1(h_{it}), \Theta_2(h_{it})$. We note this formally below.

Lemma 2 *Given any configuration of communication strategies* $(c_1(\theta_1), c_2(\theta_2), c_P)$, *and given any history* h_{it} *that could be generated by these strategies, the set of possible types that could have generated this history can be expressed as*

$$\{(\theta_1, \theta_2) \mid h_{it}(c(\theta_1, \theta_2)) = h_{it}\} = \Theta_i(h_{it}) \times \Theta_j(h_{it}). \tag{3}$$

Hence upon observing the history h_{it}, **all** *types* $\theta_i \in \Theta_i(h_{it})$ *of agent* i *will update their prior beliefs about* θ_j *in the same way, i.e., by conditioning on the event that* $\theta_j \in \Theta_j(h_{it})$.

Another implication of the rectangle property will prove useful later. To simplify exposition we shall suppress c_P, the communication plan of the Principal, since this is specified as part of the mechanism and can be taken as given by the agents.

Lemma 3 *Without loss of generality, the Principal can restrict attention to communication protocols with the property that for every possible communication plan* $c_a \in \mathcal{C}_a$ *for agent* $a \in \{1, 2\}$, *there exists type* θ_a *such that* $c_a = c_a(\theta_a)$.

In other words, attention can be restricted to protocols with no off-equilibrium or unused communication plans. This is because unused communication plans can be deleted, without affecting the set of histories generated by the chosen communication strategies. This will simplify the representation of incentive constraints subsequently: the Principal needs only to deter deviations by any type $\theta_i \in \Theta_i(h_{it})$ following history h_{it}, to mimicking the communication plan chosen by some other $\theta_i' \in \Theta_i(h_{it})$ who has pooled so far with θ_i.

This is noted in the following result, a version of which appears also in Fadel and Segal (2007, Proposition 3).

Proposition 2 *An allocation* $(t_i(\theta_i, \theta_j), q_i(\theta_i, \theta_j))$ *is attainable under centralized contracting if and only if:*

(i) There exists communication protocol $p \in \mathcal{P}$ *with production strategies* $\hat{q}_i(\theta_i, h_i)$, *and communication strategies* $c_i(\theta_i), i = 1, 2$, *such that for all* (θ_i, θ_j):

$$q_i(\theta_i, \theta_j) = \hat{q}_i(\theta_i, h_i(c_i(\theta_i), c_j(\theta_j)))$$

(i.e. $q_i(\theta_i, \theta_j)$ *is communication feasible) and every communication plan is used by some type:*

$$\mathcal{C}_i = \{c_i(\theta_i) \mid \theta_i \in \Theta_i\}.$$

(ii) Defining the set of possible histories

$$H_{it} \equiv \{h_{it}(c(\theta)) \mid \theta \in \Theta_1 \times \Theta_2\}$$

that could be generated thereby, the following incentive constraint holds for any $t \in \{0, 1, 2, ...T\}$*, any* $h_{it} \in H_{it}$ *and any* θ_i*,* $\theta_i^{'} \in \Theta_i(h_{it})$*:*

$$\begin{aligned} & E_{\theta_j}[t_i(\theta_i, \theta_j) - \theta_i q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \\ \geq \; & E_{\theta_j}[t_i(\theta_i^{'}, \theta_j) - \theta_i q_i(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \end{aligned}$$

(iii) $E_{\theta_j}[t_i(\theta_i, \theta_j) - \theta_i q_i(\theta_i, \theta_j)] \geq 0$ *for any* θ_i

As Fadel and Segal (2007) explain, this result indicates a trade-off between communication efficiency and incentive constraints. To render feasible any production allocation that depends more finely on the realized types of agents, communication protocols with more stages or larger message spaces are needed. These tend to impose more incentive constraints: enlarging the number of stages or range of message options at any stage is associated with a corresponding enlargement of the number of incentive constraints that the allocation must respect. For instance, along an iterative process of communication agents learn something about the types of other agents; this should not distort their subsequent communication or output decision choices.

We now arrive at the main result of this section, a characterization of output allocations that are attainable under centralized contracting (in combination with some transfer rules), and the associated maximum *ex ante* profit for the Principal. It is a generalization of the characterization underlying the Revenue Equivalence Theorem in auction theory (Myerson (1981)).

Proposition 3 *The output allocation* $q_i(\theta_i, \theta_j)$ *is attainable under centralized contracting (in combination with some transfer rule) if and only if*

(i) $q_i(\theta_i, \theta_j)$ *is communication feasible.*

(ii) If the associated communication protocol and strategies that implement $q_i(\theta_i, \theta_j)$ *generate the set of message histories* H_{it} *for each agent* i *at date* t*, then* $E_{\theta_j}[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})]$ *is non-increasing in* θ_i *on* $\Theta_i(h_{it})$ *for any* $h_{it} \in H_{it}$ *and any* t*.*

The maximum expected payoff for P which implements $q_i(\theta_i, \theta_j)$ is

$$E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - v_i(\theta_i)q_i(\theta_i, \theta_j) - v_j(\theta_j)q_j(\theta_i, \theta_j)] \tag{4}$$

The proof of Proposition 3 is long and detailed, and is presented in the Appendix. The necessity of conditions (i) and (ii) follows straightforwardly from the necessity of the corresponding conditions (i) and (ii) in Proposition 2. The sufficiency argument presents a number of complications. One has to construct a rule for transfers that will preserve the dynamic incentive constraints ((ii) in Proposition 2) at every date and following every message history. Moreover, the set of types $\Theta(h_{it})$ pooling until message history h_{it} need not constitute an interval. The monotonicity property (ii) for output decisions holds only 'within' $\Theta(h_{it})$, which may span two distinct intervals. The monotonicity property may therefore not hold for type ranges lying between the two intervals, which complicates the conventional argument for construction of transfers that incentivize the output allocation.

To overcome these problems, previous versions of this paper imposed strong distributional conditions (such as exponentially distributed types) to ensure that the additional dynamic incentive constraints do not bind. The construction used in the proof of Proposition 3 however works quite generally, allowing us to dispense with any special distributional conditions (apart from the monotone hazard rate condition standard in this class of models). It is based on a construction of a related output allocation which coincides with the actual allocation on equilibrium-path histories, and is globally non-increasing (with respect to types whose communication strategies could be mimicked on or off the equilibrium path). The construction is involved as it has to ensure that this property holds at every date and following every history. The transfer rule is designed to implement this related output allocation subject to incentive and participation constraints at minimum cost, as in the conventional analysis of private value auction problems. The global monotonicity (in combination with the monotone hazard rate property of the cost distribution) ensures the incentive compatibility of the rule, while the fact that the rule agrees with the actual output allocation on the equilibrium path ensures that it enables the Principal to realize her virtual expected profit.

Proposition 3 implies the mechanism design problem can be simplified as follows.

Proposition 4 *The mechanism design problem can be represented as choice of a communication protocol, communication strategies and output decision strategies to maximize the Principal's 'virtual' profit (4) subject to communication feasibility alone.*

In the case of unlimited communication, this reduces to the familiar property that an optimal output allocation can be computed on the basis of unconstrained maximization of expected virtual profits.

