Mechanism Design with Communication Constraints

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We consider mechanism design in which message sets are restricted owing to communication costs, preventing full revelation of information. A principal contracts with multiple agents each supplying a one-dimensional good at a privately known cost. We characterize optimal mechanisms subject to incentive and communication constraints, without imposing arbitrary restrictions on the number of communication rounds. We show that mechanisms that centralize production decisions are strictly dominated by those that decentralize decision-making authority to agents, and optimal communication mechanisms maximize information exchanged directly among agents. Conditions are provided for these to involve gradual release of information over multiple rounds either simultaneously or sequentially.

I. Introduction

Real-world economic organizations differ markedly from the predictions of mechanism design theory. The revelation principle (e.g., Myerson **q1**

We thank Ingela Alger, Richard Arnott, Hideshi Itoh, Ulf Lilienfeld, Bart Lipman, Michael Manove, Preston McAfee, Margaret Meyer, Alberto Motta, Andy Newman, Roy Radner, Stefan Reichelstein, Ilya Segal, Hideo Suehiro, three anonymous referees, and the editor Phil Reny. We also thank seminar participants at Boston College, Brown, Columbia, Hitotsubashi, Keio, Kobe, Nagoya, Northwestern, Oxford, Padua, Paris, Stanford, Toulouse, and Warwick for useful comments. Tsumagari is grateful for funding from Keio Gijuku Academic Development Funds.

[*Journal of Political Economy*, 2014, vol. 122, no. 5]

1982), which plays a central role in existing theory, implies that attention can be restricted to one-shot revelation mechanisms in which agents communicate everything they know to a central planner, principal, or owner, who subsequently makes all relevant production and allocation decisions. Incentive systems are designed to encourage agents to be truthful and obedient. Most real mechanisms do not involve such extreme centralization of authority and communication. Instead, decision-making authority is typically dispersed among agents, who decide their own production or consumption and are incentivized by suitable prices or transfers. Agents communicate directly with one another by participating in dynamic, time-consuming protocols involving discussions, reports, or negotiations.

In the debate on the economics of socialism, Hayek (1945) argued that the infeasibility of communication of dispersed private information by agents in an economy to a central planner was a key reason for the superiority of a decentralized market economy over a socialist economy with centralized decision making:

> If we can agree that the economic problem of society is mainly one of rapid adaptation to changes in the particular circumstances of time and place, it would seem to follow that the ultimate decisions must be left to the people who are familiar with these circumstances, who know directly of the relevant changes and of the resources immediately available to meet them. We cannot expect that this problem will be solved by first communicating all this knowledge to a central board which, after integrating all knowledge, issues its orders. We must solve it by some form of decentralization. (524)

It is not clear whether Hayek was aware of possible incentive problems associated with decentralization—wherein privately informed agents may use their discretion to pursue their own goals at the expense of the rest of society—and how this may affect the desirability of decentralization.[1]

These issues continue to be relevant to the design of internal organization of firms and design of regulatory policies. For example:

- When should firm owners delegate decisions regarding production and sourcing to managers? Should managers in turn delegate resolution of workplace problems to workers? Or should the firm be organized as a vertical hierarchy, where agents at any layer make reports to their bosses and await instructions on what to do?[2]

[1] See Caldwell (1997) for a detailed discussion of this issue.

[2] Aoki (1990) discusses key differences between American and Japanese firms in terms of these features.

- Should environmental regulations take the form of quantitative restrictions on pollution emitted by firms? Or should they take the form of tax-based incentives in which firms are authorized to make their own pollution decisions?[3]
- Should communication be vertical (from agents to principal, as in revelation mechanisms) or horizontal (between agents)? Should communication be structured as a static simultaneous process, or should it be dynamic and interactive?
- More generally, do incentive considerations justify restrictions on communication between agents or on the extent of discretion they ought to be granted?

In settings in which the revelation principle applies, these questions cannot be addressed since the principle states that a centralized revelation mechanism weakly dominates any mechanism with decentralized decision making or direct exchange of information among agents via dynamic communication processes.

In this paper we explore the role of communication costs in generating a theory that addresses these questions. Following the debates on economic socialism in the 1930s, a large literature subsequently emerged on resource allocation mechanisms that economize on communication costs.[4] Examples are the message space literature (Hurwicz 1960, 1977; Mount and Reiter 1974) and the theory of teams (Marschak and Radner 1972).[5] This early literature on mechanism design ignored incentive problems.[6] Most of the more recent literature on mechanism design, on the other hand, focuses only on incentive problems, ignoring communication costs.

There are a few papers that study mechanism design when incentive and communication costs coexist, but they impose strong ad hoc restrictions on the class of communication protocols.[7] Most authors restrict attention to mechanisms with a single round of communication, in which each agent simultaneously selects a message from an exogenously re-

[3] See discussions in Weitzman (1974, 1978) or Dasgupta, Hammond, and Maskin (1980).

[4] Caldwell (1997) provides an excellent introduction to this debate.

[5] Segal (2006) surveys recent studies of informationally efficient allocation mechanisms.

[6] A notable exception is the study by Reichelstein and Reiter (1988), who examined implications of strategic behavior for communicational requirements of mechanisms implementing efficient allocations.

[7] See Green and Laffont (1986, 1987), Melumad, Mookherjee, and Reichelstein (1992, 1997), Laffont and Martimort (1998), Blumrosen, Nisan, and Segal (2007), Kos (2012, 2013), and Blumrosen and Feldman (2013). Van Zandt (2007) and Fadel and Segal (2009) do not seek to derive optimal mechanisms given incentive and communication constraints but ask a related question: Does the communicational complexity needed to implement a given decision rule increase in the presence of incentive problems? Battigali and Maggi (2002) study a model of symmetric but nonverifiable information in which there are costs of writing contingencies into contracts. This is in contrast to the papers cited above, which involve asymmetric information with constraints on message spaces.

stricted message space. From the standpoint of informational efficiency, it is well known that dynamic communication is valuable in the presence of communication costs: they enable agents to condition their later messages on messages received at earlier stages from others, which allows more information to be exchanged. Examples have been provided in the literature in which the same is true when incentive problems also exist.[8] Hence there is no basis for restricting attention to a single round of communication, apart from problems of analytical tractability.

The key analytical problem in incorporating dynamic communication protocols into models with strategic agents is finding a suitable characterization of incentive constraints. Dynamic mechanisms enlarge the range of possible deviations available to participants, over and above those typically characterized by incentive compatibility constraints in a static revelation mechanism. Van Zandt (2007) observes that this is not a problem when the solution concept is ex post incentive compatibility, where agents do not regret their strategies even after observing all messages sent by other agents. When we use the less demanding concept of a (perfect) Bayesian equilibrium, dynamic communication protocols impose additional incentive constraints. This gives rise to a potential trade-off between informational efficiency and incentive problems.

The problem in studying this trade-off is that a precise characterization of incentive constraints for dynamic protocols is not available in existing literature. In a very general setting, Fadel and Segal (2009) provide different sets of sufficient conditions that are substantially stronger than necessary conditions. In this paper we restrict attention to contexts with single-dimensional outputs and single-crossing preferences for each agent.

Our first main result presents a set of conditions that are both necessary and sufficient for Bayesian implementation in arbitrary dynamic communication protocols (proposition 1).[9] This enables us to address the broad questions listed at the outset, without imposing ad hoc restrictions on the number of communication rounds. Our characteriza-

[8] Melumad et al. (1992, 1997), Blumrosen et al. (2007), and Van Zandt (2007, sec. 4) show the superiority of sequential over simultaneous communication protocols with limited message spaces and each agent sends a message only once. Kos (2013) studies optimal auctions with two potential buyers, a binary message set for each buyer at each round, and multiple communication rounds, where increasing the number of rounds raises the seller's welfare. We will provide some general results concerning this in Sec. VI.

[9] Celik (2013), Kos and Messner (2013), Rahman (2011), and Skreta (2006) have recently studied related problems of characterizing implementable mechanisms with restricted type spaces. The last three papers examine this question for a mechanism with a single round of communication, where type spaces are exogenous and need not be connected. In our context, the type space is connected, but types are endogenously pooled into sets of possibly nonconnected types. Moreover, we incorporate multiple communication rounds. Celik's paper deals with a problem similar to ours; the relationship is explained in more detail at the end of Sec. III.

tion of feasible mechanisms is shown to imply that the mechanism design problem reduces to selecting an output allocation rule that maximizes a payoff function of the principal (modified to include the cost of incentive rents paid to agents in a standard way with “virtual” types replacing actual types) subject to communication feasibility restrictions alone (proposition 2). This extends the standard approach to solving for optimal mechanisms with unlimited communication (following Myerson [1981]) and provides a convenient representation of the respective costs imposed by incentive problems and communication constraints. In particular, proposition 2 implies that there is no trade-off between informational efficiency and incentive compatibility, under the assumptions of our model.[10]

A number of implications of this result are then derived. The first concerns the value of delegating production decisions to agents.[11] This involves trading off benefits of delegation from enhanced informational efficiency with possible costs owing to opportunistic behavior given the presence of incentive problems.[12] Proposition 3 shows that the benefits of delegation in our model dominate: production decisions should be made by those most informed about attendant cost implications. It implies that quantitative targets for managers or workers, or pollution caps imposed by regulators, are dominated by delegation of corresponding decisions to workers, managers, and firms. These agents need to be incentivized by suitable bonus or tax formulas conditioned on reports communicated by them to the corresponding principal.

A second set of implications concern the design of optimal communication protocols. We show that if communication costs involve either material costs that are linear in the length of messages sent and in the size of the communication channel (defined by the maximum length of messages sent) or delay that is linear in the size of the communication channel, then communication should take place over multiple rounds in which agents disclose their information as slowly as possible.[13] Such dynamic protocols enable agents to exchange maximal information subject to the communication constraints. If communication costs consist only of delay, agents must report simultaneously in each round (as in dynamic auctions or budgeting systems in which agents at any given layer of a hierarchy submit

[10] The one-dimensional nature of production decisions and of cost types satisfying the single-crossing condition plays a key role. See Green and Laffont (1987) and Fadel and Segal (2009) for examples of other settings in which it is desirable to restrict the discretion of agents or their access to information in order to overcome incentive problems.

[11] Earlier literature such as Melumad et al. (1992, 1997) and Laffont and Martimort (1998) have focused on a related but different question: the value of decentralized contracting (or subcontracting) relative to centralized contracting. Here we assume that contracting is centralized and examine the value of decentralizing production decisions instead.

[12] The papers cited in n. 11 show for this reason how certain variants of delegated contracting can perform worse than centralized contracting.

[13] That is, in each round agents are assigned a small message set (consisting of the shortest possible messages).

forecasts, competing bids, or resource requests to their manager). But if they consist only of material costs, it is optimal for different agents to alternate in sending messages across successive rounds (as in price negotiations with alternating offers or meetings with interactive dialogue).

The paper is organized as follows. Section II introduces the model. Section III is devoted to characterizing feasible allocations. Section IV uses this to represent the design problem as maximizing the principal's incentive-rent-modified welfare function subject to communicational constraints alone. Section V uses this to compare centralized and decentralized allocations, while Section VI describes implications for design of optimal communication protocols. Section VII presents concluding remarks.

II. Model

There is a principal who contracts with two agents 1 and 2. Agent $i = 1, 2$ produces a one-dimensional nonnegative real-valued input q_i at cost $\theta_i q_i$, where θ_i is a real-valued parameter distributed over an interval $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ according to a positive-valued, continuously differentiable density function f_i and associated cumulative distribution function F_i.[14] The distribution satisfies the standard monotone hazard condition that $F_i(\theta_i)/f_i(\theta_i)$ is nondecreasing, implying that the "virtual cost" $v_i(\theta_i) \equiv \theta_i + F_i(\theta_i)/f_i(\theta_i)$ is strictly increasing.[15] The terms θ_1 and θ_2 are independently distributed, and the distributions F_1 and F_2 are common knowledge among the three players.

