A GENERAL EQUILIBRIUM ANALYSIS OF PERSONAL BANKRUPTCY LAW[1]

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Abstract

We analyze an economy where principals and agents match and contract subject to moral hazard. Bankruptcy law defines the limited liability constraint in these contracts. We provide a Walrasian characterization of stable allocations and use this to show that weakening bankruptcy law causes redistribution of debt and welfare from poor agents and principals to rich agents. Moreover, exemption limits Pareto-dominate other bankruptcy laws if project size is fixed, and means testing (as in the new US personal bankruptcy law) which is *ex post* pro-poor in intent makes the poor worse off *ex ante*.

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1 Introduction

Bankruptcy law plays a central role in modern economies, determining access to credit and allocation of assets. Yet, both the law and its enforcement vary widely across developed and developing countries, between developed countries, and between different states within a given country.[4] For example, personal bankruptcy law in Germany is far less lenient compared with the US: in the former country defaulting borrowers have to pay a significant portion of their earnings for six years after the default, while Chapter 7 provisions in the US have traditionally allowed most borrowers to not incur any liability against future earnings after a default. The liability of borrowers under Chapter 7 at the time of default is limited to assets owned at that point of time, in excess of an exemption limit. Those with fewer assets than the exemption limit do not incur any liability at all. These exemption limits vary widely across different states in the US (Gropp, Scholz and White (1997)).

Personal bankruptcy law affects subsequent economic fortunes of borrowers in distress, whose numbers exceeded 1 million in the US over the period 1996-2005.[5] While most popular arguments for weaker bankruptcy laws focus on their *ex post* consequences on borrowers in distress, economists draw attention to their adverse consequences on *ex ante* credit access, particularly for poor borrowers. Cross-country and cross-US-state comparisons do indicate significant positive correlation between stringency of debtor liability and *ex ante* credit access (e.g., see Djankov, McLiesh and Shleifer (2007) and Gropp, Scholz and White (1997)). Nevertheless, there are few careful theoretical analyses of the distributional incidence or optimal design of bankruptcy law in a general equilibrium setting with contracts.

We study a two-sided matching model of debt or asset lease contracts subject to moral

[4]For more details of cross-country comparisons, see Djankov, McLiesh and Shleifer (2007) and Djankov, Hart, McLiesh, and Shleifer (2006).

[5]This pertains to the annual number of non-business bankruptcy filings in the US, according to the American Bankruptcy Institute: see www.abiworld.org/AM/AMTemplate.cfm?Section=Home&TEMPLATE=/CM/ContentDisplay.cfm&CONTENTID=46621) .

hazard, where liability limits are defined by bankruptcy law. This is used to analyze general equilibrium and welfare effects of changing the law. The model is characterized by two-sided heterogeneity: agents (borrowers or tenants) may differ in wealth, and principals (lenders or asset owners) may have differing overhead or monitoring costs. The number of projects that a given agent can operate can vary, but is subject to diminishing returns. Principal-agent coalitions form *ex ante* and design financial contracts determining contributions to the project financing upfront, followed by *ex post* state-contingent transfers subject to legal liability limits. The total supply of credit (i.e., entry of principals), its allocation and pricing across different borrowers are endogenously determined.

We show that stable allocations in this setting can be characterized as Walrasian allocations in the market for contracts (corresponding to a given unit of leasing or borrowing), whose 'price' is identified with the per unit expected (operating) profit that any lender or asset owner can earn. Comparative statics with respect to weakening agent liability (e.g., raising borrower exemptions in the event of bankruptcy) show reallocation of credit (resp. assets and payoffs) across agents of differing wealth: lending to poorer agents shrink, while they expand for the richest agents. These results match cross-US-state empirical patterns identified by Gropp et al (1997).

Our theory explains these as the result of interplay between two opposing effects: (i) a *partial equilibrium (PE)* effect of weakening agent liability which restricts the set of feasible contracts by making it more difficult for borrowers (resp. tenants) to commit credibly to repaying their loans (resp. paying their rent), and (ii) a *general equilibrium (GE)* effect of a lowering of profit rates. The intensity of the adverse PE effect is greater for poorer agents, and non-existent for rich agents, as the latter do not face problems with credible payment on account of possessing sufficient assets to post collateral. On the other hand, the GE effect benefits all agents uniformly. Hence the richest agents benefit from a weakening of bankruptcy law, while the poor face greater difficulty in gaining access to the market.

Apart from explaining cross-US-state patterns, our theory generates the following normative implications. First, stronger bankruptcy laws or measures to protect lender rights are not ex ante Pareto-improving in general, as might appear from a purely partial equilibrium perspective. They hurt richer agents owing to the GE effect, while benefiting lenders (via higher profits) and poorer agents (owing to enhanced market access, due to the PE effect). Second, we show that exemption limits – where all assets above the limit (and none below) are appropriable by lenders — are an optimal form of bankruptcy law in the case where project scales are not variable (e.g., owing to strong diminishing returns).[6] These results provide a normative basis for exemption limit-based laws, and potential political-economy explanations for wide variations observed in bankruptcy law across states and countries. They predict that wealthy borrowers will demand weaker bankruptcy laws, while lenders (and poor borrowers, if they are politically organized) will demand stronger ones. The relative number and political influence of these different interest groups in a given society will affect the actual law and its enforcement. Finally, our analysis predicts that the means test of the 2005 BAPCPA reform of US bankruptcy law will have adverse effects on poor borrowers from an *ex ante* perspective, even though the law appears to be designed to protect such borrowers.

Our explanation for why bankruptcy laws are often weak or poorly enforced can be contrasted with others based on incomplete contracting, limited rationality or foresight of borrowers (including scope for manipulation or fraud by lenders), preoccupation of contract enforcers with *ex post* relief for borrowers in distress to the exclusion of their *ex ante* effects on credit access. Our theory applies in the most conventional setting employed by economists: a world of complete contracts, perfect rationality, with *ex ante* welfare judgments.

The paper is organized as follows. A detailed description of related literature is provided in Section 2. The model is set up in section 3. In section 4 we solve the model and state our main result. Section 5 shows that exemption limit laws Pareto dominate other bankruptcy

[6] In the case of a fixed project scale, weakening bankruptcy law causes agent efforts to fall or remain unchanged, implying that aggregate welfare in the economy cannot improve. In the more general case, we cannot sign the effect on aggregate welfare.

laws when project scale is not variable. The effects of the new bankruptcy law introduced by BAPCPA in 2005 is discussed in section 6. Finally, section 7 concludes.

2 Related Literature

As mentioned in the Introduction, our theory does not stress possible insurance advantages provided by bankruptcy law: borrowers are assumed to be risk neutral. In contrast most arguments for weak bankruptcy laws in existing literature rest on their role in providing insurance, where contracts are assumed to be incomplete (thus eliminating the possibility of providing insurance via state-contingent liabilities). An example is Bolton and Rosenthal (2002) who consider an agricultural economy subject to macroeconomic shocks. In their model farmers (debtors) and lenders enter into debt contracts that are not contingent on macroeconomic shocks. As a result, *ex post* intervention of the government has a potentially beneficial aspect because it helps provide insurance to farmers. This is true even if the intervention is anticipated. Similarly, the insurance advantages of a weak bankruptcy law are stressed by Gropp, Scholz and White (1997) and Fan and White (2003).[7]

Manove, Padilla and Pagano (2001) provide an alternative argument for weak bankruptcy law, in terms of the need to provide banks with incentives to screen investment projects. In their model, only lenders have the expertise to ascertain project quality by engaging in a costly screening process. They show that in a competitive credit market, equilibrium loan contracts will be designed with excessively high collateral requirements that leave lenders with insufficient incentives to screen projects. Legal restrictions on collateral mitigate this inefficiency.[8]

[7]Uninsurable income shocks are also used in recent contributions by Livshits, MacGee and Tertilt (2007) and Chatterjee et al. (2007) who analyze a dynamic formulation of credit markets to understand the impact of changing bankruptcy law.

[8]If the credit market were more monopolistic, this inefficiency also tends to be mitigated as lenders internalize the effects of superior project quality. Our theory in contrast implies that the contracting externality across borrowers owing to the GE effect is magnified when the credit market is less competitive.

Our paper differs from existing literature on private bankruptcy law primarily in its emphasis on credit reallocation effects of weakening bankruptcy provisions (some empirical evidence of which is already available). In addition, we provide a theoretical analysis of contracts with moral hazard in a tractable general equilibrium setting with two-sided heterogeneity, where the main results can be interpreted intuitively as the result of shifting patterns of Walrasian demand and supply of contracts. This model may thus prove useful in many other contexts as well. A similar methodology linking stable allocations and Walrasian equilibria in general equilibrium models with moral hazard is employed in Dam and Perez-Castrillo (2006). They consider one-sided heterogeneity and provide characterization results similar to our characterization of stable allocations. Our matching market also relates to matching problems with non-transferable utility as analyzed in Legros and Newman (2007).

Our focus on the distributional impact of the law is shared by a number of recent papers on the political economy of law and finance. In Pagano and Volpin (2005), investor protection and employment protection are determined in a political process. Perotti and von Thadden (2005) consider a similar environment and investigate the role of wealth concentration on corporate governance and labor rents. Biais and Mariotti (2006) consider defaulting firms and their access to credit when corporate bankruptcy law regulating liquidation of firms is changed. All of these papers focus on general equilibrium spillovers from the credit market to the labor market, while our focus is on general equilibrium effects *within* the credit market. The set-up of these models differs from ours since they investigate how corporate bankruptcy law affects the consumption of private benefits from the perspective of either owner/entrepreneurs (Pagano and Volpin (2005) or Biais and Mariotti (2006)) or workers (Perotti and von Thadden (2005)). In contrast we focus on personal bankruptcy law: issues such as exemption limits or the recent BAPCA reforms in the US.

Nevertheless, our main predictions of the distributional effects of softening bankruptcy law are similar in some respects, though operating via different channels. In Biais and Mariotti (2006) for instance, softer laws benefit wealthier investors as they cause entry of less wealthy investors to fall, which causes the wage rate to fall. In our context wealthy entrpreneurs benefit from softer laws owing to reduced competition for funds arising from less wealthy entrepreneurs. Therefore both theories predict wealthy borrowers will prefer soft bankruptcy laws, unlike lenders and less-wealthy borrowers.