The proof of Proposition 4 relies on the argument that the solution to the problem of maximizing expected virtual profit subject to communication feasibility alone (i.e., ignoring the monotonicity constraint (ii) in the statement of Proposition 3) must have the following property. At every date and following every possible message history, the continuation communication strategy of any agent must maximize the expected virtual profit (conditional on the information revealed by the message history so far), given the communication and production strategy of others. In particular, it should not be possible for the agent to increase (conditional expected) virtual profit by switching to the continuation communicational strategy used by some other type with whom the agent has pooled so far. A standard 'revealed preference' argument then implies (given the monotone hazard rate property for distribution of types) that the monotonicity constraint (ii) in Proposition 3 must be satisfied. Hence this constraint cannot bind, and can be dropped from the statement of the problem.

Proposition 4 implies a simple and convenient separation between costs imposed by incentive considerations, and those imposed by communicational constraints. The former is represented by the replacement of production costs of the agents by their incentive-rent-inclusive virtual costs in the objective function of the Principal, in exactly the same way as in a world with costless, unlimited communication. The costs imposed by communicational constraints are represented by the restriction of the feasible set of output allocations, which must now vary more coarsely with the type realizations of the agents. This can be viewed as the natural extension of Marschak-Radner characterization of optimal team decision problems to a setting with incentive problems. In particular, the same computational techniques can be used to solve these problems both with and without incentive problems: only the form of the objective function needs to be modified to replace actual costs by virtual costs. The 'desired' communicational strategies can be rendered incentive compatible at zero additional cost.

Van Zandt (2006) and Fadel and Segal (2007) discuss the question of 'communication cost of selfishness', which relates to a different notion of separation between incentive and communicational complexity issues. In their context they take an arbitrary social choice function (allocation in our notation) and examine whether the communicational complexity of implementing it is increased by the presence of incentive constraints. If not, communicational and incentive aspects of the problem can be separated in the sense that optimal communication protocols can be designed independently of incentive considerations. In our context we fix communicational complexity and select an allocation to maximize the Principal's expected profits (the exact representation of which depends on whether or not incentive problems are present). We do not know if there is a connection between the two separation properties.

7 Optimality of Decentralized Production Decisions

The preceding characterization of the mechanism design problem allows us to prove that an optimal mechanism must completely decentralize production decisions to agents, provided the production function satisfies Inada conditions. The intuitive reason is that with incorporation of agents' information rents in the objective function, the incentives of agents and the Principal are aligned. Hence delegation of production decisions allows greater 'flexibility' with respect to their cost realizations. Since virtual costs $v_i(\theta_i)$ are strictly increasing in θ_i, and agent i always produces a strictly positive level of output, an optimal mechanism entails output decisions which are strictly decreasing in θ_i, conditional on any given message history. This can be achieved only if the decision over q_i is delegated to i, since only i knows the exact realization of θ_i.

Proposition 5 *Suppose V satisfies the Inada condition $\frac{\partial V}{\partial q_i} \to \infty$ if $q_i \to 0$. Then in any optimal mechanism, production decisions must be completely decentralized.*

The proof is relegated to the appendix as it involves some technical details. These arise in the first step of the argument, which shows that there exist type intervals for each agent over which communication strategies and message histories are pooled. The second step of the argument then applies to any such interval of pooled types.

8 Decentralization of Communication

Proposition 4 also has useful implications for the ranking of different communication protocols. Given any set of communication strategies in a given protocol, in state (θ_i, θ_j) agent i learns that θ_j lies in the set $\Theta_j(h_i(c_i(\theta_i), c_j(\theta_j)))$, which generates an information partition for agent i over agent j's type.

Say that a protocol $p_1 \in \mathcal{P}$ is *more informative* than another $p_2 \in \mathcal{P}$ if for any set of communication strategies in the former, there exists a set of communication strategies in the latter which yields (at date T) an information partition to each agent over the type of the other agent which is more informative in the Blackwell sense in (almost) all states of the world.

It then follows that a more informative communication protocol permits a wider choice of communication feasible output allocations. Hence the Principal prefers more informative protocols. She has no interest in blocking the flow of communication among agents.

This result in turn has interesting implications for ranking of centralized and decentralized communication protocols. This is developed in detail in a previous version of this paper; we recount the main idea informally.

Say that a protocol is *completely centralized* if i and j never send or receive messages from one another directly; all communication is between agents and the Principal. Examples include tatonnement mechanisms and revelation mechanisms (in a setting without communication limits). In contrast a protocol is *completely decentralized* if neither agent communicates with the Principal at any date. In such a protocol, the Principal is not involved at all in the communication network; agents communicate directly with one another.

Suppose that the size of message sets at any date for any agent is determined by the (maximum) time it takes that agent to read and write messages, in a context where messages are represented in binary form and each agent takes a fixed amount of time to read or write one bit of information. Suppose also that there is at least one agent (i, say) who can read and write messages at least as quickly as the Principal. Then given any completely centralized protocol, there exists a completely decentralized protocol which is more informative. This can be shown by constructing a set of communication strategies in the completely decentralized protocol which mimics the communication in the completely centralized protocol. Specifically, agent i can mimic the communication strategy of the Principal in the centralized protocol *vis-a-vis* agent j. Since i does not need to communicate with himself as he knows his own state, this strategy frees up some time for agent i. This can be used by i to send some additional messages to j, in a way that generates a strict improvement in virtual profit. This argument illustrates a drawback of completely centralized protocols, in which the Principal becomes a bottleneck as all communication between agents must be channeled through her. Direct communication between agents permit greater exchange of information, leading to more flexible production decisions.

9 Limited Verifiability of Messages

We now consider the implications of limited ability of the principal to verify messages exchanged between agents. Proposition 1 relied on the assumption of perfect verifiability. We provide an example here indicating how this result no longer extends with limited verifiability.

Consider the example of a procurement auction, where V equals some large number $\bar{V}$ if $q_1 + q_2 \geq 1$, and 0 otherwise. Suppose additionally that θ_i is uniformly distributed on the unit interval. This reduces to a symmetric private value auction. Assume also that P does not have any capacity to communicate with the agents at the *interim* stage. At the same time, there are no restrictions on the ability of the agents to communicate directly with one another: each agent can send a real-valued report of his own cost to the other agent at the *interim* stage.

In this setting, it can be shown that decentralized contracting achieves the second-best outcome. Applying the results of MMR (1992), the 'cost center' arrangement where P offers a contract $t_1 = K_1 + \frac{t_2}{2}$ to agent 1 induces the latter to offer a subcontract to agent 2 which involves a linear price of θ_1 for each unit of output delivered by 2, upto a maximum of one unit. This is accepted by 2 if $\theta_2 < \theta_1$. It is not accepted if $\theta_2 > \theta_1$, whence agent 1 produces the entire amount herself. The constant K_1 can be calibrated to ensure that the second-best profit is attained by P.

Now consider centralized contracting, where P has no ability to communicate with the agents at the *interim* stage, nor verify messages exchanged by the agents. As before, we assume the agents cannot side-contract with one another. If P selects a centralized communication protocol, the two agents will not be able to communicate at all at the *interim* stage (since P is unable to communicate with them, and all messages between the agents have to be routed through P). Then the coordination of production necessary to achieve the second-best cannot be attained.

