MIDDLEMEN MARGINS AND GLOBALIZATION

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Abstract

We study a competitive theory of middlemen entrepreneurs with brand-name reputations necessary to overcome product quality moral hazard problems. Agents with heterogeneous entrepreneurial abilities sort into different sectors and occupations. Middleman margins do not equalize across sectors if production of their respective goods are differentially prone to moral hazard. In a Heckscher-Ohlin setting of North-South trade, trade liberalization then increases both North-South factor price differences and inequality of returns between middlemen and producers in Southern export sectors. Offshoring has opposite effects on the latter. In other cases, middleman margins are equalized across sectors and classical results of trade theory hold.

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1 Introduction

Conventional trade theory focuses mainly on sources of production costs, ignoring the role of endogenous marketing costs and margins that accrue to trade intermediaries. Yet there is considerable evidence of the importance of intermediaries and associated markups that drive large wedges between consumer and producer prices. For instance, Feenstra (1998) provides the following breakdown of the $10 retail price of a Barbie doll sold to US customers: 35 cents in wages paid to Chinese labor, material costs of 65 cents, $1 incurred for transportation, profits and overhead by Hong Kong intermediaries, and at least $1 return net of transport and distribution costs to Mattel, the US retailer. Arndt et al. (2000) estimate middleman markups of 111% in food crops, 52% in export crops, 59% in food processing and 36% in textile and leather in Mozambique. Fafchamps and Hill (2008), McMillan, Rodrik and Welch (2002) and Nicita (2004) estimate rates of pass-through less than 50% from border prices to producer prices in the case of Ugandan coffee, Mozambique cashews and a range of Mexican agricultural goods respectively. These facts suggest that only a small fraction of the benefits of export growth in developing countries following trade liberalization trickle down to farmers and workers. Consequently globalization may achieve limited impacts on poverty reduction and increase inequality in developing countries (Hertel and Winters (2005), Winters, McCulloch and McKay (2004)), contrary to the predictions of classic trade theory.

These observations motivate our interest in a theory which explains the role of middlemen in trade, which can be used to examine determinants of middleman margins, and subsequently predict distributive impacts of trade liberalization or offshoring. In this paper we explore a competitive equilibrium model in which brand-name reputations of middlemen are needed to overcome product quality moral hazard problems. Middleman margins represent reputational rents, rather than returns to market power resulting from technological increasing returns (in transport, storage or distribution). There is considerable evidence of the role of brand names and reputation in the context of trade (elaborated further in Section 5.1), ranging from consumer studies (Berges and Casellas (2007), Roth and Romeo (1992)); accounts of the role of trust and long-term relational contracting in international trade (Rauch (2001)), and econometric analyses of specific traded goods (Banerjee and Duflo (2000), Dalton and Goksel (2009), Machiavello (2010)).

Our model of middlemen builds on the theory of Biglaiser and Friedman (1994), and extends it to a general equilibrium theory of occupational choice. Middlemen are viewed as entrepreneurs who ‘manage’ enterprises, where the term ‘management’ comprises a composite of activities such as procuring goods produced by suppliers or workers, financing working capital requirements, supervising workers, and marketing the product. Repu-
tational markups form part of returns accruing to middlemen. These rents generate requisite incentives to maintain quality, since they would be sacrificed by middlemen in the event of losing their reputation. The size of these rents are proportional to the size of the enterprise they manage, which in turn is correlated with their underlying ability to ‘manage’.

Agents in the economy differ in their management ability; we take the distribution of ability as a parameter of the model. As in Lucas (1978), ability could be viewed as reflecting innate capacities to supervise workers or market products. Alternatively, ability differences could reflect differences in wealth which determine access to credit, as in theories of occupational choice based on credit market imperfections (Banerjee and Newman (1993)).

Agents with heterogeneous ability sort themselves into different occupations and sectors. Only those with ability above some (endogenously determined) threshold satisfy the incentive compatibility conditions for credible quality assurance in any given sector. Low ability agents have no choice but to do manual production work; they lack the reputation necessary to sell directly to final consumers. High ability agents become middlemen entrepreneurs in sectors where they meet required ability (i.e., size) thresholds. In order to focus exclusively on the nonconvexities arising due to reputations, we assume that the underlying production technology satisfies constant returns. Moreover, all agents are price takers. So middleman margins represent competitively determined incentive rents, rather than market power.

We embed this model in an otherwise standard Heckscher-Ohlin model of North-South trade, and explore the resulting implications for classical results of this theory concerning patterns of comparative advantage, Stolper-Samuelson, North-South factor price differences or welfare effects of trade liberalization. In particular we examine the consequences of differences in relative factor endowments alone across countries, rather than differences in technology or preferences. Our model is constructed in a way to allow distributional effects within any given sector to be summarized by a single measure of relative returns to entrepreneurial ability and production workers. Identifying the managerial services provided by middlemen as ‘skilled’ labor, and production work as ‘unskilled’ labor, this distributional variable can be interpreted as a ‘skill premium’.

Our main finding is that the nature of equilibria and their comparative static properties depend critically on the extent to which the severity of the moral hazard problem differs across sectors. When proneness to moral hazard differs markedly, many classical results of trade theory do not apply. In this case, our model generates discontinuous increases in returns to entrepreneurial ability at thresholds corresponding to entry into lucrative sectors. This is in contrast to existing models of sorting of heterogenous workers into different sectors. Shifts in consumer towards goods with high moral hazard necessitate increasing middleman margins in those sectors, in
order to induce entry of entrepreneurs less able than existing incumbents. Hence there is an increase in intra-sectoral skill premia, i.e., inequality between middlemen and producers. If developing countries have a comparative advantage in goods that are more prone to moral hazard, trade liberalization will increase middleman margins in the export sector and lower them in the import competing sector. Inequality within the export sector will increase, and decrease within the import competing sector. Average skill premia in the economy as a whole will rise if the export sector is large enough. Hence the Stolper-Samuelson Theorem will not apply.

Nor will factor returns be equalized across countries: a higher endowment of skill in Northern countries results in lower skill premia in the North. Trade liberalization accentuates these differences in skill premia: with an expansion of the export sector if it is more prone to moral hazard, middlemen margins in these sectors grow in the South causing Southern skill premia to rise further. Conversely, skill premia fall in the North owing to growing imports of the good more prone to moral hazard. Aggregate welfare effects of trade liberalization are ambiguous in general, owing to the pecuniary externalities associated with movement of entrepreneurs across sectors.

In this setting trade liberalization increases incentives for Northern entrepreneurs to offshore production to Southern countries. However the distributive effects of offshoring are qualitatively different from trade liberalization. Offshoring allows high ability entrepreneurs from the North to compete with Southern entrepreneurs for Southern workers, lowering intra-sectoral inequality in the South.

The preceding results obtain when the South has a comparative advantage in less skill-intensive goods which are more prone to moral hazard. In the opposite case where the less skill-intensive goods are less prone to moral hazard, similar anti-Stolper-Samuelson results also hold provided the elasticity of substitution between skilled and unskilled inputs is large enough.\(^5\) The level of the skill premium in the less skill intensive good exported by the South is now lower compared to the import competing sector. Nevertheless similar results concerning changes in skill premia obtain: trade liberalization continues to increase the share of middleman vis-a-vis producers in the export sector. The underlying reason is the same: less able entrepreneurs need to be induced to move into the export sector.

In the case where both sectors are equally prone to moral hazard, equilibria turn out to display qualitatively different properties.\(^6\) In this case, skill premia are equalized across sectors, and all classical results of standard

5The latter condition is additionally required in order to ensure that the relationship between skill premia in the two sectors needed to clear the factor market is downward sloping, in the case where they are unequal to start with. It ensures that changes in within-firm employment outweigh effects of cross-sector employment differences resulting from some entrepreneurs switching sectors.

6Similar results obtain when intersectoral differences in moral hazard are neutralized by TFP differences in the opposite direction, yielding equality of equilibrium skill premia.
Heckscher-Ohlin theory obtain. Entrepreneurs are indifferent between which sector to operate in, so they can switch into the export sector following trade liberalization despite a fall in the common skill premium in the economy. An increase in skill premium in the export sector is no longer necessary, and does not happen in equilibrium for exactly the same reason as in the standard theory.

It is hard to judge whether proneness to moral hazard differs substantially across export and import competing sectors, as there is relatively direct evidence concerning the extent of moral hazard in any given sector. Section 5.1 reviews the fragmentary evidence available concerning this. It is easier instead to check evidence concerning the division of product revenues between middleman margins and producers. Our theory indicates that non-classical results apply in cases where skill premia differ substantially across sectors. Available evidence from a number of developing countries (reviewed in Section 5.1) indicates that relative returns to middlemen are substantially higher in less skill-intensive goods which tend to be exported from the South. Moreover, there is some evidence consistent with the key mechanism underlying the anti-Stolper-Samuelson result in our theory. Fafchamps and Hill (2008) and McMillan, Rodrik and Welch (2002) show in the case of Ugandan coffee and Mozambique cashew exports respectively that increases in border prices were accompanied by widening inequality between middleman margins and farmgate prices, and entry of less efficient groups of middlemen into these sectors.

Our theory therefore has the potential to explain hitherto puzzling evidence concerning the distributive impact of trade integration on developing countries, wherein Stolper-Samuelson predictions have generally not been borne out. This literature (surveyed by Goldberg and Pavcnik (2007), Harrison, McLaren and McMillan (2010), Winters, McCulloch and McKay (2004) or Wood (1997)) shows increases in skill premia resulting in some contexts and decreases in others following trade liberalization. Our theory provides detailed predictions concerning conflicting effects on export and import-competing sectors, and the role of differences in skill premia across sectors, which could be tested in future research. The surveys of empirical evidence cited above also highlight the importance of context within which trade liberalization takes place, such as policy environment, infrastructure access or local institutions. Our theory provides one possible source of context-specificity pertaining to inequality and other determinants of entry barriers into entrepreneurship in specific sectors. Economies with highly polarized wealth distributions that lack a middle class are ones where there is limited scope for entry into lucrative sectors of trade intermediation. Our theory predicts that the output response of economy to trade liberalization will then be sluggish, with high increases in inequality (see Proposition 3, part (i)). This could potentially explain differences between effects of trade liberalization in Latin America and sub-Saharan Africa during the 1980s and 90s.
compared with East Asia during earlier decades that have been highlighted

The paper is organized as follows. Section 2 introduces the model of the
closed economy. Section 3 considers the case where the less skill-intensive
good is more prone to moral hazard. We first describe the equilibrium of
the supply-side, where product prices are taken as given. This is followed by
the economy-wide equilibrium, and the main comparative static properties.
We then extend it to a two country context and studies effects of trade lib-
eralization and offshoring. Section 4 discusses how preceding results extend
to the opposite case where the more skill-intensive good is more prone to
moral hazard. Section 5 describes relation to existing literature, both theo-
retical models as well as empirical evidence in more detail. Finally, Section
6 concludes, while proofs are collected in the Appendix.

2 Closed Economy Model

2.1 Endowment and Technology

We normalize the size of the population to unity. Each agent is characterized
by a level of entrepreneurial ability or skill \( a \), a nonnegative real variable.
We provide alternative interpretations of ‘skill’ below. A fraction \( 1 - \mu \)
of agents have no skill at all: \( a = 0 \): we refer to them as unskilled. The
remaining fraction \( \mu \) are skilled; the distribution of skill is given by a cdf
\( G(a) \) on \((0,1]\). We shall frequently use the notation \( d(a) \equiv \int_0^a \tilde{a} dG(\tilde{a}) \).
The cdf \( G \) will be assumed to have a density \( g \) which is positive-valued.
Then \( d \) is a strictly decreasing and differentiable function.

There are two goods \( L \) and \( H \), and two occupations: labor or produc-
tion work, and entrepreneurship. Each entrepreneur can operate at most one
firm, which produces any one of the two goods upon combining labor with
'managerial services' provided by the entrepreneur. Labor is hired on a com-
petitive market. There is no market for hiring managerial services, owing
to moral hazard problems not explicitly modeled here.\(^7\) Hence managerial
services must be self-supplied by the entrepreneur. Examples of these ser-
ices are: provision of essential raw materials, supervising and coordinating
production undertaken by hired workers, or marketing the product. These
correspond to alternative interpretations of entrepreneurial skill: if credit
markets are imperfect, skill can be interpreted as the entrepreneurs wealth

\(^7\)In practice, of course, firms may employ more than one manager in order to grow,
but problems of managerial moral hazard and coordination across managers eventually
limit the size of firms (as emphasized by a large literature on 'organizational diseconomies
of scale', e.g., Williamson (1967), Calvo and Wellisz (1978), Keren and Levhari (1983),
Qian (1994) or van Zandt and Radner (2001)). In order to explore the industry or general
equilibrium implications of limits to the size of firms created by such problems, we adopt
the simplifying assumption that a firm is managed by a single entrepreneur, similar to
which determines her ability to finance purchase of raw materials. Or skill may refer to the entrepreneurs ability to supervise workers, as in Lucas (1978).