The inputs of the two agents combine to produce a gross return according to a production function $V(q_1, q_2)$ for the principal. We assume that it is feasible for the two agents to select their outputs independently: $(q_1, q_2) \in \mathfrak{R}_+ \times \mathfrak{R}_+$. Note that a context of team production in which both agents produce a common output q is a special case of the model in which V takes the form $W(\min\{q_1, q_2\})$. A procurement auction in which the principal seeks to procure a fixed amount $\bar{q}$ of a good from two competing suppliers is also a special case, with $V = \min\{q_1 + q_2, \bar{q}\}$. For the time being we impose no additional assumptions on the production function V. Sections V and VI will impose additional assumptions in order to derive specific implications for optimal mechanisms.

The principal makes transfer payments t_i to i. The payoff of i is $t_i - \theta_i q_i$. Both agents are risk neutral and have autarkic payoffs of zero. The principal's objective takes the form

$$V(q_1, q_2) - \lambda_1(t_1 + t_2) - \lambda_2(\theta_1 q_1 + \theta_2 q_2), \tag{1}$$

[14] We restrict attention to linear costs for the sake of expositional simplicity. The results extend to more general cost functions of the form $K_i + A_i(\theta_i)C_i(q_i)$, where K_i is a known fixed cost and variable costs are multiplicatively separable in θ_i and q_i.

[15] Our results can be extended in the absence of this assumption, employing the "ironing" technique developed by Myerson (1981) and Baron and Myerson (1982).

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, and $(\lambda_1, \lambda_2) \neq 0$, respectively, represent welfare weights on the cost of transfers incurred by the principal and cost of production incurred by the agents.

One application is to a context of internal organization or procurement, where the principal owns a firm composed of two divisions whose respective outputs combine to form revenues $V = V(q_1, q_2)$. The principal seeks to maximize profit; hence $\lambda_1 = 1$ and $\lambda_2 = 0$. The same applies when the two agents correspond to external input suppliers.

An alternative application is to environmental regulation. The principal is a regulator seeking to control outputs or abatements q_i of two firms $i = 1, 2$. The function $V(q_1 + q_2)$ is the gross social benefit, and θ_i is firm i's unit cost. Consumer welfare equals $V - (1 + \lambda)R$, where R is the total tax revenue collected from consumers and λ is the deadweight loss involved in raising these taxes. The revenue is used to reimburse transfers t_1, t_2 to the firms. Social welfare equals the sum of consumer welfare and firm payoffs, which reduces to (1) with $\lambda_1 = \lambda$, $\lambda_2 = 1$. If $\lambda = 0$, this reduces to the efficiency objective $V - \theta_1 q_1 - \theta_2 q_2$.

III. Communication and Contracting

A. *Timing*

The mechanism is designed by the principal at an ex ante stage ($t = -1$). It consists of a *communication protocol* (explained further below) and a set of contracts to each agent. There is enough time between $t = -1$ and $t = 0$ for all agents to read and understand the offered contracts.

At $t = 0$, each agent i privately observes the realization of θ_i and independently decides whether to participate or opt out of the mechanism. If either agent opts out, the game ends; otherwise they enter the planning or communication phase, which lasts until $t = T$.

Communication takes place in a number of successive rounds $t = 1, \ldots, T$. We abstract from mechanisms in which the principal seeks to limit the flow of information across agents, by either appointing mediators or regulators or scrambling devices. Later we argue that the optimal allocation is implemented with this communication structure; that is, it is not profitable to restrict or garble the flow of information across agents. Hence this restriction will turn out to entail no loss of generality. This simplifies the exposition considerably.

The principal is assumed to be able to verify all messages exchanged between agents. Equivalently, an exact copy of every message sent by one agent to another is also sent to the principal. This rules out collusion between the agents and allows the principal to condition transfers ex post on messages exchanged. Given that agents exchange messages directly with one another and the absence of any private information possessed by the principal, there is no rationale for the principal to send any messages to the agents. In what follows we will not make the principal's role explicit in the description of the communication protocol and will focus on the exchange of communication between the agents.[16]

At the end of round T, each agent $i = 1, 2$ or the principal selects production level q_i, depending on whether the mechanism is decentralized or centralized (an issue discussed further below).

Finally, after production decisions have been made, payments are made according to the contracts signed at the ex ante stage and verification by the principal of messages exchanged by agents and outputs produced by them.

B. *Communication Protocol*

A communication protocol is a rule defining T, the number of rounds of communication, and the message set M_i of each agent i in any given round, which may depend on the history of messages exchanged in previous rounds. If some agents are not supposed to communicate anything in any round, their message sets are null in those rounds. This allows us to include protocols in which agents take turns in sending messages in different rounds. Other protocols may involve simultaneous reporting by all agents in each round.

The *vocabulary* of any agent $i \in \{1, 2\}$ is a message set $\mathcal{M}_i$, which contains all messages m_i that i can feasibly send in a single round. This incorporates restrictions on the language that agents use to communicate with one another. Specific assumptions concerning such restrictions are introduced below.

The message set M_i assigned to agent i in any round is a subset of the vocabulary of that agent. Message histories and message sets are defined recursively as follows. Let m_{it} denote a message sent by i in round t. Given a history h_{t-1} of messages exchanged (sent and received) by i until round $t - 1$, it is updated at round t to include the messages exchanged at round t: $h_t = (h_{t-1}, \{m_{it}\}_{i\in\{1,2\}})$ and $h_0 = \varnothing$. The message set for i at round t is then a subset of $\mathcal{M}_i$, which depends on h_{t-1}, unless it is null.

Formally, the *communication protocol* specifies the number of rounds T, and for every round $t \in \{1, \ldots, T\}$ and every agent i, a message set $M_i(h_{t-1}) \subseteq \mathcal{M}_i$ or $M_i(h_{t-1}) = \varnothing$ for every possible history h_{t-1} until the end of the previous round.[17]

[16] As mentioned above, any mechanism in which agents send some messages to the principal but not to each other will end up being weakly dominated by a mechanism in which these messages are also sent to other agents. Hence there is no need to consider mechanisms in which agents communicate privately with the principal.

[17] We depart from Van Zandt (2007) and Fadel and Segal (2009) insofar as their definition of a protocol combines the extensive form game of communication as well as the communication strategy of each agent.

C. *Communication Costs*

We now describe communication costs. These depend on the length of messages sent, which we now explain.

We allow agents the option of not sending any message at all in any given round: hence the null message $\phi \in \mathcal{M}_i$. Let $l(m_i)$ denote the *length* of message $m_i \in \mathcal{M}_i$, which is an integer. It is natural to assume that $l(\phi) = 0$ and is positive-valued for any other message. For example, if messages are binary encoded, $l(m_i)$ could denote the total number of 0 and 1 bits included in m_i. Or if there is a finite alphabet consisting of a set of letters and messages are sent in words that are finite sequences of letters interspersed with blank spaces (i.e., null messages), the length of a message could be identified with the total number of letters.

Communication costs could involve either material costs (e.g., telephone calls, e-mails, faxes, videoconferences) or time delays (which hold up production and thereby involve delayed shipment of goods to customers and attendant loss of revenues). These costs will typically depend on the actual length of messages sent or on the maximum length of messages that could be sent across all contingencies, that is, the *capacity* of the communication channels involved. Specific models of communication costs will be provided in Section VI. For now, we avoid any such specific cost function.

We consider communication protocols whose costs amount to at most a fixed budget B, which we take as given. The communication budget will be subtracted from the primary revenues and costs of the principal to yield the net returns to the latter. The principal could decide on B at the first stage and for given B select an optimal mechanism at the second stage. We focus on the problem confronted at the second stage, corresponding to some finite level of B, which is given. The results will not depend on the specific choice of B.

For any given finite B, there will exist a set of feasible communication protocols whose cost will not exceed B. Let this set of feasible protocols given the communication constraints be denoted by $\mathcal{P}$. Under reasonable assumptions on the structure of agent vocabularies, it can be shown that any protocol in this set will involve a finite number of communication rounds and a finite message set for every agent in each round.[18]

D. *Communication Plans and Strategies*

Given a protocol $p \in \mathcal{P}$, a *communication plan* for agent i specifies for every round t a message $m_{it}(h_{t-1}) \in M_i(h_{t-1})$ for every possible history h_{t-1} that could arise for i in protocol p until round $t-1$. The set of communi-

[18] A detailed statement of assumptions and proofs is available in the working paper version of this paper (Mookherjee and Tsumagari 2012).

cation plans for i in protocol p is denoted $C_i(p)$. As explained above, for any finite communication budget, this set is finite for any feasible protocol. For the rest of the paper, it will be assumed that communication protocols have this property.

For communication plan $c = (c_1, c_2) \in C(p) \equiv C_1(p) \times C_2(p)$, let $h_t(c)$ denote the history of messages generated thereby until the end of round t. Let $H_t(p) \equiv \{h_t(c) | c \in C(p)\}$ denote the set of possible message histories in this protocol until round t. For a given protocol, let $\mathcal{H} \equiv H_T(p)$ denote the set of possible histories at the end of round T.

Given a protocol $p \in \mathcal{P}$, a *communication strategy* for agent i is a mapping $c_i(\theta_i) \in C_i(p)$ from the set $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ of types of i to the set $C_i(p)$ of possible communication plans for i. In other words, a communication strategy describes a dynamic plan for sending messages, for every possible type of the agent. The finiteness of the set of communication plans implies that it is not possible for others in the organization to infer the exact type of any agent from the messages exchanged. Nonnegligible sets of types will be forced to pool into the same communication plan.

E. *Production Decisions and Contracts*

Many authors in previous literature (Blumrosen et al. 2007; Kos 2012, 2013; Blumrosen and Feldman 2013) have limited attention to mechanisms in which output assignments and transfers are specified as a function of the information communicated by the agents. Decision-making authority is effectively retained by the principal in this case. We shall refer to such mechanisms as *centralized*. A *contract* in this setting specifies a quantity allocation $q(h) \equiv (q_1(h), q_2(h)) : \mathcal{H} \to \mathfrak{R}^2_+$, with corresponding transfers $t(h) \equiv (t_1(h), t_2(h)) : \mathcal{H} \to \mathfrak{R} \times \mathfrak{R}$. A *centralized mechanism* is then a communication protocol $p \in \mathcal{P}$ and an associated contract $(q(h), t(h)) : \mathcal{H} \to \mathfrak{R}^2_+ \times \mathfrak{R}^2$.

Some authors (Melumad et al. 1992, 1997) have explored mechanisms in which the principal delegates decision making to one of the two agents and compared their performance with centralized mechanisms. This is a pertinent question in procurement, internal organization, or regulation contexts. They consider mechanisms in which both contracts with the second agent and production decisions are decentralized (while restricting attention to communication protocols involving a single round of communication). Here we focus attention on mechanisms in which the principal retains control over the design of contracts with both agents while decentralizing decision-making authority to agents concerning their own productions. We refer to such mechanisms as *decentralized*. The potential advantage of decentralizing production decisions to agents is that these decisions can be based on information possessed by the agents that is richer than what they can communicate to the principal. Transfers can then be based on output decisions as well as messages exchanged.

Formally, a *decentralized mechanism* is a communication protocol p and a pair of contracts for the two agents, where the contract for agent i is a transfer rule $t_i(q_i, h) : \Re_+ \times \mathcal{H} \to \Re$. Such a mechanism induces a quantity allocation $q_i(\theta_i, h) : \Theta_i \times \mathcal{H} \to \Re_+$, which maximizes $t_i(q_i, h) - \theta_i q_i$ with respect to choice of $q_i \in \Re_+$.[19] To simplify exposition we specify the quantity allocation as part of the decentralized mechanism itself.