On the other hand, if P designs a decentralized communication protocol the agents can communicate with one another, and thus coordinate their production decisions. However, if P cannot verify the messages exchanged, the transfers cannot be conditioned on messages. They will depend on observed production levels alone. But these production levels cannot reveal the information necessary to design second-best transfers. For instance, consider two states (θ_1, θ_2) and (θ_1, θ_2') where $\theta_1 < \theta_2 < \theta_2'$. If the second-best could be achieved, agent 1 must produce 1 and agent 2 will produce nothing in both states. But the second-best transfer to agent 1 must vary between the two states. This is impossible if P can neither communicate with the agents nor verify messages exchanged between the two agents. Hence the result of Proposition 1 must be reversed in this setting.

The superiority of decentralized contracting here rests on its enabling the design of contract offers to agent 2 that are sensitive to agent 1's information. Centralized contracting cannot allow such flexibility, since P can neither communicate with 1, nor later verify messages that might be sent by 1 to 2 at the *interim* stage. In this setting, the argument of MMR (1992) is vindicated.

Two points deserve to be noted in light of this example. First, it could be argued that the shortcoming of centralized contracting is that it does not allow any side-contracting between the agents. It is true that centralization with side-contracting can always perform at least as well as decentralized contracting. This follows from the fact that decentralized contracting is a special case of centralized contracting with side-contracting, where P does not contract or communicate with one of the agents. In that sense the comparison between centralization and decentralization of contracting is a trivial consequence of the definition of these two respective contracting regimes. The less trivial question concerns the relative performance of 'pure' centralized contracting *sans* side-contracts, with decentralized contracting.

Second, the superiority of the decentralized regime in the example arises from its ability to condition the subcontract for agent 2 on agent 1's cost realization. If the subcontract is enforced by a third party, the latter has to be able to verify the conditions of this subcontract, which is tantamount to verifying a real-valued cost report made by agent 1. It is then not clear why the centralized regime cannot rely on such a third party to enforce a similar contract mandated by the Principal and conditioned on a cost report by agent 1. In other words, the centralized regime turns out to be inferior owing to a limitation on verifiability of messages relative to the decentralized regime. If instead the two regimes have the same capacity to verify messages, they would permit contracts to be based on similar contingencies. It is then likely that centralized contracting will again be able to replicate the performance of decentralized contracting. The argument for decentralized contracting must be based then on an innate comparative advantage with regard to the verification 'technology'. Whether or not such an argument is plausible will need to be considered in future research.

10 Summary and Concluding Comments

This paper developed a theory of mechanism design for a production team in a context where agents and Principal have limited capacity to communicate with one another. The main finding is that the argument for decentralization of decision-making based on limitations on ability of agents to communicate, extends to settings with an incentive problem. As is well-known, in a context of unlimited communication and commitment ability of the Principal, attention can be focused on revelation mechanisms every aspect of which — contracting, production decisions and communication — are centralized. This flies in the face of pervasiveness of delegation of decision-making from owners of firms to managers, or customers to primary contractors or trading intermediaries. Previous attempts to adapt mechanism design theory to contexts of limited communication (such as MMR (1992)) in order to explain the value of decentralized mechanisms were based on a number of *ad hoc* assumptions.

The approach followed in this paper avoided imposing *ad hoc* restrictions on the class of mechanisms. A partial analogue of the 'Revelation Principle' was obtained, under the assumption of perfect *ex post* verifiability of messages: attention can be restricted to centralized contracting mechanisms. However, under weak conditions on the technology ('essentiality' of both agents), production and communication systems must be decentralized. This helps clarify the nature of the precise argument for decentralization, i.e., under the stated assumptions that it pertains to production decisions and communication, rather than contracting rights. We showed by an example that decentralized contracting may outperform centralized contracting if the principal has limited ability to verify messages, as well as communicate directly with them.

In general, we obtain the following insight concerning relative advantages of centralized and decentralized contracting. Limited verifiability of variables required for P to evaluate the performance of agents in centralized contracting — either messages exchanged between agents, or their respective contributions to team output — constitutes a drawback relative to decentralized contracting. This has to be traded off against the possible 'loss of control' resulting in decentralized contracting. Future research needs to explore this trade-off in further detail, going beyond the simple example described in Section 9.

We also hope that the approach of this paper can form a foundation for posing a range of ancillary questions concerning organizational design. Under what conditions does the presence of a third party facilitate communication and coordination among production agents? This would provide insight into the role of managers who do not participate in production activities, whose only role is to process information communicated by production agents and help formulate production plans. In the model presented here, such third party 'coordinators' would have no room for strategic behavior, owing to the assumption of absence of collusion. If the model were extended to accommodate collusion, it would lead to a theory of hierarchies where intermediaries not directly involved in production play a coordinating role and behave strategically.

Another question pertains to the effects of changing communication technology on organizational design. Comparative statics of such a model with respect to information technology could generate predictions that could be tested against empirical patterns of how these have been changing in recent times (a brief overview of which is provided in Mookherjee (2006)).

References

Aoki M. (1990), "Toward an Economic Model of the Japanese Firm," *Journal of Economic Literature*, 28(1), March, 1–27.

Deneckere R. and Severinov S. (2003), "Mechanism Design and Communication Costs," mimeo, Department of Economics, University of Wisconsin and Fuqua School of Business, Duke University.

Green, J. and J. Laffont (1986), "Incentive Theory with Data Compression," in W. Heller, R. Starr and D. Starrett (Ed.), *Uncertainty, Information and Communication: Essays in Honor of Kenneth Arrow*, Cambridge: Cambridge University Press.

———— (1987), "Limited Communication and Incentive Compatibility", *Information, Incentives, and Economic Mechanisms: Essays in honor of Leonid Hurwicz*, T. Groves, Radner, R. and Reiter, S. (ed.), Minneapolis: University of Minnesota Press

Fadel R. and I. Segal (2007), "The Communication Cost of Selfishness," mimeo, Dept of Economics, Stanford University.

Hayek, F.A. (1945), "The Use of Knowledge in Society", *American Economic Review*, 35(4), 519-530.

Hurwicz L. (1960), "Optimality and Informational Efficiency in Resource Allocation Processes," in K. Arrow, S. Karlin and P. Suppes (ed.) *Mathematical Methods in the Social Sciences*, Stanford University Press, Stanford, California.

——— (1972), "On the Dimensional Requirements of Informationally Decentralized Pareto-Satisfactory Processes," presented at Decentralization Conference, Northwestern University. Later published in *Studies in Resource Allocation Processes* ed. K. Arrow and L. Hurwicz. Cambridge: Cambridge University Press 1977.

Laffont, J. and D. Martimort (1998), "Collusion and Delegation," *Rand Journal of Economics* 29(2), 280-305.

Marschak, J. and R. Radner (1972), *Economic Theory of Teams*, New Haven: Yale University Press.

Melumad, N., D. Mookherjee and S. Reichelstein (1992), "A Theory of Responsibility Centers," *Journal of Accounting and Economics*, 15, 445-484.

