# MECHANISM DESIGN WITH COMMUNICATION CONSTRAINTS[1]

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**Abstract**

We consider the problem of designing mechanisms when communication costs prevent agents from revealing their entire private information to others. The principal contracts with multiple agents each supplying a one-dimensional good at a privately known cost. We characterize optimal mechanisms subject to incentive and communicational constraints, without imposing arbitrary restrictions on the number of communication rounds. We show mechanisms which decentralize production decisions are strictly superior to those where these decisions are centralized. Optimal communication mechanisms are designed to maximize direct information exchange among agents. Conditions are provided for agents to release information gradually over multiple rounds simultaneously or sequentially.

KEYWORDS: communication, mechanism design, decentralization, incentives, organizations

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# 1 Introduction

Real world economic organizations differ markedly from the predictions of mechanism design theory. The Revelation Principle (e.g., Myerson (1982)) which plays a central role in existing theory, implies attention can be restricted to one-shot revelation mechanisms in which agents communicate everything they know to a central planner, principal or owner, who subsequently makes all relevant production and allocation decisions. Incentive systems are designed to encourage agents to be truthful and obedient. Most real mechanisms do not involve such extreme centralization of authority and communication systems. Instead, decision-making authority is typically dispersed among agents, who decide their own production or consumption and are incentivized by suitable prices, costs or payments. Agents communicate directly with one another by participating in dynamic, time-consuming protocols involving discussions, reports or negotiations.

In the debate on the economics of socialism, Hayek (1945) argued the infeasibility of communication of dispersed private information by agents in an economy to a central planner was a key reason for the superiority of a decentralized market economy over a socialist economy with centralized decision making:

> "If we can agree that the economic problem of society is mainly one of rapid adaptation to changes in the particular circumstances of time and place, it would seem to follow that the ultimate decisions must be left to the people who are familiar with these circumstances, who know directly of the relevant changes and of the resources immediately available to meet them. We cannot expect that this problem will be solved by first communicating all this knowledge to a central board which, after integrating all knowledge, issues its orders. We must solve it by some form of decentralization." (Hayek (1945, p. 524)

It is however not clear whether Hayek was aware of possible incentive problems associated with decentralization — wherein privately informed agents may use their discretion to pursue their own goals at the expense of the rest of society — and how this may affect the desirability of decentralization.

These issues continue to be relevant to the design of internal organization of firms and design of regulatory policies. For example:

- Should firm organizations be designed to delegate to divisional managers decisions regarding production and sourcing? Should managers in turn delegate resolution of workplace problems to workers? Or should the firm be organized as a vertical hierarchy, where agents at any layer make reports to their bosses and await instructions on what to do?[4]

- Analogously, should environmental regulations take the form of quantitative restrictions on polluting firms set by the government? Or should they take the form of tax-based incentives where firms are authorized to make their own pollution decisions?[5]

- Should communication be vertical (from agents to principal, as in revelation mechanisms) or horizontal (between agents)? Should communication be structured as a static simultaneous process, or should it be dynamic and interactive?

- More generally, do incentive considerations motivate restrictions on communication between agents, or the extent of discretion they are granted?

In settings where the Revelation Principle applies, these questions cannot be addressed since the Principle states that a centralized revelation mechanism weakly dominates any mechanism with decentralized decision-making or direct exchange of information among agents via dynamic communication processes.

In this paper we explore the role of communication costs in generating a theory which addresses these questions. Following the 1930s debates on economic socialism, a large literature subsequently emerged on resource allocation mechanisms that economize on communication costs. Examples are the message space literature (Hurwicz (1960, 1972), Mount and Reiter (1974)) and the theory of teams (Marschak and Radner (1972)).[6] This early literature on mechanism design ignored incentive problems.[7] The more recent literature on mechanism design on the other hand focuses only on incentive problems, ignoring communication costs entirely.

The existing literature on mechanism design in which incentive and communication costs co-exist has focused on contexts where the information of each agent is a single dimensional real-valued variable while message spaces are finite. It has been able to make progress only under strong *ad hoc* restrictions on the class of communication protocols.[8]

[4] Aoki (1990) discusses key differences between American and Japanese firms in terms of these features.

[5] See discussions in Weitzman (1974, 1978) or Dasgupta, Hammond and Maskin (1980).

[6] Segal (2006) surveys recent studies of informationally efficient allocation mechanisms.

[7] A notable exception is Reichelstein and Reiter (1988), who examined implications of strategic behavior for communicational requirements of mechanisms implementing efficient allocations.

[8] See Green and Laffont (1986, 1987), Melumad, Mookherjee and Reichelstein (1992, 1997), Laffont and Martimort (1998), Blumrosen, Nisan and Segal (2007), Blumrosen and Feldman (2006) and Kos (2011, 2012)). Van Zandt (2007) and Fadel and Segal (2009) do not seek to derive optimal mechanisms given incentive and communication constraints, but ask a related question: does the communicational complexity need to implement a given decision rule increase in the presence of incentive problems? Battigali and Maggi (2002) study a model of symmetric but nonverifiable information where there are costs of writing contingencies into contracts. This is in contrast to the papers cited above which involve asymmetric information with constraints on message spaces.

Most authors restrict attention to mechanisms with a single round of communication, in which each agent simultaneously selects a message from an exogenously restricted message space. From the standpoint of informational efficiency, it is well-known that dynamic communication is valuable in the presence of communication costs: they enable agents to condition their later messages on messages received at earlier stages from others, which allows more information to be exchanged. Examples have been provided in the literature where the same is true when incentive problems also exist.[9] Hence there is no basis for restricting attention to a single round of communication, apart from problems of analytical tractability.

The key analytical problem in incorporating dynamic communication protocols into models with strategic agents is finding a suitable characterization of incentive constraints. Dynamic mechanisms enlarge the range of possible deviations available to participants, over and above those typically characterized by incentive compatibility constraints in a static revelation mechanism. Van Zandt (2007) observes this is not a problem when the solution concept is *ex post incentive compatibility (EPIC)*, where agents do not regret their strategies even after observing all messages sent by other agents. When we use the less demanding concept of a (perfect) Bayesian equilibrium, dynamic communication protocols do impose additional incentive constraints. Then there is a potential trade-off between informational efficiency and incentive problems.

The problem in studying this trade-off is that a precise characterization of incentive constraints for dynamic protocols is not available in existing literature. In a very general setting Fadel and Segal (2009) provide different sets of sufficient conditions that are substantially stronger than necessary conditions. In this paper we restrict attention to contexts with single dimensional outputs and single-crossing preferences for each agent, and obtain a set of conditions that are both necessary and sufficient for Bayesian implementation in arbitrary dynamic communication protocols (Proposition 1).

This enables us to address the broad questions listed at the outset, without imposing *ad hoc* restrictions on the number of communication rounds. Our characterization of feasible mechanisms is shown to imply that the mechanism design problem reduces to selecting an output allocation rule which maximizes a payoff function of the Principal (modified to include the cost of incentive rents paid to agents in a standard way with 'virtual' types replacing actual types) subject to communication feasibility restrictions alone (Proposition 2). This extends the standard approach to solving for optimal mechanisms

[9] Melumad, Mookherjee and Reichelstein (1992, 1997), Blumrosen, Nisan and Segal (2007) and Van Zandt (2007, Section 4) show the superiority of sequential over simultaneous communication protocols with limited message spaces and each agent sends a message only once. Kos (2011) studies optimal auctions with two potential buyers, a binary message set for each buyer at each round, and multiple communication rounds, where increasing the number of rounds raises the seller's welfare. This paper will provide results concerning this in Section 6.

with unlimited communication (following Myerson (1981)), and provides a convenient representation of the respective costs imposed by incentive problems and communication constraints. In particular, Proposition 2 implies that *there is no trade-off between informational efficiency and incentive compatibility*, under the assumptions of our model.[10]

A number of implications of this result are then derived. The first concerns the value of delegating production decisions to agents.[11] The result is not obvious *a priori*, since delegation can generate costs owing to opportunistic behavior in the presence of incentive problems, which have to be traded off against the benefits from enhanced informational efficiency.[12] Proposition 2 implies that production decisions should be made by those who are the most informed about attendant cost implications. Quantitative targets for managers or workers, or pollution caps imposed by regulators, are dominated by delegation of decisions to workers, managers and firms. These agents need to be incentivized by suitable bonus or tax formulae conditioned on reports communicated by them to the corresponding Principal. This shows Hayek's arguments in favor of decentralized mechanisms continue to apply in contexts with incentive problems.

A second set of implications concern the design of optimal communication protocols. We show that if communication costs either involve material costs which are linear in the length of messages sent and in the size of the communication channel (defined by the maximum length of messages sent), or delay which is linear in the size of the communication channel, then communication should take place over multiple rounds in which agents disclose their information as slowly as possible.[13] Such dynamic protocols enable agents to exchange maximal information subject to the communication constraints. If communication costs consist only of delay, agents must report simultaneously in each round (as in dynamic auctions or budgeting systems where agents at any given layer of a hierarchy submit forecasts, competing bids or resource requests to their manager). But if they consist only of material costs, it is optimal for different agents to alternate in sending messages across successive rounds (as in price negotiations with alternating offers, or meetings with interactive dialogue).

[10]The one-dimensional nature of production decisions and of cost types satisfying the single-crossing condition plays a key role. See Green and Laffont (1987) and Fadel and Segal (2009) for examples of other settings where it is desirable to restrict the discretion of agents or their access to information in order to overcome incentive problems.

[11]Earlier literature such as Melumad, Mookherjee and Reichelstein (1992, 1997) and Laffont and Martimort (1998) have focused on a related but different question: the value of decentralized contracting (or subcontracting) relative to centralized contracting. Here we assume that contracting is centralized, and examine the value of decentralizing production decisions instead.

[12]The papers cited in the previous footnote show for this reason how certain variants of delegated contracting can perform worse that centralized contracting.

[13]That is, in each round agents are assigned a small message set (consisting of the shortest possible messages).

The paper is organized as follows. Section 2 introduces the model. Section 3 is devoted to characterizing feasible allocations. Section 4 uses this to represent the design problem as maximizing the Principal's incentive-rent-modified welfare function subject to communicational constraints alone. Section 5 uses this to compare centralized and decentralized mechanisms, while Section 6 describes implications for design of optimal communication protocols. Section 7 concludes.

## 2 Model

There is a Principal who contracts with two agents 1 and 2. Agent $i = 1, 2$ produces a one-dimensional nonnegative real valued input $q_i$ at cost $\theta_i q_i$, where $\theta_i$ is a real-valued parameter distributed over an interval $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ according to a positive-valued, continuously differentiable density function $f_i$ and associated c.d.f. $F_i$.[14] The distribution satisfies the standard monotone hazard condition that $\frac{F_i(\theta_i)}{f_i(\theta_i)}$ is nondecreasing, implying that the 'virtual cost' $v_i(\theta_i) \equiv \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ is strictly increasing.[15] $\theta_1$ and $\theta_2$ are independently distributed, and these distributions $F_1, F_2$ are common knowledge among the three players.