A centralized mechanism can be viewed as a special case of a decentralized mechanism in which $q_i(\theta_i, h)$ is measurable with respect to h, that is, does not depend on θ_i conditional on h. It corresponds to a mechanism in which the principal sets an output target for each agent (based on the messages communicated) and then effectively forces them to meet these targets with a corresponding incentive scheme. We can therefore treat every mechanism as decentralized, in a formal sense. Hence the distinction between centralized and decentralized mechanisms is unclear.

The distinction between centralization and decentralization can be made more clearly and simply for allocations resulting from mechanisms rather than for mechanisms themselves. Even if agents are given discretionary authority, they may not actually utilize their authority to base production decisions on private information that has not been communicated to the principal. Hence whether decision making is effectively decentralized depends not only on the mechanism (whether it is centralized or not) but also on the behavior of agents in that mechanism. It is more meaningful, therefore, to distinguish between centralization and decentralization in terms of allocations rather than mechanisms. To this end, we need to define allocations first.

F. Feasible Production Allocations

A *production allocation* is a mapping

$$q(\theta) \equiv (q_1(\theta_1, \theta_2), q_2(\theta_1, \theta_2)) : \Theta_1 \times \Theta_2 \to \Re_+^2.$$

The standard way of analyzing the mechanism design problem with unlimited communication is to first characterize production allocations that are feasible in combination with some set of transfers and then use the revenue equivalence theorem to represent the principal's objective

[19] Since i infers the other's output q_j $(j \neq i)$ only through h, we can restrict attention to contracts in which the payments to any agent depend only on his own output without loss of generality. Specifically, if t_i were to depend on q_j, the expected value of the transfer to i can be expressed as a function of q_i and h since agent i's information about q_j has to be conditioned on h.

in terms of the production allocation alone, while incorporating the cost of the supporting transfers. To extend this method we need to characterize feasible production allocations. Restrictions are imposed on production allocations owing to both communication and incentive problems.

Consider first communication restrictions. A production allocation $q(\theta)$ is said to be *communication feasible* if (a) the mechanism involves a communication protocol p satisfying the specified constraints on communication and (b) there exist communication strategies $c(\theta) = (c_1(\theta_1), c_2(\theta_2)) \in C(p)$ and output decisions of agents $q_i(\theta_i, h) : \Theta_i \times \mathcal{H} \rightarrow \Re_+$ such that $q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta))))$ for all $\theta \in \Theta \equiv \Theta_1 \times \Theta_2$. Here $h(c)$ denotes the message histories generated by the communication strategies c in this protocol.

The other set of constraints pertains to incentives. A communication-feasible production allocation $q(\theta)$ is said to be *incentive feasible* in a mechanism if there exists a perfect Bayesian equilibrium (PBE) of the game induced by the mechanism that implements the production allocation.[20] In other words, there must exist a set of communication strategies and output decision strategies satisfying condition b above in the requirement of communication feasibility, which constitutes a PBE.

G. *Centralized and Decentralized Allocations*

We are now in a position to define centralized and decentralized allocations respectively. In a centralized mechanism, output decisions are made by the principal, following receipt of messages from agents. Hence output choices can depend on the true state only through the dependence of messages sent by the agents on their private information. This is the hallmark of production allocations resulting from a centralized mechanism.

Formally, a communication-feasible production allocation $q(\theta)$ is said to be *centralized* if it is measurable with respect to the histories induced by the communication strategies of the agents, that is, $q(\theta) = (q_1(h(c(\theta))), q_2(h(c(\theta))))$ for all $\theta \in \Theta$. The allocation is said to be *decentralized* if it is not centralized.

In a decentralized allocation, knowledge of actual message histories is not sufficient to predict the actual outputs chosen. Such an allocation cannot result from any centralized mechanism: agents must be given at least some discretionary authority over their respective production decisions. Moreover, agents must actually utilize this authority.

[20] This requires both incentive and participation constraints to be satisfied. For the definition of PBE, see Fudenberg and Tirole (1991, sec. 8.2).

H. Characterization of Incentive Feasibility

We now proceed to characterize incentive-feasible production allocations. Using the single-dimensional output of each agent and the single-crossing property of agent preferences, we can obtain as a necessary condition a monotonicity property of expected outputs with respect to types at each decision node. To describe this condition, we need the following notation.

It is easily checked (see lemma 1 in the Appendix) that given any strategy configuration $c(\theta) \equiv (c_1(\theta_1), c_2(\theta_2))$ and any history h_t until the end of round t in a communication protocol, the set of types (θ_1, θ_2) that could have generated the history h_t can be expressed as the Cartesian product of subsets $\Theta_1(h_t)$, $\Theta_2(h_t)$ such that

$$\{(\theta_1, \theta_2) | h_t(c(\theta_1, \theta_2)) = h_t\} = \Theta_1(h_t) \times \Theta_2(h_t). \tag{2}$$

A necessary condition for incentive feasibility of a production allocation $q(\theta)$ that is communication-feasible in a protocol p and supported by communication strategies $c(\theta)$ is that for any $t = 0, \ldots, T$, any $h_t \in H_t$, and any $i = 1, 2$,

$$E[q_i(\theta_i, \theta_j) | \theta_j \in \Theta_j(h_t)] \text{ is nonincreasing in } \theta_i \text{ on } \Theta_i(h_t), \tag{3}$$

where H_t denotes the set of possible histories until round t generated with positive probability in the protocol when $c(\theta)$ is played, and $\Theta_i(h_t)$ denotes the set of types of i who arrive at h_t with positive probability under the communication strategies $c(\theta)$.

The necessity of this condition follows straightforwardly from the dynamic incentive constraints that must be satisfied for any history h_t on the equilibrium path. When h_t is observed, i's beliefs about θ_j are updated by conditioning on the event that $\theta_j \in \Theta_j(h_t)$. All types of agent i in $\Theta_i(h_t)$ will have chosen the same messages up to round t. Hence any type $\theta_i \in \Theta_i(h_t)$ has the opportunity to pretend to be any other type in $\Theta_i(h_t)$ from round $t + 1$ onward, without this deviation being discovered by anyone. A PBE requires that such a deviation cannot be profitable. The single-crossing property then implies condition (3).

As noted earlier, the existing literature has provided sufficient conditions for incentive feasibility that are stronger than (3). Fadel and Segal (2009) in a more general framework (with abstract decision spaces and no restrictions on preferences) provide two sets of sufficient conditions. One set (provided in their proposition 6) of conditions is based on the observation that the stronger solution concept of ex post incentive compatibility implies Bayesian incentive compatibility. In our current context, ex post incentive compatibility requires for each $i = 1, 2$

$$q_i(\theta_i,\ \theta_j) \text{ is globally nonincreasing in } \theta_i \text{ for every } \theta_j \in \Theta_j. \tag{4}$$

Another set of sufficient conditions (proposition 3 in Fadel and Segal [2009]) imposes a no-regret property with respect to possible deviations to communication strategies chosen by other types following every possible message history arising with positive probability under the recommended communication strategies. This is applied to every pair of types for each agent at nodes where it is this agent's turn to send a message. In the context of centralized mechanisms (which Fadel and Segal restrict attention to), this reduces to the condition that for any $i = 1, 2$ and any $h_t \in H_t$, $t = 0, \ldots, T-1$, where it is i's turn to move (i.e., $M_i(h_t) \neq \varnothing$),[21]

$$E[q_i(\theta_i, \theta_j)|\theta_j \in \Theta_j(h_t)] \text{ is globally nonincreasing in } \theta_i. \tag{5}$$

Our first main result is that the necessary condition (3) is also sufficient for incentive feasibility, provided that the communication protocol prunes unused messages. Suppose that p is a communication protocol in which communication strategies used are $c(\theta)$. Then p is *parsimonious relative to communication strategies* $c(\theta)$ if every possible history $h \in \mathcal{H}$ in this protocol is reached with positive probability under $c(\theta)$.

PROPOSITION 1. Consider any production allocation $q(\theta)$ that is communication-feasible in a protocol p and is supported by communication strategies $c(\theta)$, where the protocol is parsimonious with respect to $c(\theta)$. Then condition (3) is necessary and sufficient for incentive feasibility of $q(\theta)$.

Parsimonious protocols have the convenient feature that Bayes's rule can be used to update beliefs at every node, and off-equilibrium-path deviations do not have to be considered while checking incentive feasibility. Restricting attention to such protocols entails no loss of generality since any protocol can be pruned by deleting unused messages under any given set of communication strategies to yield a protocol that is parsimonious with respect to these strategies. Hence it follows that condition (3) is both necessary and sufficient for incentive feasibility.

The proof of proposition 1 is provided in the Appendix. The main complication arises for the following reason. In a dynamic protocol with more than one round of communication, no argument is available for showing that attention can be confined to communication strategies with a threshold property. Hence the set of types $\Theta_i(h_t)$ pooling into message history h_t need not constitute an interval. The monotonicity property for output decisions in (3) holds only "within" $\Theta_i(h_t)$, which may span two distinct intervals. The monotonicity property may therefore not hold for

[21] As Fadel and Segal point out, it suffices to check the following condition at the last node of the communication game at which it is agent i's turn to move. Note also that this condition is imposed on nodes of the communication game, and not at nodes at which agents make output decisions in the case of a decentralized mechanism.

type ranges lying between the two intervals. This complicates the conventional argument for construction of transfers that incentivize a given production allocation.

The proof is constructive.[22] Given a production allocation satisfying (3) with respect to a set of communication strategies in a protocol, we first prune the protocol to eliminate unused messages. Then incentivizing transfers are constructed as follows. We start by defining a set of functions representing expected outputs of each agent following any given history h_t at any stage t, expressed as a function of the type of that agent. Condition (3) ensures that the expected output of any agent i is monotone over the set $\Theta_i(h_t)$. These are the types of i that actually arrive at h_t with positive probability on the equilibrium path. The proof shows that it is possible to extend this function over all types of this agent (not just those that arrive at h_t on the equilibrium path), which is globally monotone in a way that agrees with the actual expected outputs on the set $\Theta_i(h_t)$ and maintains consistency across histories reached at successive dates. This amounts to assigning outputs for types that do not reach h_t on the equilibrium path, which can be thought of as outputs they would be assigned if they were to deviate somewhere in the game and arrive at h_t. Since this extended function is globally monotone, transfers can be constructed in the usual way to incentivize this allocation of expected output. The construction also has the feature that the messages sent by the agent after arriving at h_t do not affect the expected outputs that would thereafter be assigned to the agent, which assures that the agent does not have an incentive to deviate from the recommended communication strategy.[23]

IV. Characterizing Optimal Mechanisms

Having characterized feasible allocations, we can now restate the mechanism design problem as follows.

Note that the interim participation constraints imply that every type of each agent must earn a nonnegative expected payoff from participating. Agents that do not participate do not produce anything or receive any transfers. Hence by the usual logic it is without loss of generality that all types participate in the mechanism. The single-crossing property ensures

[22] For a geometric illustration of the argument, see the working paper version of this paper (Mookherjee and Tsumagari 2012).