————— (1997), "Contract Complexity, Incentives and the Value of Delegation," *Journal of Economics and Management Strategy*, 6(2), 257-289.

Mookherjee, D. (2006), "Decentralization, Hierarchies and Incentives: A Mechanism Design Perspective," *Journal of Economic Literature*.

Mount, K. and S. Reiter (1974), "The Informational Size of Message Spaces," *Journal of Economic Theory*, 8, 161–191.

Mount, K. and S. Reiter (1995), "A Theory of Computing with Human Agents," working paper, Department of Economics, Northwestern University.

Myerson, R. (1981), "Optimal Auction Design", *Mathematics of Operations Research* 6, 58–73.

——– (1982), “Optimal Coordination Mechanisms in Generalized Principal-Agent Problems,” *Journal of Mathematical Economics*, 10(1), 67-81.

Radner, R. (1992), “Hierarchy: The Economics of Managing,” *Journal of Economic Literature*, 30, 1382-1415.

——– (1993),“The Organization of Decentralized Information Processing,” *Econometrica*, 61(5), 1109-1146.

Reichelstein, S. and S. Reiter (1988), “Game Forms with Minimal Message Spaces,” *Econometrica*, 56(3), 661-692.

Segal I. (2006), “Communication in Economic Mechanisms,”in *Advances in Economics and Econometrics: Theory and Application, Ninth World Congress (Econometric Society Monographs)*, ed. by Richard Blundell, Whitney K. Newey, and Torsten Persson, Cambridge University Press.

van Zandt, T. (1996), “Decentralized Information Processing in the Theory of Organizations,” in *Contemporary Economic Development Reviewed, Volume 4: The Enterprise and its Environment*, Murat Sertel (Ed.), London: Macmillan Press.

——— (1999), “Real-Time Decentralized Information Processing as a Model of Organizations with Boundedly Rational Agents,” *Review of Economic Studies*, 66,(3), 633-58.

——– (2003a), “Real-time Hierarchical Resource Allocation,”mimeo, INSEAD.

——– (2003b), “Real-time Hierarchical Resource Allocation with Quadratic Costs,”mimeo, INSEAD.

——— (2006), “Communication Complexity and Mechanism Design,” mimeo, INSEAD.

van Zandt, T. and R. Radner (2001), “Real-time decentralized information processing and returns to scale,” *Economic Theory*, 17, 497-544.

Appendix: Proofs

Proof of Lemma 1: The proof is by induction. Note that $h_{i0}(c) = \phi$ for any c, so it is true at $t = 0$. Suppose the result is true for all dates up to $t-1$, we shall show it is true at t.

Note that

$$h_{it}(c_i, c_{-i}) = h_{it}(c_i^{'}, c_{-i}^{'}) = h_{it} \tag{5}$$

implies

$$h_{i\tau}(c_i, c_{-i}) = h_{i\tau}(c_i^{'}, c_{-i}^{'}) = h_{i\tau} \tag{6}$$

for any $\tau \in \{0, 1, .., t-1\}$. Since the result is true until $t-1$, we also have

$$h_{i\tau}(c_i^{'}, c_{-i}) = h_{i\tau}(c_i, c_{-i}^{'}) = h_{i\tau} \tag{7}$$

for all $\tau \leq t-1$. So under any of the configurations of communication plans (c_i, c_{-i}), $(c_i^{'}, c_{-i}^{'})$, $(c_i^{'}, c_{-i})$ or $(c_i, c_{-i}^{'})$, member i experiences the same message history $h_{i,t-1}$ until $t-1$. Then i has the same message space at t, and (5) implies that i sends the same messages to others at t, under either c_i or $c_i^{'}$.

(6, 7) also imply that under either c_{-i} or $c_{-i}^{'}$, others send the same messages to i at all dates until $t-1$, following receipt on the (common) messages sent by i until $t-1$ under these different configurations. The result now follows from the fact that messages sent by others to i depend on the communication plan of i only via the messages they receive from i. So i must also receive the same messages at t under any of these different configurations of communication plans. ■

Proof of Proposition 2: Necessity follows straightforwardly upon using Lemma 3. To prove sufficiency, given $p \in \mathcal{P}$, $c(\theta)$ and $\hat{q}_i(\theta_i, h_i)$ which satisfies (i), define $\hat{t}_i(q_i, h_i)$ by

$$\hat{t}_i(q_i, h_i) \equiv E_{(\theta_i, \theta_j)}[t_i(\theta_i, \theta_j) \mid q_i(\theta_i, \theta_j) = q_i, h_i(c(\theta)) = h_i]$$

provided

$$\{(\theta_i, \theta_j) \mid q_i(\theta_i, \theta_j) = q_i, h_i(c(\theta)) = h_i\} \neq \phi,$$

otherwise set $\hat{t}_i(q_i, h_i) = 0$. For $h_i \in H_i \equiv H_{iT}$, define

$$Q_i(h_i) \equiv \{\hat{q}_i(\theta_i, h_i) \mid \theta_i \in \Theta_i(h_i)\}$$

and

$$\Theta_i(h_i, q_i) \equiv \{\theta_i \in \Theta_i(h_i) \mid \hat{q}_i(\theta_i, h_i) = q_i\}.$$

Then for (q_i, h_i) such that $q_i \in Q_i(h_i)$,

$$\{(\theta_i, \theta_j) \mid q_i(\theta_i, \theta_j) = q_i, h_i(c(\theta)) = h_i\} = \Theta_i(h_i, q_i) \times \Theta_j(h_i)$$

implying

$$\hat{t}_i(q_i, h_i) = E_{\theta_i}[E_{\theta_j}[t_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_i)] \mid \theta_i \in \Theta_i(h_i, q_i)].$$

The incentive constraint (ii) at $t = T$ implies that $E_{\theta_j}[t_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_i)]$ must be independent of θ_i on $\Theta_i(h_i, q_i)$, since $E_{\theta_j}[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_i)] = \hat{q}_i(\theta_i, h_i)$ is independent of θ_i on the same set by definition. Therefore with $\theta_i \in \Theta_i(h_i(c(\theta_i, \theta_j)), q_i(\theta_i, \theta_j))$,

$$\hat{t}_i(q_i(\theta_i, \theta_j), h_i(c(\theta_i, \theta_j))) = E_{\tilde{\theta}_j}[t_i(\theta_i, \tilde{\theta}_j) \mid \tilde{\theta}_j \in \Theta_j(h_i(c(\theta_i, \theta_j)))].$$

For any t and any $h_{it} \in H_{it}$,

$$\Theta_j(h_{it}) = \{\Theta_j(h_i(c(\theta_i, \theta_j))) \mid \theta_j \in \Theta_j(h_{it})\}$$

for any $\theta_i \in \Theta_i(h_{it})$. This implies that for any $\theta_i \in \Theta_i(h_{it})$,

$$E_{\theta_j}[\hat{t}_i(q_i(\theta_i, \theta_j), h_i(c(\theta_i, \theta_j))) \mid \theta_j \in \Theta_j(h_{it})] = E_{\theta_j}[t_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})].$$