Any given agent makes the following decisions: (a) whether to become an entrepreneur or worker; (b) if she decides to become an entrepreneur, she selects which good to produce; (c) how many workers to employ; and (d) one of two quality levels for the good to be produced. The production function for good \( i \) is \( X_i = A_i F_i(n_i, a) \) for the high quality version of the good, and \( A_i F_i(n_i, z_i a) \) for the low quality version, where \( z_i > 1 \) is a technology parameter representing the severity of the quality moral hazard problem, \( A_i \) is a TFP parameter, \( a \) denotes the skill of the entrepreneur, and \( n_i \) the units of labor hired. \( F_i \) is a CRS, smooth and strictly concave production function.

Producing lower quality enables an entrepreneur to produce a larger quantity of the good with the same number of workers, as it increases the number of effective units of skill.

In what follows we shall refer to production work as unskilled labor, and entrepreneurship as skilled labor. Note that neither unskilled or skilled labor is specific to either sector. Given the assumption that a firm can have only one manager, there will be no market for entrepreneurs: every entrepreneur works for herself, managing her own firm. Entrepreneurial rents will correspond to imputed prices of ‘skill’ which will be equalized across all agents, which clear the market for skill. In other words, optimal employment of production workers and an entrepreneur’s own skill will be the profit-maximizing factor combinations that would have been chosen by an ‘as if’ firm owner who pays for both unskilled and skilled inputs at the (imputed) factor prices, and ends up with zero profit. Returns earned by entrepreneurs in any given sector will be linear in their own skill. This allows a simple measure of the returns to skill within any sector.

Consider the cost-minimizing factor combinations in each sector, when skill is imputed a cost \( \gamma \) relative to unskilled labor: \( (\theta_n^H(\gamma), \theta_a^H(\gamma)) \) and \( (\theta_n^L(\gamma), \theta_a^L(\gamma)) \) are defined as

\[
(\theta_n^H(\gamma), \theta_a^H(\gamma)) \equiv \arg\min \{ \theta_n^H + \gamma \theta_a^H \mid F_H(\theta_n^H, \theta_a^H) = 1 \}
\]

and

\[
(\theta_n^L(\gamma), \theta_a^L(\gamma)) \equiv \arg\min \{ \theta_n^L + \gamma \theta_a^L \mid F_L(\theta_n^L, \theta_a^L) = 1 \}
\]

8In some cases we specialize to the case of a Leontief technology, a limiting case of such a technology.

9In the imperfect capital market interpretation, producing low quality goods requires fewer raw materials per unit of output, so corresponds to a higher number of effective units of entrepreneurial skill. The same happens in the production supervision interpretation, since producing a lower quality version requires less intensive supervision by the entrepreneur. An alternative formulation of the moral hazard problem would be one where lower quality goods are produced at lower cost (i.e., with fewer workers) rather than in higher quantity corresponding to a given number of workers. This is closely related to our formulation and the two versions coincide with a Leontief technology.
The following assumption states that good $L$ is more labor intensive than good $H$: for any common ratio of factor costs, production of $L$ uses a higher ratio of unskilled labor to skilled labor in the cost-minimizing factor choice. One can think of $L$ as corresponding to agricultural products or low-end manufactured goods, while $H$ corresponds to high-tech manufactured goods or services.

**Assumption 1** For any $\gamma > 0$,

$$\frac{\theta_L^L(\gamma)}{\theta_L^H(\gamma)} > \frac{\theta_H^L(\gamma)}{\theta_H^H(\gamma)}$$

### 2.2 Entrepreneur’s Profit Maximization and Equilibrium Price-Cost Relations

Consider an entrepreneur in sector $L$, facing a product price of $p_L$ (with the $H$ good acting as numeraire, so $p_H \equiv 1$). Suppose the wage rate for unskilled labor is $w$. If this entrepreneur were to decide to produce the high quality version of product $L$, she would solve the following problem:

$$\max_{n_L} p_L A_L F_L(n_L, a) - w n_L. \tag{1}$$

The optimal employment of unskilled labor $n_L^*$ is a function of $p_L/w$, characterized by the first-order condition

$$(p_L/w) A_L \frac{\partial F_L(n_L^*, a)}{\partial n_L} = 1. \tag{2}$$

Let $\Pi_L^*(p_L, w; a)$ denote the resulting level of profit earned by the entrepreneur. Define

$$\gamma_L \equiv \frac{\Pi_L^*}{w a} \tag{3}$$

the skill premium in the $L$-sector.

Using standard analysis of the profit-maximization conditions, we obtain the following price-cost relations in each sector (using $\gamma_i$ as the imputed price of skill in sector $i$):

$$\frac{p_L}{w} \leq \frac{\theta_L^L(\gamma_L) + \gamma_L \theta_L^H(\gamma_L)}{A_L} \text{ with } = \text{ if } X_L > 0. \tag{4}$$

$$\frac{1}{w} \leq \frac{\theta_H^L(\gamma_H) + \gamma_H \theta_H^H(\gamma_H)}{A_H} \text{ with } = \text{ if } X_H > 0. \tag{5}$$

The left-hand-side of the preceding conditions are the product wage in the two sectors respectively, which are decreasing functions of their respective skill premia. Hence the skill premium is the key measure of inequality between entrepreneurs and workers in any given sector.
Equation (4, 5) yields the following equation for ratio of prices of the two goods to their respective unit costs:

$$p_L = \frac{A_H}{A_L} \frac{\theta^L_n(\gamma_L) + \gamma_L \theta^L_a(\gamma_L)}{\theta^H_n(\gamma_H) + \gamma_H \theta^H_a(\gamma_H)}$$  \hspace{1cm} (6)

in the case where both goods are produced in positive quantities, with a corresponding inequality in the case of complete specialization. Note that the right-hand-side is increasing in the skill premium in sector $L$, and decreasing in the skill premium in sector $H$. Hence (6) expresses a relation between the skill premia in the two sectors, and the price $p_L$ of product $L$ relative to $H$. This can be expressed as follows:

$$\gamma_L = \lambda(\gamma_H; p_L, \frac{A_H}{A_L}).$$  \hspace{1cm} (7)

For any given product price $p_L$ and set of TFP parameters, it expresses a monotone increasing relation between the skill premia in the two sectors. And for any given $\gamma_H$ and TFP parameters, it expresses a monotone increasing relation between $p_L$ and $\gamma_L$.

Various properties of this relationship between skill premia in the two sectors will be used subsequently. For now we note one property which plays an important role in the subsequent analysis.

**Lemma 1**  \hspace{1cm} $\frac{\partial \gamma_L}{\partial \gamma_H} = \lambda_1(\gamma_H; p_L, \frac{A_H}{A_L}) > 1$ whenever $\gamma_L \geq \gamma_H$.

Assumption 1 plays an important role here. Since sector $L$ is less skill-intensive, an equal increase in the skill premium in the two sectors will cause unit cost in the $L$-sector to increase by less than in the $H$-sector. Hence the premium must rise by more in the $L$-sector to ensure that the ratio of unit costs remains the same.

### 2.3 Quality Moral Hazard Problem

Customers do not observe the quality of the product at the point of sale. We assume they value only the high quality version of the product, and obtain no utility from the low quality version. Entrepreneurs will be tempted to produce the low quality version which enables them to produce and sell more to unsuspecting customers. The short-run benefits of such opportunism can be held in check by possible loss of the seller’s reputation. With probability $\eta_i$, an entrepreneur selling a low-quality item in sector $i$ will be publicly exposed (say by a product inspection agency or by investigating journalists). In this event the entrepreneur’s brand-name reputation is destroyed, and the agent in question is forever barred from entrepreneurship. In equilibrium, customers will purchase only from entrepreneurs for whom the threatened loss of reputation is sufficient to deter short-term opportunism. Hence in
order for an entrepreneur with skill $a$ to be able to operate in sector $i$, the following incentive constraint must be satisfied:

$$\frac{\gamma_i w a}{1 - \delta} \geq \gamma_i w z_i a + \delta[\eta_i - \frac{w}{1 - \delta} + (1 - \eta_i) \gamma_i w a],$$

(8)

where $\delta \in (0, 1)$ denotes a common discount factor for all agents. The left-hand-side of (8) is the present value of producing and selling the high quality version of good $i$ for ever. The first term on the right-hand-side, $\gamma_i w z_i a$ represents the short-term profit that can be attained by the entrepreneur upon deviating to low quality.\(^{10}\) With probability $\eta_i$, this deviation results in the entrepreneur losing her reputation for ever, in which case the agent is forced to work as an unskilled agent thereafter. With the remaining probability the entrepreneur’s reputation remains intact.

The incentive constraint can be equivalently expressed as

$$a \geq m_i/\gamma_i$$

(9)

where

$$m_i = \frac{\delta \eta_i}{\delta \eta_i + (1 - \delta)(1 - z_i)} > 1$$

is a parameter representing the severity of the moral hazard problem in sector $i$.

Equation (9) represents a reputational economy of scale, which also translates into a sector-specific entry barrier in terms of entrepreneurial skill required. Intuitively, higher skilled entrepreneurs produce and earn profits at a higher scale (conditional on being able to operate as an entrepreneur), while the consequences of losing one’s reputation are independent of the level of skill. The stake involved in losing reputation is thus proportional to the entrepreneurs skill, which has to be large enough for the agent to be a credible seller of a high-quality good.\(^{11}\)

The sector-specific entry barriers represent elements of a specific factor model. However unlike most specific-factor models, these barriers are endogenously determined rather than exogenously imposed: e.g., the skill threshold for entry into a particular sector is decreasing in the skill premium in that sector. The reason is simple: a higher skill premium means entrepreneurs have more to lose from losing their reputations.

Note also that $m_i > 1$ implies that entrepreneurs with skills above the required threshold for sector $i$ will strictly prefer to be entrepreneurs in sector $i$ rather than work as an unskilled agent. The per period profit from the former option is $\gamma_i w a \geq w m_i > w$ if (9) is satisfied.

\(^{10}\)Recall that a deviation to low quality is equivalent to an increase in the entrepreneur’s effective skill from $a$ to $z_i a$.

\(^{11}\)We assume customers can infer quality from observing the size of the corresponding firm and existing prices, by checking whether the incentive constraint is satisfied.
The results will turn out to depend critically on the relative seriousness of the moral hazard problem across the two goods. For good \( i \) this is represented by \( m_i \), which is a function of exogenous parameters. We shall contrast two cases:

(A) \( m_L > m_H \): the \( L \)-good is more prone to moral hazard.

(B) \( m_L < m_H \): the \( H \)-good is more subject to moral hazard.

Quality moral hazard problems could be larger in the less skill-intensive good, owing to problems in quality control or regulation of these goods, and the relative lack of product warranties for farm or light manufactured goods compared with high-tech durable goods. The \( H \)-good is more durable; it is produced in a more automated and regulated production process which is easier to inspect. It thus allows less scope for skimping on labor or other essential raw material requirements. An offsetting factor would be the greater technological complexity of these goods, combining a larger number of components in the production process. This may lead to high costs of ensuring high quality, as emphasized in the O-ring theory of Kremer (1993). Owing to the absence of any concrete empirical evidence on which of these two cases is more plausible, the purpose of our analysis is to examine how the results differ between the two cases. We focus first on case A. A subsequent section explains how the results extend to Case B.

3 Case A: Where the \( L \)-good is More Prone to Moral Hazard

We break up the analysis of competitive equilibrium into two steps. First we take product prices as given, and derive the resulting equilibrium of factor markets: occupational choices and the market for production workers, which comprise the supply-side of the economy. At the second step we shall bring in consumer demands and thereby determine product prices.

\textit{Definition} Given \( p_L \) the price of good \( L \) relative to \( H \), a factor market equilibrium of the economy is a wage rate \( w \) and skill-premia \( \gamma_L, \gamma_H \) such that: (i) every agent takes these prices as given and selects between different occupations (unskilled worker, \( L \)-sector entrepreneur, \( H \)-sector entrepreneur) to maximize earnings subject to incentive constraints represented by (9); (ii) entrepreneurs within each sector select employment of production workers to maximize their profits; and (iii) the market for production workers clears.

The analysis of factor market equilibria proceeds as follows. Skill-premia in the two sectors define the entry thresholds into each sector, which determine the occupations that any given agent can feasibly choose from while
respecting the incentive constraints. The maximum profit that the agent can earn in any sector is the product of her own entrepreneurial skill, the wage rate and the skill premium in that sector. Agents select between occupational options to maximize their earnings. The allocation of agents across occupations combined with output prices and the wage rate determines demand for labor from entrepreneurs in each sector. The aggregate demand for labor must equal the supply of labor, i.e., the mass of agents that do not meet the entry thresholds for entrepreneurship in either sector. The equilibrium wage rate must in turn be consistent with the skill premia in the two sectors, satisfying the price-cost equations (4, 5) characterizing profit-maximization by entrepreneurs within each sector.

We shall represent the factor market equilibrium by the intersection of two conditions involving the skill-premia in the two sectors: one which corresponds to clearing of the factor markets, the other to the profit maximization condition (6). We start with the former.