The inputs of the two agents combine to produce a gross return according to a production function $V(q_1, q_2)$ for the Principal. We assume it is feasible for the two agents to select their outputs independently: $(q_1, q_2) \in \Re_+ \times \Re_+$. Note that a context of team production where both agents produce a common output $q$ is a special case of the model where $V$ takes the form $W(\min\{q_1, q_2\})$. A procurement auction where the Principal seeks to procure a fixed amount $\bar{q}$ of a good from two competing suppliers is also a special case, with $V = \min\{q_1 + q_2, \bar{q}\}$. For the time being we impose no additional assumptions on the production function $V$. Sections 5 and 6 will impose some additional assumptions in order to derive specific implications for optimal mechanisms.

The Principal makes transfer payments $t_i$ to $i$. The payoff of $i$ is $t_i - \theta_i q_i$. Both agents are risk-neutral and have autarkic payoffs of 0. The Principal's objective takes the form

$$V(q_1, q_2) - \lambda_1(t_1 + t_2) - \lambda_2(\theta_1 q_1 + \theta_2 q_2) \quad (1)$$

where $\lambda_1 \geq 0, \lambda_2 \geq 0$ and $(\lambda_1, \lambda_2) \neq 0$ respectively represent welfare weights on the cost of transfers incurred by the Principal and cost of production incurred by the agents.

[14]We restrict attention to linear costs for the sake of expositional simplicity. The results extend to more general cost functions of the form $K + A(\theta)C(q)$ where $K$ is a known fixed cost and variable costs are multiplicatively separable in $\theta$ and $q$.

[15]Our results can be extended in the absence of this assumption, employing the 'ironing' technique developed by Myerson (1981) and Baron and Myerson (1982).

One application is to a context of internal organization or procurement, where the Principal owns a firm composed of two divisions whose respective outputs combine to form revenues $V = V(q_1, q_2)$. The principal seeks to maximize profit, hence $\lambda_1 = 1$ and $\lambda_2 = 0$. The same applies when the two agents correspond to external input suppliers.

An alternate application is to environmental regulation. The Principal is a regulator seeking to control outputs or abatements $q_i$ of two firms $i = 1, 2$. $V(q_1 + q_2)$ is the gross social benefit, and $\theta_i$ is the firm $i$'s unit cost. Consumer welfare equals $V - (1 + \lambda)R$ where $R$ is the total tax revenue collected from consumers and $\lambda$ is the deadweight loss involved in raising these taxes. The revenue is used to reimburse transfers $t_1, t_2$ to the firms. Social welfare equals the sum of consumer welfare and firm payoffs, which reduces to (1) with $\lambda_1 = \lambda, \lambda_2 = 1$. If $\lambda = 0$, this reduces to the efficiency objective $V - \theta_1 q_1 - \theta_2 q_2$.

# 3 Communication and Contracting

## 3.1 Timing

The mechanism is designed by the principal at an ex-ante stage ($t = -1$). It consists of a *communication protocol* (explained further below) and a set of contracts to each agent. There is enough time between $t = -1$ and $t = 0$ for all agents to read and understand the offered contracts.

At $t = 0$, each agent $i$ privately observes the realization of $\theta_i$, and independently decides whether to participate or opt out of the mechanism. If either agent opts out the game ends; otherwise they enter the planning or communication phase which lasts until $t = T$.

Communication takes place in a number of successive rounds $t = 1, \ldots, T$. We abstract from mechanisms in which the Principal seeks to limit the flow of information across agents, either by appointing mediators, regulators or scrambling devices. Later we argue that the optimal allocation is implemented with this communication structure, i.e., it is not profitable to restrict or garble the flow of information across agents. Hence this restriction will turn out to entail no loss of generality. This simplifies the exposition considerably.

The Principal is assumed to be able to verify all messages exchanged between agents. Equivalently, an exact copy of every message sent by one agent to another is also sent to the Principal. This rules out collusion between the agents, and allows the Principal to condition transfers *ex post* on messages exchanged. Given that agents exchange messages directly with one another and the absence of any private information possessed by the Principal, there is no rationale for the Principal to send any messages to the agents. In what follows we will not make the Principal's role explicit in the description of the communication protocol, and will focus on the exchange of communication between the agents.[16]

At the end of round $T$, each agent $i = 1, 2$ or the Principal selects production level $q_i$, depending on whether the mechanism is decentralized or centralized (an issue discussed further below).

Finally, after production decisions have been made, payments are made according to the contracts signed at the *ex ante* stage, and verification by the Principal of messages exchanged by agents and outputs produced by them.

## 3.2 Communication Protocol

A communication protocol is a rule defining $T$ the number of rounds of communication, and the message set $M_i$ of each agent $i$ in any given round, which may depend on the history of messages exchanged in previous rounds. If some agents are not supposed to communicate anything in any round, their message sets are null in those rounds. This allows us to include protocols where agents take turns in sending messages in different rounds. Other protocols may involve simultaneous reporting by all agents in each round.

The *vocabulary* of any agent $i \in \{1, 2\}$ is a message set $\mathcal{M}_i$, which contains all messages $m_i$ that $i$ can feasibly send in a single round. This incorporates restrictions on the language that agents use to communicate with one another. Specific assumptions concerning such restrictions are introduced below.

The message set $M_i$ assigned to agent $i$ in any round is a subset of the vocabulary of that agent. Message histories and message sets are defined recursively as follows. Let $m_{it}$ denote a message sent by $i$ in round $t$. Given a history $h_{t-1}$ of messages exchanged (sent and received) by $i$ until round $t-1$, it is updated at round $t$ to include the messages exchanged at round $t$: $h_t = (h_{t-1}, \{m_{it}\}_{i\in\{1,2\}})$. And $h_0 = \emptyset$. The message set for $i$ at round $t$ is then a subset of $\mathcal{M}_i$ which depends on $h_{t-1}$, unless it is null.

Formally, the *communication protocol* specifies the number of rounds $T$, and for every round $t \in \{1, \ldots, T\}$ and every agent $i$, a message set $M_i(h_{t-1}) \subseteq \mathcal{M}_i$ or $M_i(h_{t-1}) = \emptyset$ for every possible history $h_{t-1}$ until the end of the previous round.[17]

[16] As mentioned above, any mechanism in which agents send some messages to the Principal but not to each other, will end up being weakly dominated by a mechanism in which these messages are also sent to other agents. Hence there is no need to consider mechanisms where agents communicate privately with the Principal.

[17] We depart from Fadel and Segal (2009) and Van Zandt (2007) insofar as their definition of a protocol combines the extensive form game of communication as well as the communication strategy of each agent.

### 3.3 Communication Costs

We now describe communication costs. These depend on the length of messages sent, which we now explain.

We allow agents the option of not sending any message at all in any given round: hence the null message $\phi \in \mathcal{M}_i$. Let $l(m_i)$ denote the *length* of message $m_i \in \mathcal{M}_i$, which is an integer. It is natural to assume $l(\phi) = 0$, and positive-valued for any other message. For example if messages are binary-encoded, $l(m_i)$ could denote the total number of 0's and 1 bits included in $m_i$. Or if there is a finite alphabet consisting of a set of letters, and messages are sent in words which are finite sequences of letters interspersed with blank spaces (i.e., null messages), the length of a message could be identified with the total number of letters.

Communication costs could involve either material costs (e.g., telephone calls, e-mail, faxes, videoconferences) or time delays (which hold up production and thereby involve delayed shipment of goods to customers and attendant loss of revenues). These costs will typically depend on actual length of messages sent and/or on the maximum length of messages that could be sent across all contingencies, i.e., the *capacity* of the communication channels involved. Specific models of communication costs will be provided in Section 6. For now, we avoid any such specific cost function.

We consider communication protocols whose costs amount to at most a fixed budget $B$ which we take as given. The communication budget will be subtracted from the primary revenues and costs of the Principal to yield the net returns to the latter. The Principal could decide on $B$ at the first stage, and for given $B$ select an optimal mechanism at the second stage. We focus on the problem confronted at the second stage, corresponding to some finite level of $B$ which is given. The results will not depend on the specific choice of $B$.

For any given finite $B$, there will exist a set of feasible communication protocols whose cost will not exceed $B$. Let this set of feasible protocols given the communication constraints be denoted by $\mathcal{P}$. Under reasonable assumptions on the the structure of agent vocabularies, it can be shown that any protocol in this set will involve a finite number of communication rounds and a finite message set for every agent in each round.[18]

### 3.4 Communication Plans and Strategies

Given a protocol $p \in \mathcal{P}$, a *communication plan* for agent $i$ specifies for every round $t$ a message $m_{it}(h_{t-1}) \in M_i(h_{t-1})$ for every possible history $h_{t-1}$ that could arise for $i$ in

[18] A detailed statement of assumptions and proofs is available in the working paper version of this paper (Mookherjee and Tsumagari (2012)).

protocol $p$ until round $t-1$. The set of communication plans for $i$ in protocol $p$ is denoted $C_i(p)$. As explained above, for any finite communication budget, this set is finite for any feasible protocol. For the rest of the paper, it will be assumed that communication protocols have this property.

For communication plan $c = (c_1, c_2) \in C(p) \equiv C_1(p) \times C_2(p)$, let $h_t(c)$ denote the history of messages generated thereby until the end of round $t$. Let $H_t(p) \equiv \{h_t(c) \mid c \in C(p)\}$ denote the set of possible message histories in this protocol until round $t$. For a given protocol, let $\mathcal{H} \equiv H_T(p)$ denote the set of possible histories at the end of round $T$.

Given a protocol $p \in \mathcal{P}$, a *communication strategy* for agent $i$ is a mapping $c_i(\theta_i) \in C_i(p)$ from the set $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ of types of $i$ to the set $C_i(p)$ of possible communication plans for $i$. In other words, a communication strategy describes the dynamic plan for sending messages, for every possible type of the agent. The finiteness of the set of communication plans implies that it is not possible for others in the organization to infer the exact type of any agent from the messages exchanged. Non-negligible sets of types will be forced to pool into the same communication plan.

### 3.5 Production Decisions and Contracts

Many authors in previous literature (Blumrosen and Feldman (2006), Blumrosen, Nisan and Segal (2007) and Kos (2011, 2012)) have limited attention to mechanisms where output assignments and transfers are specified as a function of the information communicated by the agents. Decision-making authority is effectively retained by the Principal in this case. We shall refer to such mechanisms as *centralized.* A *contract* in this setting specifies a quantity allocation $q(h) \equiv (q_1(h), q_2(h)) : \mathcal{H} \rightarrow \Re^2_+$, with corresponding transfers $t(h) \equiv (t_1(h), t_2(h)) : \mathcal{H} \rightarrow \Re \times \Re$. A *centralized mechanism* is then a communication protocol $p \in \mathcal{P}$ and an associated contract $(q(h), t(h)) : \mathcal{H} \rightarrow \Re^2_+ \times \Re^2$.