[23] The constructed mechanism has the property that agents are indifferent across all message options at every information set of the game. It may not be the only way of implementing the desired allocation. See, e.g., Celik (2013) for a different construction in which this property need not hold. Celik considers a more general context in which the evolution of beliefs is required to follow an arbitrary martingale process, agents choose mixed strategies, the decision space need not be single-dimensional, and aggregate side transfers to agents are required to balance at each stage.

that expected payoffs are nonincreasing in θ_i for each agent i. Since $\lambda_1 \geq 0$, it is optimal to set transfers that incentivize any given production allocation rule $q(\theta)$ satisfying (3) such that the expected payoff of the highest-cost type $\bar{\theta}_i$ equals zero for each i. The expected transfers to the agents then equal (using the arguments in Myerson [1981] to establish the revenue equivalence theorem)

$$\sum_{i=1}^{2} E[v_i(\theta_i)q_i(\theta_i, \theta_j)],$$

where $v_i(\theta_i) \equiv \theta_i + F_i(\theta_i)/f_i(\theta_i)$. Consequently the expected payoff of the principal is

$$E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i)q_i(\theta_i, \theta_j) - w_j(\theta_j)q_j(\theta_i, \theta_j)], \tag{6}$$

where

$$w_i(\theta_i) \equiv (\lambda_1 + \lambda_2)\theta_i + \lambda_1 F_i(\theta_i)/f_i(\theta_i).$$

This enables us to state the problem in terms of selecting a production allocation in combination with communication protocol and communication strategies. Given the set $\mathcal{P}$ of feasible communication protocols defined by the communication constraints, the problem is to select a protocol $p \in \mathcal{P}$, communication strategies $c(\theta)$ in p, and production allocation $q(\theta)$ to maximize (6), subject to the constraint that (i) there exists a set of output decision strategies $q_i(\theta_i, h)$, $i = 1, 2$, such that $q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta))))$ for all $\theta \in \Theta$; and (ii) the production allocation satisfies condition (3).

Condition i is a communication-feasibility constraint, which applies even in the absence of incentive problems. Condition ii is the additional constraint represented by incentive problems. Note that the above statement of the problem applies since attention can be confined without loss of generality to protocols that are parsimonious with respect to the assigned communication strategies. To elaborate, note that conditions i and ii are both necessary for implementation. Conversely, given a production allocation, a communication protocol, and communication strategies in the protocol that satisfy conditions i and ii, we can prune that protocol by deleting unused messages to obtain a protocol that is parsimonious with respect to the given communication strategies. Then proposition 1 ensures that the production allocation can be implemented as a PBE in the pruned protocol with suitably constructed transfers, which generate an expected payoff (6) for the principal while ensuring that all types of both agents have an incentive to participate.

Now observe that the incentive-feasibility constraint ii is redundant in this statement of the problem. If we consider the relaxed version of the problem stated above in which ii is dropped, the solution to that problem must automatically satisfy ii since the monotone hazard rate property on the type distributions F_i ensures that $w_i(\theta_i)$ is an increasing function for each i. This generates the following result.

Proposition 2. The mechanism design problem can be reduced to the following. Given any set $\mathcal{P}$ of feasible communication protocols defined by the communication constraints, select a protocol $p \in \mathcal{P}$, communication strategies $c(\theta)$ in p, and production allocation $q(\theta)$ to maximize (6), subject to the constraint of communication feasibility alone; that is, there exists a set of production strategies $q_i(\theta_i, h)$, $i = 1, 2$, such that

$$q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta)))) \quad \forall\, \theta \in \Theta. \tag{7}$$

In the case of unlimited communication, this reduces to the familiar property that an optimal production allocation can be computed on the basis of unconstrained maximization of expected payoffs (6) of the principal that incorporate incentive rents earned by the agents. With limited communication, additional constraints pertaining to communication feasibility have to be incorporated. In the absence of incentive problems, the same constraint would apply: the only difference would be that the agents would not earn incentive rents and the objective function of the principal would be different (w_i would be replaced by $\tilde{w}_i = [\lambda_1 + \lambda_2]\theta_i$).

Proposition 2 thus shows how costs imposed by incentive considerations are handled differently from those imposed by communicational constraints. The former is represented by the replacement of production costs of the agents by their incentive-rent-inclusive virtual costs in the objective function of the principal, in exactly the same way as in a world with costless, unlimited communication. The costs imposed by communicational constraints are represented by the restriction of the feasible set of production allocations, which must now vary more coarsely with the type realizations of the agents. This can be viewed as the natural extension of the Marschak-Radner (1972) characterization of optimal team decision problems to a setting with incentive problems. In particular, the same computational techniques can be used to solve these problems both with and without incentive problems: only the form of the objective function needs to be modified to replace actual production costs by virtual costs. The "desired" communicational strategies can be rendered incentive compatible at zero additional cost.

This result does not extend when the definition of incentive feasibility replaces the solution concept of PBE by ex post incentive compatibility (EPIC). EPIC requires the allocation to be globally monotone (condition [4]). The following example shows that the optimal PBE allocation for a specific communication protocol does not satisfy this property.

Example.—Suppose that $V(q_1, q_2) = 2(\min\{q_1, q_2\})^{1/2}$. The term θ_1 is distributed uniformly on $[0, \alpha]$, where $\alpha \in (0, 2/3)$, and θ_2 is uniformly distributed on [0, 1]. The principal's objective is $V(q_1, q_2) - t_1 - t_2$,where t_i is a transfer to agent i. There is a single feasible communication protocol with two rounds, with a binary message space for each agent, and agent 1 sends a message at the first round, followed by agent 2 in the second round. The mechanism is centralized. In this context we know from Blumrosen et al. (2007) that optimal communication strategies take the following form: agent 1 sends $m_1 = 0$ for $\theta_1 \in [0, x)$ and $m_1 = 1$ for $\theta_1 \in [x, \alpha]$ for some $x \in [0, \alpha]$. Agent 2 then sends $m_2 = 0$ for $\theta_2 \in [0, y_{m_1})$ and 1 for $\theta_2 \in [y_{m_1}, 1]$, for some $y_{m_1} \in [0, 1]$, $m_1 = 0, 1$.

Defining $q(c) \equiv 1/c^2 = \arg\max_q[2q^{1/2} - cq]$ and $\Pi(c) \equiv 2q(c)^{1/2} - cq(c) = 1/c$, the optimal output choice made by the principal conditional on the information that $(\theta_1, \theta_2) \in [\theta'_1, \theta''_1] \times [\theta'_2, \theta''_2]$ is $q_1 = q_2 = q(\theta'_1 + \theta''_1 + \theta'_2 + \theta''_2)$. The maximized payoff of the principal conditional on this information is then $\Pi(\theta'_1 + \theta''_1 + \theta'_2 + \theta''_2)$. Hence the principal's problem reduces to selecting (x, y_0, y_1) to maximize

$$\frac{x}{\alpha}\frac{x + 2y_0}{(x + y_0)(x + y_0 + 1)} + \frac{\alpha - x}{\alpha}\frac{x + 2y_1 + \alpha}{(x + y_1 + \alpha)(x + y_1 + 1 + \alpha)}.$$

Given x, the optimal

$$y_0 = (x^2 + 2x)^{1/2}/2 - x/2$$

and

$$y_1 = [(x + \alpha)^2 + 2(x + \alpha)]^{1/2}/2 - (x + \alpha)/2.$$

It is evident that $y_0 < y_1$ for any $x \in [0, \alpha]$. Since $\alpha < 2/3$, it is easy to check that $y_0 + 1 > y_1 + \alpha$ holds, implying that $q(x + y_0 + 1) < q(x + y_1 + \alpha)$. This shows that the optimal output assignment is not globally monotone in θ_1: if $\theta_2 \in (y_0, y_1)$, then q is higher when $\theta_1 \in [x, \alpha]$ compared with when $\theta_1 \in [0, x)$ (see fig. 1). Hence the optimal Bayesian allocation cannot be EPIC under any set of transfer functions.

Where incentive feasibility is based on the EPIC solution concept, therefore, condition (4) must additionally be imposed on the optimization problem, in addition to the requirement of communication feasibility. Hence the optimal PBE and EPIC allocations must differ. This observation does not apply in the case of unlimited communication: in that context, optimal Bayesian and EPIC mechanisms generally coincide (Mookherjee and Reichelstein 1992; Gershkov et al. 2013).

Van Zandt (2007) and Fadel and Segal (2009) discuss a related question of the "communication cost of selfishness": whether the communicational complexity of implementing any given social choice function (production allocation in our notation) is increased by the presence of

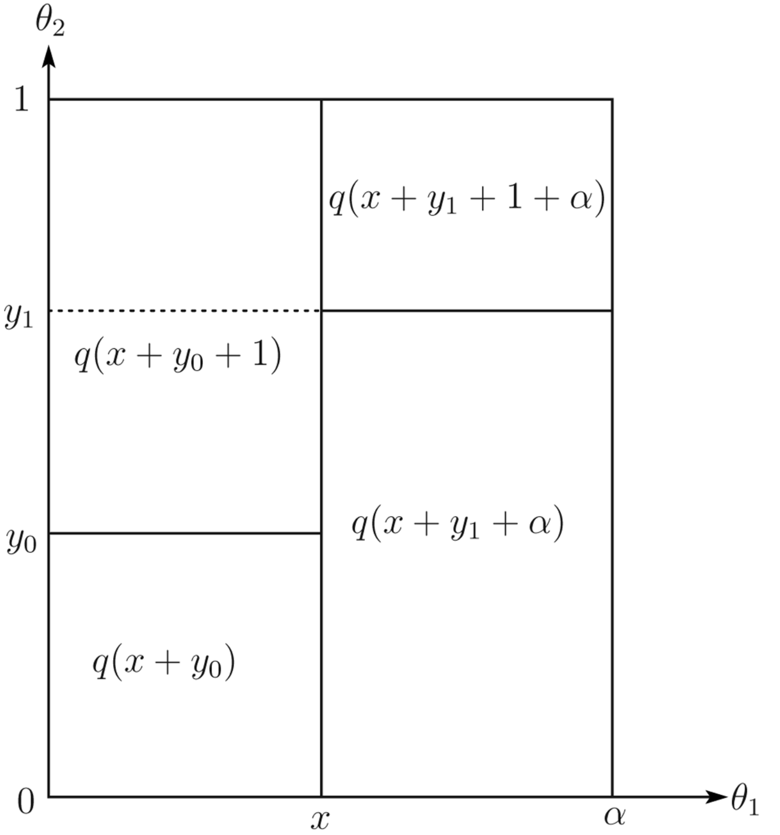

FIG. 1.—Example

incentive constraints. Van Zandt shows that this is not true when using the EPIC solution concept, while Fadel and Segal provide examples in which this is the case when using the Bayesian solution concept. In our context, where we fix communication complexity and solve for optimal mechanisms, an analogous question could be phrased as follows: Does the optimal mechanism in the presence of communication constraints alone continue to be optimal when incentive constraints are incorporated? Proposition 2 shows that the answer to this question depends on λ_1. If the principal is solely concerned with efficiency and $\lambda_1 = 0$, the objective function is the same with and without incentive constraints.[24] Then the optimal mechanism in the absence of any incentive constraints is also optimal in the presence of incentive constraints. On the other hand, if

[24] Van Zandt and Fadel and Segal do not incorporate the costs of incentivizing transfers in posing the implementation problem, so this is the appropriate case to consider when comparing with their result.

$\lambda_1 > 0$ and the principal seeks to limit transfers to the agents, the objective functions with and without incentive constraints differ. Then the optimal allocation in the absence of incentive constraints will typically not be optimal when incentive problems are present.

V. Implications for Decentralization versus Centralization of Production Decisions

We now examine implications of proposition 2 for the value of decentralized allocations compared with centralized ones. If production decisions are made by the principal, outputs are measurable with respect to the history of exchanged messages. If decisions are delegated to the agents, this is no longer true since they can be decided by the agents on the basis of information about their own true types, which is richer than what they managed to communicate to the principal. Unlike settings of unlimited communication, centralized mechanisms cannot replicate the outcomes of decentralized ones. Contracts are endogenously incomplete, owing to communication constraints. This gives rise to a meaningful question of how to trade off the costs and benefits of delegation.