First we take the skill premia in different sectors as given, and derive occupational choices of agents in the economy. The equilibrium of the factor market is illustrated graphically in Figure 1, in terms of relationship between skill premia in the two sectors. There are four different situations to consider, depending on the relation between skill premia and moral hazard parameters in different sectors.

Case A1: \( \gamma_L \geq \gamma_H \frac{m_L}{m_H} \)

Since we are in Case A and \( \frac{m_L}{m_H} > 1 \), it follows that in this situation \( \gamma_L > \gamma_H \) also holds. This implies that entrepreneurship in sector \( L \) is more profitable than in sector \( H \). The entry threshold for this sector is also lower, as \( \frac{m_L}{\gamma_L} < \frac{m_H}{\gamma_H} \). Hence all those with skill above \( \frac{m_L}{\gamma_L} \) will enter the \( L \)-sector, and those below will become production workers. The economy specializes in production of good \( L \). In Figure 1, this case corresponds to the range where \( \gamma_H < \gamma_L \).

Case A2: \( \gamma_H < \gamma_L < \gamma_H \frac{m_L}{m_H} \)

Here \( \gamma_L > \gamma_H \) implies that the \( L \)-sector is more profitable. On the other hand, the entry threshold is higher in the \( L \)-sector: \( a_L = m_L/\gamma_L > m_H/\gamma_H = a_H \). So agents with \( a \geq a_L \) will choose to become \( L \)-sector entrepreneurs, while agents with \( a \in [a_H, a_L) \) are unable to enter the \( L \)-sector and so have to be content with becoming \( H \)-sector entrepreneurs. And agents with \( a < a_H \) become workers.

Consider the relation between skill premia in the two sectors that must hold for the labor market to clear. This relationship is downward-sloping, because an increase in the skill premium in either sector tightens the labor market condition. To see this, note that a rise in \( \gamma_H \) lowers the entry threshold into the \( H \)-sector, and raises the demand for production workers.
for any given $H$-sector entrepreneur. And on the other hand a rise in the $L$-sector skill premium causes the skill threshold for the $L$-sector to fall, motivating some entrepreneurs to switch from the $H$ to the $L$-sector. By Assumption 1, and by hypothesis $\gamma_L > \gamma_H$, $L$-sector entrepreneurs demand more production workers than the $H$-sector entrepreneurs. So the switch of entrepreneurs between the two sectors tightens the labor market. This is accentuated by the rise in demand for workers by incumbent $L$-sector entrepreneurs.

This case will be of particular interest in the subsequent analysis since skill premia are not equalized across the two sectors. Entrepreneurs in the $H$-sector would prefer to locate in the $L$-sector but cannot because they cannot offer credible quality assurance if they were to produce the $L$-good. If this situation happens to prevail, the model ends up exhibiting features of a specific factor model, owing to the restrictions on the freedom of some entrepreneurs of intermediate ability to cross sectors. These restrictions arise endogenously in the model: changes in skill premia will cause entry thresholds to change, allowing some (but not all) entrepreneurs to move across sectors.

**Case A3**: $\gamma_H = \gamma_L$

$\gamma_L = \gamma_H = \gamma$, say, implies that entrepreneurs are indifferent between the two sectors. The $L$-sector is harder to enter, as $a_L = m_L / \gamma > m_H / \gamma = a_H$. Hence agents with $a \in [a_H, a_L)$ have no option but to enter sector $H$, while those with $a \geq a_L$ can enter either of the two sectors. The equilibrium in this case will involve a fraction of those with $a \geq a_L$ who will go to sector $L$, the remaining going to sector $H$. This fraction must be such as to permit the factor market to clear. This in turn translates into an upper and lower bound for the common skill premium $\gamma$, as shown in the proof of Lemma 2 of the Appendix.

This situation involves equal skill premia across the two sectors, thus corresponding to a non-specific factor setting. The relationship between the skill premia is upward-sloping (in contrast to Case A2): it coincides with part of the 45 degree line of equality in Figure 1.

**Case A4**: $\gamma_H > \gamma_L$

$\gamma_H > \gamma_L$ implies that sector $H$ is more profitable. Also the entry threshold in sector $H$ is lower. In this case no entrepreneur enters sector $L$. Those with skill $a \geq a_H$ enter sector $H$, the rest become workers. Hence the economy specializes in production of the $H$-good here.

Figure 1 shows the relationship between skill premia in the two sectors consistent with clearing of the factor market. For future reference, we shall denote this relationship by the equation

$$\gamma_L = \psi(\gamma_H).$$  \hspace{1cm} (10)
It can be checked (see the detailed proof of Lemma 2 presented in the Appendix) that this function depends on parameters $\mu, m_L, m_H$ but is independent of $p_L$ or TFP parameters $A_L, A_H$. This function is well defined for $\gamma_H < \gamma_3^H$, and is not a monotone relationship: it is decreasing below $\gamma_2^H$ but increasing thereafter. The downward-sloping part corresponding to Case A2 is the ‘non-classical’ region where skill premia are not equalized across sectors. The upward-sloping part corresponding to Case A3 coincides with the line of equality, so this is the ‘classical’ region where skill premia are equalized. The greater the relative severity $\frac{m_L}{m_H}$ of the moral hazard problem in the $L$-sector, the greater the range occupied by the non-classical region.

### 3.1 Factor Market Equilibrium

We are now in a position to characterize the factor market equilibrium, by putting together the condition that the labor market clears (which incorporates reputation effects, occupational and sectoral choices by entrepreneurs), with the relation between prices and costs representing profit maximization by active entrepreneurs in each sector.

The former is represented by the relation between skill premia that clears the factor markets. The latter is represented by the upward-sloping relation (6) between premia in the two sector for any given product price $p_L$. Geometrically it is represented by the intersection of the corresponding relations between the two skill premia. This is shown in Figure 2 for different values of $p_L$. 

---

Figure 1: Relation Between Skill Premia for Factor Market Clearing

---

\[
\gamma_L = \frac{m_L}{m_H} \gamma_H
\]
Lemma 2 Consider Case A, where the $L$-good is more prone to moral hazard: $m_L > m_H$.

(a) For any given $p_L > 0$, a factor market equilibrium exists and is unique.

(b) There exist thresholds $p_1^L > p_2^L > p_3^L$ such that:

(i) Below $p_3^L$ the economy specializes in producing good $H$ while above $p_1^L$ it specializes in good $L$. Between $p_1^L$ and $p_2^L$ both goods are produced.

(ii) When $p_L$ is between $p_2^L$ and $p_3^L$, skill premia are equalized in the two sectors (i.e., Case A3 arises).

(iii) When $p_L$ is between $p_1^L$ and $p_2^L$, the skill premium is strictly higher in the $L$-sector (i.e., Case A2 arises).

The distribution of income across agents with varying skills in case A2 is illustrated in Figure 3. Agents with skill below the entry threshold $a_H$ for the $H$-sector earn the unskilled wage $w$. Between the thresholds $a_H$ and $a_L$ for the two sectors, the agents are $H$-sector entrepreneurs, earning $\gamma_H w a_H$. By definition of the threshold $a_H = \frac{m_H}{\gamma_H}$, it follows that the earning of a $H$-sector entrepreneur at this threshold equals $w m_H$, which strictly exceeds $w$ as $m_H > 1$. Hence there is a discrete upward jump in earnings at the entry threshold for entrepreneurship. There is a similar discrete jump in earnings at the threshold $a_L$ for entry into the $L$-sector, owing to the difference in skill premia between the two sectors. The highest incomes accrue to entrepreneurs in the $L$-sector, who manage the largest firms in the economy. They are followed by $H$-sector entrepreneurs, who manage smaller firms, and finally workers who work as unskilled employees in both sectors.

3.2 Effect of Changes in Skill Endowment on Factor Market Equilibrium: Validity of the Rybczynski result

Now consider the effect of an increase in $\mu$, the proportion of agents in the economy with skills. As shown in Figure 4, the frontier between skill premia corresponding to the factor market-clearing condition (10) shifts inwards, owing to the resulting tightening of the labor market. Excepting the case that $\gamma_H = \gamma_L$ is maintained before and after the change in $\mu$, both skill premia fall. What is the effect on the relative production levels $X_L/X_H$? This determines the pattern of comparative advantage in an open economy setting, an issue addressed by the Rybczynski Theorem in classical trade theory.

Since the $H$-good is more skill-intensive, one would intuitively expect an increase in endowment of skilled labor in the economy to raise the production of $H$ relative to the $L$-good, an implication of the classical Rybczynski
\[ \gamma_L = \frac{m_L}{m_H} \]

Figure 2: Factor Market Equilibrium: Case A

Figure 3: Income Distribution across Agents with Varying Skills: Case A2
Figure 4: Effect of Increase in $\mu$ on Factor Market Equilibrium

theorem. This indeed is true in the ‘classical’ region A3 with equal skill premia in the two sectors. From (23) which is independent of $\mu$, it is evident that a rise in $\mu$ leaves the skill premium unchanged. Hence the entry thresholds into the two sectors and the demand for unskilled labor from each active entrepreneur of the same skill are unaffected. Since the $L$-sector is less skill-intensive, it follows that the production of the $L$-good must fall, in order to allow the labor market to clear.

In the ‘non-classical’ region corresponding to case A2, there will be an additional effect of a change in $\mu$ on skill premia in the two sectors. What turns out to matter is the change in the relative skill premia in the two sectors. An increase in $\mu$ tightens the labor market and thus tends to drive the unskilled wage higher. Since the $L$-sector is less skill-intensive, this tends to lower the skill premium in the $L$-sector by more than in the $H$-sector. However, the relative skill premium $\frac{\gamma_L}{\gamma_H}$ may still go up, if it was high enough to start with. In that case, we obtain a countervailing effect which can raise $\frac{X_L}{X_H}$.

To see this concretely, we consider the following example, where the density of the skill distribution does not fall too fast, and the production functions for both sectors have constant and equal elasticity of substitution between skilled and unskilled labor.

**Proposition 1** Consider Case A, and assume that $\frac{a^2g(a)}{d(a)}$ is increasing in $a$ and production function for $i = L, H$ exhibit constant, equal elasticity of substitution between skilled and unskilled labor.

\[\frac{d}{d\gamma_H}[\frac{\gamma_L}{\gamma_H}] = \frac{1}{\gamma_H} [\frac{d\gamma_L}{\gamma_H} - \frac{\gamma_L}{\gamma_H}] \text{ which is negative if } \frac{d\gamma_L}{\gamma_H} < \frac{\gamma_L}{\gamma_H}, \]

i.e., the initial value of the relative premium is high enough.

\[12\] Specifically, the tighter labor market tends to lower $\gamma_H$, and the effect on the relative premium $\frac{\gamma_L}{\gamma_H}$ of a change in $\gamma_H$ is $\frac{d[\frac{\gamma_L}{\gamma_H}]}{d\gamma_H} = \frac{1}{\gamma_H} [\frac{d\gamma_L}{\gamma_H} - \frac{\gamma_L}{\gamma_H}]$ which is negative if $\frac{d\gamma_L}{\gamma_H} < \frac{\gamma_L}{\gamma_H}$,
substitution $\kappa(\geq 0)$:

$$X_i = A_i F_i(n_i, a_i) = A_i \left( k_i^{1/\kappa} a_i^{\frac{n-1}{\kappa}} + n_i^{\frac{\kappa-1}{\kappa}} \right)^{\frac{\kappa}{\kappa-1}} \quad (11)$$

with $k_H > k_L$ (to ensure Assumption 1 is satisfied). Then in the factor market equilibrium:

(i) If $\kappa \geq 1 - \frac{\log[k_H/k_L]}{\log[m_H/m_L]}$, an increase in $\mu$ has the effect of decreasing $X_L/X_H$ for any $p_L \in (p^1_L, p^2_L)$.

(ii) If $0 \leq \kappa < 1 - \frac{\log[k_H/k_L]}{\log[m_H/m_L]}$, the increase in $\mu$ has the effect of reducing $X_L/X_H$ for any $p \in (p^3_L, A_H/A_L)$. Moreover, there exists $\bar{p}_L \in (A_H/A_L, p^1_L)$ such that $X_L/X_H$ is increasing in $\mu$ for any $p > \bar{p}_L$.

The proof is provided in the Appendix. The Proposition shows that $X_L/X_H$ falls if the elasticity of substitution is large (case (i)) and otherwise for values of $p_L$ below $A_H/A_L$, but not for values of $p_L$ close enough to $p^1_L$. In the latter case the relative skill premium in the $L$-sector is sufficiently high to start with that it increases as a result of the increase in skill endowment. This is strong enough to cause the relative production of the less skill-intensive good to rise. Figure 5 provides an illustration of the effect on $X_L/X_H$. In the context of the open economy, this will provide an instance where the Leontief paradox appears, if a North and South country differ only in their skill endowments.
3.3 Comparative Static Properties of the Factor Market Equilibrium: Validity of the Stolper-Samuelson Result

We are now in a position to consider the first key question of the paper: when does the Stolper-Samuelson relation hold? Specifically, what are the implications of changing product prices on returns to different factors? We focus on cases corresponding to lack of complete specialization in either good, i.e., where $p_L$ lies between $p^1_L$ and $p^3_L$. Presuming that the South with a shortage of skill will have comparative advantage in the $L$-good, trade integration will induce a rise in $p_L$ in that country.

