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A General Equilibrium Analysis of Personal Bankruptcy Law

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We analyse an economy where principals and agents match and contract subject to moral hazard. Bankruptcy law defines the limited liability constraint in these contracts. We analyse Walrasian allocations to generate the following predictions: (i) weakening bankruptcy law causes redistribution of debt and welfare from poor agents and principals to rich agents; (ii) exemption limits Pareto-dominate other bankruptcy laws if project size is fixed; (iii) means-testing (as in recent US personal bankruptcy law) that is *ex post* pro-poor in intent makes the poor worse off *ex ante*.

INTRODUCTION

Bankruptcy law plays a central role in modern economies, determining access to credit and allocation of assets. Yet both the law and its enforcement vary widely across developed and developing countries, between developed countries, and between different states within a given country.[1] For example, personal bankruptcy law in Germany is far less lenient compared with the USA: in the former country, defaulting borrowers have to pay a significant portion of their earnings for six years after the default, while Chapter 7 provisions in the USA have traditionally allowed most borrowers to not incur any liability against future earnings after a default. The liability of borrowers under Chapter 7 at the time of default is limited to assets owned at that point in time, in excess of an exemption limit. Those with fewer assets than the exemption limit incur no liability. These exemption limits vary widely across different states in the USA (Gropp *et al.* 1997).

Personal bankruptcy law affects subsequent economic fortunes of borrowers in distress, whose numbers typically exceeded 1 million filings per year in the USA over the period 1996–2014.[2] While most popular arguments for weaker bankruptcy laws focus on their *ex post* consequences on borrowers in distress, economists draw attention to their adverse consequences on *ex ante* credit access, particularly for poor borrowers. Cross-country and cross-US-state comparisons do indicate significant positive correlation between stringency of debtor liability and *ex ante* credit access (e.g. see Djankov *et al.* 2007; Gropp *et al.* 1997). Nevertheless, there are few theoretical analyses of the distributional incidence or optimal design of bankruptcy law in a general equilibrium setting with contracts.

We study a two-sided matching model of debt or asset lease contracts subject to moral hazard, where liability limits are defined by bankruptcy law. This is used to analyse general equilibrium and welfare effects of changing the law. The model is characterized by two-sided heterogeneity: agents (borrowers or tenants) differ in wealth, and principals (lenders or asset owners) have limited capacities to lend or contract, and may have differing overhead or monitoring costs. Capacity constraints of principals effectively create an ‘upward sloping supply curve’, which generates general equilibrium (GE) effects of changes in bankruptcy law via their effects on lender profits and interest rates. Borrowers of heterogeneous wealth coexist, and the GE effects on interest rates end up generating pecuniary externalities across them. Our principal focus is on how these distributive effects modify conventional wisdom based on partial equilibrium reasoning.

In our model, the number of projects that a given agent can operate can vary, but is subject to diminishing returns. Principal–agent coalitions form *ex ante* and design financial contracts determining contributions to the project financing up front, followed by *ex post* state-contingent transfers subject to legal liability limits. The total supply of credit (i.e. entry of principals), and its allocation and pricing across different borrowers, are endogenously determined.[3]

We show that raising borrower exemptions stipulated in bankruptcy law induce reallocation of credit (respectively, assets and payoffs) across borrowers of heterogenous wealth: lending to poorer agents shrinks, while it expands for the richest agents. At the same time, the (default-adjusted) cost of borrowing decreases. The redistribution of credit matches the cross-US-state empirical patterns identified by Gropp *et al.* (1997) or Cerqueiro and Penas (2014). Absent any GE effects, default-adjusted profit rates are fixed, and interest rates must increase as is shown in Gropp *et al.* (1997). In our model, interest rates need not increase due to higher exemption limits. Owing to general equilibrium effects, interest rates may decrease or remain constant since an increase in exemption limits will lead to a decrease in the profit rate. This makes the findings in our paper consistent with the puzzling effects found in Gropp *et al.* (1997): rich borrowers are able to borrow more in high exemption limit states without having to pay higher interest rates.

Our theory explains this as the result of interplay between two opposing effects: (i) a *partial equilibrium* (PE) effect of weakening agent liability that restricts the set of feasible contracts by making it more difficult for borrowers (respectively, tenants) to commit credibly to repaying their loans (respectively, paying their rent), and (ii) a *general equilibrium* (GE) effect of a lowering of profit rates. The intensity of the adverse PE effect is greater for poorer agents, and non-existent for rich agents, as the latter do not face problems with credible payment on account of possessing sufficient assets to post collateral. On the other hand, the GE effect benefits all agents uniformly. Hence the richest agents benefit from a weakening of bankruptcy law, while the poor face greater difficulty in gaining access to the market.