Some authors (Melumad, Mookherjee and Reichelstein (1992, 1997)) have explored mechanisms where the Principal delegates decision-making to one of the two agents, and compared their performance with centralized mechanisms. This is a pertinent question in procurement, internal organization or regulation contexts. They consider mechanisms where both contracting with the second agent as well as production decisions are decentralized (while restricting attention to communication protocols involving a single round of communication). Here we focus attention on mechanisms where the Principal retains control over the design of contracts with both agents, while decentralizing decision-making authority to agents concerning their own productions. We refer to such mechanisms as *decentralized.* The potential advantage of decentralizing production decisions to agents is that these decisions can be based on information possessed by the agents which is richer than what they can communicate to the Principal. Transfers can then be based on output decisions as well as messages exchanged.

Formally, a *decentralized mechanism* is a communication protocol $p$ and a pair of contracts for the two agents, where the contract for agent $i$ is a transfer rule $t_i(q_i, h) : \Re_+ \times \mathcal{H} \rightarrow \Re$. Such a mechanism induces a quantity allocation $q_i(\theta_i, h) : \Theta_i \times \mathcal{H} \rightarrow \Re_+$ which maximizes $t_i(q_i, h) - \theta_i q_i$ with respect to choice of $q_i \in \Re_+$.[19] To simplify exposition we specify the quantity allocation as part of the decentralized mechanism itself.

A centralized mechanism can be viewed as a special case of a decentralized mechanism in which $q_i(\theta_i, h)$ is measurable with respect to $h$, i.e., does not depend on $\theta_i$ conditional on $h$. It corresponds to a mechanism in which the Principal sets an output target for each agent (based on the messages communicated) and then effectively forces them to meet these targets with a corresponding incentive scheme. We can therefore treat every mechanism as decentralized, in a formal sense.

In view of this, say that a mechanism is *strictly decentralized* if it is not centralized. We shall in due course evaluate the relative merits of centralized and strictly decentralized mechanisms.

### 3.6 Feasible Production Allocations

The standard way of analysing the mechanism design problem with unlimited communication is to first characterize production allocations that are feasible in combination with some set of transfers, and then use the Revenue Equivalence Theorem to represent the Principal's objective in terms of the production allocation alone, while incorporating the cost of the supporting transfers. To extend this method we need to characterize feasible production allocations.

A *production allocation* is a mapping $q(\theta) \equiv (q_1(\theta), q_2(\theta)) : \Theta_1 \times \Theta_2 \rightarrow \Re_+^2$. Restrictions are imposed on production allocations owing both to communication and incentive problems.

Consider first communication restrictions. A production allocation $q(\theta)$ is said to be *communication-feasible* if: (a) the mechanism involves a communication protocol $p$ satisfying the specified constraints on communication, and (b) there exist communication strategies $c(\theta) = (c_1(\theta_1), c_2(\theta_2)) \in C(p)$ and output decisions of agents $q_i(\theta_i, h) : \Theta_i \times \mathcal{H} \rightarrow \Re_+$, such that $q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta))))$ for all $\theta \in \Theta \equiv \Theta_1 \times \Theta_2$. Here $h(c)$ denotes the message histories generated by the communication strategies $c$ in this protocol.

The other set of constraints pertains to incentives. A communication-feasible production

[19] Since $i$ infers the other's output $q_j$ ($j \neq i$) only through $h$, we can restrict attention to contracts where the payments to any agent depend only on his own output without loss of generality. Specifically, if $t_i$ were to depend on $q_j$, the expected value of the transfer to $i$ can be expressed as a function of $q_i$ and $h$, since agent $i$'s information about $q_j$ has to be conditioned on $h$.

allocation $q(\theta)$ is said to be *incentive-feasible* in a mechanism if there exists a Perfect Bayesian Equilibrium (PBE) of the game induced by the mechanism which implements the production allocation.[20] In other words, there must exist a set of communication strategies and output decision strategies satisfying condition (b) above in the requirement of communication-feasibility, which constitutes a PBE.

### 3.7 Characterization of Incentive Feasibility

We now proceed to characterize incentive-feasible production allocations. Using the single-dimensional output of each agent and the single crossing property of agent preferences, we can obtain as a necessary condition a monotonicity property of expected outputs with respect to types at each decision node. To describe this condition, we need the following notation.

It is easily checked (see Lemma 1 in the Appendix) that given any strategy configuration $c(\theta) \equiv (c_1(\theta_1), c_2(\theta_2))$ and any history $h_t$ until the end of round $t$ in a communication protocol, the set of types $(\theta_1, \theta_2)$ that could have generated the history $h_t$ can be expressed as the Cartesian product of subsets $\Theta_1(h_t), \Theta_2(h_t)$ such that

$$\{(\theta_1, \theta_2) \mid h_t(c(\theta_1, \theta_2)) = h_t\} = \Theta_1(h_t) \times \Theta_2(h_t). \tag{2}$$

A necessary condition for incentive-feasibility of a production allocation $q(\theta)$ which is communication-feasible in a protocol $p$ and supported by communication strategies $c(\theta)$ is that for any $t = 0, \dots, T$, any $h_t \in H_t$ and any $i = 1, 2$:

$$E[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)] \text{ is non-increasing in } \theta_i \text{ on } \Theta_i(h_t), \tag{3}$$

where $H_t$ denotes the set of possible histories until round $t$ generated with positive probability in the protocol when $c(\theta)$ is played, and $\Theta_i(h_t)$ denotes the set of types of $i$ who arrive at $h_t$ with positive probability under the communication strategies $c(\theta)$.

The necessity of this condition follows straightforwardly from the dynamic incentive constraints which must be satisfied for any history $h_t$ on the equilibrium path. Upon observing $h_t$, $i$'s beliefs about $\theta_j$ are updated by conditioning on the event that $\theta_j \in \Theta_j(h_t)$. Any type of agent $i$ in $\Theta_i(h_t)$ will have chosen the same messages up to round $t$. Hence any type $\theta_i \in \Theta_i(h_t)$ has the opportunity to pretend to be any other type in $\Theta_i(h_t)$ from round $t+1$ onward, without this deviation being discovered by anyone. A PBE requires that such a deviation cannot be profitable. The single-crossing property then implies condition (3).

[20] This requires both incentive and participation constraints be satisfied. For the definition of PBE, see Fudenberg and Tirole (1991, Section 8.2).

As noted earlier, the existing literature has provided sufficient conditions for incentive-feasibility that are stronger than (3). Fadel and Segal (2009) in a more general framework (with abstract decision spaces and no restrictions on preferences) provide two sets of sufficient conditions. One set (provided in their Proposition 6) of conditions is based on the observation that the stronger solution concept of ex post incentive compatibility implies Bayesian incentive compatibility. In our current context ex post incentive compatibility requires for each $i = 1, 2$:

$$q_i(\theta_i, \theta_j) \text{ is globally non-increasing in } \theta_i \text{ for every } \theta_j \in \Theta_j. \tag{4}$$

Another set of sufficient conditions (Proposition 3 in Fadel and Segal (2009)) imposes a no-regret property with respect to possible deviations to communication strategies chosen by other types following every possible message history arising with positive probability under the recommended communication strategies. This is applied to every pair of types for each agent at nodes where it is this agent's turn to send a message. In the context of centralized mechanisms (which Fadel and Segal restrict attention to), this reduces to the condition that for any any $i = 1, 2$ and any $h_t \in H_t, t = 0, \ldots, T-1$ where it is $i$'s turn to move (i.e., $M_i(h_t) \neq \emptyset$):[21]

$$E[q_i(\theta_i, \theta_j) | \theta_j \in \Theta_j(h_t)] \text{ is globally non-increasing in } \theta_i. \tag{5}$$

Our first main result is that the necessary condition (3) is also sufficient for incentive feasibility, provided the communication protocol prunes unused messages. Suppose that $p$ is a communication protocol in which communication strategies used are $c(\theta)$. Then $p$ is *parsimonious relative to communication strategies* $c(\theta)$ if every possible history $h \in \mathcal{H}$ in this protocol is reached with positive probability under $c(\theta)$.

**Proposition 1** *Consider any production allocation $q(\theta)$ which is communication-feasible in a protocol $p$ and is supported by communication strategies $c(\theta)$, where the protocol is parsimonious with respect to $c(\theta)$. Then condition (3) is necessary and sufficient for incentive-feasibility of $q(\theta)$.*

Parsimonious protocols have the convenient feature that Bayes rule can be used to update beliefs at every node, and off-equilibrium-path deviations do not have to be considered while checking incentive feasibility. Restricting attention to such protocols entail no loss of generality since any protocol can be pruned by deleting unused messages under any given set of communication strategies, to yield a protocol which is parsimonious with

[21] As Fadel and Segal point out, it suffices to check the following condition at the last node of the communication game at which it is agent $i$'s turn to move. Note also that this condition is imposed on nodes of the communication game, and not at nodes where agents make output decisions in the case of a decentralized mechanism.

respect to these strategies. Hence it follows that condition (3) is both necessary and sufficient for incentive-feasibility.

The proof of Proposition 1 is provided in the Appendix. The main complication arises for the following reason. In a dynamic protocol with more than one round of communication, no argument is available for showing that attention can be confined to communication strategies with a threshold property. Hence the set of types $\Theta_i(h_t)$ pooling into message history $h_t$ need not constitute an interval. The monotonicity property for output decisions in (3) holds only 'within' $\Theta_i(h_t)$, which may span two distinct intervals. The monotonicity property may therefore not hold for type ranges lying between the two intervals. This complicates the conventional argument for construction of transfers that incentivize a given output allocation.

The proof is constructive.[22] Given a production allocation satisfying (3) with respect to set of communication strategies in a protocol, we first prune the protocol to eliminate unused messages. Then incentivizing transfers are constructed as follows. We start by defining a set of functions representing expected outputs of each agent following any given history $h_t$ at any stage $t$, expressed as a function of the type of that agent. Condition (3) ensures the expected output of any agent $i$ is monotone over the set $\Theta_i(h_t)$. These are the types of $i$ that actually arrive at $h_t$ with positive probability on the equilibrium path. The proof shows it is possible to extend this function over all types of this agent (not just those that arrive at $h_t$ on the equilibrium path) which is globally monotone, in a way that agrees with the actual expected outputs on the set $\Theta_i(h_t)$, and which maintains consistency across histories reached at successive dates. This amounts to assigning outputs for types that do not reach $h_t$ on the equilibrium path, which can be thought of as outputs they would be assigned if they were to deviate somewhere in the game and arrive at $h_t$. Since this extended function is globally monotone, transfers can be constructed in the usual way to incentivize this allocation of expected output. The construction also has the feature that the messages sent by the agent after arriving at $h_t$ do not affect the expected outputs that would thereafter be assigned to the agent, which assures that the agent does not have an incentive to deviate from the recommended communication strategy.