A companion paper (von Lilienfeld-Toal and Mookherjee (2007)) uses a similar approach to examine laws pertaining to bonded labor provisions, wherein future labor income can be used as collateral. While banned in most countries, such bans are not well enforced in many poor countries. Our companion paper proposes an explanation of why more developed countries tend to ban (or enforce) bonded labor contracts. Bond and Newman (2007) consider imprisonment for debt and argue this creates an externality across lenders, arising from the fact that imprisoned debtors cannot contract with other lenders. Banning debtor imprisonment then enables efficiency improvements owing to the removal of this externality.

Our analysis rests on the supposition that profit rates from principals adjust locally due to a change in the bankruptcy law. We believe that this is a reasonable assumption even with globalized financial markets. What we need is some scarce local input that is required for agents to succeed. This input can be the fixed supply of assets (land, housing, taxi cab licences, or franchise outlets) or limited monitoring capacity of local financial intermediaries. Existing evidence for the US is consistent with this supposition. For example, Black and Strahan (2002), Cetorelli and Strahan (2004), and Petersen and Rajan (1995) show that local credit market conditions affect firms' access to credit and entry of young firms.

3 Model

3.1 Technology, Endowments

The economy has a population of $m \geq 2$ principals denoted $j = 1, \ldots, m$ and n agents denoted $i = 1, \ldots, n$. Each principal owns an asset such as a plot of land, equipment (real estate, taxicabs) or franchise that requires the effort of an agent to generate income, in combination with working capital funded either from the agent's wealth, or loans provided by the principal. Agents are prospective tenants who do not own the asset themselves; principals are asset owners who are unable to provide the labor necessary to generate income from these assets. Agents are wealth-constrained, thus unable to purchase the asset from principals. They are ordered in terms of their *ex ante* wealth: $w_1 \geq w_2 \geq w_3 \ldots$; the wealth distribution is given. Some results in the paper are nonvacuous only if the wealth upper bound w_1 of borrowers is sufficiently large.[9] Principals are not wealth-constrained.

Each agent can work at a scale of $\gamma = 1, 2, \ldots$, which represents the number of assets leased and operated. For instance, a tenant farmer may lease in multiple plots of land. An entrepreneur may start a project at one of many different scales. There are diminishing returns to project scale, described in more detail below.

In order to simplify the exposition, we assume that each principal owns a single unit of the relevant asset. Principal j is subject to a fixed (overhead or monitoring) cost of f_j: its net profit is its operating profit (defined by net transfers, as explained below) less this fixed cost. The results extend straightforwardly to a wider class of asset ownership patterns among principals. Specifically, if a principal owns $q \geq 2$ assets and is subject to overhead cost of r per asset, the same results obtain if there are at least q other principals owning one asset each with a overhead cost of r. The existence of such a 'competitive fringe' of small asset owners will eliminate any possible monopoly power of large asset owners. Our analysis applies to contexts where the supply side is competitive in this particular sense.

The model also applies to a pure credit context, where entrepreneurs or borrowers do not need to lease assets from principals, and only need to borrow funds from the latter. In such a case the agents own or have free access to all other assets required to execute the project. In the simple version we exposit below, each principal has the capacity to lend enough to finance a single project at unit scale; the analysis applies with more general distributions of loanable funds among lenders satisfying an analogous 'competitive fringe' property.[10]

[9]For instance, our main result (Proposition 8 below) concerning redistributional effects of changing bankruptcy law, requires existence of borrowers wealthy enough that they attain first-best project scales.

[10]One complication in the pure credit context arises if $\sigma(w) \equiv 0$ for all w. In that case borrowers wealthy enough to achieve the first-best will be able to entirely self-finance their projects, and will thus not need to

An agent leasing γ assets will form a coalition with γ principals. In order to operate each asset, an upfront (working capital or investment) cost of I has to be incurred, so the total upfront financing need is $\gamma \cdot I$, which must be distributed between the agent and the principals in the coalition.[11] If the agent leases γ assets, we say that the project scale is γ. The agent subsequently selects effort $e \in [0, 1]$, whence the project results in a success with probability e, and failure otherwise. If successful (outcome s), the return is $\gamma^\beta y_s$; if failure (outcome f), it is $\gamma^\beta y_f$, where $y_s > I > y_f$ and $\beta \in (0, 1)$ represents the extent of diminishing returns with respect to scale. Effort e entails a nonpecuniary cost of $D(e)$ for the agent, where $D(0) = 0, D'(e) > 0, D''(e) > 0, D'''(e) > 0$ for all $e > 0$. The assumption $y_s > I > y_f$ ensures the income from the project will be negative if unsuccessful, and positive if successful. In addition, we assume there exists $e \in (0, 1)$ such that $e(y_s - y_f) + y_f > I + D(e)$, i.e., at unit scale the project returns an expected net income in excess of the effort cost of the agent. Without such an assumption the technology does not allow any agent to be viable at any scale, even in a first-best setting.

3.2 Contracts and Default

An agent with *ex ante* wealth w_i can contribute any or part of it ($d \leq w_i$) towards the upfront financing cost $\gamma \cdot I$, borrowing the remainder from the principals in the coalition (denoted C_i). They design the contract defining contributions of each member of the coalition towards the upfront cost (d for the agent, I_j for each principal $j \in C_i$) and mandated financial transfers t_{kj} from the agent to each principal $j \in C_i$ after the project is completed, conditional on the outcome ($k = s, f$). After the project is completed, the agent obtains the return ($\gamma^\beta y_k$ in state k) from the project, in addition to an exogenously determined income $\sigma(w)$ from

borrow. Then Proposition 8 concerning redistributive effects of altering bankruptcy law may become vacuous. However, if *ex post* incomes are positive, this would no longer be true: there can be wealthy borrowers who achieve the first-best and yet would want to borrow (against their future incomes).

[11] We ignore the possibility of outside financing on the grounds of the benefits of 'interlinked' contracts (see Braverman and Stiglitz 1982): any financial contract offered by outsiders can be replicated by insiders as the latter are not wealth-constrained, with the benefit that common-agency externalities can be avoided.

other sources.[12] We assume $\sigma(.)$ is a strictly increasing function. The *ex post* wealth of the agent will be the sum of: (a) $w - d$, portion of *ex ante* wealth remaining after the upfront contribution; (b) project return $\gamma^\beta y_k$ in state k, and (iii) outside income $\sigma(w)$. From this wealth, the agent decides what transfers to make to each principal, and consumes the rest. Consumption must be nonnegative; hence physical feasibility requires aggregate transfers made to not exceed the *ex post* wealth of the agent.

Default occurs following outcome k if the agent fails to make the required transfer t_{kj} to some principal $j \in C_i$. Liability rules then specify a penalty of $p(W)$ to be incurred by the agent, where W denotes *ex post* wealth of the latter at the point of default. In the event of default, the project returns accrue to the principals in C_i: principal j is entitled to s_{kj} where $\sum_{j \in C_i} s_{kj} = \gamma^\beta y_k$. Feasibility dictates that $W \geq p(W)$. We also assume that $p(W)$ and $W - p(W)$ are both nondecreasing in W. The former assumption seems natural: increases in the capacity of contract enforcers to impose punishments should not result in lower punishments. The latter assumption is also a natural consequence of agents having the option of destroying their own wealth.

We assume that $p(W)$ either accrues to the government or outside parties, or represents pure social deadweight losses (e.g., court or lawyer fees, costs of imprisonment or other penalties). Part of these could also represent mandated punitive transfers to the principals in the coalition. The exact destination of $p(W)$ will make no difference, as default will not actually occur in equilibrium.

In the event that the project outcome is $k \in \{s, f\}$ and the agent does not default, the agent's net payoff will be

$$w_i + \sigma(w_i) - d - \sum_{j \in C_i} t_{kj} + \gamma^\beta y_k - D(e) \tag{1}$$

and of principal $j \in C_i$ will be

$$t_{kj} - I_j - f_j. \tag{2}$$

[12]Much of our analysis can be extended to a setting where *ex post* income is stochastic, but positively correlated with initial wealth.

And if the agent defaults, the agent will earn

$$w_i + \sigma(w_i) - d - p(w_i + \sigma(w_i) - d) - D(e) \tag{3}$$

and j will earn

$$s_{kj} - I_j - f_j. \tag{4}$$

It follows that default occurs if and only if (assuming the agent does not default unless strictly advantageous):

$$\sum_{j \in C_i} t_{kj} > \gamma^\beta y_k + p(w_i + \sigma(w_i) - d) \tag{5}$$

Once outcome k is realized, the agent and the principals in the coalition can renegotiate the contractual payments t_{kj} provided they are all better off from that point onwards.

The exact sequence of events thus is as follows: (1) each agent is matched with a coalition of principals; (2) contracts are written; (3) the agent selects effort e; (4) outcome k is realized; (5) mandated transfers are renegotiated if there is scope for an *ex post* Pareto improvement; (6) the agent decides whether or not to default, following which payoffs are realized.

Lemma 1 *Given any coalition C_i of principals associated with agent i, without loss of generality, attention can be restricted to contracts in which:*

(i) there is no default in equilibrium, i.e., (5) does not hold;

(ii) $d = w_i$, i.e., the agent contributes his entire ex ante wealth as downpayment;

(iii) principal j acquires a constant share δ_j of the project, in the sense that she contributes $I_j = \delta_j(\gamma \cdot I - w_i)$ upfront and receives transfer $t_{kj} = \delta_j \sum_{l \in C_i} t_{kl}$.

The proof of this and many subsequent results are relegated to the Appendix. The underlying idea is simple. Default does not arise in equilibrium, since any contract inducing default generates deadweight losses which can be avoided via an *ex post* Pareto improving renegotiation. Hence attention can be restricted to contracts that do not provide borrowers with an incentive to default *ex post*. This imposes an incentive compatibility restriction on the agent's default incentive, apart from the conventional incentive constraint associated with *ex ante* effort choice. The no-default constraint is defined by the level of transfers that the bankruptcy law would allow *ex post*; a weaker bankruptcy law would lower such transfers, thus restricting the set of feasible default-free contracts. Increasing downpayments made by the borrower helps relax these constraints, since these reduce *ex post* wealth of the borrower by more than they increase *ex post* transfers in the event of default. Finally, risk-neutrality of principals implies that they care only about their expected net returns. Hence their contribution to the financing needs of the borrower and their allocation of the aggregate repayments can both be equal to a common share (in turn equal to their share in total expected returns aggregated across all lenders).