Now consider the mechanism with protocol p, where agent i is paid $\hat{t}_i(q_i, h_i)$ only if messages sent by i at every date are consistent with the communication plan associated with some type θ_i; otherwise the agent is paid nothing. In this mechanism, suppose that agent i with type θ_i uses the communication plan $c_i(\theta_i)$ until date $t-1$, and h_{it} is realized at t. Then possible deviations by i at date t can be restricted to switching to the communication plan of by some other $\theta_i^{'} \in \Theta_i(h_{it})$ from date t onwards. Using Lemma 2, the resulting expected payoff of i is given by

$$E_{\theta_j}[t_i(\theta_i^{'}, \theta_j) - \theta_i q_i(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})].$$

Condition (ii) ensures no such deviation is profitable for the agent. And condition (iii) ensures participation of i at $t = 0$. Hence this mechanism implements $(t_i(\theta_i, \theta_j), q_i(\theta_i, \theta_j))$ as a PBE allocation. ■

Proof of Proposition 3: Necessity of conditions (i) and (ii) are straightforward. So we prove sufficiency of these two conditions for an output allocation to be attainable under centralized contracting in combination with a transfer rule, which generates an expected profit of

$$E[V(q_i(\theta_i,\theta_j),q_j(\theta_i,\theta_j)) - v_i(\theta_i)q_i(\theta_i,\theta_j) - v_j(\theta_j)q_j(\theta_i,\theta_j)]$$

For the proof, the following notation will be useful.

(i) $h_{it} \succ h_{is}$ if $t \geq s$ and $\Theta_i(h_{it}) \times \Theta_j(h_{it}) \subset \Theta_i(h_{is}) \times \Theta_j(h_{is})$

(ii)$H_{it}(\theta_i, h_{is}) \equiv \{h_{it} \mid h_{it} \succ h_{is}, \theta_i \in \Theta_i(h_{it})\}$ for (θ_i, h_{is}) such that $\theta_i \in \Theta_i(h_{is})$.

(iii) $h_{it}(\theta_i,\theta_j) \equiv \{h_{it} \in H_{it} \mid (\theta_i,\theta_j) \in \Theta_i(h_{it}) \times \Theta_j(h_{it})\}$.

In (i), the history h_{it} at date t is a successor of history h_{is} at date s: it results from further exchange of messages between dates $s+1$ and t after h_{is} has occurred. The set $H_{it}(\theta_i, h_{is})$ is the set of all possible histories that type θ_i could observe at t, following history h_{is} observed at date s. And $h_{it}(\theta)$ is the history observed by i at t in state θ.

The following Lemma will prove useful.

Lemma 4 *Choose arbitrary* $t \in \{1,..,T\}$, $h_{i,t-1} \in H_{i,t-1}$ *and* $\theta_i \in \Theta_i(h_{i,t-1})$. *Then for any* $h_{it}, h_{it}^{'} \in H_{it}(\theta_i, h_{i,t-1})$,

$$\Theta_i(h_{it}) = \Theta_i(h_{it}^{'}).$$

The argument is the following. Messages sent by i at t depend only on θ_i and $h_{i,t-1}$. Hence conditional on θ_i and $h_{i,t-1}$, the succeeding histories $h_{it}, h_{it}^{'}$ can differ only because of differing messages *received* by i at t, in turn owing to different realizations of θ_j. Hence the set of types θ_i that observe the history $h_{i,t-1}$ and send the same messages as θ_i equals both $\Theta_i(h_{it})$ and $\Theta_i(h_{it}^{'})$.

Lemma 4 implies that the set $\{h_{it} \mid h_{it} \succ h_{i,t-1}\}$ of states succeeding $h_{i,t-1}$ can be partitioned into a collection of subsets of h_{it} with the property that $\Theta_i(h_{it})$ is equal among all h_{it} included in the same subset. Let these subsets be numbered from 1 to $L(h_{i,t-1})$ where $L(h_{i,t-1})$ is the total number of these subsets, and let $H_{it}(k, h_{i,t-1})$ denote the subset corresponding to $k \in \{1, .., L(h_{i,t-1})\}$. Then

$$\Theta_i(h_{it}) = \Theta_i(h_{it}^{'})$$

(which can also be expressed as $\Theta_i(k, h_{i,t-1})$) for any h_{it}, $h_{it}^{'} \in H_{it}(k, h_{i,t-1})$. Moreover

$$\cup_{h_{it} \in H_{it}(k, h_{i,t-1})} \Theta_j(h_{it}) = \Theta_j(h_{i,t-1})$$

for any k and

$$\cup_{k=1}^{L(h_{i,t-1})} H_{it}(k, h_{i,t-1}) = \{h_{it} \mid h_{it} \succ h_{i,t-1}\}.$$

We are now in a position to set out the key steps of the proof of sufficiency of conditions (i) and (ii) of the proposition.

Claim 1

For $q_i(\theta_i, \theta_j)$ which satisfies conditions (i) and (ii) in the proposition, there exists $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ defined on $[\underline{\theta}_i, \bar{\theta}_i] \times \cup_{\tau=0}^{T} H_{i\tau}$ which satisfies the following conditions (a), (b) and (c).

(a) For any $h_{it} \in \cup_{\tau=0}^{T} H_{i\tau}$, $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ is non-increasing in $\tilde{\theta}_i$ on $[\underline{\theta}_i, \bar{\theta}_i]$.

(b) For any $h_{it} \in \cup_{\tau=0}^{T} H_{i\tau}$ and any $\tilde{\theta}_i \in \Theta_i(h_{it})$,

$$\tilde{q}_i(\tilde{\theta}_i, h_{it}) = E_{\theta_j}[q_i(\tilde{\theta}_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})]$$

(c) For any $h_{i,t-1} \in \cup_{\tau=0}^{T} H_{i\tau}$ and any $k \in \{1, .., L(h_{i,t-1})\}$,

$$\Sigma_{h_{it}^{'} \in H_{it}(k, h_{i,t-1})} \frac{\Pr(\Theta_j(h_{it}^{'}))}{\Pr(\Theta_j(h_{i,t-1}))} \tilde{q}_i(\tilde{\theta}_i, h_{it}^{'}) = \tilde{q}_i(\tilde{\theta}_i, h_{i,t-1})$$

Claim 1 states that there exists an 'auxiliary' output rule $\tilde{q}_i$ as a function of type $\tilde{\theta}_i$ and message history which is globally non-increasing in type (property (a)) following any history h_{it}, and nevertheless equals the expected value (conditional on h_{it}) of output in the allocation rule that is being sought to be implemented (property (b)). Property (ii) in the statement of the Proposition only allows the latter to be non-increasing over the set of types that arrive at that history h_{it} on the equilibrium path. This auxiliary output rule will be used later in the construction of transfer payments that efficiently incentivize the desired output allocation.

In order to establish Claim 1, the following Lemma is needed.