# 4 Characterizing Optimal Mechanisms

Having characterized feasible allocations, we can now restate the mechanism design problem as follows.

Note that the interim participation constraints imply that every type of each agent must

[22]For a geometric illustration of the argument, see the working paper version of this paper (Mookherjee and Tsumagari (2012)).

earn a non-negative expected payoff from participating. Agents that do not participate do not produce anything or receive any transfers. Hence by the usual logic it is without loss of generality that all types participate in the mechanism. The single crossing property ensures that expected payoffs are nonincreasing in $\theta_i$ for each agent $i$. Since $\lambda_1 \geq 0$ it is optimal to set transfers that incentivize any given output allocation rule $q(\theta)$ satisfying (3) such that the expected payoff of the highest cost type $\bar{\theta}_i$ equals zero for each $i$. The expected transfers to the agents then equal (using the arguments in Myerson (1981) to establish the Revenue Equivalence Theorem):

$$\Sigma_{i=1}^{2} E[v_i(\theta_i)q_i(\theta_i, \theta_j)]$$

where $v_i(\theta_i) \equiv \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$. Consequently the expected payoff of the Principal is

$$E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i)q_i(\theta_i, \theta_j) - w_j(\theta_j)q_j(\theta_i, \theta_j)] \tag{6}$$

where $w_i(\theta_i) \equiv (\lambda_1 + \lambda_2)\theta_i + \lambda_1 \frac{F_i(\theta_i)}{f_i(\theta_i)}$.

This enables us to state the problem in terms of selecting an output allocation in combination with communication protocol and communication strategies. Given the set $\mathcal{P}$ of feasible communication protocols defined by the communication constraints, the problem is to select a protocol $p \in \mathcal{P}$, communication strategies $c(\theta)$ in $p$ and output allocation $q(\theta)$ to maximize (6), subject to the constraint that (i) there exists a set of output decision strategies $q_i(\theta_i, h), i = 1, 2$ such that $q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta))))$ for all $\theta \in \Theta$, and (ii) the output allocation satisfies condition (3).

Condition (i) is essentially a communication-feasibility constraint, which applies even in the absence of incentive problems. Condition (ii) is the additional constraint represented by incentive problems. Note that the above statement of the problem applies since attention can be confined without loss of generality to protocols that are parsimonious with respect to the assigned communication strategies. To elaborate, note that conditions (i) and (ii) are both necessary for implementation. Conversely, given an output allocation, a communication protocol, and communication strategies in the protocol that satisfy conditions (i) and (ii), we can prune that protocol by deleting unused messages to obtain a protocol that is parsimonious with respect to the given communication strategies. Then Proposition 1 ensures that the output allocation can be implemented as a PBE in the pruned protocol with suitably constructed transfers, which generate an expected payoff (6) for the Principal while ensuring all types of both agents have an incentive to participate.

Now observe that constraint (ii) is redundant in this statement of the problem. If we consider the relaxed version of the problem stated above where (ii) is dropped, the solution to that problem must automatically satisfy (ii), since the monotone hazard rate property on the type distributions $F_i$ ensure that $w_i(\theta_i)$ is an increasing function for each $i$. This generates our main result.

**Proposition 2** *The mechanism design problem can be reduced to the following. Given any set $\mathcal{P}$ of feasible communication protocols defined by the communication constraints, select a protocol $p \in \mathcal{P}$, communication strategies $c(\theta)$ in $p$ and output allocation $q(\theta)$ to maximize (6), subject to the constraint of communication feasibility alone, i.e., there exists a set of output decision strategies $q_i(\theta_i, h), i = 1, 2$ such that*

$$q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta)))), \forall \theta \in \Theta. \tag{7}$$

In the case of unlimited communication, this reduces to the familiar property that an optimal output allocation can be computed on the basis of unconstrained maximization of expected payoffs (6) of the Principal which incorporate incentive rents earned by the agents. With limited communication additional constraints pertaining to communication feasibility have to be incorporated. In the absence of incentive problems, the same constraint would apply: the only difference would be that the agents would not earn incentive rents and the objective function of the Principal would be different ($w_i$ would be replaced by $\tilde{w}_i = (\lambda_1 + \lambda_2)\theta_i$).

Proposition 2 thus shows how costs imposed by incentive considerations are handled differently from those imposed by communicational constraints. The former is represented by the replacement of production costs of the agents by their incentive-rent-inclusive virtual costs in the objective function of the Principal, in exactly the same way as in a world with costless, unlimited communication. The costs imposed by communicational constraints are represented by the restriction of the feasible set of output allocations, which must now vary more coarsely with the type realizations of the agents. This can be viewed as the natural extension of the Marschak-Radner (1972) characterization of optimal team decision problems to a setting with incentive problems. In particular, the same computational techniques can be used to solve these problems both with and without incentive problems: only the form of the objective function needs to be modified to replace actual production costs by virtual costs. The 'desired' communicational strategies can be rendered incentive compatible at zero additional cost.

This result does not extend when the definition of incentive-feasibility replaces the solution concept of PBE by ex post incentive compatibility (EPIC). EPIC requires the allocation to be globally monotone (condition (4)). The following example shows that the optimal PBE allocation for a specific communication protocol does not satisfy this property.

*Example.* Suppose $V(q_1, q_2) = 2(\min\{q_1, q_2\})^{1/2}$. $\theta_1$ is distributed uniformly on $[0, \alpha]$ where $\alpha \in (0, 2/3)$, and $\theta_2$ is uniformly distributed on $[0, 1]$. The Principal's objective is $V(q_1, q_2) - t_1 - t_2$ where $t_i$ is a transfer to agent $i$. There is a single feasible communication protocol with two rounds, with a binary message space for each agent, and agent 1 sends a message at the first round, followed by agent 2 in the second round. The mechanism

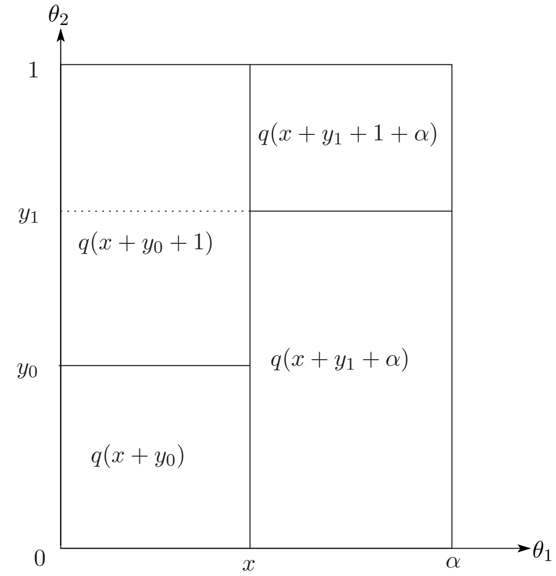


Figure 1: Example

is centralized. In this context we know from Blumrosen, Nisan and Segal (2007) that optimal communication strategies take the following form: agent 1 sends $m_1 = 0$ for $\theta_1 \in [0, x)$ and $m_1 = 1$ for $\theta_1 \in [x, \alpha]$ for some $x \in [0, \alpha]$. Agent 2 then sends $m_2 = 0$ for $\theta_2 \in [0, y_{m_1})$ and 1 for $\theta_2 \in [y_{m_1}, 1]$, for some $y_{m_1} \in [0, 1], m_1 = 0, 1$.

Defining $q(c) \equiv 1/c^2 = \arg\max_q [2q^{1/2} - cq]$ and $\Pi(c) \equiv 2q(c)^{1/2} - cq(c) = 1/c$, the optimal output choice made by the Principal conditional on the information that $(\theta_1, \theta_2) \in [\theta_1^{'}, \theta_1^{''}] \times [\theta_2^{'}, \theta_2^{''}]$ is $q_1 = q_2 = q(\theta_1^{'} + \theta_1^{''} + \theta_2^{'} + \theta_2^{''})$. The maximized payoff of the Principal conditional on this information is then $\Pi(\theta_1^{'} + \theta_1^{''} + \theta_2^{'} + \theta_2^{''})$. Hence the Principal's problem reduces to selecting $x, y_0, y_1$ to maximize

$$\frac{x}{\alpha} \frac{x + 2y_0}{(x + y_0)(x + y_0 + 1)} + \frac{(\alpha - x)}{\alpha} \frac{x + 2y_1 + \alpha}{(x + y_1 + \alpha)(x + y_1 + 1 + \alpha)}.$$

Given $x$, the optimal $y_0 = \frac{-x + (x^2 + 2x)^{1/2}}{2}$ and $y_1 = \frac{-(x+\alpha) + ((x+\alpha)^2 + 2(x+\alpha))^{1/2}}{2}$. It is evident that $y_0 < y_1$ for any $x \in [0, \alpha]$. Since $\alpha < \frac{2}{3}$, it is easy to check that $y_0 + 1 > y_1 + \alpha$ holds, implying that $q(x + y_0 + 1) < q(x + y_1 + \alpha)$. This shows that the optimal output assignment is not globally monotone in $\theta_1$: if $\theta_2 \in (y_0, y_1)$ then $q$ is higher when $\theta_1 \in [x, \alpha]$ compared with when $\theta_1 \in [0, x)$. See Figure 1. Hence the optimal Bayesian allocation cannot be EPIC under any set of transfer functions.

Where incentive feasibility is based on the EPIC solution concept, therefore, condition (4) must additionally be imposed on the optimization problem, in addition to the requirement of communication feasibility. Hence the optimal PBE and EPIC allocations must differ. This observation does not apply in the case of unlimited communication: in that context optimal Bayesian and EPIC mechanisms generally coincide (Mookherjee and Reichelstein (1992), Gershkov *et al.* (2013)).

Van Zandt (2007) and Fadel and Segal (2009) discuss a related question of the 'communication cost of selfishness': whether the communicational complexity of implementing any given social choice function (production allocation in our notation) is increased by the presence of incentive constraints. Van Zandt shows this is not true when using the EPIC solution concept, while Fadel and Segal provide examples where this is the case when using the Bayesian solution concept. In our context where we fix communication complexity and solve for optimal mechanisms, an analogous question could be phrased as follows: is the optimal mechanism in the presence of communication constraints alone, continue to be optimal when incentive constraints are incorporated? Proposition 2 shows that the answer to this question depends on $\lambda_1$. If the Principal is solely concerned with efficiency and $\lambda_1 = 0$, the objective function is the same with and without incentive constraints.[23] Then the optimal mechanism in the absence of any incentive constraints is also optimal in the presence of incentive constraints. On the other hand, if $\lambda_1 > 0$ and the Principal seeks to limit transfers to the agents, the objective function with and without incentive constraints differ. Then the optimal allocation in the absence of incentive constraints will typically not be optimal when incentive problems are present.