**Proposition 2** Consider Case A, where the $L$-good is more prone to moral hazard.

(a) If the economy is operating in Case A2, i.e., the $L$-good has a higher skill-premium than the $H$-good, a small increase in $p_L$ will raise the skill premium in $L$-sector and lower the skill premium in $H$-sector.

(b) If the economy is operating in Case A3, i.e., the two sectors have the same skill premium, a small increase in $p_L$ will lower the skill premia equally in both sectors.

Part (a) shows that the Stolper-Samuelson result is reversed in the ‘non-classical’ region where skill premia are unequal, while it continues to hold in the classical region where they are equal. The relation between output price $p_L$ and skill premia in the two sectors is illustrated in Figure 6. Focusing on the former region, it is evident that a rise in $p_L$ shifts the skill premium relation characterizing price-cost equality in the two sectors to the left. Since the relation between them characterizing the labor market clearing condition is downward-sloping in case A2, it follows that the skill premium must rise in $L$-sector and fall in $H$-sector. The price-cost relations (4,5) then imply that both $w$ and $\frac{p_L}{w}$ rise. Hence the wage rate expressed in units of the $H$-good rises, but expressed in units of the $L$-good falls.\(^{13}\)

The intuitive explanation of the increase in inequality in the $L$-sector is the following. The increase in $p_L$ induces initially a rise in profitability of the $L$-sector, lowering entry thresholds into the $L$-sector, which allows some entrepreneurs to move from the $H$ to the $L$-sector. This tightens the labor market, both because the $L$-sector employs more workers than the $H$-sector, and each $L$-sector firm employs more workers. The resulting upward pressure on the wage rate tends to reduce the skill premium in both sectors. The drop in $H$-sector profits will cause the labor market to slacken, as some low-ability $H$-sector entrepreneurs will switch to become production workers, and each

\(^{13}\)The effect on utility of workers thus depends on relative preferences in their consumption for the two goods: if biased in favor of the $L$-good sufficiently, workers will be worse off.
*H*-sector firm hires fewer workers. But the decline in skill premium in *L*-sector firms caused by increased wages cannot reverse the initial increase caused by the increase in the product price. Otherwise a lower skill premium in the *L*-sector would slacken the labor market, accentuating the effect of the decline in the *H*-sector skill premium. For the labor market to clear, the *L*-sector skill premium must rise overall.

The result resembles that in an exogenous specific factor model. However, a key difference is that in our model entrepreneurs do move between sectors in response to output price changes, as observed empirically in the Ugandan and Mozambique contexts cited in the Introduction. The newly entering entrepreneurs are of lower skill than incumbents in the sector: for them to be able to function in the *L*-sector while meeting the moral hazard constraint, entrepreneurial returns must rise relative to worker earnings, since the latter serves as the punishment payoff associated with a loss of reputation.

The other difference from an exogenous specific factor model is that the Stolper-Samuelson result holds in the classical region where skill premia are equalized across sectors. The relation between skill premia in the two sectors consistent with labor market clearing is upward-sloping: hence a leftward shift in the relation between skill premia consistent with price-cost equality implies that skill premia must fall in both sectors. The logic is similar to that in the mobile-factor version of the Heckscher-Ohlin model, arising from the ability of (some) entrepreneurs to move freely between sectors. Entrepreneurs with skill above the threshold for the *L*-sector are indifferent between operating in the *L* and *H* sectors. A positive fraction of them are already operating in either sector. Hence it is possible for a subset
of these high skill entrepreneurs to move into the $L$-sector out of the $H$-sector, without any change in the skill thresholds for sector $L$. Changes in skill premia result from a rise in the wage rate, which owes to the shift of entrepreneurs into the $L$-sector which is more labor intensive. Skill premia go down in step in both sectors.

The detailed distributional effects of a rise in $p_L$ in the anti-Stolper-Samuelson case A2 are illustrated in Figure 7. This shows the distribution of income across agents with varying skills, and how it changes as a result of an increase in $p_L$. The distribution of income is altered following a rise in $p_L$ in the following way: a rise in incomes at the top ($L$-sector entrepreneurs) and the bottom (workers), and a fall in incomes in the middle ($H$-sector entrepreneurs). Within the $L$-sector, inequality in earnings between entrepreneurs and workers rises. On the other hand inequality falls within the $H$-sector. Note that these inequality effects appear within firms.

The output and distributive impact of a rise in $p_L$ depend on induced entry and exit effects of entrepreneurs, which in turn depends on the local behavior of the ability distribution. To illustrate this, consider the limiting case of a Leontief technology.

**Proposition 3** Suppose the production function in each sector $i$ exhibits perfect complementarity: $X_i = A_i \min\{n_i/\theta^a_i, a/\theta^a_a\}$ for the high-quality good, and $X_i = A_i \min\{n_i/\theta^z_i, z(a/\theta^z_a)\}$ for the low-quality good. Suppose also that case A2 applies. Then small increase in $p_L$ results in:
(i) no change in \( w \) or outputs \( X_L, X_H \), while \( \gamma_L \) rises and \( \gamma_H \) remains constant, if \( g(\frac{m_L}{\gamma_L}) = 0 \) while \( g(\frac{m_H}{\gamma_H}) > 0 \).

(ii) no change in \( \gamma_L \), while \( w \) rises and \( \gamma_H \) falls, if \( g(\frac{m_H}{\gamma_H}) = 0 \) while \( g(\frac{m_L}{\gamma_L}) > 0 \).

This shows that relative rates of entry into the \( H \) and \( L \)-sectors, which depend on relative densities at the corresponding thresholds, affect the distribution of benefits between entrepreneurs and workers. In case (i) where there is no entry into the \( L \)-sector following a rise in \( \gamma_L \) owing to ‘thinness’ of the ability distribution at the threshold \( m_L \), changes in \( p_L \) will be associated with a zero output response, and none of the benefits of the rise in \( p_L \) are passed on to workers. In case (ii) on the other hand, the entire benefits of rising \( p_L \) are passed on to workers, as the rise in \( w \) does not choke off the output expansion in the \( L \)-sector owing to shrinking entry into the \( H \)-sector at the entry threshold for that sector (combined with lack of substitution within firms owing to a Leontief technology).

### 3.4 Autarky Equilibrium

Now we close the model of the autarkic economy by specifying the demand side. There is a representative consumer with a homothetic utility function \( U = U(D_H, D_L) \), where \( D_H, D_L \) denote consumption of the two goods. The relative demand function is then given by

\[
\frac{D_L}{D_H} = \phi(p_L)
\]

where \( \phi(p_L) \) is continuous and strictly decreasing in \( p_L \). We assume that \( \lim_{p_L \to 0} \phi(p_L) = \infty \) and \( \lim_{p_L \to \infty} \phi(p_L) = 0 \).

The economy-wide equilibrium is represented by equality of relative supply and relative demand:

\[
\frac{D_L}{D_H} = \phi(p_L) = \frac{X_L}{X_H}
\]

where the dependence of relative supply \( \frac{X_L}{X_H} \) on \( p_L \) is provided by the factor market equilibrium described in the previous section.

**Lemma 3** Consider Case A, where the \( L \)-good is more prone to moral hazard. An autarkic equilibrium always exists, and is unique. It must satisfy \( p_L \in (p_L^3, p_L^1) \).

This follows from the fact that relative demand is continuous and strictly decreasing in \( p_L \). An autarky equilibrium \( (p_L, \gamma_L, \gamma_H, w) \) is characterized as follows: for \( p_L \) in the range \( (p_L^3, p_L^1) \), and over this range is continuous and strictly increasing in \( p_L \). Moreover, as \( p_L \) tends to \( p_L^3 \), relative supply of the \( L \)-good tends to 0 while relative demand is bounded away from zero. And as \( p_L \) tends to \( p_L^1 \), relative supply of \( L \) tends to \( \infty \), while relative demand is bounded.
by conditions of profit-maximization (4), (5); the labor market clearing condition (10), and the product-market clearing condition (12). It is illustrated in Figure 8.

Now consider the effect on the autarky equilibrium of increasing $\mu$, which will be helpful in determining patterns of comparative advantage when we extend the model to an open economy setting. While the effects of varying $\mu$ on $\frac{X_L}{X_H}$ in the factor market equilibrium were seen above to be quite complicated, it turns out that the distributional effect on the autarkic equilibrium is quite simple: skill premia in both sectors fall.

Lemma 4 Suppose Case A applies. A small increase in skill endowment $\mu$ lowers skill premia in both sectors, while $w$ and $\frac{w}{p_L}$ both rise.

3.5 Free Trade Equilibrium and Lack of Factor Price Equalization

Suppose there are two countries South $S$ and North $N$, the former corresponding to the less developed country. They are identical in all respects, except that country $N$ has a higher $\mu$ the proportion of skilled agents ($\mu^S < \mu^N$). Lemma 4 then implies that in autarky Northern country has a lower skill premium in both sectors.

In a free trade equilibrium (with zero transport costs), there will be a

15 Similar results obtain when $N$ has a higher relative TFP in the $H$-sector.
common equilibrium price $p_T^L$ in the two countries, determined by

$$\frac{D_S^L + D_N^L}{D_S^H + D_N^H} = \frac{X_S^L + X_N^L}{X_S^H + X_N^H}$$

(13)

where both relative demand and supplies in each country will depend on the common price. Once $p_T^L$ is determined, the respective factor market equilibrium of each country will determine the remaining variables in each country.

If the South has a comparative advantage in the $L$-good, trade integration will induce a rise in $p_L$ in the South, with distributive effects as described in Proposition 2. If skill premia differ across sectors, the skill premium will rise within the $L$-sector and fall in the $H$-sector in the South, and the opposite happens in the North. Hence the initial gap in skill premia in the $L$-sector across the two countries will be accentuated, while that in the $H$-sector will shrink. On the other hand, if both countries are operating in the classical region with equal skill premia in the $L$ and $H$-sectors, they will decline in the South and rise in the North: in this case factor prices tend to equalize.

We summarize these results below.

**Proposition 4** Suppose the $L$-good is more prone to moral hazard, and the South has a comparative advantage in the $L$-good.

(a) If skill premia differ across sectors (i.e., Case A2 applies) within both countries under autarky and trade integration, the gap between skill premia in the $L$-sector in the two countries grows while that between skill premia in the $H$-sector narrows as a result of trade integration. In this case free trade must be associated with unequal skill premia in each sector across countries.

(b) If skill premia are equal across the two sectors (i.e., Case A3 applies) under autarky and trade integration in both countries, the gap between skill premia in either sector across countries narrows as a result of trade integration. In this case free trade must be associated with equalization of skill premia across countries.

3.6 Welfare Effects of Trade

The effects of trade on the equilibrium outcomes of each country are represented by the effect of trade on relative product price $p_L$ and thereafter on the resulting factor market equilibrium. Hence it suffices to examine the welfare effect of changes in $p_L$, which can be shown to be equivalent to the following expression.
Lemma 5 The aggregate welfare effect in country $j$ of a change in $p_j^L$ has the same sign as

$$(X_j^L - D_j^L) + w^j \mu(1 - m_J) g(a_H^j) da_H^j / dp_L^j - (\gamma_L^j - \gamma_H^j) a_L^j g(a_L^j) da_L^j / dp_L^j$$

(14)

In addition to the standard allocative effect $X_j^L - D_j^L$, there is an additional set of welfare effects operating through the change in entry thresholds $a_L$ and $a_H$. This owes to the upward jumps in incomes at these thresholds, owing to the binding incentive constraints operating at these thresholds. A relaxation of these thresholds enables agents to transfer occupations (from being a worker to an entrepreneur, when $a_H$ falls) or sectors (from sector $H$ to sector $L$, when $a_L$ falls) and experience a discrete income gain. This explains the second and third terms in the expression above. When $a_H$ falls, the change in income is $-w^j (1 - m_H)$ for every agent at the threshold, who is switching from being a worker to a $H$-sector entrepreneur. When $a_L$ falls, the change in income is proportional to the difference in skill premia between the two sectors.

In general, it is difficult to sign the sum of these income effects resulting from a change in entry thresholds following a relaxation of trade barriers. For instance, consider the cases described above where country $S$ has a comparative advantage in the $L$-good, and it is operating in case A2. Trade causes an expansion in the $L$-sector (a fall in $a_L$) which raises aggregate entrepreneurial incomes. It also causes a contraction in the $H$-sector (a rise in $a_H$) which reduces aggregate income, as some $H$-sector entrepreneurs switch to becoming workers. The net effect is ambiguous. However, in the case where the moral hazard problem in the $H$-sector is negligible (i.e., $m_H$ approaches one), the pecuniary externality at the $a_H$ threshold vanishes. In that case the aggregate income effect of trade is positive for country $S$, which adds to the standard allocative benefits of trade. Conversely, the aggregate income effect is negative for country $N$, which subtracts from the allocative benefits. Hence starting from autarky, a small expansion of trade can be welfare-reducing for country $N$.