The typical trade-off associated with delegation of decision rights to better-informed agents compares the benefit of increased flexibility of decisions with respect to the true state of the world, with the cost of possible use of discretion by the agent to increase his own rents at the expense of the principal. Proposition 2, however, shows that once the incentive rents that agents will inevitably earn have been factored into the principal's objective, incentive considerations can be ignored. The added flexibility that decentralization allows then ensures that it is superior. The following proposition shows that this is true as long as V satisfies some standard regularity conditions that ensure that optimal production allocations are always interior.[25]

Proposition 3. Suppose that V is twice continuously differentiable, strictly increasing, and strictly concave and that each agent's marginal product $\partial V/\partial q_i \to \infty$ as $q_i \to 0$. Then given any centralized production allocation that is feasible in some centralized mechanism, there exists a decentralized production allocation that is feasible in a decentralized mechanism using the same communication protocol and generates a strictly higher payoff to the principal.

[25] These regularity conditions are not satisfied in the contexts of team production or a procurement auction. For these contexts, the output allocation decision reduces to choice of q_1 alone, with $q_2 = q_1$ in the case of team production and $q_2 = \bar{q} - q_1$ in the case of a principal trying to procure a fixed quantity $\bar{q}$ from the two sellers combined. We can analogously show that any centralized mechanism is inferior to some mechanism that delegates to agent 1 the choice of q_1.

It follows that the solution to the mechanism design problem cannot involve a centralized production allocation. The agents must be delegated authority over production decisions, and they must effectively utilize this authority. The underlying argument as as follows. Consider the restricted version of the problem described in proposition 2 corresponding to a given communication protocol; that is, find optimal communication strategies and production allocation subject to communication feasibility alone. The finiteness of the set of feasible communication plans for every agent implies the existence of nonnegligible type intervals over which communication strategies and message histories are pooled. Consequently, if production decisions are centralized, the production decision for i must analogously be pooled. Instead, if production decisions were delegated to agent i, the production decision could be based on agent i's knowledge of its own true type. Under the regularity conditions assumed in proposition 3, optimal production allocations are always interior. Delegation will then enhance "flexibility" of the production allocation, which will allow a strict increase in the principal's objective (6) while preserving communication feasibility.

This result can be contrasted to the demonstration that variants of delegated contracting can be inferior to centralized mechanisms (see Melumad et al. 1992, 1997), owing to "control loss" from incentive problems (which aggravate the problem of double marginalization of rents) that can overwhelm improvements in flexibility. Such variants of delegation allow the principal contractor to choose payments made to the subcontractor, which are unobserved by the principal. Once these payments can be observed and used by the principal to evaluate the performance of the principal contractor, delegation is shown in the papers cited above to perform superior to centralized mechanisms. In the context of our model, the principal contracts directly with and thus controls payments to both agents, enabling problems of double marginalization to be avoided. This explains the relation to the results of Melumad et al. Proposition 3 shows that the superiority of decentralized allocations obtains without imposing any restrictions on the communication protocol (apart from being finite).

In the context of internal organization, this result implies the optimality of decentralizing production decisions to workers when communication constraints prevent them from fully describing shop floor contingencies to upper management, as in the prototypical "Japanese" firm (Aoki 1990), where the central headquarters contracts directly with all workers. This is in contrast to subcontracting settings considered in Melumad et al. (1992, 1997), where centralization can dominate delegation to prime contractors if the procuring firm does not monitor payments or allocation of production between subcontractors and the prime contractor.

In the environmental regulation context, Weitzman (1974) compared "price" and "quantity" regulation of pollution by firms without allowing for any communication of private information held by firms concerning abatement costs to the regulator. The outcomes of the price regulation mode correspond to a decentralized allocation with a linear incentive mechanism, while the quantity regulation mode corresponds to a centralized mechanism in which the regulator imposes a cap on emissions. In this context, Weitzman showed that either form of regulation could be superior, depending on parameters. In later work, however, Weitzman (1978) and Dasgupta et al. (1980) characterized optimal nonlinear incentive mechanisms that could be viewed as a combination of price and quantity regulation, while continuing to assume that it is infeasible for firms to communicate any information to regulators. This results in a decentralized allocation, as regulated firms select their own emission levels. The demonstration that it dominates pure quantity regulation can be viewed as a version of our result that every centralized allocation is dominated by decentralized ones if communication is limited. Proposition 3 generalizes this result to contexts in which firms communicate their information to regulators, but the extent of such communication is restricted owing to costs associated with communication of excessively detailed information.

VI. Implications for Choice of Communication Protocol

Proposition 2 has useful implications for the ranking of different communication protocols. Given any set of communication strategies in a given protocol, in state (θ_i, θ_j) agent i learns that θ_j lies in the set $\Theta_j(h(c_i(\theta_i), c_j(\theta_j)))$, which generates an information partition for agent i over agent j's type.

Say that a protocol $p_1 \in \mathcal{P}$ is *more informative* than another $p_2 \in \mathcal{P}$ if, for any set of communication strategies in the former, there exists a set of communication strategies in the latter that yields (at round T) an information partition to each agent over the type of the other agent that is more informative in the Blackwell sense in (almost) all states of the world.

It then follows that a more informative communication protocol permits a wider choice of communication-feasible production allocations. Proposition 2 implies that the principal prefers more informative protocols and would not benefit by restricting or scrambling the flow of communication among agents.

This is the reason we assumed that all messages are addressed to everyone else in the organization. If the transmission and processing of messages entail no resource or time costs, this ensures maximal flow of information between agents. In contrast, much of the literature on informational efficiency of resource allocation mechanisms (in the tradition of Hurwicz [1960, 1977] or Mount and Reiter [1974]) has focused on centralized communication protocols in which agents send messages to the principal rather than to one another. Such protocols restrict the flow of information among agents. Marschak and Reichelstein (1998) have extended this to network mechanisms in which agents communicate directly with one another and examine the consequences of such decentralized "network" mechanisms for communication costs (in the absence of incentive problems). In our approach the principal plays no active role in the communication process.[26]

Within the class of such decentralized communication protocols, more can be said about the nature of optimal protocols, depending on the precise nature of communication costs. We turn to this now.

We limit attention to agent vocabularies consisting of *letters* or messages of unit length, in which longer messages are *words* that are combinations of letters. Hence if there are L_i letters of unit length in agent i's vocabulary, then there are at most L_i^k words or messages of length not exceeding k, for any integer k. For instance, if the agents communicate using binary code, there are two letters or unit bits 0 and 1. Any longer message consists of a string of unit bits, with the length of the message identified by the number of bits. The same is true for most languages that have an alphabet of letters, words are composed of a string of letters, and the length of a word is measured by the number of letters contained in that word. In what follows, we use M_i^* to denote the set of letters in i's vocabulary in conjunction with the null message, that is, $M_i^* \equiv \{m_i \in \mathcal{M}_i | l(m_i) \leq 1\}$.

Communication costs can involve either material costs or time delays. Material costs could include variable (e.g., depending on the length of messages sent) or fixed (depending on communication capacity) costs. The communication capacity of each agent i is defined as the longest message contained in M_i: $\bar{l}(M_i) \equiv \max_{m_i \in M_i} l(m_i)$.

We assume that material communication costs for any given round are linear in length of messages and communication capacity:

$$\Phi_m = \phi_v l(m_i) + \phi_f \bar{l}(M_i) \tag{8}$$

for some constants $\phi_v \geq 0$, $\phi_f > 0$, while delay costs per round takes the form

$$\Phi_d = \phi_d \max\{\bar{l}(M_1), \bar{l}(M_2)\} \tag{9}$$

[26] If the only costs of communication involve writing or sending messages, this is without loss of generality since the principal has no private information to report to the agents, and any messages that an agent sends to the principal that are in turn sent to the other agent could be sent directly to the latter at no additional cost.

for some $\phi_d > 0$. The constraint imposed by a given budget B for communication cost pertains to the total cost incurred across different rounds in the protocol. The results reported below extend as long as there are no increasing returns to scale with respect to length of messages or communication capacity.

Our first result shows that under the above assumptions, information ought to be released "slowly" by agents across multiple rounds of communication. If any agent has a "large" message set in any given round, the agent can communicate more information at the same cost by breaking this up into a sequence of smaller messages in successive rounds. Suppose, for instance, that communication is in binary code, and an agent has the following message set in some round: $\{\phi, 0, 1, 00, 01, 10, 11\}$. This round can be broken up into two successive rounds in each of which the agent is given the message set $\{\phi, 0, 1\}$. The agent can communicate at least as much information across these two rounds as she could previously (e.g., a null message in both rounds corresponds to a null message previously, a null message in one round combined with a single-bit message 0 [or 1] in the other corresponds to a previous message of 0 [or 1], and so on). Communication costs do not increase since capacity costs are the same: the maximal length of a message was 2 previously with a single round, while it is now 1 in each of the two rounds. The aggregate length of messages remains the same in every state of the world. The agent now has a total of nine possible message combinations across the two rounds, as against seven possible messages previously. Hence the agent can now send strictly more information; for example, she has the choice of the order in which a null message is sent in one round and a single-bit message in the other. This allows a strict improvement in the principal's payoff.

Proposition 4. Suppose that agent vocabularies and communication costs are as specified above. Also suppose that the production function satisfies the regularity conditions specified in proposition 3 and, in addition, $V_{12}(q_1, q_2) \neq 0$ for every $(q_1, q_2) \gg 0$. Then any nonnull message set assigned to any agent (in any round following any history arising with positive probability in any optimal protocol) must consist of letters (messages of unit length) alone, that is, $M_i(h_{t-1}) = M_i^*$ if it is nonnull.

Our final result concerns the contrast between material costs and time delay formulations of communication cost for the nature of optimal protocols.

Proposition 5. Suppose that the same conditions as in proposition 4 hold. In addition:

i. Suppose that communication is constrained only by total material cost (i.e., $\phi_d = 0$, $\phi_f > 0$). Then there exists an optimal protocol with the feature that only one agent sends messages in any given communication round.

ii. Suppose that communication is constrained only by the total time delay (i.e., $\phi_v = \phi_f = 0 < \phi_d$), and the upper bound on total delay is denoted by D. Then every optimal protocol involves a number of communication rounds equal to the largest integer not exceeding D/ϕ_d, and both agents send messages simultaneously in each round.

The reasoning is the following. If communication entails only material costs, any round with simultaneous communication by both agents (from the set of messages of unit length or less) can be broken down into two successive rounds in which the agents alternate in sending messages from this set. Each agent has the option of sending the same message in this round when it is his turn to report. The agent now moving second has the additional option of conditioning his message on the message just sent by the other agent moving first (while restricted to sending a message of the same or shorter length as he did previously). The rest of the protocol is left unchanged. Material costs of communication are unchanged, as the communication capacity of each remains the same and the length of messages sent does not increase. Hence the principal's payoff weakly increases. The total delay of the mechanism is increased owing to the sequencing of messages across the two agents, but this is not costly by assumption.

In contrast, when communication costs consist only of delay, both agents must send messages in every round. Otherwise there would be a round in which one of the agents (i, say) does not send any messages, while the other agent j does (if neither does, then the entire round can be dispensed with). Allowing i to select a message from M_i^* in this round allows him to communicate more information than previously. As there are no material costs of communication, this does not cause any problem with the communication constraint, so a strict improvement is now possible.

VII. Concluding Comments

An obvious limitation of our approach is that it restricted attention to contexts with one-dimensional outputs and type spaces. However, the objective of the paper was to show that the special structure of this context can be exploited to obtain strong results concerning optimality of decentralized decision making and absence of trade-offs between incentives and informational efficiency. The extent to which these results can be extended to richer settings remains to be examined in future work.

