

Pareto-Improving Education Subsidies with Financial Frictions and Heterogenous Ability

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Abstract

Previous literature has shown that missing credit or insurance markets do not necessarily provide an efficiency-based argument for educational subsidies, starting with a laissez faire competitive equilibrium. We show that they do, provided learning abilities vary idiosyncratically. In an OLG economy with ability heterogeneity and missing financial markets, we prove: (a) every competitive equilibrium is interim Pareto dominated (and ex post Pareto dominated with sufficient parental altruism) by a policy providing education subsidies financed by income taxes, and (b) transfers conditional on educational investments similarly dominate unconditional transfers. The policies also result in macroeconomic improvements (higher per capita income and upward mobility)

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1 Introduction

A central issue in discussions of a welfare state concerns its normative rationale, when incomplete financial markets restrict the ability of households to finance education of their children, or insure against idiosyncratic risks. Arguments for provision of social insurance by the government have frequently been justified on the basis of notions of ex ante efficiency in an ‘original’ position prior to the realization of idiosyncratic ability or income shocks. Since such social insurance involves redistributive income or wealth taxes, such a consensus no longer holds once ability and income shocks have been realized. However, we typically observe relatively less disagreement across income classes or political parties concerning the need for public education subsidies, particularly for children from poor families.

Existing models of human capital accumulation in environments with financial frictions however do not provide a clear rationale for interim or ex post efficiency of educational subsidies. Mookherjee and Ray (2003) show the existence of fully (i.e., first-best) efficient equilibrium steady states in an OLG occupational choice model without ability heterogeneity or income risk, where parents have a Barro-Becker bequest motive, and cannot borrow to finance their children’s education. With a continuum of occupations, their model has a unique steady state which turns out to be fully efficient. In a two period model with uninsurable income risk but no borrowing constraints or ability heterogeneity, Jacobs, Schindler and Yang (2012) show that the ex ante efficient optimal subsidy on education is zero, starting from a laissez faire competitive equilibrium.

In this paper we show there is a robust Pareto-based argument for educational subsidies in the presence of ability heterogeneity, relative to a laissez faire competitive equilibrium of an OLG economy with missing credit and insurance markets. Specifically, we show that there exists a policy of educational subsidies financed by income taxes which generates an interim Pareto improvement at the stage where parents have not yet learnt the ability realization of their children. If parents exhibit sufficient altruism, such a policy is also ex post Pareto improving (i.e., even after children abilities have been observed). These policy interventions induce macroeconomic improvements, i.e., higher per capita education,

income, upward mobility and lower inter-occupation wage differences. Moreover, we show that any progressive fiscal policy not involving educational subsidies is similarly dominated by a policy involving educational subsidies. Hence transfers conditioned on educational enrollment of children dominate unconditional transfers, and educational subsidies are a necessary feature of any fiscal policy which maximizes a utilitarian welfare function, given ability heterogeneity and financial frictions.

The basic logic is illustrated most simply in a model with two occupations, skilled and unskilled, where education is needed to enter the skilled occupation, and a child's learning ability is represented (inversely) by the cost parents must incur to educate the child. This is drawn independently from an identical distribution with support over the nonnegative reals. Parents cannot borrow to finance the child's education. They decide whether or not to educate their child after observing the latter's ability. Wages in each occupation in any given generation depend on the relative supply of skilled and unskilled adults, i.e., on the aggregate of educational decisions made by parents of the previous generation. In the base model, parents are assumed to supply labor inelastically, not bear any income risk, have perfect foresight over future wages, exhibit Barro-Becker paternalistic altruism towards their children, and be unable to transfer wealth via financial bequests. Starting with an initial distribution of skills, a competitive equilibrium is characterized by a threshold ability above which parents in any given occupation and generation decide to invest in their children's education. Those that invest end up consuming less than those in the same generation-occupation pair who do not invest.

Starting with a laissez faire equilibrium, the government can offer a small subsidy to any parent that educates its child, which is financed by a tax paid by all parents in the same generation and occupation. This effectively redistributes consumption within a given occupation-generation pair from parents that do not invest in education, to parents that invest. This smooths parental consumption with respect to ability shocks of their children, besides lowering the ability threshold for education. *Ceteris paribus* this raises interim utility of parents in the given occupation-generation pair, as well as the proportion of the population in the next generation that are educated. In other words, education incentives and

insurance of parental consumption against ability risk happen to increase together. This forms the primary source of the efficiency improvement.

What about effects on educational investments and utilities in other generations? Anticipation of this intervention will affect educational incentives in previous generations. We show the latter can be neutralized at zero first order cost by a set of corresponding interventions for every occupation-generation pair. For instance, if the primary intervention involved educational transfers for unskilled parents in generation t , a similar intervention for skilled parents in generation t would generate an increase in interim utility of skilled parents of the same magnitude for unskilled parents at generation t , leaving the difference in utilities between skilled and unskilled parents unchanged. Hence educational incentives in preceding generations would be unaffected. At the same time interim utilities and educational incentives of skilled parents would also increase.

Moreover, the intervention will generate ripple effects for succeeding generations, by increasing the supply of skilled agents at $t+1$. Owing to general equilibrium effects, this would lower skilled wages at $t+1$, which leave skilled agents at $t+1$ worse off. We show these can be neutralized by occupation-specific transfers that leave post-tax wages unchanged from $t+1$ onwards, at zero first order budgetary cost. This would involve regressive income taxes which would offset the reduction in wage differences between the skilled and unskilled occupations. The eventual fiscal intervention overlays all these different interventions together, in a way that raises interim utilities of parents in every occupation-generation pair. *Ex post* parents who do not invest in their children's education end up cross-subsidizing the consumption of those who do. With sufficient altruism, the costs of this are outweighed by the gain in expected utilities of their descendants, resulting in an *ex post* Pareto improvement.

A similar argument works for any tax-distorted competitive equilibrium which does not involve educational subsidies, provided the initial taxes are progressive (which is required to ensure that an increase in the supply of skilled agents does not increase fiscal deficits). Note that the argument does not require the government to borrow or lend across generations; nor does it require it to observe the ability realizations of children in the population. We

subsequently show the argument is robust to incorporation of endogenous labor supply, paternalistic altruism, and an arbitrary number of occupations.

However, the results do not generalize quite as straightforwardly when human capital investments can be supplemented by financial bequests. In particular, they do not apply to households wealthy (and altruistic) enough that they *always* make financial bequests, irrespective of how much they invest in education. Within such a wealth class, those who do not invest in education end up spending more on their children overall, and thus consume less than parents who do invest in education. This reverses the pattern of consumption variation with respect to the realization of children's ability risk – the educational subsidy policy described above would now impose additional consumption risk, and thereby create a welfare loss. Laissez faire competitive equilibria continue to be constrained Pareto inefficient, however. The nature of a Pareto improving policy is now reversed: requiring education for the wealthy (as defined above) to be taxed, and these taxes to fund income transfers to the same class. The nature of the Pareto improving policy is unchanged for poor households who invest if at all only in education and leave no financial bequests. The aggregate macro-economic effects of such a Pareto improving policy are unclear, as the increased educational investments among the poor will be countered by falling investments among the wealthy.

This paper relates to a wide literature spanning macroeconomics, public finance, occupational choice and development economics on the policy implications of financial frictions. We present a detailed review of related literature in Section 5. We argue there that the distinguishing characteristic of this paper is a qualitative point which is simple but has not yet been made in this literature: ability heterogeneity implies that educational (or more generally investment) subsidies are interim Pareto improving under a wide range of circumstances.

The remainder of the paper is organized as follows. Section 2 introduces the baseline model, followed by the main results for this model in Section 3. Extensions are discussed in Section 4. The relation to existing literature is described in Section 5, and Section 6 concludes.

2 Baseline Model

We first describe the dynamic economy in the absence of any government intervention.

There are two occupations: unskilled and skilled (denoted 0 and 1 respectively). There is a continuum of households indexed by $i \in [0, 1]$. Generations are denoted $t = 0, 1, 2, \dots$. Each household has one adult and one child in each generation. The utility of the adult in household i in generation t is denoted $V_{it} = u(c_{it}) + \delta V_{i,t+1}$ where c_{it} denotes consumption in household i in generation t , $\delta \in (0, 1)$ is a discount factor and measure of the intensity of parental altruism, and u is a strictly increasing, strictly concave and C^2 function defined on the real line. There is no lower bound to consumption, while u tends to $-\infty$ as c tends to $-\infty$.

Household i earns y_{it} in generation t , and divides this between consumption at t and investment in child education. Education investment I_{it} is indivisible, either 1 or 0. An educated adult has the option of working in either occupation, while an uneducated adult can only work in the unskilled occupation. The ability of the child in household i is represented by how little its parent needs to spend in order to educate it. The cost of education x_{it} in household i in generation t is drawn randomly and independently according to a common distribution function F defined on the nonnegative reals. F is C^2 and strictly increasing; its density is denoted f . The household budget constraint is $y_{it} = c_{it} + x_{it}I_{it}$. Every parent privately observes the realization of education cost of its child before deciding on whether to invest in education.²

The key market incompleteness is that parents cannot borrow to finance their children's education. Neither can they insure against the risk that their child has low learning ability, the main source of (exogenous) heterogeneity in the model. The former arises owing to inability of parents to borrow against their children's future earnings. The latter could be

²Findings would not change if we assumed that parents receive a noisy signal \hat{x}_{it} of true education costs provided that the signal's precision is non-decreasing in parental status and u exhibits non-increasing absolute risk aversion.

due to privacy of information amongst parents regarding the realization of their children's ability.

Household earnings are defined by occupational wages: $y_{it} = w_{0t} + I_{i,t-1} \cdot (w_{1t} - w_{0t})$, where w_{ct} denotes the wage in occupation c in generation t obtaining in a competitive labor market.

Wages are determined as follows at any given date (so we suppress the t subscript for the time being). There is a CRS production function $G(\lambda, 1 - \lambda)$ which determines the per capita output in the economy in any generation t if the proportion of the economy that works in the skilled and unskilled occupations equal λ and $1 - \lambda$ respectively. We assume G is a C^2 , strictly increasing, linearly homogenous and strictly concave function. Let $g_c(\lambda)$ denote the marginal product of occupation $c = 0, 1$ workers when λ proportion of adults work in the skilled occupation. So g_1 is decreasing and g_0 is an increasing function. Moreover, $g_1(0) > g_0(0)$ while $g_1(1) < g_0(1)$. To avoid some technical complications we assume the functions g_i are bounded over $[0, 1]$. In other words, the marginal product of each occupation is bounded above even as its proportion in the economy becomes vanishingly small.³

Let $\bar{\lambda}$ denote the smallest value of λ at which $g_1(\lambda) = g_0(\lambda)$. Then in any given generation t , all educated workers will prefer to work in the skilled occupation, with $w_{1t} = g_1(\lambda_t)$ and $w_{0t} = g_0(\lambda_t)$, if the proportion of educated adults is $\lambda_t < \bar{\lambda}$. And if $\lambda_t \geq \bar{\lambda}$, equilibrium in the labor market at t will imply that exactly $\bar{\lambda}$ fraction of adults will work in the skilled occupation, as educated workers will be indifferent between the two occupations, and $w_{1t} = w_{0t} = g_1(\bar{\lambda}) = g_0(\bar{\lambda})$. See Figure 1. When more than $\bar{\lambda}$ fraction of adults in the

³When the production function satisfies Inada conditions, i.e., marginal products are unbounded, we obtain the same results if every household is able to resort to a subsistence self-employment earnings level \underline{w} which is positive and exogenous. As the proportion of unskilled workers tends to one, the labor market will clear at an unskilled wage equal to \underline{w} , and the proportion of skilled households working for others will be fixed at a level where the marginal product of the unskilled equals this wage. The only difference is that wages in either occupation as a function of the skill ratio become kinked at the point where the marginal product of the unskilled equals \underline{w} . Except at this single skill ratio, the wage functions are smooth, and our results continue to apply with an 'almost everywhere' proviso.