With different parameter values, these welfare results can get reversed. For instance, suppose that the moral hazard problem in the $H$-sector is non-negligible, and approximately the same as in sector $L$ (i.e., $m_H$ is bounded away from 1 and $m_L - m_H$ is negligible). Then only movements in the entry threshold $a_H$ will generate non-negligible income effects. If country $S$ has comparative advantage in the $L$-good, trade will generate negative income effects for country $S$ and positive income effects for country $N$.

3.7 Offshoring

If skill premia in the North are lower as a result of failure of factor prices to equalize with trade, Northern entrepreneurs will have an incentive to offshore production to the South. The incentive to offshore can be measured by the
difference in profits between the two countries earned by an entrepreneur of
given ability.\footnote{Without loss of generality, a Northern entrepreneur producing in the South will sell in Southern markets. This is obvious if transport costs are high enough to render trade unprofitable. If transport costs are low enough to generate trade, the difference in prices of any good across countries will equal the transport cost, implying that entrepreneurs will be indifferent between selling in either country.} Our preceding results imply that trade integration will cause the incentive for North-South offshoring in the $L$-sector to go up, and in the $H$-sector to go down, when Case A2 applies and the South has a comparative advantage in the $L$-good.\footnote{We have shown in this case that the South-North difference in skill premia in the $L$-sector rises and the $H$-sector falls, as a result of trade integration. It is easy to check that the same property holds for the difference in profits in each sector: e.g., profits in sector $L$ equals $\gamma_L w = \frac{\mu^L}{\gamma_L + \gamma_H} \mu^N$, which rises in the South because $\gamma_L$ rises while $\gamma_H$ falls, and conversely falls in the North.} Hence our model predicts complementarity between trade integration and North-South offshoring in the $L$-sector, and substitutability in the $H$-sector.

We now examine the equilibrium implications of this type of offshoring, when there are zero costs to offshore, in addition to free trade in goods. The following proposition shows that the resulting equilibrium is identical to that in the completely integrated economy with $\mu^G \equiv \frac{\mu^S + \mu^N}{2}$, with factor prices equal across the two economies.

**Proposition 5** With free trade and costless offshoring, the equilibrium is equivalent to that in the completely integrated economy with $\mu^G$ proportion of skilled agents. In this equilibrium, the skill premium in each sector are equalized across countries. If the Southern country has comparative advantage in the $L$-good under autarky, complete integration relative to autarky causes skill premia to fall (resp. rise) in each sector in the South (resp. North).

In the integrated equilibrium, the absence of any trade or offshoring costs implies that entrepreneurs are indifferent which country to locate their operations. This implies that the structure of trade is indeterminate. This indeterminacy would be resolved in the presence of small trading and offshoring costs. Since the North has a higher endowment of skill, the net outsourcing from the North must be larger.

Proposition 5 indicates that the distributional effect of full integration differs sharply from trade integration when the latter is associated with factor price disequalization. If the South operates in Case A2 under autarky, trade integration raises the skill premium in the $L$-sector while complete integration lowers it. The reason is that in Case A2 there are restrictions on entry of entrepreneurs into sector $L$, who must come from the pool of Southern entrepreneurs. These entry restrictions are relaxed under trade integration only if the skill premium in this sector increases. With complete integration on the other hand, high skill entrepreneurs from the North can
enter the $L$-sector in the South. So the Southern $L$-sector skill premium does not have to rise to induce this entry. The fact that it is higher than skill premia in the North motivates Northern entrepreneurs to offshore operations to the South, which drives down skill premia there.

The comparison of full integration with free trade is somewhat more complicated. Let us continue to suppose that under autarky the South has comparative advantage in the $L$-good, and Case A2 applies in both countries. Under autarky skill premia are higher in both sectors in the South; trade integration causes the skill premium in the Southern (resp. Northern) $L$-sector to rise (resp. fall) even further. Under complete integration the $L$-sector skill premium must be equalized, and lie between the autarkic skill premia in the two countries. Hence starting with free trade, offshoring must cause the Southern $L$-sector skill premium to fall. But the effect on the $H$-sector skill premium is ambiguous.

If relative product prices were to remain unchanged, offshoring would lower the $H$-sector skill premium in the South. In that case, offshoring unambiguously tends to reduce Southern inequality between unskilled and skilled agents. However, offshoring could have another effect, by causing a change in $p_L$. This effect is not easy to sign. If terms-of-trade effects of offshoring are insignificant we can infer that it would generally improve Southern income distribution in favor of production workers in both sectors.

4 Extension to Case B, where $H$-good is More Prone to Moral Hazard

Now consider what happens if the $H$-good is more subject to moral hazard: $m_H > m_L$. Now if skill premia differ, they will be higher in the $H$-sector. Consequently occupational patterns will be different: the most skilled entrepreneurs will locate in the $H$-sector, which will be associated with a higher entry threshold.

As we have seen previously, the relation between skill premia that ensures factor market clearing plays a central role in determining the effects of integration. The following cases can be distinguished:

**Case B1:** $\gamma_L > \gamma_H$. Here the entry threshold in the $H$-sector is higher, while the $L$-sector is more lucrative, so all entrepreneurs with skill above $m_L$ enter the $L$-sector, and the rest become workers. The economy specializes in the $L$-good. The factor market clearing condition is the same as in case A1.

**Case B2:** $\gamma_L = \gamma_H$. The entry threshold into the $L$-sector is lower but both sectors are equally profitable so entrepreneurs are indifferent between the two sectors. Those with skills intermediate between the entry thresholds of the two sectors will enter the $L$-sector; others of higher skill will divide themselves between the two sectors so as to ensure that the labor market
clears. The factor market clearing condition is similar to that in case A3.

Case B3: $\gamma_H > \gamma_L$, $\frac{m_L}{\gamma_L} < \frac{m_H}{\gamma_H}$. Now everyone with skill between $\frac{m_L}{\gamma_L}$ and $\frac{m_H}{\gamma_H}$ will enter the $L$-sector, and those above $\frac{m_H}{\gamma_H}$ will enter the $H$-sector. The skill premia must satisfy the condition

$$\mu H = \mu L \left[ d\left(\frac{m_L}{\gamma_L}\right) - d\left(\frac{m_H}{\gamma_H}\right) \right] + \mu \left[ \frac{H}{\gamma_H} \right] d\left(\frac{m_H}{\gamma_H}\right) = (1 - \mu) + \mu G\left(\frac{m_L}{\gamma_L}\right). \quad (15)$$

Case B4: $\gamma_H > \gamma_L$, $\frac{m_L}{\gamma_L} > \frac{m_H}{\gamma_H}$. Everyone with skill above $\frac{m_H}{\gamma_H}$ enters the $H$-sector, everyone else becomes a worker, the economy specializes in producing the $H$-good. The factor market clearing condition is similar to that in case A4.

Case B3 is the region where both goods are produced and unequal skill premia across the two sectors. Unlike the case where the $L$-good is more prone to moral hazard, the slope of the skill-premium-frontier is ambiguous in general. The reason is that the effect of raising $\gamma_H$ on the tightness of the labor market is subject to two conflicting effects. Increasing $\gamma_H$ lowers the entry threshold into the $H$-sector, motivating the most able entrepreneurs previously operating in the $L$-sector to now enter the $H$-sector. Since the $L$-sector is less skill-intensive, this lowers the demand for production workers by an amount that depends on differences in skill intensity between the two sectors. On the other hand the rise in $\gamma_H$ induces each $H$-sector entrepreneur to hire more workers, which tightens the labor market; the strength of this effect depends on the elasticity of substitution between skilled and unskilled factors. To show that the effect can go either way, we specialize to the case of a CES production function considered earlier in Proposition 1, and a Pareto distribution for ability.\footnote{The following result extends to more general ability distributions, provided the associated $d(a)$ function has a bounded elasticity. In such cases, the term $(2 - \delta)$, which is the corresponding (constant) elasticity for the $d(a)$ function associated with the Pareto distribution, will be replaced by upper and lower bounds of the elasticity.}

**Lemma 6** Suppose $m_H > m_L$, the production function in each sector is given by (11), and the density of the ability distribution $g(a)$ is proportional to $a^{-\delta}$ for some parameter $\delta > 2$. The relation between skill premia $\gamma_L, \gamma_H$ in case B3 is downward-sloping if

$$\kappa > \left( \frac{k_H}{k_L} - 1 \right) (\delta - 2) \quad (16)$$

and is upward-sloping if

$$\kappa < \left( \frac{m_H}{m_L} \right)^{-\delta} \left( \frac{k_H}{k_L} - 1 \right) (\delta - 2) \quad (17)$$

For sufficiently high elasticity of substitution (condition 16), the relation between skill-premia that ensures clearing of the labor market continues...
to be downward-sloping. In this case, all results of the preceding section continue to apply (with region B3 substituting for region A2, where skill premia are unequal and both goods are produced). The only difference is that skill premia are now higher in the H-sector, since the H-good is more prone to moral hazard. The key reason is that in this case a rise in the skill premium in either sector tightens the labor market. It continues to be true for the L-sector for the same reasons as before. Consider the effects of a ceteris paribus decline in $\gamma_H$ (which may have been induced by rising wages, in turn the effect of rising $p_L$ and the resulting tightening of the labor market). This induces some entrepreneurs to move from the H-sector into the L-sector.\footnote{This does not happen in Case A, where a fall in $\gamma_H$ induces some low-ability entrepreneurs in the H-sector to shift to being a production worker. It happens in case B because now the highest skill premium is in the H-sector, so entrepreneurs in the H-sector that fall below the threshold for the H-sector enter the L-sector.} This movement tightens the labor market, since the L-sector is less skill intensive. On the other hand, a high elasticity of substitution implies a sharp drop in workers hired within H-sector firms, owing to the fall in $\gamma_H$. This latter effect dominates under assumption (16), and the labor market slackens. In order to restore demand for labor, the L-sector skill premium must rise.

On the other hand for sufficiently low elasticity of substitution (condition 17), the relation between skill-premia is upward-sloping in region B3. An increase in $p_L$ will lower the skill premium in both sectors, whenever both goods are produced. Hence the Stolper-Samuelson result will always hold in this case: trade integration will move skill premia closer together across countries.\footnote{Some additional technical qualifications are necessary, however, for this statement to be true. First, note that factor market equilibrium need not be unique, as both the labor market clearing condition and the price-cost relation generate upward sloping relationship between skill premia. Hence comparative static propositions pertain to local effects of small changes in $p_L$, starting with a locally unique equilibrium. Moreover, the statement is valid provided we start at a locally stable equilibrium, where the slope of the skill premium relationship corresponding to the price-cost condition is steeper than for the factor market clearing condition.} The movement of entrepreneurs from the H-sector to the L-sector following a decline in $\gamma_H$ now ensures a tightening of the labor market, which causes the skill premium in the L-sector to decrease. However, factor prices will not get completely equalized with free trade, as long as at least one of the countries is operating in region B3.

5 Related Literature

5.1 Related Empirical Evidence

Berges and Casellas (2007) provide evidence from Argentinian consumer surveys showing the greater importance of brand names compared with labels, seals and certification in consumer perceptions of food quality. Roth and

Relatively little direct evidence is available concerning how product quality moral hazard problems vary across different categories of goods. Scandals over safety of Chinese exports of farm goods and toys have erupted in recent years, highlighting quality concerns for less skill-intensive goods exported from developing countries. Using data spanning a large number of countries, Hudson and Jones (2003) show ISO-9000 certification rates are highest in electrical and optical equipment, basic metal and fabricated metal products, machinery and equipment sectors. Conversely, agriculture and farm products, textiles, wood and pharmaceuticals have the lowest accreditation rates. Accreditation take up rates are also lowest in less developed countries. In similar vein, Dobrescu (2009, Table 1) shows that that 14% of Slovenian manufacturing exporting firms in textiles/tobacco, wearing apparel, leather/shoes, wood between 1995-2005 had ISO certification, compared with 36% in more capital intensive sectors (chemicals, rubber, machinery, communication equipment, instruments, motor vehicles). However, these facts are only suggestive of the relative extent of moral hazard problems in different sectors, since direct evidence concerning this is intrinsically difficult to obtain.

An alternative is to examine the extent to which the ratio of middleman margins to retail prices vary across sectors, as our theory indicates this to be a key determinant of whether classical trade theory results are valid. Arndt et al. (2000) provide evidence of high middleman margin rates in agriculture, food processing, textiles and leather in Mozambique ranging from 36% to 111%. In contrast these ranged between 11 and 26% in machinery, metals, fuels and chemicals, paper, wood and mining. Nicita (2004) finds substantially lower rates of pass-through of border prices to producer prices in the case of cereals (32%), fruits (22%), vegetables (14%), oils and fats (22%), and sugar (26%), compared with manufactured goods (67%), textiles and apparel (54%). These suggest that middlemen margins are substantially higher in less skill intensive goods which are typically exported from South to North. However, most of these do not adjust for transport, storage and distribution costs which also tend to be higher for farm goods produced in remote areas. Nor do they adjust for risks borne by middlemen owing to price volatility, or quality defects in procured farm goods.
Corrections for transport and storage costs are made by Fafchamps and Hill (2008) in their study of gaps between border and farmgate prices for coffee in Uganda. Using monthly data they show only a small fraction of increases in export prices during 2002-03 was passed on to coffee farmers, and that the rising shares of middlemen could not be explained by accompanying changes in transport or storage costs. Instead, the main explanation they advance is exactly consistent with the predictions of our theory: rising demand for coffee exports induced entry of a less efficient set of middlemen. Similar findings are reported by McMillan, Rodrik and Welch (2002) in the context of rising trader margins for cashews in Mozambique during the 1990s: a falling ratio of farmgate to export prices was accompanied by an increase in the number of traders, especially informal, unlicensed traders buying in smaller quantities directly from farmers’ homes.