Our formulation of decentralized decision making pertained only to production decisions. We ignored the possibility of delegating responsibility of contracting with other agents to some key agents. A broader concern is that we ignored the communicational requirements involved in contracting itself by focusing only on communication in the process of implementation of the contract, which takes place after parties have negotiated and accepted a contract. Under the assumption that precontracting communication is costless and messages exchanged between agents are verifiable by the principal, it can be shown that delegation of contracting cannot dominate centralized contracting if both are equally constrained in terms of communicational requirements. Subcontracting may thus be potentially valuable in the presence of costs of precontract communication or if agents can directly communicate with one another in a richer way than the way they can communicate with the principal. Exploring the value of delegation of contracting remains an important task for future research.

Appendix

LEMMA 1. Consider any communication protocol $p \in \mathcal{P}$. For any $h_t \in H_t(p)$ and any $t \in \{1, \ldots, T\}$,

$$\{c \in C(p) | h_t(c) = h_t\}$$

is a rectangle set in the sense that if $h_t(c_i, c_j) = h_t(c_i', c_j') = h_t$ for $(c_i, c_j) \neq (c_i', c_j')$, then

$$h_t(c_i', c_j) = h_t(c_i, c_j') = h_t.$$

Proof of Lemma 1

The proof is by induction. Note that $h_0(c) = \phi$ for any c, so it is true at $t = 0$. Suppose that the result is true for all dates up to $t - 1$; we shall show it is true at t.

Note that

$$h_t(c_i, c_j) = h_t(c_i', c_j') = h_t \tag{A1}$$

implies

$$h_\tau(c_i, c_j) = h_\tau(c_i', c_j') = h_\tau \tag{A2}$$

for any $\tau \in \{0, 1, \ldots, t - 1\}$. Since the result is true until $t - 1$, we also have

$$h_\tau(c_i', c_j) = h_\tau(c_i, c_j') = h_\tau \tag{A3}$$

for all $\tau \leq t - 1$. So under any of the configurations of communication plans (c_i, c_j), (c_i', c_j'), (c_i', c_j), or (c_i, c_j'), agent i experiences the same message history h_{t-1} until $t - 1$. Then i has the same message set at t, and (A1) implies that i sends the same messages to j at t, under either c_i or c_i'.

Equations (A2) and (A3) also imply that under either c_j or c_j', j sends the same messages to i at all dates until $t - 1$, following receipt of the (common) messages sent by i until $t-1$ under these different configurations. The result now follows from the fact that messages sent by j to i depend on the communication plan of i only via the messages j receives from i. So i must also receive the same messages at t under any of these different configurations of communication plans. QED

Proof of Proposition 1

Let $q_i(\theta_i, \theta_j)$ be a production allocation satisfying (3), which is supported by a communication strategy vector $c(\theta)$ in a protocol p that is parsimonious with respect to these strategies. In this protocol all histories are reached with positive probability on the equilibrium path; hence beliefs of every agent with regard to the types of the other agent are obtained by applying Bayes's rule.

Define $\hat{q}_i(\theta_i, h_t)$ by

$$\hat{q}_i(\theta_i, h_t) \equiv E[q_i(\theta_i, \theta_j) | \theta_j \in \Theta_j(h_t)]$$

for any $h_t \in H_t$ and any $t \in \{0, 1, \ldots, T\}$. Condition (3) requires $\hat{q}_i(\theta_i, h_t)$ to be nonincreasing in θ_i on $\Theta_i(h_t)$. Note that

$$\begin{aligned}\hat{q}_i(\theta_i, h(c(\theta_i, \theta_j))) &= E_{\tilde{\theta}_j}[q_i(\theta_i, \tilde{\theta}_j) | \tilde{\theta}_j \in \Theta_j(h(c(\theta_i, \theta_j)))] \\ &= q_i(\theta_i, \theta_j)\end{aligned}$$

since $q_i(\theta_i, \tilde{\theta}_j) = q_i(\theta_i, \theta 1_j)$ for any $\tilde{\theta}_j \in \Theta_j(h(c(\theta_i, \theta_j)))$.

Step 1: The relationship between $\hat{q}_i(\theta_i, h_t)$ and $\hat{q}_i(\theta_i, h_{t+1})$. Suppose that i observes h_t at the end of round t. Given selection of $m_{i,t+1} \in M_i(h_t)$ where $M_i(h_t)$ is the message set for h_t in protocol p, agent i's history at round $t+1$ is subsequently determined by messages received by i in round t. Let the set of possible histories h_{t+1} at the end of round $t+1$ be denoted by $H_{t+1}(h_t, m_{i,t+1})$. Evidently for $j \neq i$, $\{\Theta_j(h_{t+1}) | h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})\}$ constitutes a partition of $\Theta_j(h_t)$:

$$\cup_{h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})} \Theta_j(h_{t+1}) = \Theta_j(h_t)$$

and

$$\Theta_j(h_{t+1}) \cap \Theta_j(h'_{t+1}) \neq \phi$$

for $h_{t+1}, h'_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$ such that $h_{t+1} \neq h'_{t+1}$. The probability of $h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$ conditional on $(h_t, m_{i,t+1})$ is represented by

$$\Pr(h_{t+1} | h_t, m_{i,t+1}) = \Pr(\Theta_j(h_{t+1})) / \Pr(\Theta_j(h_t)).$$

From the definition of $\hat{q}_i(\theta_i, h_t)$ and $\hat{q}_i(\theta_i, h_{t+1})$, for any $m_{i,t+1} \in M_i(h_t)$ and any $\theta_i \in \Theta_i$,

$$\sum_{h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})} \Pr(h_{t+1} | h_t, m_{i,t+1}) \hat{q}_i(\theta_i, h_{t+1}) = \hat{q}_i(\theta_i, h_t).$$

Step 2: For any $h_{t+1}, h'_{t+1} \in H_{t+1}(h_t, m_{it+1})$, $\Theta_i(h_{t+1}) = \Theta_i(h'_{t+1}) \subset \Theta_i(h_t)$. By definition

$$\Theta_i(h_{t+1}) = \{\theta_i | m_{i,t+1}(\theta_i, h_t) = m_{i,t+1}\} \cap \Theta_{it}(h_t),$$

where $m_{i,t+1}(\theta_i, h_t)$ denotes i's message choice corresponding to the strategy $c_i(\theta_i)$. The right-hand side depends only on $m_{i,t+1}$ and h_t. It implies that the set $\Theta_i(h_{t+1})$ does not vary across different $h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$. To simplify exposition, we denote this set henceforth by $\Theta_i(h_t, m_{i,t+1})$.

Step 3: Construction of $\tilde{q}_i(\theta_i, h_t)$. We construct $\tilde{q}_i(\theta_i, h_t)$ for any $h_t \in H_t$ on the basis of claim 1.

Claim 1. For arbitrary $q_i(\theta_i, \theta_j)$ satisfying (3), there exists $\tilde{q}_i(\theta_i, h_t)$ for any $h_t \in H_t$ and any $t \in \{0, \dots, T\}$ so that

a. $\tilde{q}_i(\theta_i, h_t) = \hat{q}_i(\theta_i, h_t)$ for $\theta_i \in \Theta_i(h_t)$,

b. $\tilde{q}_i(\theta_i, h_t)$ is nonincreasing in θ_i on Θ_i, and

c.

$$\sum_{h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})} \Pr(h_{t+1} | h_t, m_{i,t+1}) \tilde{q}_i(\theta_i, h_{t+1}) = \tilde{q}_i(\theta_i, h_t)$$

for any $\theta_i \in \Theta_i$ and any $m_{i,t+1} \in M_i(h_t)$, where $M_i(h_t)$ is the message set for h_t in protocol p.

Claim 1 states that there exists an "auxiliary" output rule $\tilde{q}_i$ as a function of type θ_i and message history that is globally nonincreasing in type (property *b*) following any history h_t, and $\tilde{q}_i(\theta_i, h_t)$ equals the expected value of $\tilde{q}_i(\theta_i, h_{t+1})$ conditional on (h_t, m_{it+1}) for any $m_{it+1} \in M_i(h_t)$ (property *c*).

In order to establish claim 1, the following lemma is needed.

Lemma 2. For any $B \subset \mathfrak{R}_+$ that may not be connected, let A be an interval satisfying $B \subset A$. Suppose that $F_i(a)$ for $i = 1, \dots, N$ and $G(a)$ are real-valued functions defined on A, each of which has the following properties:

- $F_i(a)$ is nonincreasing in a on B for any i;
- $\sum_i p_i F_i(a) = G(a)$ for any $a \in B$ and for some p_i so that $p_i > 0$ and $\sum_i p_i = 1$;
- $G(a)$ is nonincreasing in a on A.

Then we can construct real-valued function $\bar{F}_i(a)$ defined on A for any i so that

- $\bar{F}_i(a) = F_i(a)$ on $a \in B$ for any i;
- $\sum_i p_i \bar{F}_i(a) = G(a)$ for any $a \in A$ and for the same p_i;
- $\bar{F}_i(a)$ is nonincreasing in a on A for any i.

This lemma says that we can construct functions $\bar{F}_i(a)$ so that the properties of functions $F_i(a)$ on B are also maintained on the interval A that covers B.

Proof of lemma 2. If this statement is true for $N = 2$, we can easily show that this also holds for any $N \geq 2$. Suppose that this is true for $N = 2$:

$$\sum_{i=1}^{N} p_i F_i(a) = p_1 F_1(a) + (p_2 + \cdots + p_N) F^{-1}(a),$$

with

$$F^{-1}(a) = \sum_{i \neq 1} \frac{p_i}{p_2 + \cdots + p_N} F_i(a).$$

Applying this statement for $N = 2$, we can construct $\bar{F}_1(a)$ and $\bar{F}^{-1}(a)$, which keeps the same property on A as on B. Next using the constructed $\bar{F}^{-1}(a)$ instead of $G(a)$, we can apply the statement for $N = 2$ again to construct desirable $\bar{F}_2(a)$ and $\bar{F}^{-2}(a)$ on A based on $F_2(a)$ and $F^{-2}(a)$ that satisfy

$$\frac{p_2}{p_2 + \cdots + p_N} F_2(a) + \left(1 - \frac{p_2}{p_2 + \cdots + p_N}\right) F^{-2}(a) = F^{-1}(a)$$

on B. We can use this method recursively to construct $\bar{F}_i(a)$ for all i.

Next let us show that the statement is true for $N = 2$. For $a \in A \setminus B$, define $\underline{a}(a)$ and $\bar{a}(a)$, if they exist, so that

$$\underline{a}(a) \equiv \sup\{a' \in B | a' < a\}$$

and

$$\bar{a}(a) \equiv \inf\{a' \in B | a' > a\}.$$

It is obvious that at least one of either $\underline{a}(a)$ or $\bar{a}(a)$ exists for any $a \in A \setminus B$.