# 5 Implications for Decentralization versus Centralization of Production Decisions

We now examine implications of Proposition 2 for the value of strictly decentralized mechanisms compared with centralized ones. If production decisions are made by the Principal, outputs are measurable with respect to the history of exchanged messages. If decisions are delegated to the agents, this is no longer true, since they can be decided by the agents on the basis of information about their own true types, which is richer than what they managed to communicate to the Principal. Unlike settings of unlimited communication, centralized mechanisms cannot replicate the outcomes of decentralized ones. Contracts are endogenously incomplete, thus permitting a nontrivial comparison of centralized and decentralization decision rights.

The typical tradeoff associated with delegation of decision rights to better informed

[23] Van Zandt and Fadel and Segal do not incorporate the costs of incentivizing transfers in posing the implementation problem, so this is the appropriate case to consider when comparing with their result.

agents compares the benefit of increased flexibility of decisions with respect to the true state of the world, with the cost of possible use of discretion by the agent to increase his own rents at the expense of the Principal. Proposition 2 however shows that once the incentive rents that agents will inevitably earn have been factored into the Principal's objective, incentive considerations can be ignored. The added flexibility that decentralization allows then ensures it is superior. The following Proposition shows this is true as long as $V$ satisfies some standard regularity conditions that ensure optimal production allocations are always interior.[24]

**Proposition 3** *Suppose $V$ is twice continuously differentiable, strictly increasing, strictly concave and each agent's marginal product $\frac{\partial V}{\partial q_i}$ tends to $\infty$ as $q_i \rightarrow 0$. Then given any feasible centralized mechanism, there exists a corresponding strictly decentralized mechanism which generates a higher payoff to the Principal.*

An outline of the argument is as follows. The finiteness of the set of feasible communication plans for every agent implies the existence of non-negligible type intervals over which communication strategies and message histories are pooled. Consequently if decisions are centralized, the production decision for $i$ must be pooled in the same way. Instead if production decisions are left to agent $i$, the production decision can be based on the agent's knowledge of its own true type. Under the regularity conditions assumed in Proposition 3, optimal production allocations are always interior. Then this added 'flexibility' will allow a strict increase in the Principal's objective (6) while preserving communication feasibility.

This result can be contrasted to the demonstration that variants of delegated contracting can be inferior to centralized mechanisms (see Melumad, Mookherjee and Reichelstein (1992, 1997)), owing to 'control loss' from incentive problems (which aggravate the problem of double marginalization of rents) that can overwhelm improvements in flexibility. Such variants of delegation allow the principal contractor to choose payments made to the subcontractor, which are unobserved by the Principal. Once these payments can be observed and used by the Principal to evaluate the performance of the principal contractor, delegation is shown in the papers cited above to perform superior to centralized mechanisms, with a single round of communication with restricted message spaces. In the context of our model, the principal contracts directly with and thus controls payments to both agents, enabling problems of double marginalization to be avoided. This

[24]These regularity conditions are not satisfied in the contexts of team production or a procurement auction. For these contexts, the output allocation decision reduces to choice of $q_1$ alone, with $q_2 = q_1$ in the case of team production, and $q_2 = \bar{q} - q_1$ in the case of a Principal trying to procure a fixed quantity $\bar{q}$ from the two sellers combined. we can analogously show that any centralized mechanism is inferior to some mechanism which delegates to agent 1 the choice of $q_1$.

explains the relation to the results in Melumad *et al.* Proposition 3 shows the superiority of the decentralized mechanism obtains without imposing any restrictions on the communication protocol (apart from the fact that it must be finite).

In the context of internal organization, this result implies the optimality of decentralizing production decisions to workers when communication constraints prevent them from fully describing shop-floor contingencies to upper management, as in the prototypical 'Japanese' firm (Aoki (1990)) where the central headquarters contracts directly with all workers. This is in contrast to subcontracting settings considered in Melumad *et al.* (1992, 1997) where centralization can dominate delegation to prime contractors if the procuring firm does not monitor payments or allocation of production between subcontractors and the prime contractor.

In the environmental regulation context, Weitzman (1974) compared 'price' and 'quantity' regulation of pollution by firms without allowing for any communication of private information held by firms concerning abatement costs to the regulator. The 'price' regulation mode corresponded to a strictly decentralized mechanism with a linear incentive mechanism, while the 'quantity' regulation mode corresponded to a centralized mechanism where the regulator imposed a cap on emissions based on its prior information concerning abatement costs. In this context, Weitzman showed that either form of regulation could be superior, depending on parameters. In later work, however, Weitzman (1978) and Dasgupta, Hammond and Maskin (1980) characterized optimal nonlinear incentive mechanisms which could be viewed as a combination of 'price' and 'quantity' regulation, while continuing to assume that it is infeasible for firms to communicate any information to regulators. This mechanism is strictly decentralized, as regulated firms select their own emission levels. The demonstration that it dominates pure 'quantity' regulation can be viewed as a version of our result that centralized mechanisms are dominated by strictly decentralized ones if communication is limited. Proposition 3 generalizes this result to contexts where firms communicate their information to regulators, but the extent of such communication is restricted owing to delay or material costs associated with communication of excessively detailed information.

## 6 Implications for Choice of Communication Protocol

Proposition 2 has useful implications for the ranking of different communication protocols. Given any set of communication strategies in a given protocol, in state $(\theta_i, \theta_j)$ agent $i$ learns that $\theta_j$ lies in the set $\Theta_j(h(c_i(\theta_i), c_j(\theta_j)))$, which generates an information partition for agent $i$ over agent $j$'s type.

Say that a protocol $p_1 \in \mathcal{P}$ is *more informative* than another $p_2 \in \mathcal{P}$ if for any set of communication strategies in the former, there exists a set of communication strategies in the latter which yields (at round $T$) an information partition to each agent over the type of the other agent which is more informative in the Blackwell sense in (almost) all states of the world.

It then follows that a more informative communication protocol permits a wider choice of communication feasible output allocations. Proposition 2 implies that the Principal prefers more informative protocols, and would not benefit by restricting or scrambling the flow of communication among agents.

This is the reason we assumed all messages are addressed to everyone else in the organization. If the transmission and processing of messages entail no resource or time costs, this ensures maximal flow of information between agents. In contrast much of the literature on informational efficiency of resource allocation mechanisms (in the tradition of Hurwicz (1960, 1972) or Mount and Reiter (1974)) has focused on centralized communication protocols where agents send messages to the Principal rather than one another. Such protocols restrict the flow of information among agents. Marschak and Reichelstein (1998) have extended this to network mechanisms where agents communicate directly with one another, and examine the consequences of such decentralized 'network' mechanisms for communication costs (in the absence of incentive problems). In our approach the Principal plays no active role in the communication process.[25]

Within the class of such decentralized communication protocols, more can be said about the nature of optimal protocols, depending on the precise nature of communication costs. We turn to this now.

We limit attention to agent vocabularies consisting of *letters* or messages of unit length, in which longer messages are *words* which are combinations of letters. Hence if there are $L_i$ letters of unit length in agent $i$'s vocabulary, then there are at most $L_i^k$ words or messages of length not exceeding $k$, for any integer $k$. For instance, if the agents communicate using binary code, there are two letters or unit bits 0 and 1. Any longer message consists of a string of unit bits, with the length of the message identified by the number of bits. The same is true for most languages which have an alphabet of letters, words are composed of a string of letters and the length of a word is measured by the number of letters contained in that word. In what follows, we use $M_i^*$ to denote the set of letters in $i$'s vocabulary in conjunction with the null message, i.e., $M_i^* \equiv \{m_i \in \mathcal{M}_i | l(m_i) \leq 1\}$.

Communication costs can involve either material costs or time-delays. Material costs could include variable (e.g., depending on the length of messages sent) or fixed (depending on communication capacity) costs. The communication capacity of each agent $i$ is defined

[25] If the only costs of communication involve writing or sending messages this is without loss of generality, since the Principal has no private information to report to the agents, and any messages that an agent sends to the Principal which are in turn sent to the other agent could be sent directly to the latter at no additional cost.

as the longest message contained in $M_i$: $\bar{l}(M_i) \equiv \max_{m_i \in M_i} l(m_i)$.

We assume that material communication costs for any given round are linear in length of messages and communication capacity:

$$\Phi_m = \phi_v l(m_i) + \phi_f \bar{l}(M_i) \tag{8}$$

for some constants $\phi_v \geq 0, \phi_f > 0$, while delay costs per round takes the form

$$\Phi_d = \phi_d \max\{\bar{l}(M_1), \bar{l}(M_2)\} \tag{9}$$

for some $\phi_d > 0$. The constraint imposed by a given budget $B$ for communication cost pertains to the total cost incurred across different rounds in the protocol. The results reported below extend as long as there are no increasing returns to scale with respect to length of messages or communication capacity.

Our first result shows that under the above assumptions, information ought to be released 'slowly' by agents across multiple rounds of communication. If any agent has a 'large' message set in any given round, the agent can communicate more information at the same cost by breaking this up a sequence of smaller messages in successive rounds. Suppose for instance that communication is in binary code, and an agent has the following message set in some round: $\{\phi, 0, 1, 00, 01, 10, 11\}$. This round can be broken up into two successive rounds in each of which the agent is given the message set $\{\phi, 0, 1\}$. The agent can communicate at least as much information across these two rounds as she could previously (e.g., a null message in both rounds corresponds to a null message previously, a null message in one round combined with a single-bit message 0 (or 1) in the other corresponds to a previous message of 0 (or 1), and so on). Communication costs do not increase since capacity costs are the same: the maximal length of a message was 2 previously with a single round, while it is now 1 in each of the two rounds. The aggregate length of messages remains the same in every state of the world. The agent now has a total of nine possible message combinations across the two rounds, as against seven possible messages previously. Hence the agent can now send strictly more information, e.g., she has the choice of the order in which a null message is sent in one round and a single-bit message in the other. This allows a strict improvement in the Principal's payoff.

**Proposition 4** *Suppose that agent vocabularies and communication costs are as specified above. Also suppose that the production function satisfies the regularity conditions specified in Proposition 3, and in addition* $V_{12}(q_1, q_2) \neq 0$ *for every* $(q_1, q_2) >> 0$*. Then any non-null message set assigned to any agent (in any round following any history arising with positive probability in any optimal protocol) must consist of letters (messages of unit length) alone, i.e.,* $M_i(h_{t-1}) = M_i^*$ *if it is non-null.*

Our final result concerns the contrast between material costs and time delay formulations of communication cost for the nature of optimal protocols.