The only paper we are aware of that examines differences in effects of trade liberalization on wages in exporting and import-competing firms is Amiti and Davis (2008). Using Indonesian manufacturing census data for 1991-2000, they find average wages were lower in import-competing firms and higher in exporting firms following a cut in tariffs. This is consistent with our model, provided average wages include (at least some fraction of) earnings of entrepreneurs. Amiti and Davis do not, however, examine effects on intra-firm inequality.

5.2 Related Models

Antras and Costinot (2010a,b) and Chau, Goto and Kanbur (2010) develop similar models of middlemen margins based on an alternative model of search. These models exogenously assume that producers cannot sell to consumers directly, and must search for middlemen who can. These models are more appropriate for matching and trade in anonymous markets, rather than contexts involving repeat transactions and long-term supplier relationships. The allocation of bargaining power between producers and middlemen in these theories depends on the relative number of agents on either side of the market. The number of active middlemen is either exogenous, in which case trade liberalization has no effect on intra-sector inequality. Alternatively, the number of active middlemen is endogenously determined by a free entry condition, where middlemen exercise market power and must earn margins to cover their fixed costs. In this case trade liberalization result in increasing entry of middlemen into the export sector which lowers their bargaining power, implying that the share of middlemen declines as border prices rise. This is opposite to what our model predicts, and contrary to the evidence in Fafchamps and Hill (2008) or McMillan, Rodrik and Welch (2002). Moreover, Antras and Costinot show effects of offshoring by Northern traders may render Southern producers worse off, if the bargaining power of the former is large enough. This is in contrast to our model where offshoring always makes Southern producers better off. A common prediction, on the other
hand, is that Southern traders will be worse off as a result of offshoring. Clearly, models based respectively on search and on reputations have different implications for distributive effects of integration, thus providing an opportunity for using empirical evidence to discriminate among them.

Similar to many recent trade models and consistent with empirical evidence (e.g., see the survey by Harrison, McLaren and McMillan (2010)), our model predicts Southern entrepreneurs locating in the export sector are of higher ability than other entrepreneurs, and earn correspondingly more. An obvious extension of our model wherein reputations depend not just on the product characteristics but also the markets in which they are sold — specifically, where international reputations are harder to build than domestic ones — would imply that the productivity thresholds for exporting would be higher than for domestic production in all countries, not just in the South. In such a context, trade integration would generally raise inequality between exporting and non-exporting firms. In this respect our approach is similar to Helpman, Itzhoki and Redding (2010), in which trade liberalization may raise inequality by inducing high productivity firms to search more intensively for high ability workers.

In a similar vein, Costinot and Vogel (2010) model matching between heterogeneous productive tasks and workers of heterogeneous abilities. However their analysis generates generalized Stolper-Samuelson predictions when the source of trade is differences in factor endowments across countries. Matsuyama (2007) provides an alternative approach wherein the activity of exporting — involving transport, finance, marketing and communication — is itself more skill-intensive than production. A rise in export activities owing to improved technology of transport or communication can then end up increasing the demand for skilled workers, and eventually the skill premium. Verhoogen (2008) provides an alternative theory and supporting empirical evidence that cars marketed in the US are of higher quality than those in Mexico (owing to non-homothetic preferences), whence increased exports of Mexican-produced cars to the US following an exchange rate devaluation of the Mexican peso generated higher demand and relative wages of skilled Mexican workers. Zhu and Trefler (2005) provide evidence with a cross-country panel wherein the rise in skill premia across middle income and developing countries was positively correlated with a shift in export shares towards more skill-intensive goods. All these approaches stress the correlation between firm productivity and export activities, which generates rising inequality as an outcome of trade integration. This feature is shared by our approach, though it operates through a different (reputational) mechanism.

Other trade models with endogenous sorting of agents of heterogeneous abilities into different sectors include Mussa (1982), Matsuyama (1992), Ohnsorge and Trefler (2007) and Grossman (2004). Our theory differs in one important qualitative respect from all of these models: there is a discontinuous rise in profits of entrepreneurs at the thresholds for entry into
each sector in our theory, while agent returns vary continuously with ability in the latter. This has implications for welfare effects of changes in trade costs or policies.

Differences between effects of trade integration and offshoring have been stressed by a number of recent papers, for reasons quite different from those in this paper. Feenstra and Hanson (1996) pioneered the literature on offshoring and inequality, showing how inequality could rise in both North and South as a consequence of offshoring low-skill tasks in the North to the South where these are relatively high-skilled. Such a mechanism relies on heterogeneity of production worker skills, something our model abstracts from. In a model with a continuum of worker skills, Grossman and Rossi-Hansberg (2008) elaborate how offshoring can benefit domestic workers via employer cost-savings through better matching, that are passed on to workers in a competitive labor market. Antras, Garicano and Rossi-Hansberg (2006) and Kremer and Maskin (2003) study related models in which agents of heterogeneous abilities sort into hierarchical teams. Inequality rises in the South in these models owing to the matching of high ability agents in the South with worker teams from the North. Karabay and McLaren (2010) examine effects on risk and long term employer-employee contracting that coexist with spot markets. Our theory abstracts from risk considerations, or the possibility of some production tasks within any given sector being offshored while others are not. Instead, we emphasize how offshoring and trade integration may have opposite effects on inequality between entrepreneurs and workers, owing to differences in the associated entry patterns and pools of potential entrepreneurs that can enter any given sector.

Wynne (2005) and Antras and Caballero (2009, 2010) present trade models with financial frictions which affect production of one good more than another, with North countries less subject to financial frictions than South countries. Our model is based instead on frictions arising from quality moral hazard which affect different goods in different ways, where the nature of the moral hazard problem is assumed to be the same between North and South. These give rise to some features which are similar, though there are many differences in the detailed way in which these appear. Other shared features include the possibility of a Leontief paradox, complementarity between trade and capital flows, and the role of wealth distributions.

6 Concluding Comments

We have constructed a a general equilibrium model of trade based on middlemen margins which arise endogenously to provide incentives to maintain product quality reputations. Entry thresholds, occupational and sectoral choices of agents are endogenously determined in an otherwise fully competitive model. The allocation of agents between production work and entrepreneurship is explained by their underlying endowment of entrepreneurial
skill. In particular, the model explains why producers cannot directly sell to consumers — their lack of a credible reputation for quality — and must sell to intermediaries instead, those who have the requisite reputation.

If the severity of moral hazard problem differs markedly between different goods, equilibrium skill premia must also vary in a corresponding way. The lack of equalization of skill premia is associated with restrictions on movement of entrepreneurs, and the distributive effects of trade liberalization end up resembling a Ricardo-Viner specific factor model. Otherwise, there is enough intersectoral mobility to ensure that classical results of the mobile factor Heckscher-Ohlin model results obtain. Empirical evidence from some African countries where rising export prices were accompanied by rising gaps between export and farmgate prices are consistent with the predictions of the model, suggesting the need for fuller empirical testing of the model in future research.

The model explains incentives for Northern countries to offshore their production to Southern countries, and predicts the distributive implications of such offshoring to be the opposite of trade liberalization. Normative implications for trade policy include the possibility of trade liberalization reducing welfare in the North owing to reduced entrepreneurial margins in import-competing sectors. Pass-through and output responsiveness to trade liberalization depends on underlying distribution of entrepreneurial ability which determines responsiveness of entry into entrepreneurship in response to increasing profit margins. The model suggests that policies encouraging entry responsiveness, such as regulatory reforms, or development of entrepreneurial abilities may thus enhance growth and pro-poor effects of globalization.

We abstracted from the realistic possibility that reputations may be market or country-specific in addition to being commodity-specific. For instance it may be harder to maintain a reputation in international markets compared with domestic markets, owing to the role of information networks that underlie word-of-mouth reputations. Such a model would create higher productivity thresholds for exports compared with domestic sales for any given commodity, providing an alternative to a number of recent explanations for export ‘premium’ in productivity and earnings. Yet another possible extension would involves country-specific reputation thresholds, owing to differences in product quality regulations or their enforcement across countries. Such a model could be useful in examining the general equilibrium implications of changes in regulatory policy. Clearly, a rich research agenda lies ahead.

References


Wood Adrian (1997), “Openness and Wage Inequality in Developing Countries: The Latin American Challenge to East Asian Conventional Wisdom”,

Appendix: Proofs

Proof of Lemma 1: Implicitly differentiating (6) we obtain

\[ \frac{d\gamma_L}{d\gamma_H} = \mu_L \frac{\theta^H L}{\theta^H K} = \frac{\theta^H L + \gamma_L \theta^L K}{\theta^H K + \gamma_H \theta^L H} = \frac{\gamma_L}{\gamma_H + \frac{\theta^L K}{\theta^H K}} > 1, \]

with the second equality using (6), and the last inequality following from Assumption 1, \( \gamma_L \geq \gamma_H \) and the fact that \( \frac{\theta^L K}{\theta^H K} \) is non-decreasing in \( \gamma_L \).

Proof of Lemma 2. We start by writing the labor market clearing condition in various cases, which characterizes the relationship between skill premia in the two sectors, as well as the outputs in each sector.

Clearing of the labor market in Case A1 requires

\[ \mu_L \frac{\theta^L K (\gamma_L)}{\theta^L K (\gamma_H)} d\left(\frac{m_L}{\gamma_L}\right) = \mu G \left(\frac{m_L}{\gamma_L}\right) + (1 - \mu). \]  

The production levels will be \( X_H = 0, X_L = \mu_A L \left(\frac{m_L}{\gamma_L}/\theta^L K (\gamma_L)\right) \).

In Case A2 the corresponding condition is

\[ \mu_L \frac{\theta^L K (\gamma_L)}{\theta^L K (\gamma_H)} d\left(\frac{m_L}{\gamma_L}\right) + \mu_H \frac{\theta^H K (\gamma_H)}{\theta^H K (\gamma_H)} d\left(\frac{m_H}{\gamma_H}\right) = \mu G \left(\frac{m_H}{\gamma_H}\right) + (1 - \mu). \]  

In this case, production levels are:

\[ X_H = \mu A H \left[\frac{d(a_H)}{\gamma_H}/\theta^H K (\gamma_H)\right] \]
\[ X_L = \mu A L \left[\frac{d(a_L)}{\gamma_L}/\theta^L K (\gamma_L)\right]. \]

In Case A3, the factor market clearing conditions are (denoting the production levels by \( X_L, X_H \) respectively)

\[ \left[\theta^L K (\gamma)/A_L\right] X_L + \left[\theta^H K (\gamma)/A_H\right] X_H = \mu G (a_H) + (1 - \mu) \]
\[ \left[\theta^L K (\gamma)/A_L\right] X_L + \left[\theta^H K (\gamma)/A_H\right] X_H = \mu d (a_H) \]

These equations are equivalent to

\[ X_L = A_L \frac{\theta^H K (\gamma) \left[\frac{\mu G (a_H)}{\theta^L K (\gamma)} + (1 - \mu)\right] - \theta^H K (\gamma) \mu d (a_H)}{\theta^L K (\gamma) \theta^L K (\gamma) - \theta^H K (\gamma) \theta^H K (\gamma)}, \]
\[ X_H = A_H \frac{-\theta^L K (\gamma) \left[\frac{\mu G (a_H)}{\theta^L K (\gamma)} + (1 - \mu)\right] + \theta^L K (\gamma) \mu d (a_H)}{\theta^L K (\gamma) \theta^L K (\gamma) - \theta^H K (\gamma) \theta^H K (\gamma)} \]

However since only agents with \( a \geq a_L \) have the option to become L-sector entrepreneurs,

\[ X_L \leq \mu A_L \left[\frac{d(a_L)}{\gamma_L}/\theta^L K (\gamma)\right] \]

which implies

\[ \mu_L \frac{\theta^L K (\gamma)}{\theta^L K (\gamma)} d\left(\frac{m_L}{\gamma_L}\right) + \mu_H \frac{\theta^H K (\gamma)}{\theta^H K (\gamma)} d\left(\frac{m_H}{\gamma_H}\right) \geq \mu G \left(\frac{m_H}{\gamma_H}\right) + (1 - \mu). \]  

On the other hand, \( X_L \geq 0 \) implies

\[ \mu_L \frac{\theta^H K (\gamma)}{\theta^L K (\gamma)} d\left(\frac{m_H}{\gamma_H}\right) \leq \mu G \left(\frac{m_H}{\gamma_H}\right) + (1 - \mu). \]

Inequalities (20, 21) provide lower and upper bounds on the common premium rate \( \gamma \). Note that (20) is the inequality version of the factor market clearing condition (19) in
Case A2. Hence the lower bound in Case A3 exactly equals the limiting premia in Case A2 as \( \gamma_L \) and \( \gamma_H \) approach each other (see Figure 1).