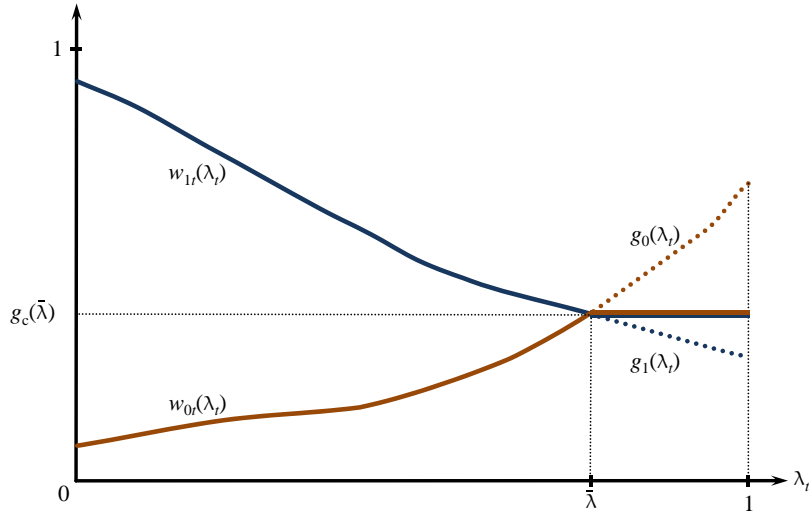


Figure 1: Labor Market

economy are educated, the returns to education are zero. Since education is costly, education incentives vanish if households anticipate that more than $\bar{\lambda}$ proportion of adults in the next generation will be educated. Hence the proportion of educated adults will always be less than $\bar{\lambda}$ in any equilibrium with perfect foresight. We can identify the occupation of each household i in generation t with its education status $I_{i,t-1}$, and refer to λ_t as the skill ratio in the economy in generation t .

2.1 Dynamic Competitive Equilibrium under Laissez Faire

Definition 1 *Given a skill ratio $\lambda_0 \in (0, \bar{\lambda})$ in generation 0, a dynamic competitive equilibrium under laissez faire (DCELF) is a sequence $\{\lambda_t\}_{t=0,1,2,\dots}$ of skill ratios and investment strategies $\{I_{ct}(x)\}_{t=0,1,2,\dots}$ for every household in occupation c in generation t when its child's education cost happens to be x such that:*

(a) *For each household and each t : $I_{ct}(x) \in \{0, 1\}$ maximizes*

$$u(w_{ct} - I_{ct}x) + \delta E_{\tilde{x}} V_{t+1}(I_{ct}, \tilde{x}) \quad (1)$$

and the resulting value is $V_t(c, x)$.

(b)

$$\lambda_t = \lambda_{t-1} \mathbf{E}_x[I_{1t}(x)] + (1 - \lambda_{t-1}) \mathbf{E}_x[I_{0t}(x)]. \quad (2)$$

(c) Every household correctly anticipates $w_{ct} = g_c(\lambda_t)$ for occupation $c = 0, 1$ in generation t .

It is useful to note the following features of a DCELF.

Lemma 1 *In any DCELF and at any date t :*

(i) $V_t(1, x) > V_t(0, x)$ for all x if and only if $\lambda_t < \bar{\lambda}$.

(ii) $\lambda_t < \bar{\lambda}$, $w_{1t} > w_{0t}$.

(iii) $I_{ct}(x) = 1$ iff $x < x_{ct}$, where threshold x_{ct} is defined by

$$u(g_c(\lambda_t)) - u(g_c(\lambda_t) - x_{ct}) = \delta[W_{1,t+1} - W_{0,t+1}] \quad (3)$$

and $W_{ct} \equiv \mathbf{E}_x V_t(c, x)$

(iv) The investment thresholds satisfy $x_{0t} < x_{1t}$, are uniformly bounded away from 0, and uniformly bounded above, while λ_t is uniformly bounded away from 0 and $\bar{\lambda}$ respectively. Consumptions of all agents are uniformly bounded.

This Lemma shows that skilled wages always exceed unskilled wages, and those agents in skilled occupations always have higher utility. There is inequality of educational opportunity: children born to skilled parents are more likely to be educated. There is also upward and downward mobility: some talented children born to unskilled parents receive an education, while some untalented children born to skilled parents fail to receive an education. Finally, equilibrium consumptions and utility differences are bounded, which will be useful in our subsequent analysis.

2.2 Competitive Equilibrium with Taxes

We now extend the model to incorporate fiscal policies. The government observes the occupation/income of parents as well as the education decisions they make for their children. Transfers can accordingly be conditioned on these. Fiscal policy is represented by four variables in any generation t : income transfers τ_{1t}, τ_{0t} based on parental occupation, and transfers e_{1t}, e_{0t} based additionally on the parent's education investment decision. In particular, the government does not observe directly nor indirectly the ability realization of any given child.⁴ This is the key informational constraint that prevents attainment of a first-best utilitarian optimum. We are also focusing on transfers that depend only on the current status of the household, thus ruling out educational loans and schemes which condition on a family's transfer or decision history. Similar to private agents, the government will also not be able to lend or borrow across generations, and will hence have to balance its budget within each generation.

Definition 2 *Given a skill ratio $\lambda_0 \in (0, \bar{\lambda})$ in generation 0, a dynamic competitive equilibrium (DCE) given fiscal policy $\{\tau_{0t}, \tau_{1t}, e_{0t}, e_{1t}\}_{t=0,1,2,\dots}$ is a sequence $\{\lambda_t\}_{t=0,1,2,\dots}$ of skill ratios and investment strategies $\{I_{ct}(x)\}_{t=0,1,2,\dots}$ for every household in occupation c in generation t when its child's education cost happens to be x such that for each c, t :*

(a) $I_{ct}(x) \in \{0, 1\}$ maximizes

$$u(w_{ct} + \tau_{ct} - I_{ct} \cdot (x - e_{ct})) + \delta \mathbf{E}_{\tilde{x}} V_{t+1}(I_{ct}, \tilde{x}) \quad (4)$$

and the resulting value is $V_t(c, x)$.

(b)

$$\lambda_t = \lambda_{t-1} \mathbf{E}_x I_{1t}(x) + (1 - \lambda_{t-1}) \mathbf{E}_x I_{0t}(x). \quad (5)$$

⁴Indirect observability of children's abilities from the parental education expenses or test results would allow policy to realize efficiency gains from explicit improvements in the talent composition of investors. We think of education costs x as having a major unverifiable component, possibly also reflecting parental time that would be dedicated to a child's education and training in a more sophisticated model of labor supply.

(c) Every household correctly anticipates $w_{ct} = g_c(\lambda_t)$ for occupation $c = 0, 1$ in generation t .

The government has a balanced budget if at every t it is the case that

$$\lambda_t \{ \tau_{1t} + e_{1t} \mathbf{E}[I_t(1, x)] \} + (1 - \lambda_t) \{ \tau_{0t} + e_{0t} \mathbf{E}[I_t(0, x)] \} \leq 0. \quad (6)$$

A DCELF with a (trivially) balanced budget obtains as a special case of a DCE when the government selects zero income transfers and educational subsidies.

It is easy to check that a DCE can also be described by investment thresholds x_{ct} satisfying the following conditions. Define the interim expected utility of consumption of a parent in occupation c in generation t as follows:

$$\mathcal{U}_{ct} \equiv u(w_{ct} + \tau_{ct}) [1 - F(x_{ct})] + \int_0^{x_{ct}} u(w_{ct} + \tau_{ct} + e_{ct} - x) dF(x) \quad (7)$$

The thresholds must then satisfy

$$u(w_{ct} + \tau_{ct}) - u(w_{ct} + \tau_{ct} + e_{ct} - x_{ct}) = \delta \cdot \Delta W_{t+1} \quad (8)$$

where

$$\Delta W_t \equiv W_{1,t} - W_{0,t} = \sum_{k=0}^{\infty} \nu_k [\mathcal{U}_{1,t+k} - \mathcal{U}_{0,t+k}] \quad (9)$$

with $\nu_0 = 1$, $\nu_k = \delta^k \prod_{l=0}^{k-1} [F(x_{1,t+l}) - F(x_{0,t+l})]$ for $k \geq 1$. A DCE is then described by a sequence $\{\lambda_t, w_{1t}, w_{0t}, x_{1t}, x_{0t}, \mathcal{U}_{1t}, \mathcal{U}_{0t}\}_{t=0,1,2,\dots}$ which satisfies equalities (5) and (7)–(9).

3 Results for the Baseline Model

Our first result is an efficiency as well as a macroeconomic role for fiscal policy. The efficiency criterion is *interim Pareto dominance*, which requires parental expected utility $W_{ct} \equiv \mathbf{E}_x V_t(c, x)$ to be higher for every c, t . The criterion of *macroeconomic dominance* is that the skill ratio λ_t must be higher at every t , and the investment threshold x_{ct} must be

higher for every c, t . This ensures higher per capita skill and output at every date, as well as greater educational opportunity in the sense of a higher probability for every child to become educated (both conditional on parent's occupation, and unconditionally).

Theorem 1 *Consider any DCELF starting from an arbitrary skill ratio $\lambda_0 \in (0, \bar{\lambda})$ at $t = 0$. There exists a balanced budget fiscal policy with educational subsidies for each occupation funded by income taxes, and an associated DCE which interim Pareto as well as macroeconomically dominates the original DCELF.*

Our second result is that any fiscal policy involving income transfers alone is dominated in a similar way by a policy with educational subsidies.

Theorem 2 *Consider any DCE given an initial skill ratio $\lambda_0 \in (0, \bar{\lambda})$ and a balanced budget fiscal policy consisting of income transfers alone ($e_{ct} = 0$ for all c, t), satisfying the following conditions:*

- (a) $\tau_{0t} \geq \tau_{1t}$ for all t ;
- (b) there exists $\kappa > 0$ such that $-\tau_{1t} + \tau_{0t} < [g_1(0) - g_0(0)] - \kappa$ for all t ;
- (c) τ_{ct} is uniformly bounded.

Then there exists another balanced budget fiscal policy consisting of income transfers combined with educational subsidies ($e_{ct} > 0$ for all c, t) and an associated DCE which interim Pareto as well as macroeconomically dominates the original DCE.

Condition (a) of Theorem 2 requires the income transfers to be progressive in the weak sense that unskilled parents receive a higher transfer (or pay a lower tax), while (b) restricts the marginal tax rate to be less than (and bounded away from) 100%. Condition (c) is a technical restriction needed to ensure that competitive equilibria always involve bounded

consumptions and investment thresholds. The role of (b) is to ensure that skilled households earn more both before and after government transfers, so agents always have investment incentives (that are bounded away from zero). Condition (a) ensures that the direct effect of any reduction in the proportion of unskilled households is to weaken the government budget balance constraint. These conditions imply that equilibria with fiscal policy continue to satisfy the same properties as equilibria under *laissez faire* that were shown in Lemma 1.

It is evident that Theorem 1 is a special case of Theorem 2. The latter result can be generalized further to situations in which the status quo policy includes educational taxes. In this case we need to impose the additional condition that these educational taxes are not so large that they destroy investment incentives (i.e., give rise to $x_{ct} = 0$ for some t and $c = 0, 1$). Specifically, the same dominance result can be established when the status quo policy and DCE satisfy the following conditions:

- (a) the fiscal policy is *progressive* in the sense that $\tau_{0t} + F(x_{0t})e_{0t} \geq \tau_{1t} + F(x_{1t})e_{1t}$ for all t ;
- (b) there exists $\kappa > 0$ such that $-\tau_{1t} - \tau_{0t} < [g_1(0) - g_0(0)] - \kappa$ for all t ;
- (c) τ_{ct}, e_{ct} are uniformly bounded;
- (d) for some c : $e_{ct} \leq 0$ for all t ,⁵ while $\sup_{c,t}\{-e_{ct}\}$ is small enough to bound x_{ct} uniformly away from zero.