**Proposition 5** *Suppose the same conditions as in Proposition 4 hold. In addition*

*(i) Suppose that communication is constrained only by total material cost (i.e., $\phi_d = 0$, $\phi_f > 0$). Then there exists an optimal protocol with the feature that only one agent sends messages in any given communication round.*

*(ii) Suppose that communication is constrained only by the total time delay (i.e., $\phi_v = \phi_f = 0 < \phi_d$), and the upper bound on total delay is $D$. Then every optimal protocol involves a number of communication rounds equal to the largest integer not exceeding $D/\phi_d$, and both agents send messages simultaneously in each round.*

The reasoning is the following. If communication entails only material costs, any round with simultaneous communication by both agents (from the set of messages of unit length or less) can be broken down into two successive rounds in which the agents alternate in sending messages from this set. Each agent has the option of sending the same message in this round when it is their turn to report. The agent now moving second has the additional option of conditioning his message on the message just sent by the other agent moving first (while restricted to sending a message of the same or shorter length as he did previously). The rest of the protocol is left unchanged. Material costs of communication are unchanged, as the communication capacity of each remains the same and the length of messages sent do not increase. Hence the Principal's payoff weakly increases. The total delay of the mechanism is increased owing to the sequencing of messages across the two agents, but this is not costly by assumption.

In contrast when communication costs consist only of delay, both agents must send messages in every round. Otherwise there would be a round in which one of the agents ($i$, say) does not send any messages, while the other agent $j$ does (if neither does then the entire round can be dispensed with). Allowing $i$ to select a message from $M_i^*$ in this round allows him to communicate more information than previously. As there are no material costs of communication this does not cause any problem with the communication constraint, so a strict improvement is now possible.

## 7 Concluding Comments

An obvious limitation of our approach is that it limits attention to contexts with one-dimensional outputs and type spaces. However, the object of the paper was to show how the special structure of this context can be exploited to obtain strong results concerning optimality of decentralized decision-making and absence of trade-offs between incentives and informational efficiency. The extent to which these results can be extended to richer settings remains to be examined in future work.

Our formulation of decentralized decision-making pertained only to production decisions. We ignored the possibility of delegating responsibility of contracting with other agents to some key agents. A broader concern is that we ignored the communicational requirements involved in contracting itself, by focusing only on communication in the process of implementation of the contract, which takes place after parties have negotiated and accepted a contract. Under the assumption that pre-contracting communication is costless, and messages exchanged between agents are verifiable by the Principle, it can be shown that delegation of contracting cannot dominate centralized contracting if both are equally constrained in terms of communicational requirements. Subcontracting may thus be potentially valuable in the presence of costs of pre-contract communication, or if agents can directly communicate with one another in a richer way than the way they can communicate with the Principal. Exploring the value of delegation of contracting remains an important task for future research.

## Appendix: Proofs

**Lemma 1** *Consider any communication protocol $p \in \mathcal{P}$. For any $h_t \in H_t(p)$ and any $t \in \{1, \ldots, T\}$:*

$$\{c \in C(p) \mid h_t(c) = h_t\}$$

*is a rectangle set in the sense that if $h_t(c_i, c_j) = h_t(c_i^{'}, c_j^{'}) = h_t$ for $(c_i, c_j) \neq (c_i^{'}, c_j^{'})$, then*

$$h_t(c_i^{'}, c_j) = h_t(c_i, c_j^{'}) = h_t.$$

**Proof of Lemma 1**: The proof is by induction. Note that $h_0(c) = \phi$ for any $c$, so it is true at $t = 0$. Suppose the result is true for all dates up to $t-1$, we shall show it is true at $t$.

Note that

$$h_t(c_i, c_j) = h_t(c_i^{'}, c_j^{'}) = h_t \tag{10}$$

implies

$$h_\tau(c_i, c_j) = h_\tau(c_i^{'}, c_j^{'}) = h_\tau \tag{11}$$

for any $\tau \in \{0, 1, .., t-1\}$. Since the result is true until $t-1$, we also have

$$h_\tau(c_i^{'}, c_j) = h_\tau(c_i, c_j^{'}) = h_\tau \tag{12}$$

for all $\tau \leq t-1$. So under any of the configurations of communication plans $(c_i, c_j)$, $(c_i^{'}, c_j^{'})$, $(c_i^{'}, c_j)$ or $(c_i, c_j^{'})$, agent $i$ experiences the same message history $h_{t-1}$ until $t-1$. Then $i$ has the same message set at $t$, and (10) implies that $i$ sends the same messages to $j$ at $t$, under either $c_i$ or $c_i^{'}$.

(11) and (12) also imply that under either $c_j$ or $c_j^{'}$, $j$ sends the same messages to $i$ at all dates until $t-1$, following receipt on the (common) messages sent by $i$ until $t-1$ under these different configurations. The result now follows from the fact that messages sent by $j$ to $i$ depend on the communication plan of $i$ only via the messages $j$ receives from $i$. So $i$ must also receive the same messages at $t$ under any of these different configurations of communication plans. ■

**Proof of Proposition 1:**

Let $q_i(\theta_i, \theta_j)$ be a production allocation satisfying (3), which is supported by a communication strategy vector $c(\theta)$ in a protocol $p$ which is parsimonious with respect to these strategies. In this protocol all histories are reached with positive probability on the equilibrium path, hence beliefs of every agent with regard to the types of the other agent are obtained by applying Bayes rule.

Define $\hat{q}_i(\theta_i, h_t)$ by

$$\hat{q}_i(\theta_i, h_t) \equiv E[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)].$$

for any $h_t \in H_t$ and any $t \in \{0, 1, .., T\}$. Condition (3) requires $\hat{q}_i(\theta_i, h_t)$ to be non-increasing in $\theta_i$ on $\Theta_i(h_t)$. Note that

$$\hat{q}_i(\theta_i, h(c(\theta_i, \theta_j))) = E_{\tilde{\theta}_j}[q_i(\theta_i, \tilde{\theta}_j) \mid \tilde{\theta}_j \in \Theta_j(h(c(\theta_i, \theta_j)))] = q_i(\theta_i, \theta_j),$$

since $q_i(\theta_i, \tilde{\theta}_j) = q_i(\theta_i, \theta_j)$ for any $\tilde{\theta}_j \in \Theta_j(h(c(\theta_i, \theta_j)))$.

**Step 1**: The relationship between $\hat{q}_i(\theta_i, h_t)$ and $\hat{q}_i(\theta_i, h_{t+1})$

Suppose that $i$ observes $h_t$ at the end of round $t$. Given selection of $m_{i,t+1} \in M_i(h_t)$ where $M_i(h_t)$ is the message set for $h_t$ in protocol $p$, agent $i$'s history at round $t+1$ is subsequently determined by messages received by $i$ in round $t$. Let the set of possible histories $h_{t+1}$ at the end of round $t+1$ be denoted by $H_{t+1}(h_t, m_{i,t+1})$. Evidently for $j \neq i$, $\{\Theta_j(h_{t+1}) | h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})\}$ constitutes a partition of $\Theta_j(h_t)$:

$$\cup_{h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})} \Theta_j(h_{t+1}) = \Theta_j(h_t)$$

and

$$\Theta_j(h_{t+1}) \cap \Theta_j(h_{t+1}^{'}) \neq \phi$$

for $h_{t+1}, h^{'}_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$ such that $h_{t+1} \neq h^{'}_{t+1}$. The probability of $h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$ conditional on $(h_t, m_{i,t+1})$ is represented by

$$\Pr(h_{t+1} \mid h_t, m_{i,t+1}) = \Pr(\Theta_j(h_{t+1}))/\Pr(\Theta_j(h_t)).$$

From the definition of $\hat{q}_i(\theta_i, h_t)$ and $\hat{q}_i(\theta_i, h_{t+1})$, for any $m_{i,t+1} \in M_i(h_t)$ and any $\theta_i \in \Theta_i$,

$$\Sigma_{h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})} \Pr(h_{t+1} \mid h_t, m_{i,t+1}) \hat{q}_i(\theta_i, h_{t+1}) = \hat{q}_i(\theta_i, h_t).$$

**Step 2**: For any $h_{t+1}, h^{'}_{t+1} \in H_{t+1}(h_t, m_{it+1})$, $\Theta_i(h_{t+1}) = \Theta_i(h^{'}_{t+1}) \subset \Theta_i(h_t)$

By definition

$$\Theta_i(h_{t+1}) = \{\theta_i \mid m_{i,t+1}(\theta_i, h_t) = m_{i,t+1}\} \cap \Theta_{it}(h_t)$$

where $m_{i,t+1}(\theta_i, h_t)$ denotes $i$'s message choice corresponding to the strategy $c_i(\theta_i)$. The right hand side depends only on $m_{i,t+1}$ and $h_t$. It implies that the set $\Theta_i(h_{t+1})$ does not vary across different $h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$. To simplify exposition, we denote this set henceforth by $\Theta_i(h_t, m_{i,t+1})$.

**Step 3**: Construction of $\tilde{q}_i(\theta_i, h_t)$

We construct $\tilde{q}_i(\theta_i, h_t)$ for any $h_t \in H_t$ based on the following Claim 1.

**Claim 1:**

For arbitrary $q_i(\theta_i, \theta_j)$ satisfying (3), there exists $\tilde{q}_i(\theta_i, h_t)$ for any $h_t \in H_t$ and any $t \in \{0, .., T\}$ so that

(a) $\tilde{q}_i(\theta_i, h_t) = \hat{q}_i(\theta_i, h_t)$ for $\theta_i \in \Theta_i(h_t)$

(b) $\tilde{q}_i(\theta_i, h_t)$ is non-increasing in $\theta_i$ on $\Theta_i$

(c) $\Sigma_{h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})} \Pr(h_{t+1} \mid h_t, m_{i,t+1}) \tilde{q}_i(\theta_i, h_{t+1}) = \tilde{q}_i(\theta_i, h_t)$ for any $\theta_i \in \Theta_i$ and any $m_{i,t+1} \in M_i(h_t)$ where $M_i(h_t)$ is the message set for $h_t$ in protocol $p$.

Claim 1 states that there exists an 'auxiliary' output rule $\tilde{q}_i$ as a function of type $\theta_i$ and message history which is globally non-increasing in type (property (b)) following any history $h_t$, and $\tilde{q}_i(\theta_i, h_t)$ equals the expected value of $\tilde{q}_i(\theta_i, h_{t+1})$ conditional on $(h_t, m_{it+1})$ for any $m_{it+1} \in M_i(h_t)$ (property (c)).

In order to establish Claim 1, the following Lemma is needed.

**Lemma 2** *For any $B \subset \Re_+$ which may not be connected, let $A$ be an interval satisfying $B \subset A$. Suppose that $F_i(a)$ for $i = 1, ..., N$ and $G(a)$ are real-valued functions defined on $A$, each of which has the following properties:*

- *$F_i(a)$ is non-increasing in $a$ on $B$ for any $i$.*
- *$\Sigma_i p_i F_i(a) = G(a)$ for any $a \in B$ and for some $p_i$ so that $p_i > 0$ and $\Sigma_i p_i = 1$.*
- *$G(a)$ is non-increasing in $a$ on $A$.*

*Then we can construct real-valued function $\bar{F}_i(a)$ defined on $A$ for any $i$ so that*

- *$\bar{F}_i(a) = F_i(a)$ on $a \in B$ for any $i$.*
- *$\Sigma_i p_i \bar{F}_i(a) = G(a)$ for any $a \in A$ and for the same $p_i$*
- *$\bar{F}_i(a)$ is non-increasing in $a$ on $A$ for any $i$.*

This lemma says that we can construct functions $\bar{F}_i(a)$ so that the properties of functions $F_i(a)$ on $B$ are also maintained on the interval $A$ which covers $B$.