In what follows we restrict attention to contracts depicted in the above Lemma. It helps to denote contracts in terms of wealth v_k of the agent following outcome k:

$$v_k \equiv \gamma^\beta y_k + \sigma(w_i) - T_k \tag{6}$$

where $T_k \equiv \sum_{j\in C_i} t_{kj}$ denotes the aggregate transfer paid by the agent in state k. The agent's net payoff in state k following effort choice e is $v_k - D(e)$, and expected payoff is $ev_s + (1-e)v_f - D(e)$. Denoting aggregate (expected) operating profit of the principals by

$$\Pi \equiv eT_s + (1-e)T_f - [\gamma \cdot I - w_i], \tag{7}$$

the expected payoff of principal $j \in C_i$ is $\delta_j\Pi - f_j$. The contract can then be represented by the aggregate financial transfers and shares of different principals: $(T_s, T_f, \{\delta_j\}_{j\in C_i})$, besides project scale γ and effort e. Equivalently we can represent it in terms of $(v_s, v_f, \{\delta_j\}_{j\in C_i}, \gamma, e)$, using (6). The no-default condition in state k requires

$$T_k \leq \gamma^\beta \cdot y_k + p(\sigma(w_i)) \tag{8}$$

which reduces to

$$\sigma(w_i) \leq v_k + p(\sigma(w_i)). \tag{9}$$

The contract $(v_s, v_f, \{\delta_j\}_{j\in C_i}, \gamma, e)$ with shares $\delta_j \geq 0, \sum_{j\in C_i} \delta_j = 1$ is **feasible** if it satisfies the following constraints:

$$v_s - v_f = D'(e) \tag{IC}$$

$$v_k \geq \sigma(w_i) - p(\sigma(w_i)), k = s, f \tag{LL}$$

$$\Pi \equiv \gamma^{\beta}[ey_s + (1-e)y_f] - [ev_s + (1-e)v_f] + \sigma(w_i) + w_i - \gamma \cdot I \geq \sum_{j \in C_i} f_j \tag{PCP}$$

$$ev_s + (1-e)v_f - D(e) \geq w_i + \sigma(w_i) \tag{PCA}$$

Here (IC) refers to the effort incentive constraint, (LL) to the no-default constraint, and (PCA) to the participation constraint for the agent. The constraint (PCP) is clearly necessary for each principal to break even. It is also sufficient in the sense that one can find shares $\delta_j, j \in C_i$ such that j breaks even in expectation (i.e., $\delta_j \cdot \Pi \geq f_j$). Accordingly we can simplify the definition of a contract to a tuple $(v_s, v_f, \gamma \geq 1, e)$, and call it feasible for coalition C_i of principals if it satisfies (IC), (LL), (PCP) and (PCA). We can then call an agent with wealth w_i *viable* if the set of feasible contracts is nonempty for that agent, for some coalition C_i of principals.

Note the role of bankruptcy law, in its stipulation of default penalties imposed on the agent: these define the limit of the agent's liability, as represented by (LL). A stronger bankruptcy law pertains to a penalty function $\tilde{p}(.)$ which uniformly dominates another $p(.)$, i.e., if $\tilde{p}(W) \geq p(W)$ for all W. An example of a specific bankruptcy law is one involving an exemption limit E, with a zero marginal tax rate below the limit and 100% above the limit: $p(W; E) = \max\{0, W - E\}$. A lower exemption limit then corresponds to a strengthening of bankruptcy law, and a corresponding weakening of the (LL) constraint. This enlarges the set of feasible contracts for any given agent.

It is easy to check that viability of agents is positively related to their wealth. Intuitively, this occurs because wealthier agents require less external finance, and thus need to repay less to the lenders: this effect outweighs the higher payoff option available to them if they default.[13]

[13] This relies on the assumption that the slope of $p(.)$ is less than one.

Lemma 2 *Suppose agent i with wealth i is viable. Then every agent l with $w_l > w_i$ is also viable.*

4 Stable Allocations

An *allocation* is a matching of each agent $i = 1, \ldots, n$ with a coalition C_i of principals such that $C_i \bigcap C_l = \emptyset$ when $i \neq l$, and a contract for each matched agent i which is feasible for the coalition C_i. An agent i is *unmatched* if $C_i = \emptyset$: such an agent gets payoff $w_i + \sigma(w_i)$. A principal j is *unmatched* if j does not belong to C_i for any agent i. Such a principal earns a payoff of 0. Payoffs for matched agents and principals are defined by the contracts they enter into.

An allocation is *stable* if there does not exist any agent i and a coalition $\hat{C}_i$ of principals that can enter into a feasible contract with i which generates a higher payoff for i and every principal in $\hat{C}_i$.

We now show that stable allocations can be characterized as a Walrasian equilibrium in the market for contracts for leasing one unit of the asset, with the 'price' π of such contracts represented by the rate of operating profit per asset leased. Principals and agents take this profit rate as given: each principal decides whether to enter the market and offer her asset for lease. Each agent decides on how many assets to lease, and designs a contract to maximize her own utility, subject to the constraint of generating a profit of at least π for each asset leased in.

The Walrasian demand for contracts by an agent i corresponds to the solution of the following problem, given the profit rate π: select contract (v_s, v_f, e, γ) to maximize expected payoff $ev_s + (1-e)v_f - D(e)$ subject to (IC), (LL), (PCA) and the following 'budget constraint':

$$\gamma^{\beta}[ey_s + (1-e)y_f] - [ev_s + (1-e)v_f] + w_i + \sigma(w_i) \geq \gamma \cdot (I + \pi). \qquad (BC)$$

If the feasible contract set is empty, set $\gamma = 0$. We shall call any solution to this problem as an A-optimal contract for agent i, given profit rate π per asset leased.

A *Walrasian allocation* is an allocation and an operating profit rate π per asset such that:

(a) for any agent i, the contract $(v_s^i, v_f^i, e^i, \gamma^i)$ assigned to agent i is A-optimal for i relative to π.

(b) the 'supply' of assets (or number of active principals) is determined as follows: any principal with $f_j > \pi$ is inactive; any principal with $f_j < \pi$ is active. Every active principal receives the same expected operating profit π.

(c) the total demand for assets $\sum_i \gamma^i$ equals the supply. Agent i is assigned a coalition C_i consisting of γ^i principals, arbitrarily selected from the set of active principals.

Proposition 3 *An allocation is stable if and only if it is Walrasian.*

The intuitive reasoning underlying this result is as follows. Owing to competition among lenders, they must all attain the same rate of operating profit per unit leased. Any principal that can cover its fixed costs with the common rate of operating profit will be willing to enter, others will not be willing to enter. Hence supply decisions are as if lenders take the rate of operating profit as given and decide on their profit-maximizing responses. Moreover, every borrower must select an optimal contract, subject to the 'budget' constraint of paying the going expected rate of return to all its lenders (apart from effort and no-default incentive constraints). Otherwise it is possible to find a Pareto-improving coalition: a contract can be designed to provide the borrower with higher expected utility, and its lenders a higher rate of profit. In this sense, the demand for projects is also Walrasian. Finally, matching implies that supply and demand are balanced.

This characterization is convenient as it allows us to focus on the comparative static effect of altering bankruptcy law on Walrasian equilibria. Since the supply side of the market is simple, we need to understand how A-optimal demands for project scale for borrowers of differing wealths are affected. We turn to this next.

4.1 A-Optimal Contracts

The A-optimal problem for an agent with wealth w can be represented more simply as follows. Using (IC) to substitute for v_s in terms of v_f and e, the problem is to choose (v_f, e, γ) to maximize

$$v_f + eD'(e) - D(e)$$

subject to

$$\gamma^\beta R(e) - eD'(e) - \gamma \cdot (I + \pi) + \sigma(w) + w \geq v_f \geq \sigma(w) - p(\sigma(w))$$

where $R(e) \equiv ey_s + (1-e)y_f$, the first constraint is the budget constraint, and the second constraint is (LL). The feasible set in this problem is non-empty if there exists (e, γ) such that

$$\gamma^\beta R(e) - eD'(e) - \gamma \cdot (I + \pi) \geq -w - p(\sigma(w)) \qquad (F)$$

If the feasible set is empty we can set $\gamma = e = 0$. Otherwise, for any (e, γ) satisfying (F), it is optimal to set

$$v_f = \gamma^\beta R(e) - eD'(e) - \gamma \cdot (I + \pi) + w + \sigma(w)$$

so we can restate the A-optimal problem as selection of (e, γ) to maximize

$$\{\gamma^\beta R(e) - D(e) - \gamma \cdot (I + \pi)\} + w + \sigma(w) \qquad (AO)$$

subject to constraint (F).

Denote the solution to the A-optimal problem (AO) by $\gamma(\pi, w), e(\pi, w)$, with the convention that $\gamma(\pi, w) = e = 0$ if the maximized value of (AO) falls below the autarkic payoff of $w + \sigma(w)$. And denote the corresponding problem of maximizing (AO) without any constraints the *first-best problem*, with solution $\gamma^*(\pi), e^*(\pi)$. Note that the discreteness of project scale implies that the optimal contract may be non-unique in either first-best or second-best situations; hence $(\gamma(\pi, w), e(\pi, w))$ and $(\gamma^*(\pi), e^*(\pi))$ are correspondences.

Note also that the first-best generates positive surplus to the agent (i.e., above the autarkic payoff of $w+\sigma(w)$) if $\pi = 0$, since there exists e (with $\gamma = 1$) such that $R(e)-D(e) >$

I. On the other hand for π sufficiently large, a positive surplus cannot be generated. This will impose an upper bound $\bar{\pi}$ to the profit rate π consistent with a positive demand for projects from the agent.

Lemma 4 *(a) For any given profit rate $\pi \geq 0$, there exists $\bar{w}(\pi)$, a nondecreasing function of π, such that every A-optimal contract for an agent with wealth w above $\bar{w}(\pi)$ is first-best: $(\gamma(\pi, w), e(\pi, w)) = (\gamma^*(\pi), e^*(\pi))$. Conversely, $w < \bar{w}(\pi)$ implies that the first-best cannot be attained.*

(b) The first-best contract $(\gamma^*(\pi), e^*(\pi))$ is nonincreasing in π, in the sense that $\pi_2 > \pi_1$ implies that $\gamma_2 \leq \gamma_1$ and $e_2 \leq e_1$ for any first-best choice $(\gamma_m, e_m) \in (\gamma^*(\pi_m), e^*(\pi_m)), m = 1, 2$.