In all these versions, the new policy provides educational subsidies to a given occupation which are funded by income taxes levied on the same occupation. The proof constructs a small increase $\epsilon(1 - \mu_t)$ in education subsidy for occupation c which is financed by an increase $\epsilon F(x_{ct})$ in income taxes on this occupation.

The key observation is that parents' consumptions vary with the realizations of their children's abilities. The nature of this variation is shown in Figure 2. If the child is a genius

⁵We can also relax the requirement 'for all t ' to 'for some t '.

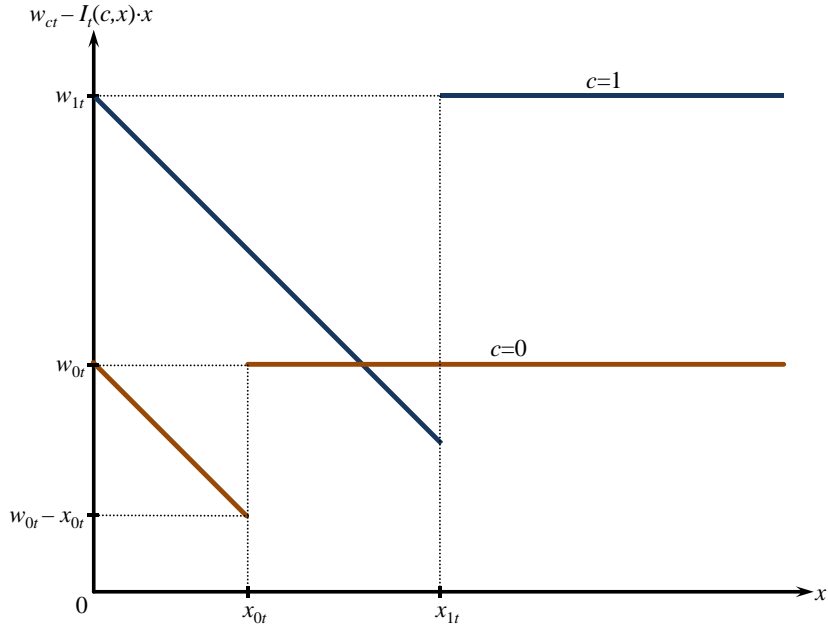


Figure 2: Variation of Parental Consumption with Education Cost

and can be costlessly educated, the parent's consumption equals his earning. The same is true when the child has low enough ability that it is not educated. For intermediate abilities where the child is educated, the parent invests a positive amount, lowering consumption. Hence parental consumption varies non-monotonically with respect to the cost necessary to educate the child.

The educational subsidy increase $\epsilon(1 - \mu_t)$ raises the consumption of the investors, while financing it by income taxes on the same occupation lowers the consumption of the non-investors.⁶ See Figure 3. If μ_t were zero, average consumption would be unaffected but differences in consumption associated with heterogeneity of the children's education costs would be reduced (assuming that education is not subsidized to start with and hence non-investing households consume more than every household in the same occupation that

⁶Lowered consumption utility of non-investors may – despite a universal increase in the dynastic utility component – preclude the policy from achieving also an *ex post* Pareto improvement. By restricting tax-funded education subsidies to the minority occupation defined by $\lambda_t \geq \frac{1}{2}$, a government could prevent any political commitment problems which might derive from having too many ex post losers.

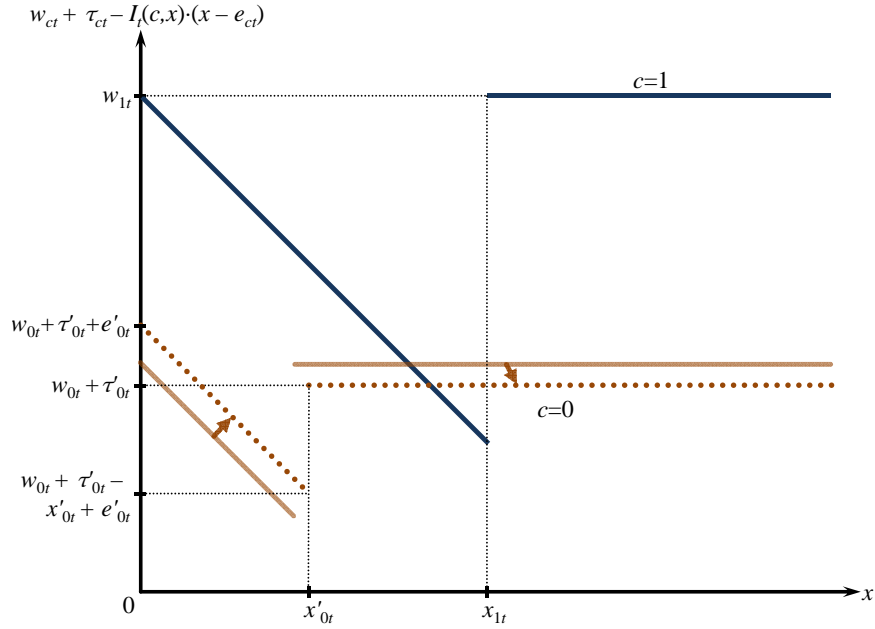


Figure 3: Effects of Steps 1 and 2 of Fiscal Policy Variation on Parental Consumption

does invest). This would result in a mean-preserving reduction of the variation in parental consumption, thus raising the interim expected utility of current consumption in occupation c .

The parameter μ_t , however, is set so as to reduce the mean consumption enough that there is no change in the expected utility of current consumption at date t for each occupation. Assuming wages are unchanged, this implies that dynastic utilities of both occupations are unchanged. Hence the future benefit of investment is unchanged. The subsidization of education in occupation c on the other hand lowers the sacrifice parents must endure to educate their children. Hence households invest more often.

Aggregate investment in the economy will then rise, which will tend to lower skilled wages and raise unskilled wages. These general equilibrium changes would reduce the benefits of investment and therefore fiscal policy is adjusted further to neutralize the wage changes. This results in a new competitive equilibrium sequence with a higher skill ratio at every date, and a zero first order effect on interim utilities. However, the government has a first

order improvement in its surplus, owing to the rise in the skill ratio and the extraction of resources from households by setting $\mu_t > 0$. The progressivity of the original fiscal policy implies that the government budget surplus also improves as a result of the decline in the proportion of unskilled households.

In the last step of the argument the government constructs another variation in its tax-subsidy policy. It distributes the additional revenues so as to achieve a strict interim Pareto improvement, while preserving investment incentives. Note that by construction the dispersion in utility of consumption between occupations is unchanged, while a fraction of agents move up from the unskilled to the skilled occupation in every generation.

The consumption losses which the policy imposes on non-investing parents stay bounded while the dynastic gains which are created for all parents grow without bound as $\delta \rightarrow 1$. So with a sufficiently high degree of parental altruism, parameterized by δ , the policy-induced gains in expected utility of descendants outweigh any loss in own consumption relative to laissez faire. The constructed policy then achieves an ex post Pareto improvement. Formally, we can show:

Theorem 3 *Let a collection of economies with identical consumption utility function u , production function G and ability distribution F but different parental discount factors $\delta \in (0, 1)$ be given. For each corresponding DCELF that starts from skill ratio $\lambda_0 \in (0, \bar{\lambda})$ at $t = 0$, consider fiscal policies $\{\tilde{\tau}_{0t}^\delta(\epsilon), \tilde{\tau}_{1t}^\delta(\epsilon), \tilde{e}_{0t}^\delta(\epsilon), \tilde{e}_{1t}^\delta(\epsilon)\}_{t=0,1,2,\dots}$ which induce an interim Pareto improvement according to Theorem 1. Then there exist $\underline{\delta} \in (0, 1)$ and $\bar{\epsilon} > 0$ such that for any $\epsilon \in (0, \bar{\epsilon})$ and $\delta \in (\underline{\delta}, 1)$ the fiscal policy also ex post Pareto dominates the respective DCELF for all t .*

4 Extensions

4.1 Endogenous Labor Supply

A first extension of the baseline model could be to consider households who choose how many hours of labor they supply, together with the binary decision whether to invest in education or not. That is, each household in occupation c selects $I_{ct}(x) \in \{0, 1\}$ and $l_{ct}(x) \geq 0$ to maximize

$$u(l_{ct} w_{ct} - I_{ct} x) - d(l_{ct}) + \delta \mathbf{E}_{\tilde{x}} V_{t+1}(I_{ct}, \tilde{x}) \quad (10)$$

for strictly increasing and convex disutility of labor d . The optimal investment strategy $I_{ct}(x)$ in this case is of the same threshold form as in the baseline model. Namely, if we define

$$v(w_{ct}, x, I_{ct}) \equiv \max_{l_{ct}} \left[u(l_{ct} w_{ct} - I_{ct} x) - d(l_{ct}) \right] \quad (11)$$

then a parent in occupation c in period t who faces education cost x will invest iff $x < x_{ct}$, where threshold x_{ct} is defined by

$$v(w_{ct}, x_{ct}, 0) - v(w_{ct}, x_{ct}, 1) = \delta [W_{1,t+1} - W_{0,t+1}] \quad (12)$$

and $W_{ct} \equiv \mathbf{E}_{\tilde{x}} V_t(c, \tilde{x})$. Parents in occupation c with cost $x = 0$ or cost $x \geq x_{ct}$ have identical (indirect) utilities of consumption $v(w_{ct}, 0, 1) = v(w_{ct}, x, 0)$, while those with cost $x \in (0, x_{ct})$ consume less. In particular, from (11) and the envelope theorem, we have

$$\frac{\partial v(w_{ct}, x, I_{ct}(x))}{\partial x} = -u'(l_{ct}(x)w_{ct} - x) < 0 \text{ for each } x \in (0, x_{ct}). \quad (13)$$

It follows that consumption utilities $v(w_{ct}, x, I_{ct}(x))$ are decreasing on $[0, x_{ct})$, jump back to $v(w_{ct}, 0, 1)$, and then stay at this level. That is, they exhibit a non-monotonic pattern with respect to education cost x just like in the baseline model (cf. Figure 2). A variation of the baseline policy intervention can therefore be applied in order to raise interim utility.

4.2 Paternalistic Altruism

Next suppose parents do not have Barro-Becker dynastic preferences. Instead, they value (only) the earnings of their children according to a given increasing function $Y(w_{t+1})$, as in Becker and Tomes (1979) or Mookherjee and Napel (2007) – perhaps incorporating parental concern for their own old age security. A parent in occupation $c \in \{0, 1\}$ at date t with a child who costs x to educate then selects $I \in \{0, 1\}$ to maximize $u(w_{ct} - Ix) + IY(w_{1,t+1}) + (1 - I)Y(w_{0,t+1})$. Theorems 1 and 2 continue to extend with this formulation of parental altruism. The wage neutralization policy preserves after-tax wages in each occupation, whence the altruistic benefit of investments remain unchanged. The costs of investing are lowered by providing educational subsidies, and at the same time the variation of parental consumption is lowered. So investment incentives continue to rise, while enhancing interim expected utilities.

4.3 Continuous Investment Choices

What if educational investments can be varied continuously, rather than being indivisible? Our results extend straightforwardly to this context, too, as we now explain.

Let the extent of education be described by a compact interval $E \equiv [0, \bar{e}]$ of the real line. Assume that the relation between wage earnings and education is given by a real-valued continuous function $w(e)$ defined on E . If the earnings function depends endogenously on the supply of workers with varying levels of education, the analysis can be extended using a similar strategy of following up on educational subsidy policies that increase the supply of more educated workers with a wage-neutralization policy that leaves the after-tax remuneration pattern unchanged. To illustrate how our results extend, it therefore suffices to take the earnings–education pattern in the status quo equilibrium as given.

Let $I(e'; x)$ denote the expenditure that must be incurred by a parent to procure education $e' \geq 0$ for its child whose learning ability gives rise to a learning cost parameter x .