Apart from explaining cross-US-state patterns, our theory generates the following normative implications. First, stronger bankruptcy laws or measures to protect lender rights are not *ex ante* Pareto-improving in general, as might appear from a purely partial equilibrium perspective. They hurt richer agents owing to the GE effect, while benefiting lenders (via higher profits) and poorer agents (owing to enhanced market access, due to the PE effect). Second, we show that exemption limits—where all assets above the limit (and none below) are appropriable by lenders—are an optimal form of bankruptcy law in the case where project scales are not variable (e.g. owing to strong diminishing returns).[4] These results provide a normative basis for exemption-limit-based laws, and potential political-economy explanations for wide variations observed in bankruptcy law across states and countries. They predict that wealthy borrowers will demand weaker bankruptcy laws, while lenders (and poor borrowers, if they are politically organized) will demand stronger ones. The relative number and political influence of these different interest groups in a given society will affect the actual law and its enforcement. Finally, our analysis predicts that the means test of the 2005 Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) reform of US bankruptcy law will have adverse effects on poor borrowers from an *ex ante* perspective, even though the law appears to be designed to protect such borrowers.

Our paper also makes a methodological contribution by providing a new and simple way of analysing markets where principals and agents match and enter into contracts. The typical approach in the literature to modelling the effects of bankruptcy law on a competitive credit market is to use a Walrasian approach, with the interest rate as the 'price' (e.g. see Gropp *et al.* 1997). One problem with this approach is that both supply and demand for credit depend on the default rate, which is endogenously determined. Changes in the law that induce changes in default then cause both the supply and demand curves to shift, which obscures comparative static effects on equilibrium allocations. The microfoundations of this approach are also unclear; neither is it obvious how to integrate credit rationing into it.

We use an alternative way of modelling outcomes in the market for credit contracts that overcomes these problems, as a Walrasian allocation with a different notion of 'price' that is taken as given by all market participants: the per unit (default-risk-adjusted) expected return on loans received by lenders. The demand curve is obtained by solving for an optimal contract from the standpoint of each borrower, subject to incentive constraints for the borrower, and a participation constraint for each lender that they attain at least the going rate of return on monies lent. This problem can be solved using tools of conventional contract theory, and incorporates credit limits that may be imposed on the borrower for incentive reasons, as well as the attendant default risks. The market demand curve is obtained by aggregating the individual demand curves across all borrowers, and the equilibrium rate of return is obtained by equating supply and demand. In this formulation, the supply curve is unaffected by default rates and hence by the law. The latter affects only the demand in ways that can be obtained by application of standard contract-theoretic techniques.

Besides generating strong predictions that are easy to understand and interpret, this approach can be provided a secure microfoundation: we show that the set of Walrasian allocations as defined above is equivalent to the set of stable allocations (in the sense of Gale and Shapley (1962) or Roth and Sotomayor (1990)) in the two-sided matching game with lenders and borrowers. This provides a new way of extending conventional partial equilibrium contract theory to a general equilibrium setting.

As Section I elaborates, our explanation for why bankruptcy laws are often weak or poorly enforced is in contrast to others based on incomplete contracting, limited rationality or foresight of borrowers (including scope for manipulation or fraud by lenders), or preoccupation of contract enforcers with *ex post* relief for borrowers in distress to the exclusion of their *ex ante* effects on credit access. Our theory applies in the most conventional setting employed by economists: namely, a world of complete contracts, perfect rationality, with *ex ante* welfare judgments. Moreover, it does not rely on effects of bankruptcy law on insurance provided to borrowers, as all agents in our model are assumed to be risk-neutral.

The paper is organized as follows. A detailed description of related literature is provided in Section I. The model is set up in Section II. In Section III we solve the model, provide a microeconomic foundation for the use of our Walrasian equilibrium concept, and state our main result. Section IV discusses the simpler case of fixed project size in more detail: we show that exemption limit laws Pareto-dominate other bankruptcy laws in this setting. The effects of the new bankruptcy law introduced by BAPCPA in 2005 is discussed in Section V. Finally, Section VI concludes.

I. Related Literature

Supporting empirical papers

There are two closely related empirical papers that test our theory. A direct test of the impact of exemption limits on small entrepreneurs in the USA is provided in Cerqueiro and Penas (2014).[5] They focus on small businesses covered in the Kauffman Firm Survey for whom personal bankruptcy law applies. In their empirical specification, they run difference-of-differences regressions using the staggered state-by-state changes of exemption limits across US states. They show that an increase in exemption limits had a negative impact on low-wealth entrepreneurs, and a positive impact on high-wealth entrepreneurs. These findings are motivated by and consistent with our theory.