(c) If $w < \bar{w}(\pi)$, every A-optimal contract involves lower effort than the first-best ($e(\pi, w) < e^*(\pi)$), and $\gamma(\pi, w) \leq \gamma^*(\pi)$.

This Lemma states that for agents with wealth above some threshold, A-optimal contracts will be first-best. For the first-best contract which is an unconstrained maximizer of (AO) is independent of the wealth of the agent, and it satisfies constraint (F) if this wealth is sufficiently high. As the required profit rate to be paid increases, it reduces the desired project scale, and in turn this reduces the borrower's *ex ante* effort. For those borrowers not wealthy enough to be able to implement the first-best contract, the scale of the project has to be reduced in order to meet constraint (F). While these results appear reasonable enough, proving them is somewhat complicated because the problem of selecting an A-optimal contract is not a convex optimization problem (owing to the complementarity between project scale and effort).

The next set of results show that A-optimal demand for project scale is nondecreasing in the agent's wealth, and nonincreasing in the going profit rate.

Lemma 5 *If $w_1 < w_2 < \bar{w}(\pi)$ then $e(\pi, w_1) \leq e(\pi, w_2)$ and $\gamma(\pi, w_1) \leq \gamma(\pi, w_2)$ for any selection of A-optimal contracts.*

Lemma 6 *If $\pi_1 < \pi_2$ then $e(\pi_1, w) \geq e(\pi_2, w)$ and $\gamma(\pi_1, w) \geq \gamma(\pi_2, w)$ for any selection of A-optimal contracts.*

Moreover, weakening bankruptcy law tightens the no-default constraint, which in turn reduces A-optimal demand for project scale.

Lemma 7 *Consider a weakening of bankruptcy rules from $p_2(.)$ to $p_1(.)$ in the sense that $p_1(W) \leq p_2(W)$ for all W. Consider any (π, w) and let γ_l denote the A-optimal project scale under rule $p_l, l = 1, 2$. Then if the A-optimal payoff differs under the two rules, $\gamma_1 \leq \gamma_2$.*

4.2 Distributional impact of changing bankruptcy law

These results enable us to derive our central result.

Proposition 8 *Consider a weakening of bankruptcy rule from $p_2(.)$ to $p_1(.)$ (in the sense that $p_2(W) \geq p_1(W)$ for all W). Consider an allocation which is Walrasian at p_2 but is not Walrasian at p_1, in the following sense: there does not exist a Walrasian allocation at p_1 with the same total number of assets leased and the same profit rate. Let $\pi_l, l = 1, 2$ denote the corresponding profit rates with rule p_l. Then:*

(a) the profit rate is lower with the weaker rule: $\pi_1 \leq \pi_2$;

(b) for agents with $w > \bar{w}(\pi_2; p_1)$, project scale γ, effort e and payoff are higher (or remain the same) in the Walrasian allocation corresponding to the weaker rule p_1;

(c) total number of assets leased by agents with $w < \bar{w}(\pi_2; p_1)$ are lower (or the same) under the weaker rule p_1.

Proof of Proposition 8: Let S_2 be the total number of assets leased in the Walrasian allocation under p_2. By hypothesis, when the rule is changed to p_1, there is no Walrasian allocation with S_2 projects leased and the same profit rate π_2. In other words, we cannot find A-optimal project scales $\gamma(\pi_2, w_i, p_1)$ under rule p_1 such that $\sum_i \gamma(\pi_2, w_i, p_1) = S_2$. But there were A-optimal project scales $\gamma(\pi_2, w_i, p_2)$ under rule p_2 such that $\sum_i \gamma(\pi_2, w_i, p_2) = S_2$. By Lemma 7 we know that $\gamma(\pi_2, w_i, p_1) \leq \gamma(\pi_2, w_i, p_2)$. So it must be the case that $\sum_i \gamma(\pi_2, w_i, p_1) < S_2$ for any set of A-optimal project choices at p_1.

Suppose $\pi_1 > \pi_2$. Then any principal that was active under p_2 will continue to be active under p_1. Hence the supply of assets under p_1 cannot be smaller than under p_2: $S_1 \geq S_2$. On the other hand, Lemma 6 ensures that $\gamma(\pi_1, w_i, p_1) \leq \gamma(\pi_2, w_i, p_1)$. This implies $S_1 > \sum_i \gamma(\pi_1, w_i, p_1)$, i.e., there cannot be a Walrasian allocation at π_1 under p_1. Therefore $\pi_1 \leq \pi_2$, establishing (a).

Wealthy agents with $w > \bar{w}(\pi_2, p_1)$ will achieve the first-best in both allocations. Since $\pi_1 \leq \pi_2$, they are (weakly) better off, and strictly better off if the profit rate falls. By Lemma 4 their A-optimal project scale and effort will increase or remain the same.

The total supply of assets cannot increase because $\pi_1 \leq \pi_2$. Therefore the total number of assets leased by agents with wealth below $\bar{w}(\pi_2, p_1)$ cannot increase. ■

The intuitive reasoning is as follows. A weaker bankruptcy rule lowers the demand for projects from poorer agents that cannot achieve the first-best, and leaves the demand of wealthier agents unchanged. This reduces the total demand for assets, creating an excess supply, which lowers the profit rate. This in turn *increases* the demand from wealthy agents, and raises their payoffs. The reduction in the profit rate restricts supply of assets. Hence there is a reallocation of assets from poorer to wealthier agents. The poorer the agent, the more important is the (LL) constraint, so they tend to be the most adversely impacted by the weakening of the bankruptcy rule. Some of them may be excluded from the market altogether.

Figure 1 highlights the driving forces behind the redistribution of assets leased.[14] Individual demand for assets of an agent with wealth w' under law $p_2(\cdot)$ is given as $\gamma_2(\pi, w' < \bar{w})$ and depicted in the left graph of figure 1. Weakening liability law shifts demand downward to $\gamma_1(\pi, w')$. In contrast, demand of rich enough agents is unaffected and given as $\gamma(\pi, w \geq \bar{w})$

[14]Note that figure 1 depicts the continuous limit of our results and abstracts from minor issues that arise due to the discreteness of project scale.

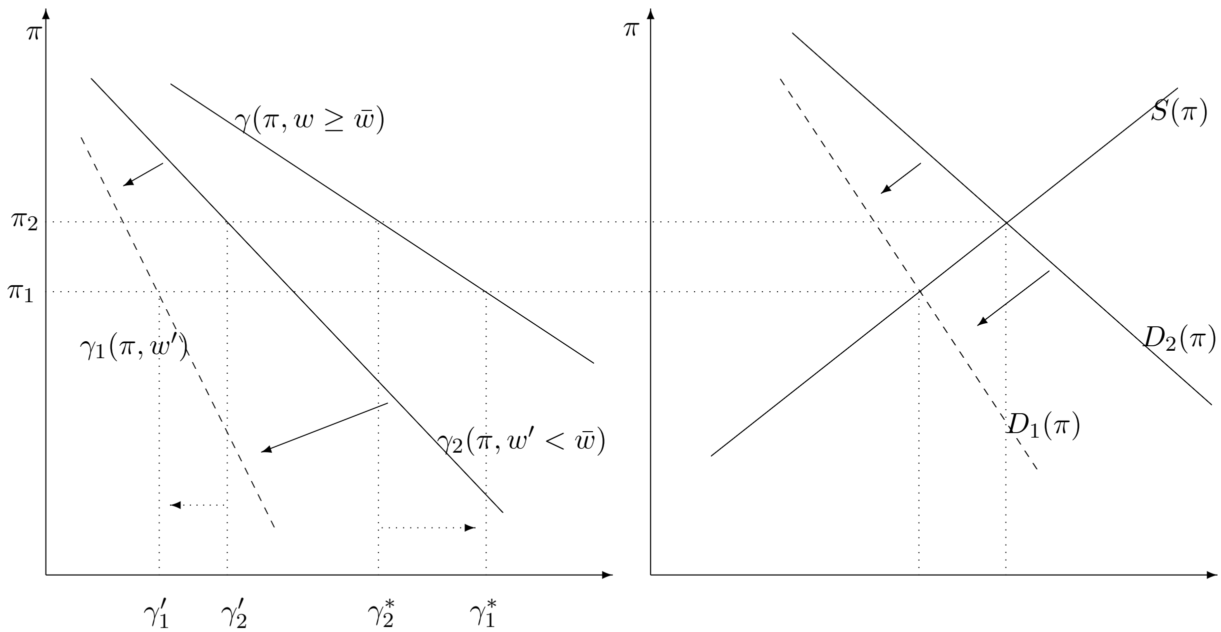

Figure 1: Impact of changing liability law on poor and rich agents.

under either law. As a result, aggregate demand D_2 shifts downward to D_1 (right graph of figure 1). A reduction in aggregate demand leads to a reduction of the profit rate from π_2 to π_1. Now, individual equilibrium demand for assets is affected differently for rich and poor agents. For rich agents, equilibrium demand *increases* from γ_2^* to γ_1^* which is due to the reduced profit rate. However, for poor agents demand *decreases* from γ_2' to γ_1' because the partial equilibrium impact of weakening liability law overrules the general equilibrium effect of lower profit rates.

These results are in line with empirical evidence found by Gropp, Scholz and White (1997). Consistent with proposition 8 part c, Gropp et al. (1997, p. 238 table III) report that the amount of debt is decreasing in the exemption limit for poor borrowers in the lowest two quartiles of the wealth distribution. As implied by part (b) of Proposition 8, debt increases with exemption limits for all agents situated in the highest two quartiles of the wealth distribution. These differences are significant and economically meaningful. Gropp et al. (1997, p. 242, table V) compare the predicted value of debt for an observationally equivalent household living in two hypothetic states with different exemption limits.[15] If the household is "poor" with assets worth \$47,000, belonging to the second quartile of the wealth distribution, the estimated debt holding is \$28,105 in a state with low exemption limit (\$6000), which *decreases* to \$10,551 if the same household lives in a state with a high exemption limit (\$50,000). These differences change their sign if the household is rich, with \$150,000 worth of assets belonging to the highest wealth quartile. The rich household has a predicted debt of \$36,136 in the low exemption limit state which *increases* to \$72,076 in the high exemption limit state.