The latter varies according to a continuous distribution with full support on $[0, \infty)$, similar to the preceding section. The function I is strictly increasing and differentiable in both arguments. It satisfies $I(0; x) = 0$ for all x , while for any given $e' \geq 0$ the marginal cost $\frac{\partial I}{\partial e'}$ is increasing in x , approaching ∞ as $x \rightarrow \infty$.

The value function of a parent with education e and a child whose learning cost parameter is x is then

$$V(e|x) \equiv \max_{0 \leq e' \leq \bar{e}} [u(w(e) - I(e'; x)) + \delta W(e')] \quad (14)$$

where $W(e') \equiv \mathbb{E}_{\tilde{x}} V(e'|\tilde{x})$. Let the corresponding policy function be $e'(e; x)$. Given that wages are bounded above by $w(\bar{e})$, consumptions are also bounded above. Given this and the feature that u is unbounded below, consumptions can be bounded from below almost surely.⁷ Hence the marginal utility of consumption is bounded almost surely, implying that $W'(0) \equiv \mathbb{E}_{\tilde{x}} [u'(w(0) - I(e'(0; \tilde{x})))]$ is bounded.

We can therefore define $x^*(e)$ as the solution for x in the equation $\frac{\partial I(e'; x)}{\partial e'} \Big|_{e'=0} = \frac{\delta W'(0)}{u'(w(e))}$. Then the optimal policy function takes the form $e'(e; x) = 0$ if $x \geq x^*(e)$ and positive otherwise.⁸ In other words, parents decide to acquire no education for their children if and only if their learning cost parameter is larger than a threshold $x^*(e)$. These ‘non-investors’ consume their entire earnings $w(e)$ – just like those parents with the same education e whose children have a learning cost parameter of $x = 0$. For those whose children have intermediate learning ability, parents spend a positive amount on education.

We thus have a similar non-monotone pattern of variation of parental consumption with their children’s learning costs as in the two-occupation case. This ensures that a similar policy of educational subsidies funded by income taxes on all parents with the same education will reduce the riskiness of parental consumption, and thereby permit a Pareto improvement.

⁷Any policy where consumption approaches $-\infty$ with positive probability will be dominated by a policy where parents never invest.

⁸This follows since the value function is concave, owing to a direct argument.

The essential argument is thus simple. Non-investing parents within any given occupation will by definition consume more than investing parents. The educational subsidy funded by the income tax on this occupation then redistributes consumption away from those consuming high amounts to those consuming less. Since these consumption variations arise from the ‘ability lottery’ of their children, the policy increases interim expected utilities of each occupation. The preceding analytical details were needed to ensure that there is a positive mass of investors and non-investors respectively, so as to allow a strict Pareto improvement.

4.4 Financial Bequests

There is however one important assumption underlying the above reasoning: that educational investments constitute the sole means by which parents transfer wealth to their children. In practice parents have other means as well, such as leaving them financial bequests or physical assets. The simple logic then breaks down: a parent that does not invest in his child’s education owing to low learning ability of the latter could provide financial bequests instead. It no longer follows that education non-investors invest less when we aggregate across different forms of intergenerational transfers.

We now consider the consequences of allowing parents to leave financial bequests besides investing in their children’s education. To simplify matters, suppose that the rate of return $(1 + r)$ on financial bequests is exogenously given, as in Becker and Tomes (1979) or Mookherjee and Ray (2010). This could correspond to a globalized capital market where the savings of any given country leave the interest rate unaffected. Even if the interest rate depends on the supply of savings, a ‘neutralization’ policy allows policy-makers to ensure that the after-tax interest rate is unchanged. For the same reason we here abstract from general equilibrium effects in the labor market and suppose that wages of different occupations are exogenously given.

Let us further simplify to the case of two occupations, skilled and unskilled, where the

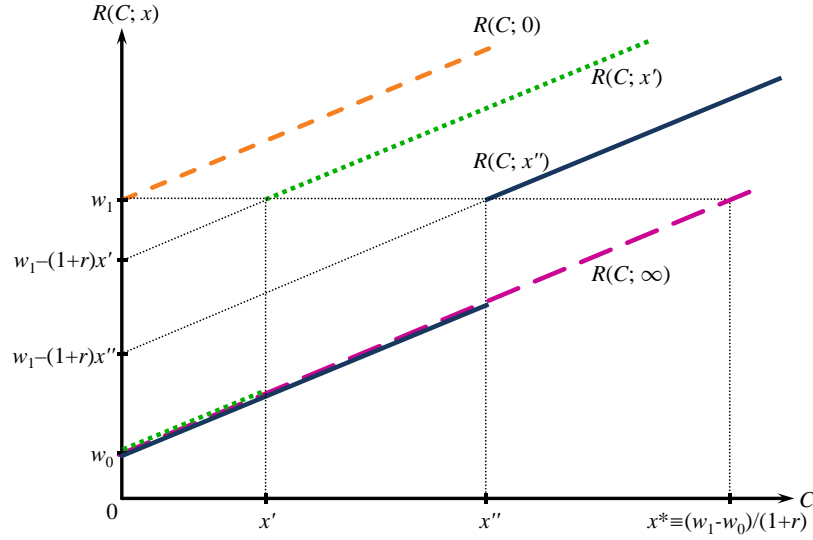


Figure 4: Child wealth as function of total investment expenditure C , given cost x

education cost of the former is denoted x and the latter equals zero. And suppose that parental altruism is paternalistic, where a parent with lifetime wealth W and education cost x chooses financial bequest $b \geq 0$ and education investment $I \in \{0, 1\}$ to maximize $u(W - b - Ix) + \delta Y(W')$ where Y is a strictly increasing and strictly concave function of the child's future wealth $W' = (1 + r)b + Iw_1 + (1 - I)w_0$.

This problem can be reformulated as follows. Let $C \equiv b + Ix$ denote the total parental investment expenditure on his child. An efficient way to allocate C across financial bequest and educational expenses is the following: $I = 0$ if either $C < x$, or $C \geq x$ and the rate of return on education is dominated by the return on financial assets: $\frac{w_1 - w_0}{x} < 1 + r$. Conversely, if the rate of return on education exceeds r and $C \geq x$, then $I = 1$, and $b = C - x$. Then the child ends up with wealth $W' \equiv R(C; x)$ given by

$$R(C; x) = \begin{cases} (1 + r)C + w_0 & \text{if } C < x, \text{ or } C \geq x \text{ and } \frac{w_1 - w_0}{x} \leq 1 + r, \\ (1 + r)C + w_1 - (1 + r)x & \text{if } C > x \text{ and } \frac{w_1 - w_0}{x} > 1 + r. \end{cases} \quad (15)$$

It is illustrated in Figure 4.

Define the *BT (Becker-Tomes) bequest* as the optimal bequest of a parent in the absence

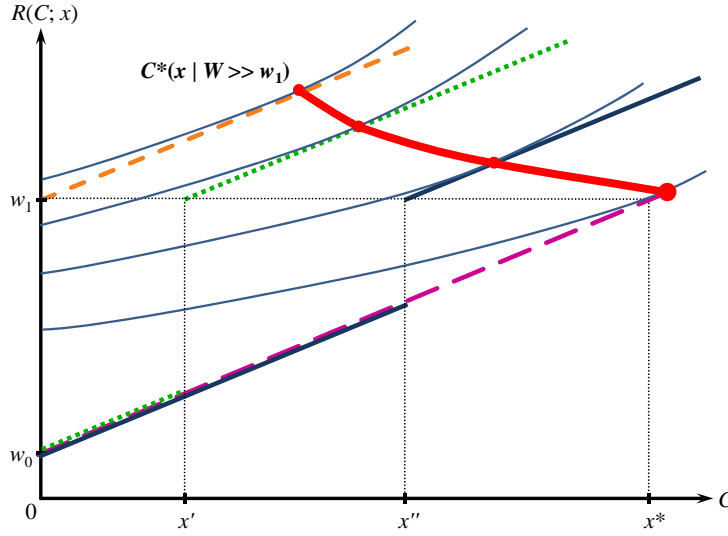


Figure 5: Investment expenditures of sufficiently wealthy parents (case A)

of any opportunity to invest in education, with a given flow earning w of the child when the parent leaves a zero bequest. This is the problem of choosing $C \geq 0$ to maximize $u(W - C) + \delta Y((1 + r)C + w)$. Denote the BT bequest by $C^{BT}(W; w)$. It is easily checked that this is increasing in parental wealth W and decreasing in w .

Recall that a parent will invest in education only if the child has enough ability to ensure that $x \leq x^* \equiv \frac{w_1 - w_0}{1 + r}$. Whenever $x > x^*$, there will be no investment in education, and the optimal bequest equals the BT bequest $C^{BT}(W; w_0)$. When $x < x^*$, the optimization problem entails a nonconvexity and the solution is more complicated. The dotted and solid lines in Figure 4, for instance, respectively represent the nonconvex sets of feasible (C, W') -combinations for parents with children whose education costs x' and x'' lie below x^* .

Nevertheless we can illustrate the solution for some extreme cases, corresponding to different parental wealths.

Case A. W sufficiently large: Suppose W is large enough that $C^{BT}(W; w_1 - (1 + r)x) > x$

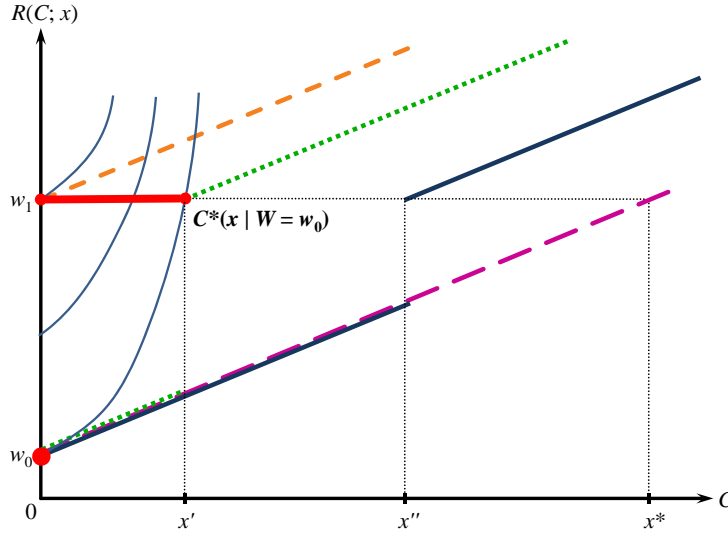


Figure 6: Investment expenditures of poor parents (case B)

for all $x \leq x^*$.⁹ In words, irrespective of where x lies below x^* , the parent will always supplement education investments with a financial bequest. See Figure 5.

Case B. *W sufficiently small*: Suppose $W = w_0$, $\delta(1+r) \leq 1$ and $Y \equiv u$. Then the BT bequest $C^{BT}(w_0; w) = 0$ for all $w \geq w_0$, and the parent will never make a financial bequest. If however the child learning cost x is sufficiently small, the parent will invest in education. The optimal choice of expenditure C^* is illustrated in Figure 6, where the low parental wealth is reflected by steep indifference curves.

The implied consumption patterns of sufficiently wealthy and poor households are illustrated in Figure 7. For parents with very small wealth W , investment decisions are exactly as in our simple model without any financial bequests, and ‘non-investors’ consume more than the ‘investors’. The situation is very different, however, for sufficiently wealthy parents. Their parental consumption (conditional on wealth W) is strictly decreasing in x over $x \in [0, x^*]$, and constant thereafter. The ‘non-investors’ (those with $x > x^*$) now consume less than the ‘investors’, opposite to the pattern in the model without any financial bequests.

⁹A sufficient condition for this is $C^{BT}(W; w_1) > x^*$.