*Proof of Lemma 2:*

If this statement is true for $N = 2$, we can easily show that this also holds for any $N \geq 2$. Suppose that this is true for $N = 2$.

$$\Sigma_{i=1}^{N} p_i F_i(a) = p_1 F_1(a) + (p_2 + ... + p_N) F^{-1}(a)$$

with

$$F^{-1}(a) = \Sigma_{i \neq 1} \frac{p_i}{p_2 + ... + p_N} F_i(a).$$

Applying this statement for $N = 2$, we can construct $\bar{F}_1(a)$ and $\bar{F}^{-1}(a)$ which keeps the same property on $A$ as on $B$. Next using the constructed $\bar{F}^{-1}(a)$ instead of $G(a)$, we can apply the statement for $N = 2$ again to construct desirable $\bar{F}_2(a)$ and $\bar{F}^{-2}(a)$ on $A$ based on $F_2(a)$ and $F^{-2}(a)$ which satisfy

$$\frac{p_2}{p_2 + ... + p_N} F_2(a) + [1 - \frac{p_2}{p_2 + ... + p_N}] F^{-2}(a) = F^{-1}(a).$$

on $B$. We can use this method recursively to construct $\bar{F}_i(a)$ for all $i$.

Next let us show that the statement is true for $N = 2$. For $a \in A \backslash B$, define $\underline{a}(a)$ and $\bar{a}(a)$, if they exist, so that

$$\underline{a}(a) \equiv \sup\{a^{'} \in B \mid a^{'} < a\}$$

and

$$\bar{a}(a) \equiv \inf\{a^{'} \in B \mid a^{'} > a\}.$$

It is obvious that at least one of either $\underline{a}(a)$ or $\bar{a}(a)$ exists for any $a \in A \backslash B$.
Let's specify $\bar{F}_1(a)$ and $\bar{F}_2(a)$ so that $\bar{F}_1(a) = F_1(a)$ and $\bar{F}_2(a) = F_2(a)$ for $a \in B$, and for $a \in A \backslash B$ as follows.

(i) For $a \in A \backslash B$ so that only $\underline{a}(a)$ exists,

$$\bar{F}_1(a) = F_1(\underline{a}(a))$$

$$\bar{F}_2(a) = \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}$$

(ii)For $a \in A \backslash B$ so that both $\underline{a}(a)$ and $\bar{a}(a)$ exist,

$$\bar{F}_1(a) = \min\{F_1(\underline{a}(a)), \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1}\}$$

$$\bar{F}_2(a) = \max\{F_2(\bar{a}(a)), \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}\}$$

(iii)For $a \in A \backslash B$ so that only $\bar{a}(a)$ exists,

$$\bar{F}_1(a) = \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1}$$

$$\bar{F}_2(a) = F_2(\bar{a}(a))$$

It is easy to check that $\bar{F}_i(a)$ is non-increasing in $a$ on $A$ for $i = 1, 2$ and

$$p_1 \bar{F}_1(a) + p_2 \bar{F}_2(a) = G(a)$$

for $a \in A$. This completes the proof of the lemma. ■

*Proof of Claim 1:*

Choose arbitrary $t \in \{0, ..., T\}$ and $h_t \in H_t$. Suppose that $\tilde{q}_i(\theta_i, h_t)$ satisfies (a) and (b) in Claim 1. Then for any $m_{i,t+1} \in M_i(h_t)$, we can construct a function $\tilde{q}_i(\theta_i, h_{t+1})$ for any $h_{t+1} \in H_t(h_t, m_{it+1})$ so that (a), (b) and (c) are satisfied. This result is obtained upon applying Lemma 2 with

$$B = \Theta_i(h_t, m_{i,t+1})$$

$$A = \Theta_i$$

$$
\begin{aligned}
a &= \theta_i \\
G(\theta_i) &= \hat{q}_i(\theta_i, h_t) \\
F_{h_{t+1}}(\theta_i) &= \hat{q}_i(\theta_i, h_{t+1}) \\
p_{h_{t+1}} &= \frac{\Pr(\Theta_j(h_{t+1}))}{\Pr(\Theta_j(h_t))}
\end{aligned}
$$

for any $h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$ where each element of the set $H_{t+1}(h_t, m_{i,t+1})$ corresponds to an element of the set $\{1, ..., N\}$ in Lemma 2. This means that for $\tilde{q}_i(\theta_i, h_t)$ which satisfies (a) and (b) for any $h_t \in H_t$, we can construct $\tilde{q}_i(\theta_i, h_{t+1})$ which satisfies (a)-(c) for any $h_{t+1} \in H_{t+1}$.

With $h_0 = \phi$, since $\tilde{q}_i(\theta_i, h_0) = \hat{q}_i(\theta_i, h_0)$ satisfies (a) and (b), $\tilde{q}_i(\theta_i, h_1)$ is constructed so that (a)-(c) are satisfied for any $h_1 \in H_1$. Recursively $\tilde{q}_i(\theta_i, h_t)$ can be constructed for any $h_t \in \cup_{\tau=0}^{T} H_\tau$ so that (a)-(c) are satisfied. ■

**Step 4**

We are now in a position to complete the proof of sufficiency. We focus initially on the case where the mechanism is decentralized so agents select their own outputs independently.

Given $\tilde{q}_i(\theta_i, h)$ (with $h = h_T$) constructed in Claim 1, construct transfer functions $t_i(q_i, h)$ as follows:

$$t_i(q_i, h) = \hat{\theta}_i(q_i, h) q_i + \int_{\hat{\theta}_i(q_i,h)}^{\bar{\theta}_i} \tilde{q}_i(x, h) dx.$$

for $q_i \in Q_i(h) \equiv \{\tilde{q}_i(\theta_i, h) \mid \theta_i \in \Theta_i\}$, and $t_i(q_i, h) = -\infty$ for $q_i \notin Q_i(h)$ where $\hat{\theta}_i(q_i, h)$ is defined as follows:

$$\hat{\theta}_i(q_i, h) \equiv \sup\{\theta_i \in \Theta_i \mid \tilde{q}_i(\theta_i, h) \geq q_i\}.$$

We show that the specified communication strategies $c(\theta)$ and output choices $(\tilde{q}_i(\theta_i, h), \tilde{q}_j(\theta_j, h))$ constitute a PBE (combined with beliefs obtained by applying Bayes rule at every history). By construction, $\tilde{q}_i(\theta_i, h)$ maximizes $t_i(q_i, h) - \theta_i q_i$ for any $h \in \mathcal{H} \equiv H_T$ and any $\theta_i \in \Theta_i$, where

$$t_i(\tilde{q}_i(\theta_i, h), h) - \theta_i \tilde{q}_i(\theta_i, h) = \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(x, h) dx.$$

Now turn to the choice of messages. Start with round $T$. Choose arbitrary $h_{T-1} \in H_{T-1}$ and arbitrary $m_{iT} \in M_i(h_{T-1})$. The expected payoff conditional on $\theta_j \in \Theta_j(h_{T-1})$ (i.e., conditional on beliefs given by $\Pr(h \mid h_{T-1}, m_{iT}) = \frac{\Pr(\Theta_j(h))}{\Pr(\Theta_j(h_{T-1}))}$ for $h \in H_T(h_{T-1}, m_{iT}))$ is

$$\begin{aligned} & E_h[t_i(\tilde{q}_i(\theta_i, h), h) - \theta_i \tilde{q}_i(\theta_i, h) \mid h_{T-1}, m_{iT}] \\ = & \int_{\theta_i}^{\bar{\theta}_i} E_h[\tilde{q}_i(x, h) \mid h_{T-1}, m_{iT}] dx \\ = & \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(x, h_{T-1}) dx. \end{aligned}$$

This does not depend on the choice of $m_{iT} \in M_i(h_{T-1})$. Therefore agent $i$ does not have an incentive to deviate from $m_{iT} = m_{iT}(\theta_i, h_{T-1})$.

The same argument can recursively be applied for all previous rounds $t$, implying that $m_{i,t+1} = m_{i,t+1}(\theta_i, h_t)$ is an optimal message choice for any $h_t \in H_t$ and any $t$. It is also evident that at round 0, it is optimal for agent $i$ to accept the contract. This establishes that participation, followed by the communication strategies $c(\theta)$ combined with output choices $(\tilde{q}_i(\theta_i, h), \tilde{q}_j(\theta_j, h))$ constitute a PBE.

The same argument applies to a centralized mechanism, since this is a special case of the previous mechanism where the assigned outputs $\hat{q}_i(\theta_i, h) = \hat{q}_i(h)$ are measurable with respect to $h$, i.e., are independent of $\theta_i$ conditional on $h$. Then $\tilde{Q}_i(h) \equiv \{\tilde{q}_i(\theta_i, h) \mid \theta_i \in \Theta_i(h)\} = \hat{q}_i(h)$. Agent $i$ can effectively be forced to choose output $\hat{q}_i(h)$ following history $h$ at the end of the communication phase with a transfer $\hat{t}_i(q_i, h)$. ■

**Proof of Proposition 2**:

We show that the solution of the relaxed problem where (ii) is dropped satisfies (ii). Suppose not. Let the solution of the relaxed problem be represented by a (parsimonious) communication protocol $p$, communication strategies $c(\theta)$ and output allocation $(q_1(\theta_1, \theta_2), q_2(\theta_1, \theta_2))$. $H_t$, $\Theta_i(h_t)$ and $\Theta_j(h_t)$ are well defined for $(p, c(\theta))$. Then there exists $t \in \{0, \ldots, T\}$, $h_t \in H_t$ and $\theta_i, \theta_i^{'} \in \Theta_i(h_t)$ with $\theta_i > \theta_i^{'}$ so that

$$E_{\theta_j}[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)] > E_{\theta_j}[q_i(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_t)].$$

This implies that at least either one of

$$\begin{aligned} & E[V(q_i(\theta_i^{'}, \theta_j), q_j(\theta_i^{'}, \theta_j)) - w_i(\theta_i) q_i(\theta_i^{'}, \theta_j) - w_j(\theta_j) q_j(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_t)] \\ > \; & E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i) q_i(\theta_i, \theta_j) - w_j(\theta_j) q_j(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)] \end{aligned}$$

or

$$\begin{aligned} & E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i^{'}) q_i(\theta_i, \theta_j) - w_j(\theta_j) q_j(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)] \\ > \; & E[V(q_i(\theta_i^{'}, \theta_j), q_j(\theta_i^{'}, \theta_j)) - w_i(\theta_i^{'}) q_i(\theta_i^{'}, \theta_j) - w_j(\theta_j) q_j(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_t)] \end{aligned}$$

holds. This means that if at least one type of either $\theta_i$ or $\theta_i^{'}$ takes other type of communication plan and output decision rule, the Principal's payoff is improved. This is a contradiction. ■

**Proof of Proposition 3:**

Consider any communication-feasible centralized mechanism with protocol $p$ and communication strategies $c(\theta)$ that result in an output allocation $q^*(\theta) = q(h(c(\theta)))$. Consider any history $h$ that arises from these communication strategies with positive probability, and let the corresponding set of types be $\Theta_i(h) \times \Theta_j(h)$. Then $q^*(\theta)$ must be constant over $\Theta_i(h) \times \Theta_j(h)$.