Proposition 8 does not describe the impact of weakening bankruptcy law on any given wealth-constrained borrower; it states a reduction in the scale of projects aggregating across all wealth-constrained borrowers. A more detailed result is possible if we impose the additional restriction that the bankruptcy law is weakened more for poorer borrowers.[16] Hence a relaxation which appears to be 'progressive' *ex post* ends up having a 'regressive' impact *ex ante*.

Proposition 9 *Consider a weakening of bankruptcy rule from $p_2(.)$ to $p_1(.)$ with the property that $p_2(W) \geq p_1(W)$ for all W, and $p_2(W) - p_1(W)$ is nonincreasing in W. Then there exists $\tilde{w} \leq \bar{w}(\pi_2; p_2)$ such that the payoff, effort and project scale of all borrowers with wealth above (resp. below) $\tilde{w}$ (weakly) increases (resp. decreases).*

Proof of Proposition 9: By the previous Proposition we have $\pi_1 \leq \pi_2$. Since equilibrium project scale is nondecreasing in w, the fact that $p_2(W) - p_1(W)$ is nonincreasing implies that

$$\gamma(w; p_2)[\pi_2 - \pi_1] - [p_2(\sigma(w)) - p_1(\sigma(w))] \tag{10}$$

is nondecreasing in w. Define $\tilde{w}$ to be the smallest w such that (10) is nonnegative. Then weakening the bankruptcy law to p_1 causes constraint (F) to be relaxed at the A-optimal

[15] They are doing the exercise for a family with a 45 year male head, \$75,000 yearly income, college degree and varying financial wealth.

[16] This condition is however not met when bankruptcy laws take the form of exemption limits: when the exemption limit is raised the bankruptcy law is weakened more for wealthier borrowers.

contract under p_2 for any $w > \tilde{w}$, and strengthened otherwise. Using arguments analogous to those in previous results, it follows that project scale, effort and payoff of agents will rise (weakly) above $\tilde{w}$ and fall otherwise. From Proposition 8 we know that those above $\bar{w}(\pi_2; p_2)$ are better off, so it must be the case that $\tilde{w} \leq \bar{w}(\pi_2; p_2)$. ■

The next section considers the special case of a fixed project size, and obtains some additional results in that context.

5 Fixed Project Size

We now consider the special case where the returns to scale diminish fast enough that project scale is at most one for any borrower. This is a special case of our model where β is sufficiently small.[17]

Fixing project scale restricts the scope of asset reallocations across borrowers: it is no longer possible for wealthy borrowers to borrow more when bankruptcy laws are eased. However, this case is nevertheless of practical interest in many situations: e.g., tenant households rarely want to rent more than one apartment, and taxicab drivers can rarely drive more than one taxi. In such contexts, we can obtain some additional results concerning the optimal shape of bankruptcy law, and more detailed distributional and incentive effects of changing the law.

Recall that the bankruptcy law is represented by the function $p(.)$ of *ex post* assets of a defaulting borrower, which was restricted to be nondecreasing with a slope less than unity. We can think of this as an *ex post* tax function. In practice, bankruptcy law often takes the form of an exemption limit, with a marginal tax rate of 0 upto the limit and 1 above the limit. We now show that such forms of bankruptcy law are Pareto optimal:

[17] When $\beta = 0$, this is obviously true: there are no returns at all to increasing project scale beyond one unit. It is also true in a positive neighborhood of 0. Such a neighborhood can be found by imposing the requirement that the first-best project scale at $\pi = 0$ equals one. Since A-optimal project scales are nonincreasing in the profit rate, and bounded above by the first-best scale, the A-optimal scale for every agent at any nonnegative profit rate will be 0 or 1.

Proposition 10 *Suppose β is small enough that the first-best project size is at most one. Then, any bankruptcy law is (weakly) Pareto-dominated by an exemption limit law in the following sense. For any allocation that is Walrasian under law $p(\cdot)$ with operating profit rate π per asset, there exists an exemption limit E^* such that with the bankruptcy law changed to $p(W) = \max\{W - E^*, 0\}$, there is a Walrasian allocation with the same operating profit rate π per asset, in which every borrower is weakly better off.*

Proof. Let $\underline{n}$ be the poorest matched agent in the Walrasian allocation under law $p(\cdot)$ with operating profit rate π per asset and contracts $(v_s^i, v_f^i, e^i, \gamma^i)$ for all i. From Lemma 5 it follows that all richer agents are also matched: $\gamma^i = 1$ for all $i \leq \underline{n}$. Choose $E^* = \sigma(w_{\underline{n}}) - p(\sigma(w_{\underline{n}}))$.

For every agent $i \leq \underline{n}$ the set of contracts obeying constraint (F) increases due to a change from $p(\cdot)$ to E^* if π is constant. In contrast, for every agent $i \geq \underline{n}$, the set of contracts obeying (F) shrinks. This is true due to the corresponding change in $\sigma - p(\sigma)$ as depicted in Figure 2.[18]

Construct a new Walrasian allocation as follows, with the same profit rate π. Borrowers with $i \geq \underline{n}$ do not demand any project. Those with $i \leq \underline{n}$ demand one project, and are assigned an A-optimal contract corresponding to profit rate π and exemption limit law E^*. The same principals are active. This is a Walrasian allocation as long as the assigned project scales are A-optimal for every agent.

For an agent with $i \geq \underline{n}$, zero is an A-optimal project scale, because (F) has become tighter for them. For $i \leq \underline{n}$, (F) has become more relaxed. See Figure 2. Therefore by Lemma 7, their A-optimal project scale cannot decrease (as the profit rate is the same). The A-optimal project scale for them was one previously, so it must continue to be one. Therefore the constructed allocation is Walrasian.

Note finally that agents with $i \geq \underline{n}$ are as well off as before. Those with $i \leq \underline{n}$ with wealth high enough to attain the first-best will also be left unaffected. Others will be better

[18]Note that $\sigma - p(\sigma)$ is increasing and below the 45 degree line for the class of liability laws we consider here.

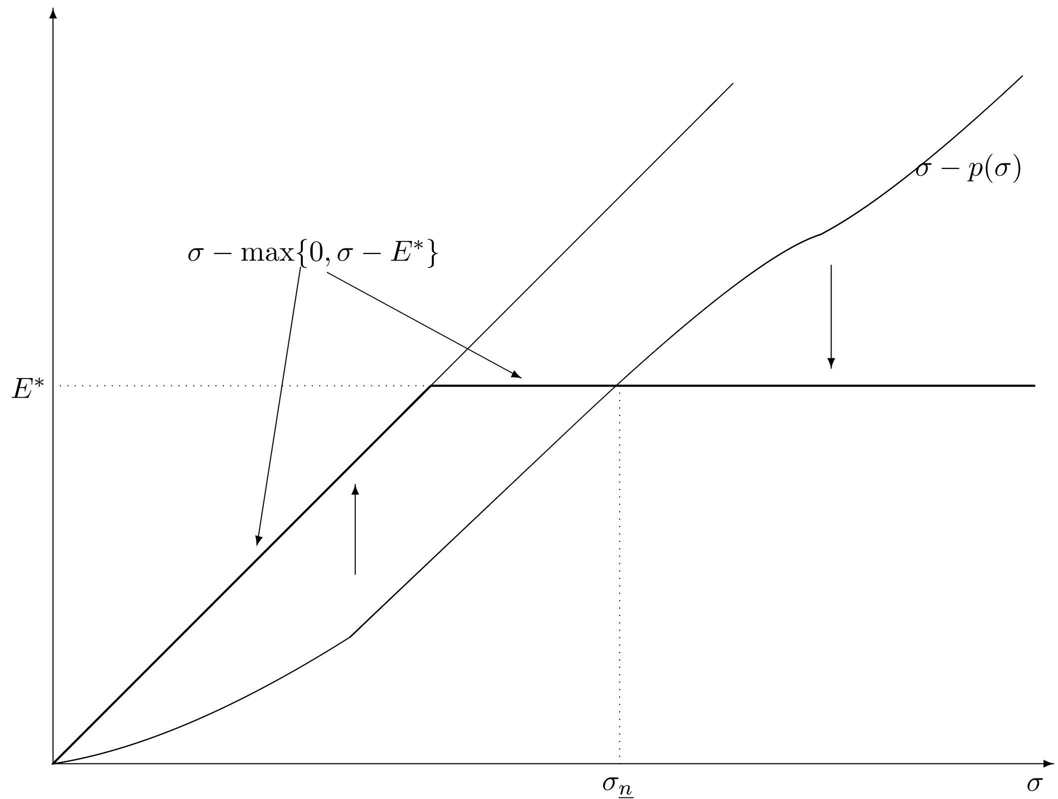

Figure 2: Replacing $p(\cdot)$ with exemption limit law E^*

off, as (F) was binding to start with, and has been relaxed. ■

Figure 2 conveys the underlying idea: fixing the exemption limit to equal the liability limit of the marginal agent active in the market ensures that liability is raised for excluded agents, and lowered for intramarginal active agents. Then the demand pattern is unaffected: excluded agents continue to demand no project, while intramarginal agents demand a single project. Hence there is an equilibrium with the same profit rate and the same allocation of projects; active agents can now commit credibly to higher repayments in time of distress and obtain credit on easier terms as a result. The logic does not extend if project scales could exceed unity: intramarginal agents may then demand more projects, creating excess demand and raising the profit rate. This may cause some marginal agents to get excluded from the market, so a Pareto improvement no longer results.

The case of fixed project scale also permits a more detailed description of the distributional impact of weakening bankruptcy law, if we further assume that all principals are identical, i.e., have the same overhead cost f. Let the bankruptcy law be represented by exemption limit E. An agent is viable at the exemption limit E if there exists a contract for that agent feasible with this bankruptcy law, that generates an operating profit at least f. Let $n(E)$ denote the number of viable agents at exemption limit E; this is a nonincreasing function by virtue of Lemma 7.

Walrasian allocations can be computed as follows. Without loss of generality, equilibrium π must be at least f.[19] Compute the A-optimal demand for each viable agent when $\pi = f$ and the exemption limit is E. If the resulting aggregate A-optimal demand exceeds m, the number of principals, the Walrasian allocation must involve $\pi > f$. In that case all principals will be active, and some viable agents will be excluded from the market. In this case, the principals are on the short-side of the market.