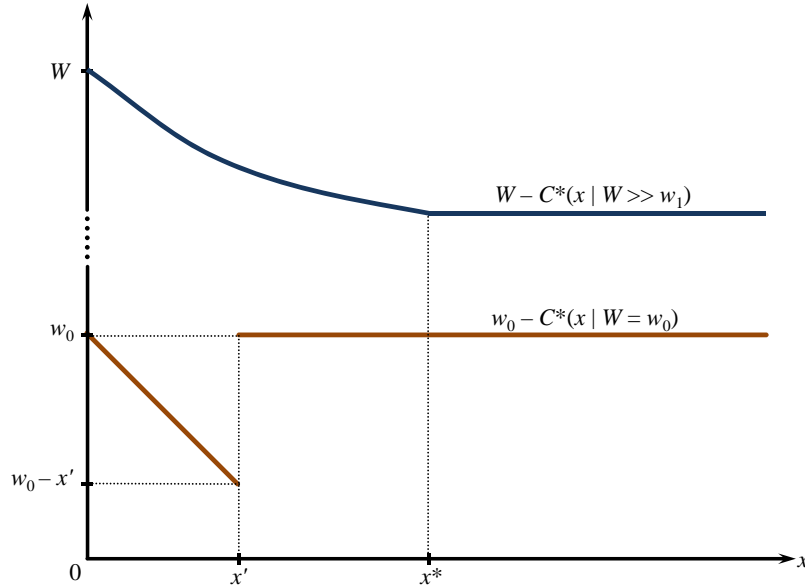


Figure 7: Consumption of sufficiently wealthy and poor parents

The argument that educational subsidies (financed by income or wealth taxes) lower consumption risk no longer applies to wealthy households falling under case A. They would instead raise risk. *So an opposite result holds here: an educational tax for parents with wealths falling in case A which funded a wealth subsidy (or income tax break) on the same set of households would reduce risk.* Starting with laissez faire, such a policy would be Pareto improving. It would, however, have opposite macroeconomic effects, as educational investments among such parents would fall. The resulting decline in skilled agents implies that the result about superiority of conditional transfers may not apply if the status quo policy is progressive, as this would worsen the government's fiscal balance.

On the other hand, our previous arguments would continue to apply for poor households in case B, who never make any financial bequests, and behave exactly as described in previous sections. For such poor households, therefore, our previous results remain unchanged: educational subsidies funded by income taxes would be Pareto improving as well as generate macro improvements.

For other classes of households, whether parents make financial bequests typically de-

depends on the child's ability: they are made when the child is of sufficiently high ability, as well as when ability is low. For intermediate abilities, they make no financial bequests and make educational investments alone. The comparison of consumptions across 'investors' and 'non-investors' can go either way depending on the child's ability.

This suggests that arguments for educational subsidies should be limited to household wealth classes which make little or no financial bequests. The exact range of such households is an empirical matter. In the model of Abbott, Gallipoli, Meghir and Violante (2013) calibrated to fit the NLSY 1997 data, all parents in the bottom quartile of the wealth distribution make inter-vivos transfers (inclusive of imputed value of rent when children lived with parents) to their children (when the latter were between ages of 16–22) which were smaller than what the latter spent on educational tuitions. The same was true for most of the second quartile as well. On the other hand, many parents in the top quartile transferred more than education tuition costs, and this happened to be true for *all* parents in the top 5%. This suggests case A applies to the top 5% of the US population, while case B applies to the bottom third of the population.

Indeed, our results suggest that it may be optimal for the government to use mixed policies of the following form: educational taxes for the population in case A, and subsidies for those in case B. The effects on educational investments in these two classes could then offset each other, leaving aggregate education investments unaltered. The composition of the educated would however change: since marginal children in case B are likely to be of higher ability than those in case A, there would be a rise in the average returns to education which would augment the efficiency benefits from the risk effects. We conjecture that it is generally possible to construct such mixed policies which Pareto dominate a policy consisting of income-based transfers alone.

5 Relation to Literature

Sinn (1995, 1996) and Varian (1980) evaluate incentive and insurance effects of social insurance provided by a progressive fiscal policy in a setting with ex ante representative households and missing credit and/or insurance markets. Typically, ex ante efficiency entails an interventionist fiscal policy which trades off incentive and insurance effects. At the interim or ex post stage, however, unanimous agreement is generally unlikely. Agents with positive income shocks who are required to subsidize others with negative shocks will prefer *laissez faire* to the interventions, with the opposite true for those with negative shocks. In contrast, the cross-subsidization in our context occurs across parents with the same ex post income across different ability realizations of their children, generating agreement at the interim stage (and also ex post, with sufficient altruism). Our paper thus helps explain the substantially larger consensus typically observed across classes and political parties regarding the desirability of government subsidization of elementary schooling, compared with fiscal policies that redistribute from rich to poor. Moreover, we show that education subsidies can be designed to offset any possible redistribution or adverse incentive effects.

Subsequent literature in public economics has examined implications of redistributive tax distortions for education subsidies.¹⁰ Bovenberg and Jacobs (2005) argue redistributive taxes and education subsidies are ‘Siamese twins’: the latter are needed to counter the effects of the former in dulling educational incentives. Alternatively, the presence of progressive income taxes implies a ‘fiscal externality’ associated with education which enables agents to earn higher incomes and thereby pay higher taxes. Educational subsidies are needed to ensure agents internalize this externality. They consider a static model without any borrowing constraints or income risk. Here the argument for educational subsidies arises from tax distortions, and disappears if the status quo involves *laissez faire*, unlike our

¹⁰A large part of the dynamic public finance literature (e.g., Golosov, Tsyvinski and Werning (2006), or Golosov, Troshkin and Tsyvinski (2015)) is unrelated, as it abstracts from human capital investments and assumes that skills follow an exogenous Markov process. The focus of most of this literature is to extend the Mirrlees (1971) optimal income tax model to a dynamic setting and examine consequences for optimal taxation of labor and savings.

paper. Jacobs, Schindler and Yang (2012) show the same result obtains when the model is extended to a context with uninsurable income risk. None of these models incorporate ability heterogeneity and missing credit markets, which create an efficiency role for educational subsidies in our model, even in the absence of any progressive income taxes.

Dynamic models of investment in physical and/or human capital which incorporate missing credit and insurance markets and agent heterogeneity have been studied in the literature on macroeconomics and fiscal policy (Loury (1981), Aiyagari (1994), Aiyagari, Greenwood and Sheshadri (2002), Bénabou (1996, 2002)). Loury (1981) provided a pioneering analysis of human capital investments by altruistic parents in an environment with ability shocks and no financial markets. Most of his analysis concerned the characterization of dynamic properties of competitive equilibria. He showed that redistributive policies could raise aggregate output and welfare, but did not explore the efficiency properties of laissez faire equilibria. Aiyagari, Greenwood and Sheshadri (2002) study a model of human capital investment where education entails fixed and variable resource costs, besides child care. Education takes the form of increasing efficiency units of homogenous labor acquired by the child, as a function of the child's ability realization, parental resource and child care expenses. Apart from incorporating child care, the model is more general than Bénabou's or ours by incorporating physical capital and financial bequests. But the main focus of their paper is different: to characterize first-best Pareto efficient allocations which can be decentralized with complete markets, and contrast these to laissez faire allocations that result when there are no credit or insurance markets. They do not consider the effects of fiscal policy.

Efficiency properties of competitive equilibria with endogenous human capital and missing financial markets are studied in Bénabou (1996). His model is more general than ours regarding parental labor supply decisions. On the other hand, attention is restricted to particular functional forms for utility and production, and specific distributions are assumed for ability and productivity shocks. These are realized after investment choices are made, which removes the key heterogeneity in our model: all agents with a given income level take identical investment decisions and have identical consumptions. In this case there is

no point in redistributing within the same income/occupational class. However, given that incomes follow an ergodic process in equilibrium, individual investments are complements in production and utility is strictly concave in consumption, every dynasty prefers a positive amount of inter-occupational redistribution from a long-term perspective. In the short run, tax-funded education subsidies reduce consumption for rich dynasties but the greater the discount factor, the more agents find that the long-term gains dominate. So analogous to our result concerning ex post Pareto improving policies that redistribute within occupations, Bénabou finds that collective financing of education becomes a Pareto improvement in a sufficiently patient society.¹¹ The main contrast with our paper is that we focus on redistribution among parents in the same occupation who have children with different abilities, which achieves interim Pareto improvements without any constraints on intensity of parental altruism.

Versions of these models have been calibrated to fit data of real economies in order to evaluate the welfare and macroeconomic effects of various fiscal policies in numerical simulations (Heathcote (2005), Bohacek and Kapicka (2008), Cespedes (2014), Berriel and Zilberman (2011), Abbott, Gallipoli, Meghir and Violante (2013), Findeisen and Sachs (2013, 2014)). These studies rely on specific functional forms for technology and preferences, and focus on aggregate measures of welfare. Apart from the need to understand the source of these welfare effects (e.g., evaluating attendant insurance effects), these papers leave open the question whether there may exist other policies which could have resulted in a Pareto improvement, or what the effects might be in economies with different preferences and technology. Our paper complements this literature by providing purely qualitative results concerning efficient fiscal policies which apply irrespective of the specific welfare function, technology or preferences.

¹¹Bénabou (2002) specializes the production side of his earlier model while considering aggregate efficiency properties of a richer set of redistribution schemes. In both models, the income distribution is log-normal in any period, i.e., it has unbounded support. Proposition 4 in Bénabou (1996) hence establishes a Pareto improvement asymptotically (for $\delta \rightarrow 1$), while sufficient patience could be identified with $\delta \geq \bar{\delta}$ for some $\bar{\delta} < 1$ in models with bounded incomes like ours.

Our model is related to the occupational choice literature (Banerjee and Newman (1993), Galor and Zeira (1993), Ljungqvist (1993), Freeman (1996), Aghion and Bolton (1997), Bandyopadhyay (1997), Lloyd-Ellis and Bernhardt (2000), Matsuyama (2000, 2006), Ghatak and Jiang (2002), Mookherjee and Ray (2002, 2003, 2008, 2010), Mookherjee and Napel (2007)), but this literature mostly does not examine efficiency properties of competitive equilibria. The only exception is Mookherjee and Ray (2003) who abstract from ability heterogeneity, and find *laissez faire* steady state allocations that are efficient. Our analysis reveals how ability heterogeneity has a dramatic effect on the efficiency properties of equilibria. Mookherjee and Napel (2007) study a closely related model with ability heterogeneity but do not analyze welfare properties of equilibria.

D’Amato and Mookherjee (2013) investigate efficiency properties of equilibria in another closely related model with ability heterogeneity, where the labor market is additionally characterized by signaling (i.e., productivity depends on ability in addition to education). They examine effects of educational loans provided by the government, funded by bonds released to the public. They obtain a result similar to our first result, viz. competitive equilibria are Pareto dominated by such a loan program. This intervention works differently than ours by changing the composition of the educated in favor of children from low-income families who have higher abilities than children from high income families. Per capita education and output in the economy are unchanged. Such interventions require parents to take loans on behalf of their children, which possibly involve a number of legal, moral and enforcement problems that do not arise with the interventions studied in the current paper.

Our paper also contributes to debates concerning the design of anti-poverty programs (e.g., Mookherjee (2006), Mookherjee and Ray (2008), Ghatak (2014)): whether transfers to poor households should be uniform/cash/unconditional rather than in kind/conditional on investments in human capital of children. While there are general arguments based on the Pareto criterion in favor of the former in static contexts – as in the Mirrlees (1971) or Atkinson-Stiglitz (1976) models – this no longer applies in dynamic settings when effects on investments need to be incorporated. Our results provide a general argument for superiority

of conditional transfers in such settings.

6 Concluding Observations

We have provided theoretical arguments for Pareto-superiority of fiscal policies involving educational subsidies funded by income taxes imposed on the same income/occupational class. These dominate laissez faire outcomes, as well as policies where transfers are not conditioned on education decisions. The results apply quite generally, provided parents do not supplement education investments with financial bequests. In the presence of financial bequests, laissez faire outcomes continue to be Pareto dominated by similar policies applied only to poor households that do not leave financial bequests. For wealthy household classes that always leave financial bequests, Pareto optimality requires an opposite policy involving educational taxes or fees which fund unconditional transfers within the same class.