In Case A4 the labor market clearing condition is

\[
\mu \left| \frac{\partial \theta^H (\gamma_H)}{\partial \gamma_H} \right| d(a_H) = \mu G(a_H) + (1 - \mu).
\]

(22)

The production levels are

\[
X_L = 0 \\
X_H = \mu A_H d(a_H)/\theta^H (\gamma_H).
\]

The entry thresholds depicted \( \gamma^1_H, \gamma^2_H \) and \( \gamma^3_H \) in Figure 1 are defined by the solutions to the following equations.

\[
\mu \left| \frac{\partial \theta^L (\gamma^1_H)}{\partial \gamma^1_H} \right| d(m_H/\gamma^1_H) = \mu G(m_H/\gamma^1_H) + (1 - \mu) - \mu \left| \frac{\partial \theta^H (\gamma^1_H)}{\partial \gamma^1_H} \right| d(m_H/\gamma^1_H) - d(m_L/\gamma^2_H)
\]

\[
\mu \left| \frac{\partial \theta^L (\gamma^2_H)}{\partial \gamma^2_H} \right| d(m_H/\gamma^2_H) + \mu \left| \frac{\partial \theta^H (\gamma^2_H)}{\partial \gamma^2_H} \right| \theta_L (\gamma^2_H) = \mu G(m_H/\gamma^2_H) + (1 - \mu) - \mu \left| \frac{\partial \theta^H (\gamma^2_H)}{\partial \gamma^2_H} \right| d(m_H/\gamma^2_H).
\]

The price thresholds which mark the transition between Cases A1, A2, A3 and A4 are calculated as follows:

\[
p^*_L = (A_L/A_H) \frac{\theta^L_n (m_L/m_H \gamma^1_H) + m_L/m_H \gamma^1_H \theta^L_n (m_L/m_H \gamma^1_H)}{m_L/m_H \gamma^2_H + \gamma^2_H \theta^L_n (m_L/m_H \gamma^2_H)}
\]

\[
p^2_L = (A_L/A_H) \frac{\theta^L_n (\gamma^2_H) + \gamma^2_H \theta^L_n (\gamma^2_H)}{m_L/m_H \gamma^2_H + \gamma^2_H \theta^L_n (m_L/m_H \gamma^2_H) + \gamma^2_H \theta^L_n (\gamma^2_H)}
\]

\[
p^*_L = (A_L/A_H) \frac{\theta^L_n (\gamma^1_H) + \gamma^1_H \theta^L_n (\gamma^1_H)}{m_L/m_H \gamma^1_H + \gamma^1_H \theta^L_n (m_L/m_H \gamma^1_H) + \gamma^1_H \theta^L_n (\gamma^1_H)}
\]

Now we are in a position to describe the factor market equilibrium, where the price-cost conditions characterizing profit maximization must be satisfied along with clearing of the labor market. We consider the following price ranges A1, A2, A3, A4, and refer to Figure 1.

**Case A1:** \( p_L \geq p^*_L \)

In this case, there is an equilibrium with \( \gamma_H \leq \gamma^1_H \), with complete specialization in product \( L \), and production levels \( X_L = \mu d(m_L/m_H) \theta^L_n (\gamma_H), X_H = 0 \). Since the price-cost relation (6) between skill premia in the two sectors is upward-sloping, it is evident there cannot be any other equilibrium. In the interior of this range, equilibrium outputs are locally independent of \( p_L \).

**Case A2:** \( p^*_L > p_L \geq p^2_L \)

Here there is an equilibrium corresponding to the downward sloping stretch in the relation between \( \gamma_H, \gamma_L \) expressing labor market clearing. This follows from the fact that at \( p^*_L \),

\[\text{If the price-cost relation does not intersect the horizontal segment of the skill-premium-frontier associated with labor market clearing, the skill premium in the } L-\text{sector is defined by the point where this horizontal segment intersects the vertical axis. Here the price-cost relation takes the form of an inequality for the } H-\text{sector and equality for the } L-\text{sector.} \]
there is an equilibrium corresponding to \( \gamma^*_H \), and at \( p_L^2 \) there is an equilibrium corresponding to \( \gamma^*_H \). Moreover in this case there cannot be any other equilibrium owing to Lemma 1. For if there were another equilibrium, it would have to lie in the range \( \gamma_H > \gamma^*_H \). But this would require the slope of the skill premium relationship expressing (6) to have a slope smaller than one somewhere above the 45 degree line, which is ruled out by Lemma 1.

In the interior of this range of prices, increasing \( p_L \) results in an increase in \( X_L \) and \( \gamma_L \), and a decrease in \( X_H \) and \( \gamma_H \).

Case A3: \( p_L^2 > p_L \geq p_L^3 \)

Now there will be an equilibrium in which skill premia are equalized across the two sectors. The same argument as in Case A2 ensures the equilibrium is unique. Note in particular that Lemma 1 ensures that the slope of the relation between skill premia expressing (6) strictly exceeds unity even on the 45 degree line. Hence a tangency of this relation with the 45 degree line is ruled out. The equilibrium skill premium in this case
\[ \gamma^*_L = \gamma^*_H = \gamma^* \]
is determined by the condition
\[ p_L = \left( \frac{A_H}{A_L} \theta^L(\gamma^*) + \gamma^* \theta^L(\gamma^*) \right) \theta^H(\gamma^*) + \gamma^* \theta^H(\gamma^*) \]  
(23)

It is evident that an increase in \( p_L \) will increase \( X_L \), reduce \( X_H \) and the common skill premium \( \gamma^* \). The latter results as the shift in production towards the \( L \)-sector raises the demand for labor, inducing a rise in the wage rate.

Case A4: \( p_L < p_L^3 \)

In this case, there is a unique equilibrium with perfect specialization in sector \( H \).\(^{22} \) The production level is \( X_L = 0 \) and
\[ X_H = \mu A_H d(a_H) / \theta^H(\gamma_H) \]  
An increase in \( p_L \) in this region will raise \( \gamma_L \), while leaving \( X_H, \gamma_H \) unchanged.

This concludes the proof of Lemma 2.

Proof of Proposition 1

Step 1

(i) If \( \kappa \geq 1 \), \( d(\gamma_L, \gamma_H) / d\gamma_H = d(\lambda(\gamma_H; p_L, A_H / A_L)) / d\gamma_H > 0 \) for any \( \gamma_H \) so that \( \lambda(\gamma_H; p_L, A_H / A_L) / \gamma_H \geq 1 \)

(ii) If \( \kappa < 1 \), \( d(\gamma_L, \gamma_H) / d\gamma_H = d(\lambda(\gamma_H; p_L, A_H / A_L)) / d\gamma_H > 0 \) if and only if \( p_L < A_H / A_L \) (and equivalently \( \gamma_L / \gamma_H < (\frac{A_H}{A_L}) ^{\frac{1}{1-\kappa}} \)).

Proof of Step 1

\(^{22}\) Again, if the price-cost relation does not intersect the vertical segment of the skill-premium-frontier represent labor market clearing, the equilibrium is located at a skill premium of \( \gamma^*_H \) in the \( H \)-sector and 0 in the \( L \)-sector. Here prices equal cost in the \( H \)-sector while they fall below cost in the \( L \)-sector.
From (6),
\[ d[\gamma_L/\gamma_H]/d\gamma_H = (1/\gamma_H)[\gamma_L + \frac{\phi_L}{\phi_H} - \frac{\gamma_L}{\gamma_H}] \]
which means that \( d[\gamma_L/\gamma_H]/d\gamma_H > 0 \) if and only if
\[ \frac{\gamma_L\theta_L^H(\gamma_L)}{\theta_L^H(\gamma_L)} < \frac{\gamma_H\theta_L^H(\gamma_H)}{\theta_L^H(\gamma_H)} \]
Under this production function in the proposition,
\[ \frac{\theta_L^H(\gamma_L)}{\theta_L^H(\gamma_L)} = (\gamma)^{-\kappa} \]
and
\[ p_L = A_H \frac{\theta_L^L(\gamma_L) + \gamma_L\theta_L^L(\gamma_L)}{A_L\theta_L^L(\gamma_L)} = A_H \frac{kL\gamma_L^{1-n} + 1}{k_H\gamma_L^{1-n} + 1}. \]
In the case of \( \kappa \geq 1 \) and \( \gamma_L \geq \gamma_H \),
\[ \frac{\gamma_L\theta_L^L(\gamma_L)}{\theta_L^L(\gamma_L)} = (\gamma_L)^{1-n}k_L < (\gamma_H)^{1-n}k_H = \frac{\gamma_H\theta_L^H(\gamma_H)}{\theta_L^H(\gamma_H)} \]
implies \( d[\gamma_L/\gamma_H]/d\gamma_H > 0 \). In the case of \( \kappa < 1 \),
\[ \frac{\gamma_L\theta_L^L(\gamma_L)}{\theta_L^L(\gamma_L)} < \frac{\gamma_H\theta_L^H(\gamma_H)}{\theta_L^H(\gamma_H)} \]
if and only if \( \gamma_L/\gamma_H < (\frac{\lambda H}{A_L})^{1-n} \) which is equivalent to \( p_L < A_H/A_L \).

**Step 2**

(i) If \( \kappa \geq 1 - \frac{\log(\frac{\lambda H}{A_L})}{\log(\frac{A_H}{A_L})} \),
\[ d[\lambda(\gamma_H; p_L; A_H/A_L)/\gamma_H]/d\gamma_H > 0 \]
holds for \( p_L \in [p_L^2, p_L^2] \).

(ii) If \( 0 \leq \kappa < 1 - \frac{\log(A_H/A_L)}{\log(\frac{A_H}{A_L})} \), for any \( p_L < A_H/A_L \),
\[ d[\lambda(\gamma_H; p_L; A_H/A_L)/\gamma_H]/d\gamma_H > 0 \]
and for any \( p_L > A_H/A_L \),
\[ d[\lambda(\gamma_H; p_L; A_H/A_L)/\gamma_H]/d\gamma_H < 0. \]

**Proof of Step 2**

First suppose that \( \frac{m_L}{m_H} \leq [\frac{m_H}{m_L}]^{1-(1-n)} \) and \( \kappa < 1 \), which are equivalent to \( 1 > \kappa \geq 1 - \frac{\log(\frac{\lambda H}{A_L})}{\log(\frac{A_H}{A_L})} \). If \( p_L \in (p_L^2, p_L^1) \), since \( m_L/m_H > \gamma_L/\gamma_H \geq 1 \) is satisfied in an equilibrium of supply-side, it implies \( \gamma_L/\gamma_H < (\frac{m_H}{m_L})^{1-(1-n)} \) (or \( p_L < \frac{A_H}{A_L} \)). From (ii) of Step 1, this means that
\[ d[\lambda(\gamma_H; p_L; A_H/A_L)/\gamma_H]/d\gamma_H > 0 \]
holds for \( p_L \in [p_L^2, p_L^1] \). From (i) of Step 1, this inequality also holds for \( \kappa \geq 1 \). This completes the proof of (i).
Next take $0 \leq \kappa < 1 - \log\left(\frac{4\mu}{\mu\lambda}\right)$. From (ii) in Step 1, for any $p_L < A_H/A_L$,
\[
d[\lambda(\gamma_H; p_L, \frac{A_H}{A_L})]/\gamma_H > 0
\]
and for any $p_L > A_H/A_L$,
\[
d[\lambda(\gamma_H; p_L, \frac{A_H}{A_L})]/\gamma_H < 0
\]
This completes the proof of (ii).