Lemma 5 *For any $B \subset R_+$ which may not be connected, let A be an interval satisfying $B \subset A$. Suppose that $F_i(a)$ for $i = 1, ..., N$ and $G(a)$ are functions defined on A, each of which has the following properties:*

- *$F_i(a)$ is non-increasing in a on B for any i.*
- *$\Sigma_i p_i F_i(a) = G(a)$ for any $a \in B$ and for some p_i so that $p_i > 0$ and $\Sigma_i p_i = 1$.*
- *$G(a)$ is non-increasing in a on A.*

Then we can construct $\bar{F}_i(a)$ defined on A for any i so that

- *$\bar{F}_i(a) = F_i(a)$ on $a \in B$ for any i.*
- *$\Sigma_i p_i \bar{F}_i(a) = G(a)$ for any $a \in A$ and for the same p_i*
- *$\bar{F}_i(a)$ is non-increasing in a on A for any i.*

This lemma says that we can construct functions $\bar{F}_i(a)$ so that the properties of functions $F_i(a)$ on B are also maintained on the interval A which covers B.

Proof of Lemma 5

If this statement is true for $N = 2$, we can easily show that this also holds for any $N \geq 2$. Suppose that this is true for $N = 2$.

$$\Sigma_{i=1}^N p_i F_i(a) = p_1 F_1(a) + (p_2 + ... + p_N) F^{-1}(a)$$

with

$$F^{-1}(a) = \Sigma_{i \neq 1} \frac{p_i}{p_2 + ... + p_N} F_i(a).$$

Applying this statement for $N = 2$, we can construct $\bar{F}_1(a)$ and $\bar{F}^{-1}(a)$ which keeps the same property on A as on B. Next using the constructed $\bar{F}^{-1}(a)$ instead of $G(a)$, we can apply the statement for $N = 2$ again to construct desirable $\bar{F}_2(a)$ and $\bar{F}^{-2}(a)$ on A based on $F_2(a)$ and $F^{-2}(a)$ which satisfy

$$\frac{p_2}{p_2 + ... + p_N} F_2(a) + [1 - \frac{p_2}{p_2 + ... + p_N}] F^{-2}(a) = F^{-1}(a).$$

on B. We can use this method recursively to construct $\bar{F}_i(a)$ for all i.

Next let us show that the statement is true for $N = 2$. For $a \in A \backslash B$, define $\underline{a}(a)$ and $\bar{a}(a)$, if they exists, so that

$$\underline{a}(a) \equiv \sup\{a^{'} \in B \mid a^{'} < a\}$$

and

$$\bar{a}(a) \equiv \inf\{a^{'} \in B \mid a^{'} > a\}.$$

It is obvious that at least one of either $\underline{a}(a)$ or $\bar{a}(a)$ exists for any $a \in A \backslash B$.

Let's specify $\bar{F}_1(a)$ and $\bar{F}_2(a)$ so that $\bar{F}_1(a) = F_1(a)$ and $\bar{F}_2(a) = F_2(a)$ for $a \in B$, and for $a \in A \backslash B$ as follows.

(i) For $a \in A \backslash B$ so that only $\underline{a}(a)$ exists,

$$\bar{F}_1(a) = F_1(\underline{a}(a))$$

$$\bar{F}_2(a) = \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}$$

(ii)For $a \in A \backslash B$ so that both $\underline{a}(a)$ and $\bar{a}(a)$ exist,

$$\bar{F}_1(a) = \min\{F_1(\underline{a}(a)), \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1}\}$$

$$\bar{F}_2(a) = \max\{F_2(\bar{a}(a)), \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}\}$$

(iii)For $a \in A \backslash B$ so that only $\bar{a}(a)$ exists,

$$\bar{F}_1(a) = \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1}$$

$$\bar{F}_2(a) = F_2(\bar{a}(a))$$

It is easy to check that $\bar{F}_i(a)$ is non-increasing in a on A for $i = 1, 2$ and

$$p_1 \bar{F}_1(a) + p_2 \bar{F}_2(a) = G(a)$$

for $a \in A$. This completes the proof of the lemma. ∎

Proof of Claim 1:

Choose arbitrary $t \in \{1, ..., T\}$ and $h_{i,t-1} \in H_{i,t-1}$, and $k \in \{1, 2, ..., L(h_{i,t-1})\}$. Lemma 5 implies that for $\tilde{q}_i(\tilde{\theta}_i, h_{i,t-1})$ which satisfies (a) and (b) for this $h_{i,t-1}$, we can construct a function $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ for any $h_{it} \in H_{it}(k, h_{i,t-1})$ so that (a), (b) and (c) are satisfied. This result is obtained upon applying the Lemma with

$$B = \Theta_i(k, h_{i,t-1})$$

$$A = [\underline{\theta}_i, \bar{\theta}_i]$$

$$a = \tilde{\theta}_i$$

$$G(\tilde{\theta}_i) = \tilde{q}_i(\tilde{\theta}_i, h_{i,t-1})$$

$$F_{h_{it}}(\tilde{\theta}_i) = \tilde{q}_i(\tilde{\theta}_i, h_{it})$$

$$p_{h_{it}} = \frac{\Pr(\Theta_j(h_{it}))}{\Pr(\Theta_j(h_{i,t-1}))}$$

for any $h_{it} \in H_{it}(k, h_{i,t-1})$ where each element of the set $H_{it}(k, h_{i,t-1})$ corresponds one-to-one to each one of the set $\{1, ..., N\}$ in Lemma 5. This means that for $\tilde{q}_i(\tilde{\theta}_i, h_{i,t-1})$ which satisfies (a) and (b) for any $h_{i,t-1} \in H_{i,t-1}$, we can construct $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ which satisfies (a)-(c) for any $h_{it} \in H_{it}$.

With $h_{i0} = \phi$, since $\tilde{q}_i(\tilde{\theta}, h_{i0}) = E_{\theta_j}[q_i(\theta_i, \theta_j)]$ satisfies (a) and (b), $\tilde{q}_i(\tilde{\theta}_i, h_{i1})$ is constructed so that (a)-(c) are satisfied for any $h_{i1} \in H_{i1}$. Recursively $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ can be constructed for any $h_{it} \in \cup_{\tau=0}^{T} H_{i\tau}$ so that (a)-(c) are satisfied. ∎

Claim 2

For $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ constructed in Claim 1,

$$E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h_{it})] = \tilde{q}_i(\tilde{\theta}_i, h_{it})$$

for any $\theta_i \in \Theta_i(h_{it})$.