The main contribution of the paper is to establish results on the constrained inefficiency of laissez faire equilibria in economies with incomplete financial markets and idiosyncratic abilities that depend little on detailed assumptions concerning preferences or technology, or on the nature of social preferences for redistribution. They provide suggestions for policies based only on the Pareto criterion, that would generate no distributional conflict and create rather than destroy incentives to invest. These results also help provide insights into the source of estimated welfare effects of educational subsidy policies in calibrated macro models.

The investigated welfare state is a rather minimal one. Some cross-occupational redistribution is involved, and is critical for achieving a Pareto improvement. But the major component of the proposed intervention operates at an only intra-occupational level: the indirect provision of within-group education financing, which lowers the existing variation of parental consumption. The same non-monotonic pattern of consumption utility in a given income/occupational class has been demonstrated to arise in several extensions of the baseline model. Further generalizations are desirable but left for future research. For instance,

a child's future wage income and financial inheritance could be subject to random shocks. Analysis of the corresponding extension of the scenario considered in Section 4.4 would be complicated by gains from diversifying the risky payoffs to financial vs. educational investment. For the very poor households who do not leave financial bequests, the identified pattern of parental consumption however should prevail, suggesting that a scheme along the indicated lines could still raise interim welfare.

The budget surplus which arises from income taxes exceeding education subsidies in a given educational class is rebated in the constructed intervention so as to neutralize general equilibrium wage effects and to raise individual consumption for all families, independently of their current occupation. Private agents would have no interest to do so. But could not the members of, say, the unskilled occupation – or profit-maximizing companies – still organize a similar kind of scheme? Why is public intervention needed? Mutual aid and benefit societies, fraternal lodges, trade unions and guilds have historically provided many private insurance services that have been taken over – and to some extent crowded out – by the welfare state (see Beito (2000)). Such societies usually have better social monitoring and enforcement possibilities than commercial companies. Still, collective education financing at more than a very localized scale seems to have been the exception. The key problem in taking the considered scheme private presumably is adverse selection: parents have an opportunistic interest in subscribing to a social or corporate scheme ex post, after gaining private information about their children's ability, rather than before the resolution of talent uncertainty. Fiscal policy can circumvent this parental commitment problem. The government is also likely to face less severe information asymmetries regarding private inheritances and wealth, which matter when different types of bequest give rise to different ability–consumption patterns also within an occupation category.

Appendix

Proof of Lemma 1: Part (i) follows from the fact that $w_{1t} > w_{0t}$ if and only if $\lambda_t < \bar{\lambda}$, and $V_t(1, x) > V_t(0, x)$ for any x if and only if $w_{1t} > w_{0t}$. If (ii) is false and $\lambda_t \geq \bar{\lambda}$ at some date, we have $V_t(1, x) = V_t(0, x)$ for all x , implying that no parent with a child with $x > 0$ will want to invest in education at $t - 1$, so $\lambda_t = 0 < \bar{\lambda}$ – a contradiction.

For (iii) note that (3) follows straightforwardly from the optimization problem faced by parents. And $x_{0t} < x_{1t}$ follows from (ii) above. To show the next claim in (iv), suppose it is not true. Then we can find a subsequence $\{x_{c,t_n}\}_{n=1,2,\dots}$ along which x_{c,t_n} for some occupation c either tends to 0 or ∞ . In the former case, (3) implies $[W_{1,t_n+1} - W_{0,t_n+1}]$ must converge to 0, which in turn requires λ_{t_n+1} to converge to $\bar{\lambda}$. Then x_{d,t_n} must tend to 0 for *both* occupations $d = 0, 1$, and (2) implies λ_{t_n+1} converges to 0 – a contradiction. In the latter case $[W_{1,t_n+1} - W_{0,t_n+1}]$ must converge to ∞ , implying x_{d,t_n} must tend to ∞ for both occupations $d = 0, 1$ by virtue of (3). Equation (2) then implies λ_{t_n+1} approaches 1. This contradicts (ii) above. Since $\lambda_t \geq F(x_{0t})$ (owing to (2) and $x_{1t} > x_{0t}$), it follows that λ_t is uniformly bounded away from 0. Moreover, the argument which ruled out that sequence $\{x_{ct}\}_{t=1,2,\dots}$ has a cluster point at 0 also ensures λ_t is bounded away from $\bar{\lambda}$. The bounds on consumption follow from the bounds on wages and on investment thresholds. ■

Proof of Theorems 1 and 2

A useful preliminary result shows that any government budget surplus can be disposed of in an ex post Pareto improving manner while leaving investment incentives unchanged.

Lemma 2 *Given any sequence of positive budgetary surpluses $\{R_t\}_{t=0,1,\dots}$ resulting from a fiscal policy $\{\tau_{ct}, e_{ct}\}_{c;t}$ and an associated DCE $\{\lambda_t, w_{ct}, x_{ct}, \mathcal{U}_{ct}\}_{c;t}$, there exists another fiscal policy $\{\tau'_{ct}, e'_{ct}\}_{c;t}$ with $\tau'_{ct} > \tau_{ct}, e'_{ct} > e_{ct}$ for all $c = 0, 1$ and $t = 0, 1, \dots$ with an associated DCE with the same skill ratios, wages and thresholds $\{\lambda_t, w_{ct}, x_{ct}\}_{c;t}$ which ex post Pareto dominates the original DCE, i.e., with $\mathcal{U}'_{ct} > \mathcal{U}_{ct}$ for all c, t .*

Proof of Lemma 2: Let the original DCE involve wages $\{w_{ct}\}_{t=0,1,2,\dots}$ and investment thresholds $\{x_{ct}\}_{t=0,1,2,\dots}$ in occupation c . For any period t and positive budgetary amount $R_{ct} \leq R_t$ to be disposed of to households in occupation c in t , select $\Delta\tau_{ct}(R_{ct}) \geq 0, \Delta e_{ct}(R_{ct}) \geq 0$ as defined by the unique solution to:

$$\begin{aligned} R_{ct} &= \alpha_{ct}[\Delta\tau_{ct} + F(x_{ct})\Delta e_{ct}] \\ u(w_{ct} + \tau_{ct}) - u(w_{ct} + \tau_{ct} + e_{ct} - x_{ct}) &= u(w_{ct} + \tau_{ct} + \Delta\tau_{ct}) \\ &\quad - u(w_{ct} + \tau_{ct} + \Delta\tau_{ct} + e_{ct} + \Delta e_{ct} - x_{ct}) \end{aligned} \quad (16)$$

where α_{ct} equals λ_t if $c = 1$ and $1 - \lambda_t$ otherwise. This results in a change in interim consumption utility of a household in occupation c in period t by

$$\begin{aligned} \Delta\mathcal{U}_{ct}(R_{ct}) &= [u(w_{ct} + \tau_{ct} + \Delta\tau_{ct}) - u(w_{ct} + \tau_{ct})](1 - F(x_{ct})) \\ &\quad + \int_0^{x_{ct}} \{u(w_{ct} + \tau_{ct} + \Delta\tau_{ct} + e_{ct} + \Delta e_{ct} - x) - u(w_{ct} + \tau_{ct} + e_{ct} - x)\} dF(x) \end{aligned}$$

provided the investment threshold remains x_{ct} .

$\Delta\tau_{ct}(R_{ct}), \Delta e_{ct}(R_{ct})$ and $\Delta\mathcal{U}_{ct}(R_{ct})$ are continuous, strictly increasing functions, taking the value 0 at $R_{ct} = 0$. By the Intermediate Value Theorem, for any $R_t > 0$ there exist R_{0t} and R_{1t} such that $R_{0t} + R_{1t} = R_t$ and $\Delta\mathcal{U}_{1t}(R_{1t}) = \Delta\mathcal{U}_{0t}(R_{0t})$. This ensures that $\mathcal{U}_{1t} - \mathcal{U}_{0t}$ is unchanged.

Because the definition of $\Delta\tau_{ct}$ and Δe_{ct} in (16) keeps investment sacrifices constant for threshold types x_{1t}, x_{0t} , the same investment strategies remain optimal for households in period t if they expect an unchanged welfare difference $W_{1,t+1} - W_{0,t+1}$. The sequence $\{W_{1t} - W_{0t}\}_{t=0,1,2,\dots}$ remains unchanged given that there is no change to the sequence of consumption utility differences $\{\mathcal{U}_{1t} - \mathcal{U}_{0t}\}_{t=0,1,2,\dots}$. The policy is constructed precisely to assure this, where preservation of the original investment thresholds also preserves skill ratios $\{\lambda_t\}_{t=1,2,\dots}$ and associated pre-tax wages $\{w_{1t}, w_{0t}\}_{t=1,2,\dots}$. The government budget is then balanced, while transfers to all households have increased. \blacksquare

The proof of Theorem 2 itself proceeds in five steps. We here prove the generalization described in the text following the statement of the result, where conditions (a)–(d) are satisfied by the status quo. Theorem 1 is a special case of Theorem 2.

Step 1: Conditions (a)–(d) imply that the status quo fiscal policy and DCE satisfy the following properties:

- (i) there exists $\bar{\lambda}_t \in (0, 1)$ such that $\lambda_t \in (0, \bar{\lambda}_t)$ for all t , and λ_t is uniformly bounded away from 0;
- (ii) x_{ct} is uniformly bounded above, and uniformly bounded away from zero;
- (iii) consumptions of all agents are uniformly bounded.

To see this, define $\bar{\lambda}_t$ by the property that $g_1(\bar{\lambda}_t) - g_0(\bar{\lambda}_t) = \tau_{1t} - \tau_{0t}$. By virtue of condition (b), $\bar{\lambda}_t \in (0, 1)$ for all t , and is uniformly bounded away from 0. Note that $\lambda_t \geq \bar{\lambda}_t$ implies that after-tax wages are equalized across the two occupations, since skilled agents can always work in the unskilled occupation. Then $W_{1,t} = W_{0,t}$, implying that parents at $t-1$ have no incentive to educate their children. This implies $\lambda_t = 0$ – a contradiction. Hence equilibrium always involves $\lambda_t \in (0, \bar{\lambda}_t)$ for all t . Since $[g_1(\lambda_t) + \tau_{1t}] - [g_0(\lambda_t) + \tau_{0t}] > \kappa > 0$ for all t , $[W_{1t} - W_{0t}]$ is bounded away from 0 for all t . Since the distribution of x has full support over $(0, \infty)$, the proportion of parents at $t-1$ investing in their children’s education is bounded away from zero in each occupation. Hence λ_t is uniformly bounded away from 0, which establishes (i). It also follows that x_{ct} is uniformly bounded above, which combined with condition (d) implies property (ii). Finally, (iii) follows from (ii) combined with condition (c).

Step 2: For arbitrary $\epsilon > 0$, construct the following policy change. Denote the status quo DCE by an * superscript. Fix an occupation c , and leave the fiscal policy $\{\tau_{dt}, e_{dt}\}_{t=0,1,2,\dots}$ for occupation $d \neq c$ unchanged. Take $\tau'_{ct}(\epsilon) = \tau_{ct} - \epsilon F(x_{ct}^*)$, $e'_{ct}(\epsilon) = e_{ct} + \epsilon(1 - \mu_t)$ where

$$\mu_t \equiv (1 - F(x_{ct}^*)) \left[1 - \frac{F(x_{ct}^*) u'(w_{ct}^* + \tau_{ct})}{\int_0^{x_{ct}^*} u'(w_{ct}^* + \tau_{ct} + e_{ct} - x) dF(x)} \right]. \quad (17)$$

It is evident that $\mu_t \in (0, 1)$ for all t . By Step 1 and the concavity of u , it is uniformly bounded away from 0 and 1 respectively.