Let us specify $\bar{F}_1(a)$ and $\bar{F}_2(a)$ so that $\bar{F}_1(a) = F_1(a)$ and $\bar{F}_2(a) = F_2(a)$ for $a \in B$ and for $a \in A \setminus B$ as follows:

i. For $a \in A \setminus B$ so that only $\underline{a}(a)$ exists,

$$\bar{F}_1(a) = F_1(\underline{a}(a)),$$
$$\bar{F}_2(a) = \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}.$$

ii. For $a \in A \setminus B$ so that both $\underline{a}(a)$ and $\bar{a}(a)$ exist,

$$\bar{F}_1(a) = \min\left\{F_1(\underline{a}(a)), \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1}\right\},$$
$$\bar{F}_2(a) = \max\left\{F_2(\bar{a}(a)), \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}\right\}.$$

iii. For $a \in A \setminus B$ so that only $\bar{a}(a)$ exists,

$$\bar{F}_1(a) = \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1},$$
$$\bar{F}_2(a) = F_2(\bar{a}(a)).$$

It is easy to check that $\bar{F}_i(a)$ is nonincreasing in a on A for $i = 1, 2$ and

$$p_1 \bar{F}_1(a) + p_2 \bar{F}_2(a) = G(a)$$

for $a \in A$. This completes the proof of the lemma. QED

Proof of claim 1. Choose arbitrary $t \in \{0, \dots, T\}$ and $h_t \in H_t$. Suppose that $\tilde{q}_i(\theta_i, h_t)$ satisfies parts *a* and *b* in claim 1. Then for any $m_{i,t+1} \in M_i(h_t)$, we can construct a function $\tilde{q}_i(\theta_i, h_{t+1})$ for any $h_{t+1} \in H_t(h_t, m_{it+1})$ so that parts *a*, *b*, and *c* are satisfied. This result is obtained upon applying lemma 2 with

$$\begin{aligned} B &= \Theta_i(h_t, m_{i,t+1}), \\ A &= \Theta_i, \\ a &= \theta_i, \\ G(\theta_i) &= \hat{q}_i(\theta_i, h_t), \\ F_{h_{t+1}}(\theta_i) &= \hat{q}_i(\theta_i, h_{t+1}), \\ p_{h_{t+1}} &= \frac{\Pr(\Theta_j(h_{t+1}))}{\Pr(\Theta_j(h_t))} \end{aligned}$$

for any $h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$, where each element of the set $H_{t+1}(h_t, m_{i,t+1})$ corresponds to an element of the set $\{1, \dots, N\}$ in lemma 2. This means that for $\tilde{q}_i(\theta_i, h_t)$ that satisfies *a* and *b* for any $h_t \in H_t$, we can construct $\tilde{q}_i(\theta_i, h_{t+1})$ that satisfies *a*–*c* for any $h_{t+1} \in H_{t+1}$.

With $h_0 = \phi$, since $\tilde{q}_i(\theta_i, h_0) = \hat{q}_i(\theta_i, h_0)$ satisfies *a* and *b*, $\tilde{q}_i(\theta_i, h_1)$ is constructed so that *a*–*c* are satisfied for any $h_1 \in H_1$. Recursively, $\tilde{q}_i(\theta_i, h_t)$ can be constructed for any $h_t \in \cup_{\tau=0}^{T} H_\tau$ so that *a*–*c* are satisfied. QED

Step 4: We are now in a position to complete the proof of sufficiency. We focus initially on the case in which the mechanism is decentralized so that agents select their own outputs independently.

Given $\tilde{q}_i(\theta_i, h)$ (with $h = h_T$) constructed in claim 1, construct transfer functions $t_i(q_i, h)$ as follows:

$$t_i(q_i, h) = \hat{\theta}_i(q_i, h) q_i + \int_{\hat{\theta}_i(q_i, h)}^{\bar{\theta}_i} \tilde{q}_i(x, h)\, dx$$

for $q_i \in Q_i(h) \equiv \{\tilde{q}_i(\theta_i, h) | \theta_i \in \Theta_i\}$, and $t_i(q_i, h) = -\infty$ for $q_i \notin Q_i(h)$, where $\hat{\theta}_i(q_i, h)$ is defined as follows:

$$\hat{\theta}_i(q_i, h) \equiv \sup\{\theta_i \in \Theta_i | \tilde{q}_i(\theta_i, h) \geq q_i\}.$$

We show that the specified communication strategies $c(\theta)$ and output choices $(\tilde{q}_i(\theta_i, h), \tilde{q}_j(\theta_j, h))$ constitute a PBE (combined with beliefs obtained by applying Bayes's rule at every history). By construction, $\tilde{q}_i(\theta_i, h)$ maximizes $t_i(q_i, h) - \theta_i q_i$ for any $h \in \mathcal{H} \equiv H_T$ and any $\theta_i \in \Theta_i$, where

$$t_i(\tilde{q}_i(\theta_i, h), h) - \theta_i \tilde{q}_i(\theta_i, h) = \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(x, h)\, dx.$$

Now turn to the choice of messages. Start with round T. Choose arbitrary $h_{T-1} \in H_{T-1}$ and arbitrary $m_{iT} \in M_i(h_{T-1})$. The expected payoff conditional on $\theta_j \in \Theta_j(h_{T-1})$, that is, conditional on beliefs given by

$$\Pr(h|h_{T-1}, m_{iT}) = \Pr(\Theta_j(h))/\Pr(\Theta_j(h_{T-1}))$$

for $h \in H_T(h_{T-1}, m_{iT})$, is

$$E_h[t_i(\tilde{q}_i(\theta_i, h), h) - \theta_i \tilde{q}_i(\theta_i, h)|h_{T-1}, m_{iT}] = \int_{\theta_i}^{\bar{\theta}_i} E_h[\tilde{q}_i(x, h)|h_{T-1}, m_{iT}]dx$$
$$= \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(x, h_{T-1})dx.$$

This does not depend on the choice of $m_{iT} \in M_i(h_{T-1})$. Therefore, agent i does not have an incentive to deviate from $m_{iT} = m_{iT}(\theta_i, h_{T-1})$.

The same argument can recursively be applied for all previous rounds t, implying that $m_{i,t+1} = m_{i,t+1}(\theta_i, h_t)$ is an optimal message choice for any $h_t \in H_t$ and any t. It is also evident that at round 0, it is optimal for agent i to accept the contract. This establishes that participation, followed by the communication strategies $c(\theta)$ combined with output choices $(\tilde{q}_i(\theta_i, h), \tilde{q}_j(\theta_j, h))$, constitute a PBE.

The same argument applies to a centralized mechanism since this is a special case of the previous mechanism in which the assigned outputs $\hat{q}_i(\theta_i, h) = \hat{q}_i(h)$ are measurable with respect to h, that is, are independent of θ_i conditional on h. Then

$$\tilde{Q}_i(h) \equiv \{\tilde{q}_i(\theta_i, h)|\theta_i \in \Theta_i(h)\} = \hat{q}_i(h).$$

Agent i can effectively be forced to choose output $\hat{q}_i(h)$ following history h at the end of the communication phase with a transfer $t_i(q_i, h)$. QED

Proof of Proposition 2

We show that the solution of the relaxed problem in which the incentive feasibility restriction ii is dropped automatically satisfies this restriction. Suppose not. Let the solution of the relaxed problem be represented by a (parsimonious) communication protocol p, communication strategies $c(\theta)$, and production allocation $(q_1(\theta_1, \theta_2), q_2(\theta_1, \theta_2))$. The functions H_t, $\Theta_i(h_t)$, and $\Theta_j(h_t)$ are well defined for $(p, c(\theta))$. Then there exist $t \in \{0, \ldots, T\}$, $h_t \in H_t$, and $\theta_i, \theta_i' \in \Theta_i(h_t)$ with $\theta_i > \theta_i'$ so that

$$E_{\theta_j}[q_i(\theta_i, \theta_j)|\theta_j \in \Theta_j(h_t)] > E_{\theta_j}[q_i(\theta_i', \theta_j)|\theta_j \in \Theta_j(h_t)].$$

This implies that at least either one of

$$E[V(q_i(\theta_i', \theta_j), q_j(\theta_i', \theta_j)) - w_i(\theta_i)q_i(\theta_i', \theta_j) - w_j(\theta_j)q_j(\theta_i', \theta_j)|\theta_j \in \Theta_j(h_t)]$$
$$> E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i)q_i(\theta_i, \theta_j) - w_j(\theta_j)q_j(\theta_i, \theta_j)|\theta_j \in \Theta_j(h_t)]$$

or

$$E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i')q_i(\theta_i, \theta_j) - w_j(\theta_j)q_j(\theta_i, \theta_j)|\theta_j \in \Theta_j(h_t)]$$
$$> E[V(q_i(\theta_i', \theta_j), q_j(\theta_i', \theta_j)) - w_i(\theta_i')q_i(\theta_i', \theta_j) - w_j(\theta_j)q_j(\theta_i', \theta_j)|\theta_j \in \Theta_j(h_t)]$$

holds. This means that if at least one type of either θ_i or θ_i' selects the communication plan and output decision rule of the other type, the principal's payoff is improved. This is a contradiction. QED

Proof of Proposition 3

Consider the restricted version of the problem described in proposition 2, where the communication protocol is fixed; that is, communication strategies $c(\theta)$ and production allocation $(q_i(\theta_i, h(c(\theta))), q_j(\theta_j, h(c(\theta))))$ are chosen to maximize the principal's expected payoff

$$E[V(q_i(\theta_i, h(c(\theta))), q_j(\theta_j, h(c(\theta)))) - w_i(\theta_i)q_i(\theta_i, h(c(\theta))) \\ - w_j(\theta_j)q_j(\theta_j, h(c(\theta)))].$$

We claim that the solution will have the property that for any history h such that $\Theta_i(h) \times \Theta_j(h)$ is nonempty, $q_i(\theta_i, h)$ will be strictly decreasing in θ_i. The reason is that it must satisfy the following necessary condition: $q_i(\theta_i, h)$ maximizes

$$E[V(q_i, q_j(\theta_j, h))|\theta_j \in \Theta_j(h)] - w_i(\theta_i)q_i$$

and $w_i(\theta_i)$ is strictly increasing.

The optimal allocation is decentralized and generates a strictly higher payoff for the principal compared to any centralized allocation that is communication feasible relative to the given protocol (since, in the latter allocation, production levels must be constant over $\Theta_i(h) \times \Theta_j(h)$ for all h). By the same argument as in the proof of proposition 2, this allocation is incentive feasible, while by construction it is communication feasible. QED

Proof of Proposition 4

Suppose that there is a round t and history h_{t-1} with $M_i(h_{t-1}) \neq \phi$ and $M_i(h_{t-1}) \neq M_i^*$ for some agent i. Without loss of generality, let $n_i \equiv \bar{l}(M_i(h_{t-1})) \geq n_j \equiv \bar{l}(M_j(h_{t-1}))$ and $n_i \geq 1$ (otherwise both agents have null message sets and the round can be deleted).

Following history h_{t-1}, we replace round t with rounds $t, t+1, \ldots, t+n_i-1$ with message set M_i^* for i in each of these rounds and message set M_j^* for j in rounds $t, t+1, \ldots, t+n_j-1$. Agent j is assigned a null message set in rounds $t+n_j, \ldots, t+n_i-1$ if $n_i > n_j$. Then notice by construction that

$$\bar{l}(M_k(h_{t-1})) = n_k = n_k\bar{l}(M_k^*)$$

for both agents $k = i, j$, implying that aggregate capacity cost or delay will remain unchanged. Moreover, for agent i we have

$$\begin{aligned}\#M_i(h_{t-1}) &\leq \#\{m_i \in \mathcal{M}_i | l(m_i) \leq n_i\} \\ &\leq 1 + L_i + \cdots + (L_i)^{n_i} \\ &< (1 + L_i)^{n_i} = \{\#M_i^*\}^{n_i}\end{aligned}$$

if $n_i \geq 2$, while

$$\#M_j(h_{t-1}) \leq 1 + L_j + \cdots + (L_j)^{n_j} \leq (1 + L_j)^{n_j} = \{\#M_j^*\}^{n_j}.$$

If $n_i = 1$, then $M_i(h_{t-1})$ is a proper subset of M_i^* and $\#M_i(h_{t-1}) < \#M_i^*$. Hence the set of messages available to each agent is now larger for both and is strictly larger for agent i. So for either agent $k = i, j$ we can select $\hat{M}_k$, which is a subset of $(M_k^*)^{n_k}$ such that $\#\hat{M}_k = \#M_k(h_{t-1})$ and for agent i it is a proper subset. In other words, there exists $\tilde{m}_i \in (M_i^*)^{n_i} \setminus \hat{M}_i$. For each $k = i, j$, we can select a one-to-one mapping μ_k from $M_k(h_{t-1})$ to $\hat{M}_k$ such that $l(\mu_k(m_k)) = l(m_k)$ for all $m_k \in M_k(h_{t-1})$. Also, $l(\tilde{m}_i) \leq n_i = \bar{l}(M_i(h_{t-1}))$, so there exists $\bar{m}_i \in M_i(h_{t-1})$ such that $l(\bar{m}_i) = n_i \geq l(\tilde{m}_i)$.