Proof of Claim 2

From the construction of $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ which satisfies (c), for any $\theta_i \in \Theta_i(h_{it})$,

$$\begin{aligned}
& \tilde{q}_i(\tilde{\theta}_i, h_{it}) = \Sigma_{h_{it+1} \in H_{it+1}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{it+1}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{it+1}) \\
= & \Sigma_{h_{it+1} \in H_{it+1}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{it+1}))}{\Pr(\Theta_j(h_{it}))} \Sigma_{h_{it+2} \in H_{it+2}(\theta_i, h_{it+1})} \frac{\Pr(\Theta_j(h_{it+2}))}{\Pr(\Theta_j(h_{it+1}))} \tilde{q}_i(\tilde{\theta}_i, h_{it+2}) \\
= & \Sigma_{h_{it+1} \in H_{it+1}(\theta_i, h_{it})} \Sigma_{h_{it+2} \in H_{it+2}(\theta_i, h_{it+1})} \frac{\Pr(\Theta_j(h_{it+2}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{it+2}) \\
= & \Sigma_{h_{it+2} \in H_{it+2}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{it+2}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{it+2}) = \cdots \\
= & \Sigma_{h_{iT} \in H_{iT}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{iT}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{iT})
\end{aligned}$$

The first and second equalities come from (c) and the fact that there exists k so that $H_{i\tau}(\theta_i, h_{i,\tau-1}) = H_{i\tau}(k, h_{i,\tau-1})$ for any $\theta_i \in \Theta_i(h_{i,\tau-1})$. The fourth equality comes from

$$H_{it+2}(\theta_i, h_{it}) = \{H_{it+2}(\theta_i, h_{it+1}) \mid h_{it+1} \in H_{it+1}(\theta_i, h_{it})\}.$$

Since

$$H_{iT}(\theta_i, h_{it}) = \{h_{iT}(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})\},$$

$$\begin{aligned}
& \Sigma_{h_{iT} \in H_{iT}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{iT}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{iT}) \\
= & \Sigma_{h_{iT} \in \{h_{iT}(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})\}} \frac{\Pr(\Theta_j(h_{iT}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{iT}) \\
= & E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h_{it})]
\end{aligned}$$

This completes the proof of the Claim. ■

We are now in a position to complete the proof of sufficiency. Based on $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ which was constructed in Claim 1, consider the following mechanism: $(t_i(\theta_i, \theta_j), q_i(\theta_i, \theta_j))$ with

$$t_i(\theta_i, \theta_j) = \theta_i q_i(\theta_i, \theta_j) + \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) d\tilde{\theta}_i$$

Then for any t, any $h_{it} \in H_{it}$ and any $\theta_i, \theta_i^{'} \in \Theta_i(h_{it})$,

$$
\begin{aligned}
& E_{\theta_j}[t_i(\theta_i^{'}, \theta_j) - \theta_i q_i(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \\
= \; & (\theta_i^{'} - \theta_i) E_{\theta_j}[q_i(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] + \int_{\theta_i^{'}}^{\bar{\theta}_i} E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i^{'}, \theta_j)) \mid \theta_j \in \Theta_j(h_{it})] d\tilde{\theta}_i \\
= \; & (\theta_i^{'} - \theta_i) \tilde{q}_i(\theta_i^{'}, h_{it}) + \int_{\theta_i^{'}}^{\bar{\theta}_i} \tilde{q}_i(\tilde{\theta}_i, h_{it}) d\tilde{\theta}_i \\
\leq \; & \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(\tilde{\theta}_i, h_{it}) d\tilde{\theta}_i \\
= \; & \int_{\theta_i}^{\bar{\theta}_i} E_{\theta_j}[q_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h_{it})] d\tilde{\theta}_i \\
= \; & E_{\theta_j}[t_i(\theta_i, \theta_j) - \theta_i q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})]
\end{aligned}
$$

The second equality comes from (b) in Claim 1 and the statement of Claim 2 and the third inequality comes from (a) in Claim 1. The fourth equality comes from the statement of Claim 2. This means that the incentive constraint is satisfied at all stages of communication.

At the interim stage with $t = 0$,

$$
\begin{aligned}
& E_{\theta_j}[t_i(\theta_i, \theta_j) - \theta_i q_i(\theta_i, \theta_j)] \\
= \; & \int_{\theta_i}^{\bar{\theta}_i} E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j))] d\tilde{\theta}_i \\
= \; & \int_{\theta_i}^{\bar{\theta}_i} E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h_{i0})] d\tilde{\theta}_i \\
= \; & \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(\tilde{\theta}_i, h_{i0}) d\tilde{\theta}_i \\
= \; & \int_{\theta_i}^{\bar{\theta}_i} E_{\theta_j}[q_i(\tilde{\theta}_i, \theta_j)] d\tilde{\theta}_i
\end{aligned}
$$

The third equality comes from the statement of Step 2, and the fourth one is from (b) in Claim 1. This implies that the interim participation constraint is satisfied and P's ex-ante payoff is equal to

$$E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - v_i(\theta_i) q_i(\theta_i, \theta_j) - v_j(\theta_j) q_j(\theta_i, \theta_j)].$$

which is the maximum possible payoff under the constraint that θ_i is private information of i. This completes the proof of the proposition. ■

Proof of Proposition 4:

We show that the solution of the problem without the constraint (ii) in Proposition 3 satisfies this constraint. Suppose not. Let $(q_1(\theta_1, \theta_2), q_2(\theta_1, \theta_2))$ be the solution without (ii). H_{it}, $\Theta_i(h_{it})$ and $\Theta_j(h_{it})$ are defined for this communication feasible productions. Then there exists t, $h_{it} \in H_{it}$ and $\theta_i, \theta_i^{'} \in \Theta_i(h_{it})$ with $\theta_i > \theta_i^{'}$ so that

$$E_{\theta_j}[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] > E_{\theta_j}[q_i(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})].$$

This implies that at least either one of

$$\begin{aligned} & E[V(q_i(\theta_i^{'}, \theta_j), q_j(\theta_i^{'}, \theta_j)) - v_i(\theta_i) q_i(\theta_i^{'}, \theta_j) - v_j(\theta_j) q_j(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \\ > \; & E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - v_i(\theta_i) q_i(\theta_i, \theta_j) - v_j(\theta_j) q_j(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \end{aligned}$$

or

$$\begin{aligned} & E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - v_i(\theta_i^{'}) q_i(\theta_i, \theta_j) - v_j(\theta_j) q_j(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \\ > \; & E[V(q_i(\theta_i^{'}, \theta_j), q_j(\theta_i^{'}, \theta_j)) - v_i(\theta_i^{'}) q_i(\theta_i^{'}, \theta_j) - v_j(\theta_j) q_j(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \end{aligned}$$

holds. This means that if at least one type of either θ_i or $\theta_i^{'}$ takes other type of communication plan, P's payoff is improved. This is a contradiction. ■

Proof of Proposition 5:

The proof relies on the following Lemma.

Lemma 6 *In an optimal mechanism, each agent i's communication strategy* $c_i^*(\theta_i)$ *is almost everywhere locally constant.*

Proof.

Step 1

In what follows fix the optimal communication plan for P and suppress it in the notation, while focusing on optimal choice of communication plan by each agent. With $h_i = h_i(c_i, c_j)$, the optimal production $\hat{q}_i^*(\theta_i, h_i)$ and communication plan $c_i^*(\theta_i)$ for agent i satisfy

$$\begin{aligned} \hat{q}_i^*(\theta_i, h_i) &\in \arg\max_{q_i} E_{\theta_j}[V(q_i, \hat{q}_j^*(\theta_j, h_j(c_i, c_j^*(\theta_j)))) \mid h_i(c_i, c_j^*(\theta_j)) = h_i] \\ &- v_i(\theta_i) q_i \end{aligned} \tag{8}$$

and

$$\begin{aligned} c_i^*(\theta_i) &\in \arg\max_{c_i \in \mathcal{C}_i} \pi(\theta_i, c_i) \equiv E_{\theta_j}[V(\hat{q}_i^*(\theta_i, h_i(c_i, c_j^*(\theta_j)), \hat{q}_j^*(\theta_j, h_j(c_i, c_j^*(\theta_j)))) \\ &- v_i(\theta_i)\hat{q}_i^*(\theta_i, h_i(c_i, c_j^*(\theta_j))) - v_j(\theta_j)\hat{q}_j^*(\theta_j, h_j(c_i, c_j^*(\theta_j)))] \end{aligned}$$

In (8) the optimal production decision conditional on a given message history h_i does not depend on i's communication plan c_i, since h_i includes all messages sent by i and the latter are a sufficient statistic for inferences made by i about the output decisions to be made by j.