For arbitrary $q_i$, denote

$$E[V(q_i, q_j^*(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h)]$$

by $V(q_i, q_j(h))$. Consider the problem of choosing $q_i$ to maximize $V(q_i, q_j(h)) - w_i(\theta_i)q_i$ for any $\theta_i \in \Theta_i(h)$. It is evident that the function $V(q_i, q_j(h))$ is strictly concave in $q_i$, and satisfies the Inada condition. Given the monotonicity of $w_i(\theta_i)$, the optimal solution to this problem, denoted by $\hat{q}_i(\theta_i, h)$, is strictly decreasing in $\theta_i$ on $\Theta_i(h)$. Hence

$$\begin{aligned} & E[V(\hat{q}_i(\theta_i, h), q_j^*(\theta)) - w_i(\theta_i)\hat{q}_i(\theta_i, h) - w_j(\theta_j)q_j^*(\theta) \mid (\theta_i, \theta_j) \in \Theta_i(h) \times \Theta_j(h)] \\ > \;& E[V(q^*(\theta)) - w_i(\theta_i)q_i^*(\theta) - w_j(\theta_j)q_j^*(\theta) \mid (\theta_i, \theta_j) \in \Theta_i(h) \times \Theta_j(h)]. \end{aligned}$$

Now replace the output allocation $(q_i^*(\theta), q_j^*(\theta))$ by $(\hat{q}_i(\theta_i, h(c(\theta))), q_j^*(\theta))$ over $\Theta_i(h) \times \Theta_j(h)$, while leaving it unchanged everywhere else. This is a decentralized mechanism which is communication-feasible, which attains a strictly higher expected payoff for the Principal compared with the centralized mechanism. ■

**Proof of Proposition 4**: Suppose there is a round $t$ and history $h_{t-1}$ with $M_i(h_{t-1}) \neq \phi$ and $M_i(h_{t-1}) \neq M_i^*$ for some agent $i$. Without loss of generality, let $n_i \equiv \bar{l}(M_i(h_{t-1})) \geq n_j \equiv \bar{l}(M_j(h_{t-1}))$, and $n_i \geq 1$ (otherwise both agents have null message sets and the round can be deleted).

Following history $h_{t-1}$, we replace round $t$ with rounds $t, t+1, \ldots, t+n_i-1$ with message set $M_i^*$ for $i$ in each of these rounds, and message set $M_j^*$ for $j$ in rounds $t, t+1, \ldots, t+n_j-1$. Agent $j$ is assigned a null message set in rounds $t+n_j, \ldots t+n_i-1$ if $n_i > n_j$. Then notice by construction that

$$\bar{l}(M_k(h_{t-1})) = n_k = n_k\bar{l}(M_k^*) \tag{13}$$

for both agents $k = i, j$, implying that aggregate capacity cost or delay will remain unchanged. Moreover for agent $i$ we have

$$\begin{aligned} \#M_i(h_{t-1}) &\leq \#\{m_i \in \mathcal{M}_i | l(m_i) \leq n_i\} \\ &\leq 1 + L_i + \ldots + (L_i)^{n_i} \\ &< (1 + L_i)^{n_i} = \{\#M_i^*\}^{n_i} \end{aligned} \tag{14}$$

if $n_i \geq 2$, while

$$\#M_j(h_{t-1}) \leq 1 + L_j + \ldots + (L_j)^{n_j} \leq (1 + L_j)^{n_j} = \{\#M_j^*\}^{n_j}. \tag{15}$$

If $n_i = 1$ then $M_i(h_{t-1})$ is a proper subset of $M_i^*$ and $\#M_i(h_{t-1}) < \#M_i^*$. Hence the set of messages available to each agent is now larger for both, and is strictly larger for agent $i$. So for either agent $k = i, j$ we can select $\hat{M}_k$ which is a subset of $(M_k^*)^{n_k}$ such that $\#\hat{M}_k = \#M_k(h_{t-1})$ and for agent $i$ it is a proper subset. In other words, there exists $\tilde{m}_i \in (M_i^*)^{n_i} \backslash \hat{M}_i$. For each $k = i, j$ we can select a one-to-one mapping $\mu_k$ from $M_k(h_{t-1})$ to $\hat{M}_k$ such that $l(\mu_k(m_k)) = l(m_k)$ for all $m_k \in M_k(h_{t-1})$. Also $l(\tilde{m}_i) \leq n_i = \bar{l}(M_i(h_{t-1}))$, so there exists $\bar{m}_i \in M_i(h_{t-1})$ such that $l(\bar{m}_i) = n_i \geq l(\tilde{m}_i)$.

Given any choice of a subset $\Theta_i^{'}$ of $\Theta_i(h_{t-1}, \bar{m}_i)$, we can construct communication plans for different types of $i$ in rounds $t, \ldots, t + n_i - 1$ as follows:

(a) If $\theta_i \in \Theta_i^{'}$ then type $\theta_i$ of $i$ reports $\tilde{m}_i$ instead of $\bar{m}_i$

(b) If $\theta_i \in \Theta_i(h_{t-1}, \bar{m}_i) \backslash \{\Theta_i^{'}\}$, type $\theta_i$ reports $\bar{m}_i$, as before

(c) If $\theta_i$ does not belong to $\Theta_i(h_{t-1}, \bar{m}_i)$ and $\theta_i$ reported $m_i \in M_i(h_{t-1})$ previously, she now selects the vector of reports $\mu_i(m_i) \in \hat{M}_i$ across the new $n_i$ rounds.

We shall describe later in the proof the method for selecting the subset $\Theta_i^{'}$.

The communication strategy for $j$ is adapted to the following. If type $\theta_j$ reported $m_j \in M_j(h_{t-1})$ in round $t$ in the previous protocol, she now selects the vector of reports $\mu_j(m_j) \in \hat{M}_j$ in rounds $t, \ldots, t + n_j - 1$.

From round $t+n_i$ onwards, the continuation of the protocol and communication strategies exactly replicates the previous protocol and communication strategies from round $t + 1$ onwards, with the continuation following $\mu_i(m_i), \mu_j(m_j)$ in the new protocol exactly matching the continuation following messages $m_i \in M_i(h_{t-1}), m_j \in M_j(h_{t-1})$ in the old protocol. Moreover, the continuation following $\tilde{m}_i, \mu_j(m_j)$ in the new protocol matches the continuation following messages $\bar{m}_i, m_j$ in the old protocol.

By construction, then, total cost of communication capacity and delay is maintained the same. The variable material cost has not increased (since $l(\tilde{m}_i) \leq l(\bar{m}_i)$ while the length of all other messages has remained the same). On the other hand, the set of available messages has expanded for each agent, and strictly for agent $i$.

It remains to describe how the set $\Theta_i^{'}$ is chosen. Consider any history $h_T$ till the end of the communication phase which is a continuation of $(h_{t-1}, \bar{m}_i)$ which arises with positive probability in the previous protocol. Following history $h_T$, agent $j$'s information about $\theta_i$ is that it is contained in $\Theta_i(h_T)$ which is a non-degenerate interval of $\Theta_i$, and is a subset of $\Theta_i(h_{t-1}, \bar{m}_i)$. Now for any $\hat{\theta}_j$ in the interior of $\Theta_j(h_T)$, we can find a subset $\Theta_i^{'}$ of $\Theta_i(h_T)$ such that both $\Theta_i^{'}$ and $\Theta_i(h_T)\backslash\{\Theta_i^{'}\}$ are non-degenerate, and

$$E[V_{q_j}(q_i(\theta_i, \hat{\theta}_j), q_j(\theta_i, \hat{\theta}_j))|\theta_i \in \Theta_i^{'}] > E[V_{q_j}(q_i(\theta_i, \hat{\theta}_j), q_j(\theta_i, \hat{\theta}_j))|\theta_i \in \Theta_i(h_T)\backslash\{\Theta_i^{'}\}] \quad (16)$$

since $V_{12} \neq 0$ and $q_i$ is strictly decreasing in $\theta_i$ over $\Theta_i(h_T)$. Since this inequality is strict, and since the production decision functions are continuous under the postulated regularity properties on $V$, it must also hold in a non-degenerate neighborhood of $\hat{\theta}_j$. This implies that optimal production decisions must change with positive probability.

Agent $i$'s information about $j$'s type remains unchanged in the new protocol. And agent $j$ has strictly better information in the new protocol concerning $i$'s type following history $h_T$. This information is strictly valuable as the agents must change their production decisions with positive probability. Hence the Principal can secure a strict improvement in her expected payoff. ■

**Proof of Proposition 5:** Given Proposition 4, we can restrict attention to the protocol where any non-null message set assigned to agent $i$ is $M_i^*$ in every round. To show (i), suppose there exists round $t$ and $h_{t-1} \in H_{t-1}$ such that $M_k(h_{t-1}) = M_k^*$ for both agents $k = i, j$. Then consider a new communication protocol $\tilde{p}$ where round $t$ (following history $h_{t-1}$) is split into two successive rounds with sequential communication: in the first, $i$ has a message set $M_i^*$ while $j$ is assigned a null message set, and in the second $j$ has a message set $M_j^*$ while $i$ is assigned a null message set. Each agent can send the same message as they did in the previous protocol when it is their turn to report. From the next round onwards the rest of the protocol continues as before. This modification does not raise total material cost (although it evidently raises total delay). In this protocol , $j$ can send messages which can depend on $m_{it}$, something that is not possible in $p$. Hence it allows a weak improvement in the Principal's payoff.

For (ii), suppose that there exists round $t$ and $h_{t-1} \in H_{t-1}$ such that $M_i(h_{t-1}) = M_i^*$ and $M_j(h_{t-1}) = \{\phi\}$. We can now construct a new communication protocol $\tilde{p}$ with $M_j(h_{t-1}) = M_i^*$ instead of $\{\phi\}$ in round $t$ (with history $h_{t-1}$). All other components of the communication protocol are preserved. This modification does not raise the total time delay (although it raises the total material cost). Here agent $j$ who was silent in round $t$ following history $h_{t-1}$ in $p$ can now send some messages in this round, thus increasing the amount of information exchanged between the agents. Using the same argument as in the proof of Proposition 4, the Principal's payoff can be strictly improved. ■

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