Conversely, if aggregate A-optimal demand at $\pi = f$ and exemption limit E does not exceed m, there will be a Walrasian allocation at $\pi = f$, where all viable agents are matched and some principals are not. This is the case where the agents are on the short-side of the market.

Now suppose the exemption limit is raised from E to E'. There are three cases to consider:

(A) *Principals are on the short side of the market at both E and E':* Then there is no effect on the total volume of leasing or credit; the allocation of credit across agents remains unaltered, but the profit rate falls or remains unchanged (since the A-optimal demand for every agent falls or remains unchanged as the exemption limit rises, at any given profit rate). Then *every active agent is better off, while every active principal is worse off.* The result is a redistribution from active principals to active agents. Moreover, it

[19] If $\pi < f$ no principal nor agent will be active; an equivalent allocation with no activity is obtained with $\pi = f$.

can be shown that wealthier active agents benefit more, while the effort level declines (weakly) for all active agents.[20] The intuitive reason is that the beneficial GE effect applies equally to all active agents, while the adverse PE effect of a higher exemption limit is less significant for wealthier agents. Nevertheless the former outweighs the latter for all active agents, not just the wealthiest ones.

(B) *Agents are on the short side of the market both before and after the change:* Then the equilibrium profit rate is unchanged (at f); there is no GE effect. All agents are (weakly) worse off, owing to the strengthening of the no-default constraint. Principals are unaffected, so the result is a Pareto-deterioration of welfare.

(C) *Principals are on the short-side at E but on the long-side at E':* Then the profit rate drops from $\pi(E) > f$ to $\pi(E') = f$; the number of assets leased falls, and the poorest agents active at E get excluded from the market at E'. On the other hand, the wealthiest agents are better off owing to the drop in the profit rate. In this case the weakening of the bankruptcy law makes lenders and poor borrowers worse off, while wealthy borrowers are better off. It can be shown that the effort level declines or remains constant for all borrowers that continue to be active. In this case, aggregate welfare also declines.

6 Chapter 7 vs. chapter 13

In this section, we discuss a simple variation of the model that helps predict the impact of the current change in US bankruptcy law undertaken in the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005. We focus on one particular aspect of BAPCPA: the abolition of free choice between chapter 7 and chapter 13.

Prior to the change in the US bankruptcy law defaulting borrowers were free to choose between the Chapter 7 code and the Chapter 13 code in most instances. The Chapter 7

[20]This implies that aggregate welfare, the sum of net payoffs across all principals and agents, decreases (weakly). This result is shown in an earlier version of this paper, available on request.

code is a much weaker bankruptcy law which allows defaulting borrowers to keep a large fraction of wealth and all of their future labor income. So prior to 2005, most debtors filed under chapter 7 (approximately 70% of al households, according to White (1987)).

Following BAPCPA, households are only allowed to file under Chapter 7 if they pass a means test, effectively requiring that their *ex post* income during the 6 month prior to filing does not exceed the median (household size adjusted) income of the state the debtor is living in (White (2007)). If the income exceeds this threshold, the household is not allowed to file for bankruptcy unless it passes a second test, the repayment test which checks whether average consumption prior to filing exceeds a certain threshold. If it does, the household must file under Chapter 13. If average consumption prior to filing does not exceed the threshold, the household may file under Chapter 7 if a certain requirement relating disposable income to secured debt is met.

In what follows we will use the following simple interpretation of US bankruptcy law before and after BAPCPA to understand the impact of the new law, especially the means test used in BAPCPA. First, we only consider the impact of the means test under BAPCPA, assuming that the procedures under Chapter 7 and Chapter 13 codes are unchanged.[21]

Second, we exaggerate the attractiveness of Chapter 7 vs. Chapter 13 from the standpoint of defaulting borrowers: what we call Chapter 7 will always be more attractive to defaulting borrowers *ex post*.[22] Alternatively we restrict attention to the majority of borrowers for whom Chapter 7 is more attractive.

The law before the change is depicted in figure (3). Ex post, Chapter 7 is more attrac-

[21] Actually there are two additional important changes in bankruptcy law due to BAPCPA. First, the administrative procedure for filing under both, Chapter 7 and Chapter 13 has become more complex, rendering either bankruptcy procedure less attractive to defaulting borrowers. Further, Chapter 13 has been made less attractive since certain kinds of debt can no longer be discharged, mainly debt obtained by fraud.

[22] This is true for at least 70% of defaulting borrowers prior to BAPCPA as White (1987) reports that 70% of defaulting borrowers opted to default under Chapter 7. Note, however, that not all remaining defaulting borrower preferred defaulting under Chapter 13. Under repeated default, borrower are no longer allowed to choose between chapter 13 and chapter 7 prior to BAPCPA.

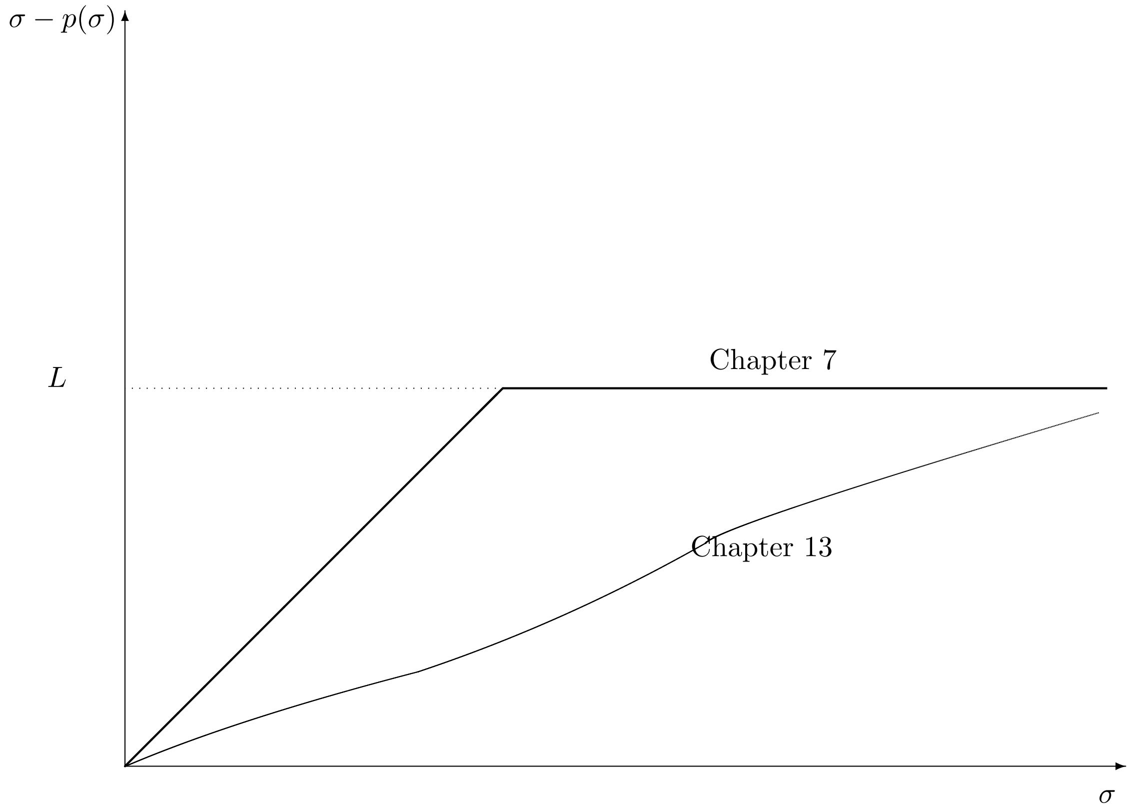

Figure 3: Chapter 7 vs. chapter 13 prior to BAPCPA

tive to defaulting borrowers than Chapter 13 since the corresponding $\sigma - p(\sigma)$ curve under Chapter 7 is always above the one corresponding to Chapter 13. In contrast, the law after BAPCPA is depicted in figure (4). Note that post BAPCPA law violates our earlier assumption that $W - p(W)$ must be non-decreasing since $W - p(W)$ makes a jump downward at σ^T. Here, σ^T corresponds to the threshold value used in the means test. In what follows we will assume that agents cannot reduce their *ex post* wealth to seek protection under Chapter 7. We expect our arguments to be valid even if we allow for such opportunistic behavior as long as reducing *ex post* income is costly.

Due to the discrete number of projects, Walrasian allocations need not be unique. In what follows, if there exist multiple Walrasian allocations, we assume that the Walrasian allocation with the highest profit rate is realized.[23]

Proposition 11 *Consider a change of bankruptcy law as described above, and restrict attention to the Walrasian allocation with the maximal profit rate* π *(across all Walrasian*

[23] We conjecture that the results hold for other rules, for example all allocations with minimal profits or allocations with profits that are a weighted average of minimal and maximal profits.