For arbitrary thresholds x_{ct}, x_{dt} define

$$\begin{aligned} \mathcal{U}_{ct}(x_{ct}, \epsilon) &\equiv u(w_{ct}^* + \tau_{ct} - \epsilon F(x_{ct}^*)) [1 - F(x_{ct})] \\ &\quad + \int_0^{x_{ct}} u(w_{ct}^* + \tau_{ct} - \epsilon F(x_{ct}^*) + e_{ct} + \epsilon(1 - \mu_t) - x) dF(x) \end{aligned} \quad (18)$$

$$\mathcal{U}_{dt}(x_{dt}, \epsilon) \equiv u(w_{dt}^* + \tau_{dt}) [1 - F(x_{dt})] + \int_0^{x_{dt}} u(w_{dt}^* + \tau_{dt} + e_{dt} - x) dF(x) \quad (19)$$

By construction of μ_t we have

$$\frac{\partial \mathcal{U}_{ct}(x_{ct}^*, 0)}{\partial \epsilon} = 0 = \frac{\partial \mathcal{U}_{dt}(x_{dt}^*, 0)}{\partial \epsilon}. \quad (20)$$

Considering bounded but otherwise arbitrary sequences of investment thresholds $\{x_{ct}\}_{t=0,1,2,\dots}$ and $\{x_{dt}\}_{t=0,1,2,\dots}$, define

$$\Delta W_t(\epsilon) \equiv \sum_{k=0}^{\infty} \nu_k \cdot [\mathcal{U}_{1,t+k}(x_{1,t+k}, \epsilon) - \mathcal{U}_{0,t+k}(x_{0,t+k}, \epsilon)]. \quad (21)$$

The associated interim welfare of a parent in the unskilled occupation at t is

$$W_{0t}(\epsilon) = \sum_{k=0}^{\infty} \delta^k \mathcal{U}_{0t}(x_{0,t+k}, \epsilon) + \delta \sum_{k=0}^{\infty} \delta^k F(x_{0,t+k}) \Delta W_{t+1+k}(\epsilon) \quad (22)$$

and that of a parent in the skilled occupation at t is

$$W_{1t}(\epsilon) = W_{0t}(\epsilon) + \Delta W_t(\epsilon). \quad (23)$$

These are the correct expressions for the value functions corresponding to the specified investment thresholds, *assuming that the policy change leaves after-tax wages for each occupation unchanged at every t* . Throughout Step 2, we shall continue to assume this.

The series in (21)–(22) converge uniformly in a non-empty interval J around $\epsilon = 0$.¹² Moreover, for any given bounded sequences $\{x_{ct}\}_{t=0,1,2,\dots}$ and $\{x_{dt}\}_{t=0,1,2,\dots}$, the partial sums

$$\sum_{k=0}^T \delta^k \frac{\partial \mathcal{U}_{ot}(x_{ot}, \epsilon)}{\partial \epsilon}$$

¹²Convergence is uniform because $\delta < 1$, $\nu_k < \delta^k$, and $|\mathcal{U}_{ot}(x_{ot}, \epsilon)|$ is bounded uniformly on $J = [-\xi, \xi]$, $\xi > 0$, by, e.g., $\kappa_o = \max_{\epsilon \in J} \{|u(g_1(\bar{\lambda}) + \tau_{ot})|, |u(\tau_{ot} + e_{ot} + \epsilon - \sup_t \{x_{1t}^*, x_{0t}^*\})|\}$ for $o = 0, 1$.

converge uniformly on J because $\left| \frac{\partial \mathcal{U}_{ot}(x_{ot}, \epsilon)}{\partial \epsilon} \right|$ is uniformly bounded on J given that u is C^2 on \mathbb{R} and wages and thresholds are uniformly bounded. We can therefore exchange the order of summation and differentiation when evaluating the welfare effects of a small change of ϵ . Moreover, applying the Envelope Theorem at each t , we can neglect induced changes in the investment thresholds at $\epsilon = 0$. So one obtains

$$\frac{\partial \Delta W_t(0)}{\partial \epsilon} = \sum_{k=0}^{\infty} \nu_k \left[\frac{\partial \mathcal{U}_{1,t+k}(x_{1t}^*, 0)}{\partial \epsilon} - \frac{\partial \mathcal{U}_{0,t+k}(x_{0t}^*, 0)}{\partial \epsilon} \right] = 0 \quad (24)$$

from (20); and combining (22)–(24) allows us to conclude that

$$\frac{\partial W_{0t}(0)}{\partial \epsilon} = 0 = \frac{\partial W_{1t}(0)}{\partial \epsilon}. \quad (25)$$

Since u is C^2 on \mathbb{R} , the second derivatives of $\mathcal{U}_{ot}(x_{ot}, \epsilon)$ are also uniformly bounded on J . Hence we can also exchange the order of summation and differentiation when considering the derivative of $\partial \Delta W_t(\epsilon)/\partial \epsilon$, and conclude that $\Delta W_t(\epsilon)$ is C^1 because $\partial^2 \Delta W_t(\epsilon)/\partial \epsilon^2$ exists.

Optimal investment thresholds $x_{ct}(\epsilon)$ and $x_{dt}(\epsilon)$ are determined by

$$u(w_{ct}^* + \tau_{ct} - \epsilon F(x_{ct}^*)) - u(w_{ct}^* + \tau_{ct} + e_{ct} + \epsilon(1 - \mu_t) - \epsilon F(x_{ct}^*) - x_{ct}(\epsilon)) = \delta \cdot \Delta W_{t+1}(\epsilon) \quad (26)$$

and

$$u(w_{dt}^* + \tau_{dt}) - u(w_{dt}^* + \tau_{dt} + e_{dt} - x_{dt}(\epsilon)) = \delta \cdot \Delta W_{t+1}(\epsilon). \quad (27)$$

Since all involved terms are C^1 functions of ϵ , we can conclude from the Implicit Function Theorem that $x_{ct}(\epsilon)$ and $x_{dt}(\epsilon)$ are C^1 on an interval J around $\epsilon = 0$.

As we vary ϵ from 0, the threshold x_{ct} undergoes a first order increase, while the first order change in x_{dt} is zero. Namely, the derivatives of the right-hand sides of (26) and (27) w.r.t. ϵ at 0 are zero, given that (24) holds for every t . Differentiating (27) directly implies $\frac{\partial x_{dt}(0)}{\partial \epsilon} = 0$. Differentiating (26) yields

$$\frac{\partial x_{ct}(0)}{\partial \epsilon} = F(x_{ct}^*) \frac{u'(w_{ct}^* + \tau_{ct})}{u'(w_{ct}^* + \tau_{ct} + e_{ct} - x_{ct}^*)} + (1 - \mu_t - F(x_{ct}^*)) \quad (28)$$

and the concavity of u implies

$$\int_0^{x_{ct}^*} u'(w_{ct}^* + \tau_{ct} + e_{ct} - x) dF(x) < F(x_{ct}^*) u'(w_{ct}^* + \tau_{ct} + e_{ct} - x_{ct}^*). \quad (29)$$

Hence

$$\mu_t < (1 - F(x_{ct}^*)) \left[1 - \frac{u'(w_{ct}^* + \tau_{ct})}{u'(w_{ct}^* + \tau_{ct} + e_{ct} - x_{ct}^*)} \right], \quad (30)$$

and substituting this into (28) we obtain

$$\frac{\partial x_{ct}(0)}{\partial \epsilon} > \frac{u'(w_{ct}^* + \tau_{ct})}{u'(w_{ct}^* + \tau_{ct} + e_{ct} - x_{ct}^*)} > 0. \quad (31)$$

So assuming that the policy change leaves after-tax wages unchanged for each occupation, it produces a first order increase in investment thresholds for parents in occupation c and a zero first order effect on thresholds for parents in occupation d at every $t \geq 1$. Also it generates a zero first order effect on the dynastic utilities at every $t = 0, 1, 2, \dots$ if after-tax wages are unchanged.¹³

Step 3: In order to ensure that after-tax wages remain at their original levels, we introduce a wage neutralization policy at each t . First, for any $\epsilon \geq 0$ and $t \geq 0$, recursively define the skill ratio that would be induced in period $t + 1$ by the investment thresholds $x_{ct}(\epsilon), x_{dt}(\epsilon)$

$$\lambda_{t+1}(\epsilon) = F(x_{1t}(\epsilon))\lambda_t(\epsilon) + F(x_{0t}(\epsilon))(1 - \lambda_t(\epsilon)) \quad (32)$$

with $\lambda_0(\epsilon) = \lambda_0$ given. Since $x_{1t}(\epsilon), x_{0t}(\epsilon)$ are C^1 functions of ϵ in a neighborhood of 0, so – by induction – must be $\lambda_{t+1}(\epsilon)$ for all t . Also note that (31) combined with $x_{1t}(\epsilon) > x_{0t}(\epsilon)$ at all t implies that $\lambda'_{t+1}(0)$ is positive (and uniformly bounded away from 0).

Now switch to the following modified policy $(\tilde{\tau}_{ct}(\epsilon), \tilde{\tau}_{dt}(\epsilon), \tilde{e}_{ct}(\epsilon), \tilde{e}_{dt}(\epsilon))$ for each period $t \geq 1$

$$\tilde{\tau}_{ct}(\epsilon) = w_{ct}^* - w_{ct}(\epsilon) + \tau'_{ct}(\epsilon) = w_{ct}^* - w_{ct}(\epsilon) + \tau_{ct} - \epsilon F'_{ct} \quad (33)$$

$$\tilde{e}_{ct}(\epsilon) = e'_{ct}(\epsilon) = e_{ct} + \epsilon(1 - \mu_t) \quad (34)$$

$$\tilde{\tau}_{dt}(\epsilon) = w_{dt}^* - w_{dt}(\epsilon) + \tau'_{dt} = w_{dt}^* - w_{dt}(\epsilon) + \tau_{dt} \quad (35)$$

$$\tilde{e}_{dt}(\epsilon) = e'_{dt}(\epsilon) = e_{dt} \quad (36)$$

¹³Interim welfares W_{0t} and W_{1t} condition on the parental occupation. So there is no contradiction between (25) and the observation that some period $t > 0$ parents *are* better off than without the policy, because they enjoy W_{1t} instead of W_{0t} due to the increase of $x_{c,t-1}$.

where $w_{ot}(\epsilon) = g_o(\lambda_t(\epsilon))$, $o = c, d$.

This modified policy induces a DCE with skill ratios $\{\lambda_t(\epsilon)\}_{t=1,2,\dots}$, investment thresholds $\{x_{ct}(\epsilon), x_{dt}(\epsilon)\}_{t=0,1,2,\dots}$ and the interim utilities $\{\mathcal{U}_{ct}(x_{ct}(\epsilon), \epsilon), \mathcal{U}_{dt}(x_{dt}(\epsilon), \epsilon)\}_{t=0,1,2,\dots}$ which were constructed in Step 2 under the assumption of unchanged after-tax wages in each occupation at each date. Given investment thresholds $x_{ct}(\epsilon), x_{dt}(\epsilon)$ the resulting skill ratio is $\lambda_{t+1}(\epsilon)$ and hence pre-tax wages are $g_c(\lambda_{t+1}(\epsilon)), g_d(\lambda_{t+1}(\epsilon))$. The transfers defined by (33)–(36), therefore, ensure that the household's optimization problem in each period corresponds to the one under original wages $\{w_{1t}^*, w_{0t}^*\}_{t=0,1,2,\dots}$ and the policy $\{\tau'_c(\epsilon), \tau'_d(\epsilon), e'_c(\epsilon), e'_d(\epsilon)\}_{t=0,1,2,\dots}$.