**Step 3**

Taking $p_L \in (p_L^1, p_L^2)$ as given, let’s consider the effect of $\mu$ on
\[
X_L/X_H = \frac{A_L}{A_H} \frac{\theta_L^H(\gamma_H)}{\theta_L^L(\gamma_L)} \frac{d(a_H)}{d(a_L)}
\]
We can use the following relationship,
\[
d\frac{\theta_L^H(\gamma_H)}{\theta_L^L(\gamma_L)}/d\mu = \theta_L^H(\gamma_H) \theta_L^L(\gamma_L) \frac{\theta_L^H(\gamma_H)}{\theta_L^L(\gamma_L)} - \theta_L^L(\gamma_L) \lambda(\gamma_H; p_L, \frac{A_H}{A_L}) d\gamma_H / d\mu
\]
\[
= \theta_L^H(\gamma_H) \theta_L^L(\gamma_L) \frac{\lambda(\gamma_H; p_L, \frac{A_H}{A_L})}{\gamma_L} - \lambda(\gamma_H; p_L, \frac{A_H}{A_L}) d\gamma_H / d\mu
\]
\[
< \theta_L^H(\gamma_H) \theta_L^L(\gamma_L) \frac{\lambda(\gamma_H; p_L, \frac{A_H}{A_L})}{\gamma_H} - \lambda(\gamma_H; p_L, \frac{A_H}{A_L}) d\gamma_H / d\mu < 0
\]
if $d[\lambda(\gamma_H; p_L, \frac{A_H}{A_L})]/\gamma_H > 0$. This relationship is using the fact that
\[
\frac{\gamma_H}{\theta_L^H(\gamma_H)} = -\kappa/(\gamma_H/\theta_L^H(\gamma_H) + 1)
\]
and $d[\lambda(\gamma_H; p_L, \frac{A_H}{A_L})]/\gamma_H > 0$ if and only if $\gamma_H \theta_L^H(\gamma_H) < \gamma_H \theta_L^H(\gamma_H)$. Similarly, we obtain
\[
d\frac{d(a_H)}{d(a_L)}/d\mu = \frac{d(a_H)}{d(a_L)} (\frac{(a_H)^2 g(a_H)}{\gamma_H} - \frac{(a_L)^2 g(a_L)}{\gamma_L} \lambda(\gamma_H; p_L, \frac{A_H}{A_L}) d\gamma_H / d\mu)
\]
\[
> \frac{d(a_H)}{d(a_L)} (\frac{(a_H)^2 g(a_H)}{\gamma_H} - \frac{\lambda(\gamma_H; p_L, \frac{A_H}{A_L})}{\gamma_H} d\gamma_H / d\mu > 0
\]
if $d[\lambda(\gamma_H; p_L, \frac{A_H}{A_L})]/\gamma_H > 0$. This is using the assumption that $\frac{a_H^2(a_H)}{d\gamma_H}$ is increasing in $a$. This implies that
\[
d(X_L/X_H)/d\mu < 0.
\]
for $p_L \in [p_L^1, p_L^2]$ if $\kappa \geq 1 - \log\left(\frac{4\mu}{\mu\lambda}\right)$ and for $p_L \in [p_L^2, A_H/A_L]$ if $0 \leq \kappa < 1 - \log\left(\frac{4\mu}{\mu\lambda}\right)$. 43
Step 4

Next suppose \( p_L \in (\hat{p}_L^2, \hat{p}_L^1) \). \( \gamma_L = \gamma_H = \gamma^* \) is determined by

\[
\frac{p_L}{(A_H \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) + \frac{L_H}{m_H})} \begin{bmatrix} \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) + \frac{L_H}{m_H} \gamma_L \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) \gamma_H \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) \gamma_H \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) \end{bmatrix} = \frac{\gamma_1^1 H^1 \gamma_H^1 H^1}{\gamma_1^1 H^1 \gamma_H^1 H^1} \frac{d \gamma_H}{d \mu} = 0.
\]

\( \gamma^* \) is independent of \( \mu \). This means that \( d \gamma^*/d \mu = 0 \). We have only the direct effect of \( \mu \) on \( X_L/X_H \), which is negative.

From step 3 and this step, this completes the proof of (i) and the first half of (ii) in the proposition.

Step 5

Finally let us show the last part of (ii). Suppose that there does not exist \( \hat{p}_L \in (A_H / A_L, p_L^1) \) so that \( X_L/X_H \) is increasing in \( \mu \) for any \( p \in (\hat{p}_L, \hat{p}_L^1) \). Then \( p_L^1 \) has to be non-decreasing in \( \mu \). However

\[
dp_L^1/d\mu = (p_L^1/\gamma_H^1) \left( \frac{\partial \gamma_H}{\partial a_L} \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) + \frac{L_H}{m_H} \gamma_L \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) \gamma_H \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) \gamma_H \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) \right) = \frac{\gamma_1^1 H^1 \gamma_H^1 H^1}{\gamma_1^1 H^1 \gamma_H^1 H^1} \frac{d \gamma_H}{d \mu} \]

is negative from step 2(ii). This is the contradiction.

Proof of Proposition 3

With perfect complementarity in production, factor intensities within firms are independent of the skill premium in that sector. Moreover, \( d(a) \) is locally constant if \( g(a) = 0 \). Then from the factor market clearing condition in case A2, we obtain

\[
d\gamma_L/d\gamma_H = \frac{\frac{\partial g(a_H)}{\partial a_H} \gamma_H + 1}{\partial g(a_H) \gamma_H} \left( \frac{\gamma_H}{\partial a_L} (\frac{m}{m_H}) \gamma_H \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) \gamma_H \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) \gamma_H \frac{\partial L}{\partial a_L} (\frac{m}{m_H}) \right) = \frac{\gamma_1^1 H^1 \gamma_H^1 H^1}{\gamma_1^1 H^1 \gamma_H^1 H^1} \frac{d \gamma_H}{d \mu} \frac{d \gamma_H}{d \mu}.
\]

This shows that \( \gamma_H \) and hence \( w \) does not change in case (i), while \( \gamma_L \) does not change in case (ii). The rise in \( \gamma_L \) in case (i) generates no change in \( X_L \) because \( g(\frac{m}{m_H}) = 0 \).

Proof of Lemma 4

Suppose that an increase in \( \mu \) raises \( p_L \) that the initial price level is in \( (\hat{p}_L^2, \hat{p}_L^1) \). Then as explained previously, taking \( p_L \) as given, the increase in \( \mu \) causes \( \gamma_H \) and \( \gamma_L \) to decrease in the factor market equilibrium. On the other hand, the increase in \( p_L \) causes \( \gamma_H \) to fall and \( \gamma_L \) to rise. Therefore the total effect on \( \gamma_H \) is negative. Since equilibrium \( p_L \) rises, the equilibrium level of \( X_L/X_H \) must be lower. However the right hand side of

\[
X_L/X_H = \frac{A_H \theta_L^1 (\gamma_L)}{A_L \theta_L^1 (\gamma_H)} \frac{d\gamma_H}{d a_H} = \frac{d(a_L)}{d(a_H)}
\]

increases with a decrease in \( \gamma_H \), which implies that the total effect on \( \gamma_L \) must be negative. From the price-cost relations, the effect on \( w \) and \( w/p_L \) must be positive. On the other hand, if the price level is in \( (\hat{p}_L^2, \hat{p}_L^1) \), the increase in \( \mu \) does not have a direct effect on \( \gamma_H \) and \( \gamma_L \) for given \( p_L \), and the effect on both through the increase in \( p_L \) is negative.

Next, consider the case where an increase in \( \mu \) is associated with a fall in \( p_L \). By Proposition 1 this is possible only if \( p_L \in (\hat{p}_L^2, \hat{p}_L^1) \). Then the direct effect of \( \mu \) taking \( p_L \) as given is negative for both \( \gamma_L \) and \( \gamma_H \). On the other hand, the indirect effect through the decrease in \( p_L \) is negative for \( \gamma_H \) and positive for \( \gamma_H \). Hence the total effect on \( \gamma_L \) is negative. A symmetric argument to that in the previous paragraph also implies that the total effect on \( \gamma_H \) is negative.
Proof of Lemma 5

Let the indirect utility function for country $j$ be $V(p^j_L, I^j)$ where national income

$$I^j = p^j_L X^j_L + X^j_H.$$  

By Roy’s identity $D^j_L = -\frac{\partial V(p^j_L, I^j)}{\partial p^j_L}$

$$dV(p^j_L, I^j)/dp^j_L = \frac{\partial V(p^j_L, I^j)}{\partial I^j} \frac{dI^j}{dp^j_L} = \frac{\partial V(p^j_L, I^j)}{\partial I^j} [-D^j_L + dI^j/dp^j_L]$$

$$= \frac{\partial V(p^j_L, I^j)}{\partial I^j} [(X^j_L - D^j_L) + p^j_L dX^j_L/dp^j_L + dX^j_H/dp^j_L]$$

Since $\frac{\partial V(p^j_L, I^j)}{\partial I^j} > 0$, the sign of the welfare effect of the change in $p^j_L$ is the same as the sign of $$(X^j_L - D^j_L) + p^j_L dX^j_L/dp^j_L + dX^j_H/dp^j_L.$$  

Owing to linear homogeneity of the production function and the common $n_L/a$ chosen by all entrepreneurs in a given sector, $X^j_L = A_L F_L(n^j_L, h^j_L)$ where $n^j_L$ (resp. $h^j_L$) is the total amount of unskilled (resp. skilled) labor used in the $L$-sector of country $j$. Since $p^j_L dX^j_L/dn^j_L = w^j$ and $\gamma^j_L = \frac{\partial F_L(n^j_L, h^j_L)}{\partial h^j_L}/\partial n^j_L$, then

$$p^j_L dX^j_L/dp^j_L = p^j_L A_L [\partial F_L(n^j_L, h^j_L)/\partial n^j_L] [dn^j_L/dp^j_L] + \partial F_L(n^j_L, h^j_L)/\partial h^j_L [dh^j_L/dp^j_L]$$

$$= w^j [dn^j_L/dp^j_L + \gamma^j_L dh^j_L/dp^j_L]$$

Similarly

$$dX^j_H/dp^j_L = A_H [\partial F_H(n^j_H, h^j_H)/\partial n^j_H] [dn^j_H/dp^j_L] + \partial F_H(n^j_H, h^j_H)/\partial h^j_H [dh^j_H/dp^j_L]$$

$$= w^j [dn^j_H/dp^j_L + \gamma^j_H dh^j_H/dp^j_L]$$

Therefore

$$p^j_L dX^j_L/dp^j_L + dX^j_H/dp^j_L = w^j [dn^j_L/dp^j_L + \gamma^j_L dh^j_L/dp^j_L] + \gamma^j_H (d(h^j_L + h^j_H)/dp^j_L + (\gamma^j_L - \gamma^j_H) dh^j_L/dp^j_L].$$

Since $h^j_L = \mu d(a^j_L)$ for $p^j_L \in (p^j_L^2, p^j_L^1)$ and $\gamma^j_H = \gamma^j_H$ for $p^j_L \in (p^j_L^3, p^j_L^2)$,

$$(\gamma^j_L - \gamma^j_H) dh^j_L/dp^j_L = \mu (\gamma^j_L - \gamma^j_H) d(d(a^j_L))/dp^j_L$$

holds for $p^j_L \in (p^j_L^3, p^j_L^1)$. With $n^j_L + n^j_H = \mu G(a^j_L) + (1 - \mu)$ and $h^j_L + h^j_H = \mu d(a^j_H)$ for $p^j_L \in (p^j_L^3, p^j_L^1)$,

$$(X^j_L - D^j_L) + p^j_L dX^j_L/dp^j_L + dX^j_H/dp^j_L$$

$$= (X^j_L - D^j_L) + w^j (1 - \mu [G(a^j_L)]) [da^j_L/dp^j_L + (\gamma^j_L - \gamma^j_H) g(a^j_H)]$$

Proof of Proposition 5

Suppose that $w^S < w^N$ with free trade and costless offshoring. If $w^S \neq w^N$, all entrepreneurs would hire only workers in country $S$. However production workers in country $N$ do not have the option to become entrepreneurs, and would thus be unemployed, implying $w^N = 0$, a contradiction. Similarly, we cannot have $w^S > w^N$. With a common product price ratio $p_L$ and the common unskilled wage, skill premia must be equalized in each sector across the two countries. Thus these premia must clear the market for production
workers in the integrated economy, i.e., satisfy (10) with $\mu^G$ representing the proportion of skilled agents.

As shown in the autarky equilibrium, in the region that $\gamma_L > \gamma_H$ holds in the equilibrium, the autarky levels of $\gamma_H$ and $\gamma_L$ are decreasing in $\mu$ regardless of its impact on $p_L$. Hence $\mu^S < \mu^G < \mu^N$ implies a fall (resp. rise) in skill premia in each sector in the South (resp. North).

Proof of Lemma 6

It is evident that increasing $\gamma_L$ tightens the labor market clearing condition (15). So the relation with $\gamma_H$ ensuring labor market clearing is downward-sloping if and only if an increase in $\gamma_H$ also tightens the labor market, i.e.,

$$\phi_L(\gamma_L) d\left(\frac{m_H}{\gamma_H}\right) - [\phi_H(\gamma_H) - \phi_L(\gamma_L)] d\left(\frac{m_H}{\gamma_H}\right) \gamma_H > 0$$

where $\phi_i(\gamma_i) \equiv \frac{\delta_i(\gamma_i)}{\delta_i'(\gamma_i)}$, which reduces to the condition

$$\left[\frac{\phi_L}{\phi_H} - 1\right] \frac{\phi_H}{\phi_H'} \gamma_H < \frac{\gamma_H}{m_H} \left(\frac{d}{d\left(\frac{m_H}{\gamma_H}\right)}\right)$$

With the CES production function, we have $\phi_i = (\gamma_i)^{\frac{1}{\kappa}}$, with $k_H > k_L$. Hence it reduces to

$$\left[\frac{\gamma_L}{\gamma_H}\right]^{\frac{k_H}{k_L}} \gamma_H - 1] \epsilon < \kappa$$

where $\epsilon$ denotes the elasticity of $d$ evaluated at $\frac{m_H}{\gamma_H}$. This elasticity equals $\delta - 2$ in the case of the Pareto distribution. The result now follows, upon observing that in region B3, the skill premium ratio $\frac{\gamma_L}{\gamma_H}$ varies between $\frac{m_L}{m_H}$ and 1.