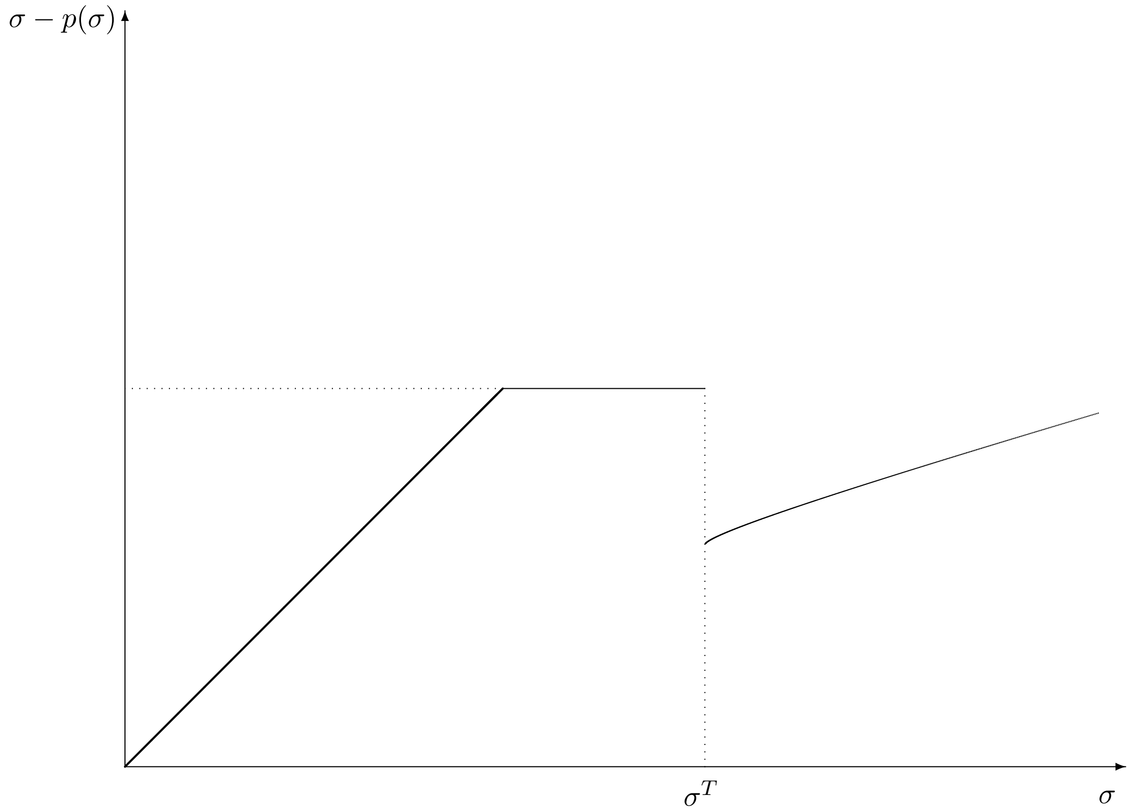

Figure 4: Chapter 7 vs. chapter 13 after BAPCPA

allocations at any given bankruptcy law). Then:

1. *all agents with wealth $\sigma < \sigma^T$ are weakly worse off due to the change. Agents with wealth $\sigma \geq \sigma^T$ may be better or worse off and all principals benefit from the change.*

2. *The total number of assets leased by agents with $\sigma < \sigma^T$ will decrease (or remain the same).*

The argument follows from an inspection of Figures 3 and 4. Prior to BAPCA, there was a unique bankruptcy law for all agents, represented by Chapter 7. Due to BAPCPA, agents with $\sigma \geq \sigma^T$ have a potentially beneficial partial equilibrium effect since their $\sigma - p(\sigma)$ curve is shifted downwards. These rich agents are effectively able to post a greater fraction of their *ex post* wealth as collateral. This leads to a (weak) increase in their A-optimal demand. For agents with $\sigma < \sigma^T$, the A-optimal demand patterns are unaltered. Hence the aggregate A-optimal demand curve shifts outwards, raising the equilibrium profit rate. This renders principals (weakly) better off. Whether or not agents with $\sigma \geq \sigma^T$ benefit depends upon the interaction of partial and general equilibrium effect which is not clear in general.

However, for agents whose *ex post* wealth is below the threshold σ^T, the effect is clear. There is no beneficial partial equilibrium effect and the (weak) increase in equilibrium profits will render these poor agents worse off. Due to Lemma 6, the allocation of credit to these agents must fall.

7 Conclusion

To summarize our principal result, weakening bankruptcy law leads to a redistribution of credit and/or assets from poor to rich borrowers. This explains the findings of cross-sectional analysis employing differences across US state bankruptcy provisions (Gropp, Scholz and White (1997)). This study showed that higher exemption limits were associated with a doubling of debt for rich households, and a halving of the debt of poor households. Hence the effects emphasized in this paper are likely to be quantitatively significant.

Our theory generated implications for the effects of the recent reform of bankruptcy law in the US that could be empirically tested in future work. Effects of changing enforcement of debt in other countries would also be interesting to investigate. For instance, some developing countries such as India have implemented reforms intended to increase the range of collaterizable assets and enforcement of debt contracts (Visaria (2006), Vig (2006)). Our theory indicates these may result in increased allocation of credit to borrowers covered by the new provisions, and reduced allocation to other classes of borrowers (i.e., poor borrowers who do not come under the ambit of the new provisions, or rich borrowers who qualified for cheap credit under the old provisions).

The model of the paper neglected dynamic effects of altering bankruptcy rules or collaterizability of assets on savings incentives of agents, and on the ownership distribution of assets in future periods. Enlarging the range of collateralizable assets may allow increased access to credit in the short run, but subsequently renders borrowers more vulnerable to downturns in the economy. Investigation of such dynamic effects remains an important task for future research.

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Appendix: Proofs

Proof of Lemma 1: Suppose there is contract which satisfies (5), and the agent defaults following outcome k. Then $j \in C_i$ receives s_{kj}, instead of t_{kj}, the transfer in the event that the agent does not default. Now the earlier contract can be replaced with one where $\tilde{t}_{kj} = s_{kj}$. This is feasible because $\sum_{j \in C_i} \tilde{t}_{kj} = \sum_{j \in C_i} s_{kj} = \gamma^\beta y_k \leq \gamma^\beta y_k + w_i - d + \sigma(w_i)$ the *ex post* wealth of the agent, as $w_i - d \geq 0, \sigma(w_i) \geq 0$. The agent will not default under this new contract as

$$\sum_{j \in C_i} \tilde{t}_{kj} = \sum_{j \in C_i} s_{kj} = \gamma^\beta y_k \leq \gamma^\beta y_k + p(w_i - d + \sigma(w_i)). \tag{11}$$

And each principal $j \in C_i$ earns the same payoff as before when default occurs, while the agent avoids the penalty associated with default. So the new contract *ex post* Pareto-dominates the previous one, which is thus vulnerable to renegotiation. This establishes (i).

For (ii), note that if $d < w_i$, the downpayment d can be raised by some $\epsilon > 0$, with a corresponding reduction of $\sum_{j \in C_i} t_{kj}$ and of $\sum_{j \in C_i} I_j$ by ϵ. In the new contract the agent will also not want to default, as the mandated transfers fall by ϵ, while default penalty $p(w_i - d + \sigma(w_i))$ falls by at most ϵ (because the slope of $p(.)$ is between 0 and 1). Then the agent's payoff, as well as that of every $j \in C_i$, in state k is unaltered. This new contract is then payoff-equivalent to the previous one.

To show (iii), fix $d = w_i$, then re-allocate transfers and contributions across $j \in C_i$ in such a way to leave their expected payoffs unchanged. The aggregate financial transfers *vis-a-vis* the agent remain unaltered, so do the agent's incentives and payoffs. Specifically, let π_j denote the expected operating profit of j, which must cover the overhead cost f_j (otherwise j would do better to leave the coalition):

$$\pi_j \equiv e t_{sj} + (1 - e) t_{fj} - I_j \geq f_j \tag{12}$$

Let $\Pi \equiv \sum_{j \in C_i} \pi_j \geq \sum_{j \in C_i} f_j$ denote aggregate operating profit. If $\Pi > 0$, define $\delta_j \equiv \frac{\pi_j}{\Pi}$ and select $\tilde{I}_j = \delta_j \sum_{l \in C_i} I_l; \tilde{t}_{kj} = \delta_j \sum_{l \in C_i} t_{kl}$. Then the payoff of j equals $\tilde{\pi}_j = \delta_j \Pi = \pi_j$.

If $\Pi = 0$, then we have $I_j = et_{sj} + (1-e)t_{fj}$ for all $j \in C_i$: select arbitrary $\delta_j \geq 0$ with $\sum_{j \in C_i} \delta_j = 1$ and repeat the above construction. This completes the proof of Lemma 1. ■

Proof of Lemma 2: Since i is viable, there exists a feasible contract (v_s, v_f, e, γ) for some coalition C_i including i. Let $\delta \equiv (w_l + \sigma(w_l)) - (w_i + \sigma(w_i)) > 0$. For agent l we can select a coalition C_l consisting of l and the same principals j that belong to C_i. Then select the following contract: $\hat{v}_s = v_s + \delta, \hat{v}_f = v_f + \delta$, combined with the same e and γ. By construction this contract satisfies (IC), (PCP) and (PCA). It also satisfies (LL): the increase in the left-hand-side of (LL) is δ, while the increase in the right-hand-side of (LL) is $[\sigma(w_l) - \sigma(w_i)] - [p(\sigma(w_l)) - p(\sigma(w_i))] \leq \sigma(w_l) - \sigma(w_i) < \delta$. ■

Proof of Proposition 3: We proceed through a sequence of steps.

Step 1.1: A Walrasian allocation is stable. If not, there will exist an agent i that deviates with a coalition $\hat{C}_i$ and a contract $(\hat{v}_s, \hat{v}_f, \hat{e}, \hat{\gamma})$ which generates an expected utility larger than that in the A-optimal contract relative to π, and an expected operating profit for every $j \in \hat{C}_i$ that exceeds π. This contradicts the definition of an A-optimal contract.

Step 1.2: In any stable allocation, all active principals attain the same (expected) operating profit. If this is false, there exist two active principals j, m with $\pi_j > \pi_m$, $\pi_j \geq f_j, \pi_m \geq f_m$. Let $ji \in C_i$. Then i can form a coalition with $\hat{C}_i \equiv C_i \setminus \{j\} \cup \{m\}$. Let n_i denote the number of principals in C_i, and select any $\epsilon \in (0, \frac{\pi_j - \pi_m}{n_i + 1})$. We can then select a contract for the deviating coalition which increases v_s, v_f by ϵ, and also increases π_h by ϵ for every $h \in \hat{C}_i$. This contract is feasible and makes everyone in the deviating coalition better off.

Step 1.3: In any stable allocation, $\pi \geq f_j$ for every active principal. If not, such a principal would be better off unmatched.

Step 1.4: In any stable allocation, $\pi \leq f_j$ for any unmatched principal. Otherwise, an unmatched principal j could make positive net profit by being matched with an active agent at the going profit rate of π. An argument analogous to Step 1.2 can now be used to show the allocation is not stable.

Step 1.5: In any stable allocation, every agent gets an A-optimal contract relative to π, the common rate of operating profit earned by active principals. Otherwise, there exists a feasible contract for i which generates a higher expected payoff to i, while paying an operating profit rate of at least π on each asset leased. We now argue there exists a contract in the neighborhood of this deviating contract which awards a profit rate greater than π for every asset leased, while still enabling the agent to attain a higher expected payoff compared with that in the stable allocation.

The original contract satisfied (PCA), so the deviating contract satisfied (PCA) with slack. We can thus ignore this aspect of feasibility in what follows.