Step 4: We next check that there is a first order improvement in government revenues at every t . Supposing that $c = 1, d = 0$ (an analogous argument works for the opposite case), the budget surplus for $t \geq 0$ is

$$\begin{aligned}
B_t(\epsilon) &= -\lambda_t(\epsilon)[\tilde{\tau}_{1t}(\epsilon) + F(x_{1t}(\epsilon))\tilde{e}_{1t}(\epsilon)] \\
&\quad - [1 - \lambda_t(\epsilon)][\tilde{\tau}_{0t}(\epsilon) + F(x_{0t}(\epsilon))\tilde{e}_{0t}(\epsilon)] \\
&= -\lambda_t(\epsilon)[w_{1t}^* - w_{1t}(\epsilon) + \tau_{1t} - \epsilon F_{1t}^* + F_{1t}(\epsilon)\{e_{1t} + \epsilon(1 - \mu_t)\}] \\
&\quad - [1 - \lambda_t(\epsilon)][w_{0t}^* - w_{0t}(\epsilon) + \tau_{0t} + F_{0t}(\epsilon)e_{0t}] \\
&= \lambda_t(\epsilon)F_{1t}(\epsilon)\epsilon\mu_t - \lambda_t(\epsilon)\epsilon[F_{1t}(\epsilon) - F_{1t}^*] \\
&\quad - \lambda_t(\epsilon)[w_{1t}^* - w_{1t}(\epsilon) + \tau_{1t} + F_{1t}(\epsilon)e_{1t}] \\
&\quad - [1 - \lambda_t(\epsilon)][w_{0t}^* - w_{0t}(\epsilon) + \tau_{0t} + F_{0t}(\epsilon)e_{0t}]
\end{aligned}$$

where $F_{ct}(\epsilon) \equiv F(x_{ct}(\epsilon))$.

Hence, recalling that $\lambda_t g'_1(\lambda_t) + (1 - \lambda_t) g'_0(\lambda_t) = 0$ for every λ_t owing to the CRS property of the production function, we obtain

$$\begin{aligned}
B'_t(0) &= F_{1t}^* \mu_t \lambda_t^* \\
&\quad - \lambda'_t(0) [\tau_{1t} + F_{1t}^* e_{1t}] + \lambda'_t(0) [\tau_{0t} + F_{0t}^* e_{0t}] \\
&\quad - \lambda_t^* f(x_{1t}^*) x'_{1t}(0) e_{1t} \\
&\geq F_{1t}^* \mu_t \lambda_t^* - \lambda'_t(0) [(\tau_{1t} + F_{1t}^* e_{1t}) - (\tau_{0t} + F_{0t}^* e_{0t})] \\
&\geq F_{1t}^* \mu_t \lambda_t^*
\end{aligned}$$

where the first inequality uses the assumption that $e_{1t} \leq 0$ and the second that the original fiscal policy is progressive in each period t .¹⁴

From Step 1, $F_{1t}^* \mu_t \lambda_t^*$ and hence $B'_t(0)$ are bounded away from 0 uniformly. Hence a sufficiently small $\epsilon > 0$ will generate a positive budget surplus at every date.

Step 5: Finally, apply Lemma 2 in order to dispose of the resulting budget surplus in an interim Pareto-improving way. \square

Proof of Theorem 3

We denote the respective investment thresholds, skill ratios, wages, etc. in the DCELF which is associated with a given discount factor δ by x_{ct}^δ , λ_t^δ , w_{ct}^δ , etc. Recall that the induced skill ratio λ_t^δ must be strictly smaller than $\bar{\lambda}$ for every δ and t (cf. Lemma 1). From this follows that if we fix an arbitrary $\delta' > 0$ there exist \underline{x} and \bar{x} such that $0 < \underline{x} \leq x_{0t}^\delta \leq x_{1t}^\delta \leq \bar{x} < \infty$ for all t and all $\delta \in (\delta', 1)$. To see this, suppose otherwise, i.e., that there exists a sequence $\{t_n, \delta_n\}_{n=1,2,\dots}$ such that (i) $x_{c,t_n}^{\delta_n} \rightarrow 0$ or (ii) $x_{c,t_n}^{\delta_n} \rightarrow \infty$. In case (i), vanishing investment by occupation c in period t_n requires that the benefit of having a skilled child in $t_n + 1$ vanishes. Then no parent in occupation $d \neq c$ would have an incentive to invest in t_n either,

¹⁴Since $\lambda'_0(0) = 0$, the second term in the penultimate line is zero rather than positive for $t = 0$: the skill ratio is unchanged and a budget surplus arises only from $\mu_0 > 0$. At all succeeding dates, the skill ratio rises and generates a higher budget improvement owing to the weak progressivity of the tax system.

implying $\lambda_{t_n+1} \approx 0$. The consequent gap between skilled and unskilled equilibrium wages in period $t_n + 1$ and $\delta_n > \delta' > 0$ would then, however, induce a non-vanishing benefit of one's child to be skilled in $t_n + 1$ – a contradiction. In case (ii), benefits of having a skilled child in $t_n + 1$ would need to grow without bound. This implies that parents in occupation $d \neq c$ will also find it optimal to invest in t_n for arbitrarily large x . But $x_{0,t_n}^\delta, x_{1,t_n}^\delta \rightarrow \infty$ implies $\lambda_{t_n+1}^\delta \rightarrow 1$, in contradiction to $\lambda_t^\delta < \bar{\lambda}$ for all t .

We next establish that there exist $\delta' > 0$ and \underline{b} such that

$$F(x_{1t}^\delta) \lambda_t^\delta \mu_t^\delta \geq \underline{b} > 0 \text{ for all } t \text{ and } \delta \in (\delta', 1) \quad (37)$$

with

$$\mu_t^\delta \equiv (1 - F(x_{ct}^\delta)) \left[1 - \frac{F(x_{ct}^\delta) u'(w_{ct}^\delta)}{\int_0^{x_{ct}^\delta} u'(w_{ct}^\delta - x) dF(x)} \right] \quad (38)$$

for a given $c \in \{0, 1\}$. To see (37) note, first, that $F(x_{1t}^\delta) \geq F(\underline{x}) > 0$ because F is strictly increasing. Second, $\lambda_t^\delta < \bar{\lambda}$ implies $w_{1t}^\delta > w_{0t}^\delta$ and $x_{1t}^\delta > x_{0t}^\delta$. From this follows $\lambda_t^\delta \geq F(x_{0t}^\delta) \geq F(\underline{x}) > 0$. Finally, note that $(1 - F(x_{ct}^\delta)) \geq (1 - F(\bar{x})) > 0$ in (38). Now suppose that

$$\frac{u'(w_{ct_n}^{\delta_n})}{\frac{1}{F(x_{ct_n}^{\delta_n})} \int_0^{x_{ct_n}^{\delta_n}} u'(w_{ct_n}^{\delta_n} - x) dF(x)} \rightarrow 1$$

for some sequence $\{t_n, \delta_n\}_{n=1,2,\dots}$. This would require $x_{ct_n}^{\delta_n} \rightarrow 0$, in contradiction to $x_{ct}^\delta \geq \underline{x} > 0$ for all t and $\delta \in (\delta', 1)$. Hence μ_t^δ is bounded away from zero.

Equation (37) implies that, by choosing $\epsilon \in (0, \bar{\epsilon})$ for a small $\bar{\epsilon} > 0$, a strictly positive budget surplus $B_t^\delta(\epsilon)$ is created for all t and $\delta \in (\delta', 1)$ in the first steps of the proof of Theorems 1 and 2. Since the full policy intervention is budget balancing, it must raise consumption in period t for at least one occupation $d \in \{0, 1\}$ by a non-vanishing amount. This must increase the respective interim consumption utility \mathcal{U}_{dt}^δ at a rate which is bounded away from zero, recalling that every agent's marginal utility of consumption is bounded below by $u'(g_1(0))$. Because the policy is constructed such that the difference $\mathcal{U}_{1t}^\delta(\epsilon) - \mathcal{U}_{0t}^\delta(\epsilon)$ is constant, we can in fact conclude that

$$\left. \frac{\partial \mathcal{U}_{0t}^\delta}{\partial \epsilon} \right|_{\epsilon=0} = \left. \frac{\partial \mathcal{U}_{1t}^\delta}{\partial \epsilon} \right|_{\epsilon=0} \geq \underline{\nu} \quad (39)$$

for some $\underline{\nu} > 0$ and all t and $\delta \in (\delta', 1)$.

The increase in interim consumption utility in (39), of course, does not imply that consumption increases for every cost type x . But every period t parent who invests in education before and after introduction of the policy enjoys a net subsidy, and so experiences an ex post increase in consumption utility (as well as everyone's policy-induced increase in dynastic utility). Parents who did not invest before but switch to investing x in response to the policy reveal that this is optimal for them. So showing that the remaining non-investing parents are rendered better off is sufficient for establishing ex post Pareto dominance.

The rate at which $\epsilon > 0$ decreases these non-investing parents' consumption utility is bounded above. Namely, there exists $L < \infty$ such that

$$\frac{\partial}{\partial \epsilon} \left[u(w_{ct}^\delta) - u(w_{ct}^\delta - \epsilon F(x_{ct}^\delta) + S_{ct}^\delta(\epsilon)) \right] \Big|_{\epsilon=0} = \left(F(x_{ct}^\delta) - \frac{\partial S_{ct}^\delta(0)}{\partial \epsilon} \right) \cdot u'(w_{ct}^\delta) \quad (40)$$

$$\leq F(\bar{x}) \cdot u'(g_0(0)) < L \quad (41)$$

where $S_{ct}^\delta(\epsilon)$ denotes the (increasing) budget surplus which is allocated to occupation c in Step 5 of the proof of Theorems 1 and 2.

In contrast, the rate at which $\epsilon > 0$ increases the non-investing parents' dynastic utility $\delta W_{0,t+1}^\delta$ is unbounded. Namely, for every $M < \infty$ there exists $\underline{\delta} \in (0, 1)$ such that for all $\delta \in (\underline{\delta}, 1)$:

$$\frac{\partial}{\partial \epsilon} \left\{ \delta \left[W_{0,t+1}^\delta(\epsilon) - W_{0,t+1}^\delta \right] \right\} \Big|_{\epsilon=0} > M \quad (42)$$

where $W_{0,t+1}^\delta = W_{0,t+1}^\delta(0)$ refers to interim welfare in the original DCELF. To see this, consider

$$\frac{\partial}{\partial \epsilon} \left[W_{0,t+1}^\delta(\epsilon) - W_{0,t+1}^\delta \right] = \frac{\partial}{\partial \epsilon} \left[\sum_{k=0}^{\infty} \delta^k \mathcal{U}_{0t}^\delta(\epsilon) + \delta \sum_{k=0}^{\infty} \delta^k F(x_{0,t+k}^\delta(\epsilon)) \Delta W_{t+1+k}^\delta(\epsilon) \right] \quad (43)$$

and note that the derivative at $\epsilon = 0$ of the second summand in the brackets is zero (cf. (24) and Lemma 2). The corresponding derivative of the first summand is

$$\sum_{k=0}^{\infty} \delta^k \frac{\partial \mathcal{U}_{0t}^\delta}{\partial \epsilon} \Big|_{\epsilon=0} \geq \sum_{k=0}^{\infty} \delta^k \underline{\nu} = \frac{\underline{\nu}}{1 - \delta}, \quad (44)$$

and so the left-hand side of (42) indeed grows without bound as $\delta \rightarrow 1$.

Combining (40) and (42), we can conclude that the total welfare change of non-investing parents satisfies

$$\frac{\partial}{\partial \epsilon} \left\{ \left[u(w_{ct}^\delta - \epsilon F(x_{ct}^\delta) + S_{ct}^\delta(\epsilon) + \delta W_{0,t+1}^\delta(\epsilon)) \right] - \left[u(w_{ct}^\delta) + \delta W_{0,t+1}^\delta \right] \right\} \Big|_{\epsilon=0} \geq \psi \quad (45)$$

for all t and $\delta \in (\underline{\delta}, 1)$ for some $\psi > 0$. We can therefore choose $\bar{\epsilon} > 0$ such that each individual's ex post welfare change is positive for any $\epsilon \in (0, \bar{\epsilon})$ for every $\delta \in (\underline{\delta}, 1)$. \square

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