Given any choice of a subset Θ_i' of $\Theta_i(h_{t-1}, \bar{m}_i)$, we can construct communication plans for different types of i in rounds $t, \ldots, t + n_i - 1$ as follows:

a. If $\theta_i \in \Theta_i'$, then type θ_i of i reports $\tilde{m}_i$ instead of $\bar{m}_i$.
b. If $\theta_i \in \Theta_i(h_{t-1}, \bar{m}_i) \setminus \{\Theta_i'\}$, type θ_i reports $\bar{m}_i$, as before.
c. If θ_i does not belong to $\Theta_i(h_{t-1}, \bar{m}_i)$ and θ_i reported $m_i \in M_i(h_{t-1})$ previously, she now selects the vector of reports $\mu_i(m_i) \in \hat{M}_i$ across the new n_i rounds.

We shall describe later in the proof the method for selecting the subset Θ_i'.

The communication strategy for j is adapted to the following. If type θ_j reported $m_j \in M_j(h_{t-1})$ in round t in the previous protocol, she now selects the vector of reports $\mu_j(m_j) \in \hat{M}_j$ in rounds $t, \ldots, t + n_j - 1$.

From round $t + n_i$ onward, the continuation of the protocol and communication strategies exactly replicates the previous protocol and communication strategies from round $t + 1$ onward, with the continuation following $\mu_i(m_i), \mu_j(m_j)$ in the new protocol exactly matching the continuation following messages $m_i \in M_i(h_{t-1})$, $m_j \in M_j(h_{t-1})$ in the old protocol. Moreover, the continuation following $\tilde{m}_i$, $\mu_j(m_j)$ in the new protocol matches the continuation following messages $\bar{m}_i$, m_j in the old protocol.

By construction, then, total cost of communication capacity and delay is maintained the same. The variable material cost has not increased (since $l(\tilde{m}_i) \leq l(\bar{m}_i)$ while the length of all other messages has remained the same). On the other hand, the set of available messages has expanded for each agent, and strictly for agent i.

It remains to describe how the set Θ_i' is chosen. Consider any history h_T till the end of the communication phase, which is a continuation of $(h_{t-1}, \bar{m}_i)$ that arises with positive probability in the previous protocol. Following history h_T, agent j's information about θ_i is that it is contained in $\Theta_i(h_T)$, which is a nondegenerate interval of Θ_i and is a subset of $\Theta_i(h_{t-1}, \bar{m}_i)$. Now for any $\hat{\theta}_j$ in the interior of $\Theta_j(h_T)$, we can find a subset Θ_i' of $\Theta_i(h_T)$ such that both Θ_i' and $\Theta_i(h_T) \setminus \{\Theta_i'\}$ are nondegenerate, and

$$\begin{aligned} &E[V_{q_j}(q_i(\theta_i, \hat{\theta}_j), q_j(\theta_i, \hat{\theta}_j)) | \theta_i \in \Theta_i'] \\ &> E[V_{q_j}(q_i(\theta_i, \hat{\theta}_j), q_j(\theta_i, \hat{\theta}_j)) | \theta_i \in \Theta_i(h_T) \setminus \{\Theta_i'\}] \end{aligned}$$

since $V_{12} \neq 0$ and q_i is strictly decreasing in θ_i over $\Theta_i(h_T)$. Since this inequality is strict and since the production decision functions are continuous under the postulated regularity properties on V, it must also hold in a nondegenerate neighborhood of $\hat{\theta}_j$. This implies that optimal production decisions must change with positive probability.

Agent *i*'s information about *j*'s type remains unchanged in the new protocol. And agent *j* has strictly better information in the new protocol concerning *i*'s type following history h_T. This information is strictly valuable as the agents must change their production decisions with positive probability. Hence the principal can secure a strict improvement in her expected payoff. QED

Proof of Proposition 5

Given proposition 4, we can restrict attention to the protocol in which any nonnull message set assigned to agent i is M_i^* in every round. To show part i, suppose that there exists round t and $h_{t-1} \in H_{t-1}$ such that $M_k(h_{t-1}) = M_k^*$ for both agents $k = i, j$. Then consider a new communication protocol $\tilde{p}$ in which round t (following history h_{t-1}) is split into two successive rounds with sequential communication: in the first, i has a message set M_i^* while j is assigned a null message set, and in the second j has a message set M_j^* while i is assigned a null message set. Each agent can send the same message as he did in the previous protocol when it is his turn to report. From the next round onward the rest of the protocol continues as before. This modification does not raise total material cost (although it evidently raises total delay). In this protocol, j can send messages that can depend on m_{it}, something that is not possible in p. Hence it allows a weak improvement in the principal's payoff.

For part ii, suppose that there exists round t and $h_{t-1} \in H_{t-1}$ such that $M_i(h_{t-1}) = M_i^*$ and $M_j(h_{t-1}) = \{\phi\}$. We can now construct a new communication protocol $\tilde{p}$ with $M_j(h_{t-1}) = M_i^*$ instead of $\{\phi\}$ in round t (with history h_{t-1}). All other components of the communication protocol are preserved. This modification does not raise the total time delay (although it raises the total material cost). Here agent j, who was silent in round t following history h_{t-1} in p, can now send some messages in this round, thus increasing the amount of information exchanged between the agents. From the same argument as in the proof of proposition 4, the principal's payoff can be strictly improved. QED

References

Aoki, Masahiko. 1990. "Toward an Economic Model of the Japanese Firm." *J. Econ. Literature* 28 (1): 1–27.

Baron, David P., and Roger B. Myerson. 1982. "Regulating a Monopolist with Unknown Costs." *Econometrica* 50 (4): 911–30.

Battigalli, Pierpaolo, and Giovanni Maggi. 2002. "Rigidity, Discretion, and the Costs of Writing Contracts." *A.E.R.* 92 (4): 798–817.

Blumrosen, Liad, and Michal Feldman. 2013. "Mechanism Design with a Restricted Action Space." *Games and Econ. Behavior* 82:424–43.

Blumrosen, Liad, Noam Nisan, and Ilya Segal. 2007. "Auctions with Severely Bounded Communication." *J. Artificial Intelligence Res.* 28:233–66.

Caldwell, Bruce. 1997. "Hayek and Socialism." *J. Econ. Literature* 35 (4): 1856–90.

Celik, Gorkem. 2013. "Sense and Suspense: Implementation by Gradual Revelation." Manuscript, ESSEC Bus. School.

Dasgupta, Partha, Peter Hammond, and Eric Maskin. 1980. “On Imperfect Information and Optimal Pollution Control.” *Rev. Econ. Studies* 47 (5): 857–60.

Fadel, Ronald, and Ilya Segal. 2009. “The Communication Cost of Selfishness.” *J. Econ. Theory* 144 (5): 1895–1920.

Fudenberg, Drew, and Jean Tirole. 1991. *Game Theory.* Cambridge, MA: MIT Press.

Gershkov, Alex, Jacob K. Goeree, Alexeya Kushnir, Benny Moldovanu, and Xianwen Shi. 2013. “On the Equivalence of Bayesian and Dominant Strategy Implementation.” *Econometrica* 81 (1): 197–220.

Green, Jerry R., and Jean-Jacques Laffont. 1986. “Incentive Theory with Data Compression.” In *Uncertainty, Information and Communication: Essays in Honor of Kenneth Arrow,* edited by Walter P. Heller, Ross M. Starr, and David A. Starrett, 239–53. Cambridge: Cambridge Univ. Press.

———. 1987. “Limited Communication and Incentive Compatibility.” In *Information, Incentives, and Economic Mechanisms: Essays in Honor of Leonid Hurwicz,* edited by Theodore Groves, Roy Radner, and Stanley Reiter, 308–29. Minneapolis: Univ. Minnesota Press.

Hayek, Friedrich A. 1945. “The Use of Knowledge in Society.” *A.E.R.* 35 (4): 519–30.

Hurwicz, Leonid. 1960. “Optimality and Informational Efficiency in Resource Allocation Processes.” In *Mathematical Methods in the Social Sciences,* edited by Kenneth J. Arrow, Samuel Karlin, and Patrick Suppes, 27–46. Stanford, CA: Stanford Univ. Press.

———. 1977. “On the Dimensional Requirements of Informationally Decentralized Pareto-Satisfactory Processes.” In *Studies in Resource Allocation Processes,* edited by Kenneth J. Arrow and Leonid Hurwicz, 413–24. Cambridge: Cambridge Univ. Press.

Kos, Nenad. 2012. “Communication and Efficiency in Auctions.” *Games and Econ. Behavior* 75:233–49.

———. 2013. “Asking Questions.” *Games and Econ. Behavior,* electronically published June 19.

Kos, Nenad, and Matthias Messner. 2013. “Extremal Incentive Compatible Transfers.” *J. Econ. Theory* 148 (1): 134–64.

Laffont, Jean-Jacques, and David Martimort. 1998. “Collusion and Delegation.” *Rand J. Econ.* 29 (2): 280–305.

Marschak, Jacob, and Roy Radner. 1972. *Economic Theory of Teams.* New Haven, CT: Yale Univ. Press.

Marschak, Thomas, and Stefan Reichelstein. 1998. “Network Mechanisms, Information Efficiency and Hierarchies.” *J. Econ. Theory* 79 (1): 106–41.

Melumad, Nahum, Dilip Mookherjee, and Stefan Reichelstein. 1992. “A Theory of Responsibility Centers.” *J. Accounting and Econ.* 15 (4): 445–84.

———. 1997. “Contract Complexity, Incentives and the Value of Delegation.” *J. Econ. and Management Strategy* 6 (2): 257–89.

Mookherjee, Dilip, and Stefan Reichelstein. 1992. “Dominant Strategy Implementation of Bayesian Incentive Compatible Allocation Rules.” *J. Econ. Theory* 56 (2): 378–99.

Mookherjee, Dilip, and Masatoshi Tsumagari. 2012. “Mechanism Design with Communication Constraints.” Working paper, Boston Univ. http://people.bu.edu/dilipm/wkpap/.

Mount, Kenneth, and Stanley Reiter. 1974. “The Informational Size of Message Spaces.” *J. Econ. Theory* 8 (2): 161–91.

Myerson, Roger B. 1981. “Optimal Auction Design.” *Math. Operations Res.* 6 (1): 58–73.

———. 1982. "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems." *J. Math. Econ.* 10 (1): 67–81.
Rahman, David. 2011. "Detecting Profitable Deviations." Manuscript, Dept. Econ., Univ. Minnesota.
Reichelstein, Stefan, and Stanley Reiter. 1988. "Game Forms with Minimal Message Spaces." *Econometrica* 56 (3): 661–92.
Segal, Ilya. 2006. "Communication in Economic Mechanisms." In *Advances in Economics and Econometrics: Theory and Application, Ninth World Congress*, vol. 1, edited by Richard Blundell, Whitney K. Newey, and Torsten Persson, 222–68. Econometric Society Monographs. Cambridge: Cambridge Univ. Press.
Skreta, Vasiliki. 2006. "Mechanism Design for Arbitrary Type Spaces." *Econ. Letters* 91 (2): 293–99.
Van Zandt, Timothy. 2007. "Communication Complexity and Mechanism Design." *J. European Econ. Assoc.* 5 (2–3): 543–53.
Weitzman, Martin L. 1974. "Prices versus Quantities." *Rev. Econ. Studies* 41 (4): 477–91.
———. 1978. "Optimal Rewards for Economic Regulation." *A.E.R.* 68 (4): 683–91.

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