If (LL) is slack in both states in the deviating contract, we can reduce v_s, v_f by some common but small ϵ, which preserves feasibility. In this case the argument is straightforward.

So consider the case where (LL) binds in some state. This must be state f, because $v_s > v_f$ is needed to induce $e > 0$, which in turn is necessary for feasibility ((PCP) requires the project to break-even in expectation, and this is not possible if $e = 0$ given that $y_f < I$). So we must have $v_f = \sigma(w_i) - p(\sigma(w_i))$. Now holding v_f fixed at this level, consider varying v_s above v_f, with e adjusted according to the (IC): i.e., with $e = e(v_s)$ that solves $v_s - \{\sigma(w_i) - p(\sigma(w_i))\} = D'(e)$. Let the corresponding aggregate profit for asset owners be denoted $\Pi(v_s) \equiv e(v_s)[\gamma^\beta y_s - v_s] + (1 - e(v_s))[\gamma^\beta y_f - \{\sigma(w_i) - p(\sigma(w_i))\}]$, where γ is the project scale in the deviating contract.

Define the P-optimal contract to be one where v_s is selected to maximize $\Pi(v_s)$, subject to $v_s \geq \sigma(w_i) - p(\sigma(w_i))$. This problem can be restated as follows (replacing e as the control variable): select $e \in [0, 1]$ to maximize $e \cdot [\gamma^\beta(y_s - y_f) - D'(e)]$. Noting that $eD'(e)$ is strictly convex, the objective function is strictly concave, and thus has a unique solution. It is also evident that $e > 0$ in the P-optimal contract.

We claim that in the original contract (in the stable allocation), the agent must have attained a utility at least as large as in the P-optimal contract. Otherwise, the agent obtained a smaller payoff, and the contract in the stable allocation was not P-optimal (as the P-optimal contract is unique, as shown above). So aggregate operating profit of the principals in C_i must be less than the profit in the P-optimal contract. Then the agent and principals in C_i could deviate to the P-optimal contract, which would make all of them strictly better off.

Since the agent received a higher payoff in the deviating contract compared with the contract in the stable allocation, it follows that the agent's payoff in the deviating contract is strictly higher than in the P-optimal contract. In both the deviating contract and in the P-optimal contract, we have $v_f = \sigma(w_i) - p(\sigma(w_i))$, so they must differ in v_s, with the deviating contract associated with a higher v_s. It follows that as v_s is lowered from that in the deviating contract to the level in the P-optimal contract, the agent's payoff is (continuously) lowered while aggregate profit of the principals in C_i is (continuously) raised (owing to the strict concavity of aggregate profit with respect to e). Therefore we can find a contract with v_s slightly below that in the deviating contract, which will allow a strictly higher aggregate profit, and a slightly lower payoff for the agent. This allows all members of C_i as well as i to be better off compared to the stable allocation, a contradiction. This completes the proof of Step 1.5.

The proof of Proposition 3 now follows from combining Steps 1.1–1.5 to infer that a stable allocation must be Walrasian. ■

Proof of Lemma 4: Define $S^*(\pi) \equiv \gamma^{*\beta} R(e^*) - e^* D'(e^*) - \gamma^*(I + \pi)$, where we drop the dependence of the first-best contract on π to avoid notational clutter. Then if $S^*(\pi) \geq -p(\sigma(0))$, the first-best satisfies (F) for all $w \geq 0$, so we can set $\bar{w}(\pi) = 0$ in that case. Otherwise $S^*(\pi) < -p(\sigma(0))$ and there exists $\bar{w}(\pi) > 0$ such that $S^*(\pi) = -w - p(\sigma(w))$, since $-w - p(\sigma(w))$ is decreasing in w and goes to $-\infty$ as w becomes arbitrarily large. Then the first-best is attainable with profit rate π if and only if $w \geq \bar{w}(\pi)$, which establishes (a).

(b) Suppose there exist $\pi_1 < \pi_2$, wealth w and choices of first-best contracts such that $\gamma_2 = \gamma^*(\pi_2, w) > \gamma_1 = \gamma^*(\pi_1, w)$. Then

$$\gamma_1^\beta R(e_1) - D(e_1) - \gamma_1(I + \pi_1) \geq \gamma_2^\beta R(e_2) - D(e_2) - \gamma_2(I + \pi_1)$$

and

$$\gamma_2^\beta R(e_2) - D(e_2) - \gamma_2(I+\pi_2) \geq \gamma_1^\beta R(e_1) - D(e_1) - \gamma_1(I+\pi_2)$$

where e_m denotes $e^*(\pi_m, w), m = 1, 2$. Adding these two inequalities we obtain

$$(\gamma_2 - \gamma_1)(I+\pi_1) \geq [\gamma_2^\beta R(e_2) - D(e_2)] - [\gamma_1^\beta R(e_1) - D(e_1)] \geq (\gamma_2 - \gamma_1)(I+\pi_2)$$

which contradicts the hypothesis that $\gamma_2 > \gamma_1$, as $\pi_1 < \pi_2$. Hence, $\gamma^*(\pi, w)$ is nonincreasing in π. This in turn implies $e^*(\pi, w)$ is nonincreasing because it maximizes ${\gamma^*}^\beta R(e) - D(e)$.

In order to establish (c), note the following.

Observation 3.1: In any A-optimal contract, $\gamma(\pi, w)$ *maximizes* $\gamma^\beta R(e(\pi, w)) - \gamma(I+\pi)$. Otherwise we can select a different γ to raise the value of $\gamma^\beta R(e(\pi, w)) - \gamma(I+\pi)$: this both raises the value of the objective function (AO), and helps relax the constraint (F).

Observation 3.2: In any first-best contract, $\gamma^*(\pi)$ *maximizes* $\gamma^\beta R(e^*(\pi)) - \gamma(I+\pi)$. Otherwise the value of the first-best objective function can be raised by switching to a different γ, while leaving $e^*(\pi)$ unchanged.

We now claim that if $w < \bar{w}(\pi)$, then $e(\pi, w) < e^*(\pi)$ for any selection of second-best and first-best contracts. If this is false, we can find contracts with $e(\pi, w) \geq e^*(\pi)$. Using Observations 3.1 and 3.2, it follows that corresponding second-best and first-best project scales satisfy $\gamma(\pi, w) \geq \gamma^*(\pi)$.

The hypothesis $w < \bar{w}(\pi)$ implies that the first-best cannot be achieved by w at profit rate π. This means that $(\gamma^*(\pi), e^*(\pi))$ violates (F), while by its very nature $(\gamma(\pi, w), e(\pi, w))$ satisfies (F). This implies that

$$\begin{aligned}
&[\gamma(\pi, w)]^\beta R(e(\pi, w)) - e(\pi, w)D'(e(\pi, w)) - \gamma(\pi, w)(I+\pi) \\
&\quad > [\gamma^*(\pi)]^\beta R(e^*(\pi)) - e^*(\pi)D'(e^*(\pi)) - \gamma^*(\pi)(I+\pi)
\end{aligned}$$

which in turn implies

$$\begin{aligned}
\{[\gamma(\pi, w)]^\beta R(e(\pi, w)) - \gamma(\pi, w)(I+\pi)\} \quad - \quad &\{[\gamma^*(\pi)]^\beta R(e^*(\pi)) - \gamma^*(\pi)(I+\pi)\} \\
&> e(\pi, w)D'(e(\pi, w)) - e^*(\pi)D'(e^*(\pi)) \\
&> D(e(\pi, w)) - D(e^*\pi))
\end{aligned}$$

the last inequality following from the fact that $\frac{\partial e D'(e)}{\partial e} = eD''(e) + D'(e) > D'(e)$. Then it must be the case that the second-best choices yield a higher utility than the first-best choice, which contradicts the definition of the first-best.

Therefore every second-best effort must always be less than any first-best effort. The rest of (c) now follows from Observations 3.1 and 3.2. ■

Proof of Lemma 5: Suppose there exist A-optimal efforts $e_m \equiv e(\pi, w_m), m = 1, 2$ such that $e_1 > e_2$. Then by Observation 3.1 corresponding A-optimal scales satisfy $\gamma_1 \geq \gamma_2$.

Since $w_m < \bar{w}(\pi)$, the first-best is not achievable at $(\pi, w_m), m = 1, 2$ and constraint (F) must be binding at (γ_m, e_m) for w_m. Since $w_2 > w_1$, the contract (γ_1, e_1) must satisfy (F) with slack at (π, w_2). To see this, note that

$$
\begin{aligned}
[\gamma_2]^\beta R(e_2) \qquad & -e_2 D'(e_2) - \gamma_2(I + \pi) \\
& = -w_2 - p(\sigma(w_2)) \\
& < -w_1 - p(\sigma(w_1)) \\
& = [\gamma_1]^\beta R(e_1) - e_1 D'(e_1) - \gamma_1(I + \pi).
\end{aligned}
$$

This implies that

$$
\begin{aligned}
\{[\gamma_1]^\beta R(e_1) - \gamma_1(I + \pi)\} \quad & -\{[\gamma_2]^\beta R(e_2) - \gamma_2(I + \pi)\} \\
& \geq e_1 D'(e_1) - e_2 D'(e_2) \\
& > D(e_1) - D(e_2)
\end{aligned}
$$

the last inequality following from the hypothesis that $e_1 > e_2$. This implies that the contract (γ_1, e_1) generates a higher payoff than (γ_2, e_2), contradicting the A-optimality of the latter, since the former is feasible at the wealth w_2. Hence $e_1 \leq e_2$. The remaining part of the result follows from Observation 3.1. ■

Proof of Lemma 6: The argument is virtually identical to that of Lemma 5 above, in the case that the first-best is not achievable in either situation. And if the first-best is achievable in either or both situations, we can use Lemma 4. ■

Proof of Lemma 7: Suppose the first-best payoff is achievable at (π, w) under rule p_1; then it is achievable under p_2 and the A-optimal payoffs coincide. So suppose that the first-best is not achievable at (π, w) under rule p_1. If the first-best is achieved at (π, w) under rule p_2 then the result follows from Lemma 4. So we need to consider the case where the first-best is not achieved under either rule p_1 or p_2.

Let e_l denote an A-optimal effort under rule p_l. An argument analogous to that used in preceding Lemmas shows that $e_1 \leq e_2$, which in turn implies that $\gamma_1 \leq \gamma_2$. ■