Step 2: *There exists an optimal communication strategy $c_i^*(\theta_i)$ for each agent i such that*

$$c_i^*(\theta_i) \in \arg\max_{c_i \in \mathcal{C}_i} \pi(\theta_i, c_i)$$

and $c_i^*(\theta_i)$ is almost everywhere locally constant.

Before we set out the argument for Step 2, note that since $v_i(\theta_i)$ is continuous, the Maximum Theorem implies that $\pi(\theta_i, c_i)$ is a continuous function of θ_i, for any c_i. Suppose Step 2 is false. Then there exists a non-degenerate interval $(\theta_i^*, \theta_i^{**})$ over which the optimal communication strategy cannot be selected to be a constant strategy for any subinterval. In other words, for any θ_i in this interval, if $\hat{c}_i(\theta_i) \in \arg\max_{c_i \in \mathcal{C}_i} \pi(\theta_i, c_i)$, then in any neighborhood of θ_i there exists $\theta_i^{'}$, and an alternative communication plan $c_i' \in \mathcal{C}_i$ such that

$$\pi(\theta_i, \hat{c}_i(\theta_i)) \geq \pi(\theta_i, c_i^{'})$$

and

$$\pi(\theta_i^{'}, c_i^{'}) > \pi(\theta_i^{'}, \hat{c}_i(\theta_i)).$$

Now choose arbitrary $\theta_i^0 \in (\theta_i^*, \theta_i^{**})$. $B(\theta_i^0)$ and $C_i(\theta_i^0)$ are constructed as follows:

- $C_i(\theta_i^0)$ is defined as $\{c_i \in C_i \mid \pi(\theta_i^0, \hat{c}_i(\theta_i^0)) = \pi(\theta_i^0, c_i)\}$. This is the set of communication plans that maximize $\pi(\theta_i^0, .)$.

- In the case that $\bar{C}_i(\theta_i^0) \equiv C_i \backslash C_i(\theta_i^0)$ is not empty: Since $\pi(\theta_i, c_i)$ is continuous for θ_i, for $c_i \in \bar{C}_i(\theta_i^0)$, there exists neighborhood $B(\theta_i^0, c_i)$ of θ_i^0 so that $\pi(\theta_i^{'}, \hat{c}_i(\theta_i^0)) > \pi(\theta_i^{'}, c_i)$ for any $\theta_i^{'} \in B(\theta_i^0, c_i)$. Since there are a finite number of elements in $\bar{C}_i(\theta_i^0)$, $\Pr(\cap_{c_i \in \bar{C}_i(\theta_i^0)} B(\theta_i^0, c_i)) > 0$. Define $B(\theta_i^0) \equiv \cap_{c_i \in \bar{C}_i(\theta_i^0)} B(\theta_i^0, c_i)$. Then for any $\theta_i^{'} \in B(\theta_i^0)$, $\pi(\theta_i^{'}, c_i^{'}) < \pi(\theta_i^{'}, \hat{c}_i(\theta_i^0))$ for any $c_i^{'} \in \bar{C}_i(\theta_i^0)$.

- In the case that $\bar{C}_i(\theta_i^0) \equiv C_i \backslash C_i(\theta_i^0)$ is empty: $B(\theta_i^0)$ is chosen as an arbitrary neighborhood of θ_i^0.

By hypothesis, there exists $\theta_i^1 \in B(\theta_i^0)$ and $c_i^{'}$ so that

$$\pi(\theta_i^1, c_i^{'}) > \pi(\theta_i^1, \hat{c}_i(\theta_i^0)).$$

Hence $c_i^{'}$ does not belong to $\bar{C}_i(\theta_i^0)$. This implies that $c_i^{'}$ belongs to $C_i(\theta_i^0)$.

Next construct $C_i(\theta_i^1)$ and $B(\theta_i^1)$ from θ_i^1 using the same procedure as in the construction of $C_i(\theta_i^0)$ and $B(\theta_i^0)$ from θ_i^0.

We claim that $C_i(\theta_i^1) \subseteq C_i(\theta_i^0)$. Suppose c_i does not belong to $C_i(\theta_i^0)$. Then $c_i \in \bar{C}_i(\theta_i^0)$. Since $\theta_i^1 \in B(\theta_i^0)$, this implies $\pi(\theta_i^1, c_i) < \pi(\theta_i^1, \hat{c}_i(\theta_i^0))$. Since $\hat{c}_i(\theta_i^0)) < \pi(\theta_i^1, c_i^{'})$, it follows that c_i does not belong to $C_i(\theta_i^1)$.

Note also that $C_i(\theta_i^1)$ does not include $\hat{c}_i(\theta_i^0)$. Hence the number of elements in $C_i(\theta_i^1)$ is strictly smaller than $C_i(\theta_i^0)$.

In a manner similar to the choice of θ_i^1 given θ_i^0, we can choose $\theta_i^2 \in B(\theta_i^1)$ and construct $C_i(\theta_i^2)$ and $B(\theta_i^2)$. This procedure can be repeated until the number of elements in $C_i(\theta_i^k)$ becomes one. Then since $\hat{c}_i(\theta_i)$ is constant for $\theta_i \in B(\theta_i^k)$, we obtain a contradiction, thus completing the proof of Step 2 and Lemma 6.

Return now to the proof of Proposition 5. Lemma 6 implies that there exist $\bar{c}_i \in \mathcal{C}_i$ and non-degenerate interval $[\theta_i^{'}, \theta_i^{''}] \subset \{\theta_i \mid c_i^*(\theta_i) = \bar{c}_i\}$. Given the Inada conditions, $q_i^*(\theta_i, \theta_j) = \hat{q}_i^*(\theta_i, h_i(\bar{c}_i, c_j^*(\theta_j)))$ is strictly decreasing in θ_i on $[\theta_i^{'}, \theta_i^{''}]$, since this solves

$$\max_{q_i} E_{\theta_j}[V(q_i, \hat{q}_j^*(\theta_j, h_j(\bar{c}_i, c_j^*(\theta_j)))) \mid h_i(\bar{c}_i, c_j^*(\theta_j)) = h_i] - v_i(\theta_i)q_i.$$

On the other hand, $h_j = h_j(\bar{c}_i, c_j^*(\theta_j))$ and $h_P = h_P(\bar{c}_i, c_j^*(\theta_j))$ are independent of θ_i on $[\theta_i', \theta_i'']$. This implies that q_i^* is not measurable with respect to h_j and h_P. ∎