Von Lilienfeld-Toal *et al.* (2012) base a model of firm bankruptcy on the theoretical framework in this paper, which allows for difference in speed in bankruptcy decisions by the court. They analyse debt recovery tribunals (DRTs) implemented in India that reduced delays in debt-recovery suits for firms with large debts. Our analysis implies that with an upward-sloping supply of capital, the reform would raise interest rates and reallocate credit from small to large firms. They verified these predictions empirically, utilizing the staggered introduction of these DRTs across different Indian states with a difference-of-differences specification.[6]

A key assumption in our analysis is the existence of capacity constraints for principals, which give rise to the GE effects of bankruptcy law on equilibrium profit rates. We believe that this is a reasonable assumption even with globalized financial markets. The critical feature needed is the existence of some scarce local input required for agents to succeed. This input can be the fixed supply of assets (land, housing, taxi licences or franchise outlets) or limited monitoring capacity of local financial intermediaries. Existing evidence for the USA is consistent with this hypothesis: for example, Black and Strahan (2002), Cetorelli and Strahan (2006), and Petersen and Rajan (1995) show that local credit market conditions affect firms' access to credit and entry of young firms. Von Lilienfeld-Toal *et al.* (2012) also provide evidence that Indian banks relocated loan officers and bank branches away from rural areas serving mainly small borrowers to urban areas serving large firms following the DRT reform.

Theoretical analyses

Many arguments for weak bankruptcy laws in existing literature rest on their role in providing insurance, a feature we abstract from. The underlying assumption in such literature is that contracts are incomplete (e.g. Gropp *et al.* 1997; Bolton and Rosenthal 2002; Fan and White 2003). An analogous point is made in the literature on general equilibrium with incomplete markets (see Zame 1993; Perri 2008). Uninsurable income shocks also play an important role in dynamic macroeconomic models calibrated to US data studying effects of changes in bankruptcy law (Athreya 2002; Livshits *et al.* 2007; Chatterjee *et al.* 2007). These papers focus on GE effects operating through different channels. For instance, Athreya (2002) focuses on effects of stronger bankruptcy laws in lowering interest rates on borrowing owing to a decline in default rates, which in turn increase interest rates on saving in order to generate required loanable funds. This induces a rise in household savings that generate welfare benefits via self-insurance. Li and Sarte (2006) obtain opposite results in a model where lower borrowing interest rates (resulting from stronger bankruptcy laws) increase household borrowing, crowding out funds for firm investment, which lowers output and welfare. They also argue that reforms such as the 2005 BAPCPA could also have similar effects owing to increased incidence of Chapter 13 filings, which act as an effective tax on labour supply. Mitman (2015) examines negative interactions between bankruptcy and foreclosure arising from household substitution between unsecured and secured credit, owing to a decline in interest rates on unsecured credit that results from stronger bankruptcy laws. Our analysis abstracts from these channels of impact (insurance, production or housing foreclosure), being concerned mainly about distributional effects of changes in bankruptcy law rather than output or efficiency impacts.

Besley *et al*. (2012) focus on GE effects in a theoretical model of credit contracts with moral hazard, limited liability and two-sided heterogeneity similar to ours. Rather than looking at bankruptcy law, they investigate the impact of changing collateralizability of assets owned by borrowers. Changing bankruptcy provisions and collateralizability are similar in some respects, by changing the ability of lenders to seize borrower assets in the event of default. Hence some of our results overlap—for example, the observation that the distribution of benefits between borrowers and lenders depends on who is on the short side of the market. However, our analyses differ on many other dimensions. Our main focus is on the distribution of benefits *across* borrowers of heterogeneous wealth, an issue ignored by their model (which abstracts from lender capacity constraints). Moreover, our paper addresses issues specific to the bankruptcy setting, such as the optimal design of bankruptcy penalties, and distributive effects of exemption limits or means-testing.

A related paper of ours (von Lilienfeld-Toal and Mookherjee 2010) uses a similar approach to this paper to examine laws pertaining to bonded labour provisions, wherein future labour income can be used as collateral. While this is banned in most countries, such bans are not well enforced in many poor countries. That paper proposes an explanation of why more developed countries tend to ban (or enforcebans on) bonded labour contracts.

Manove *et al*. (2001) provide a novel argument for weak bankruptcy law, in terms of the need to provide banks with incentives to screen investment projects. They show that in a competitive credit market, equilibrium loan contracts will be designed with excessively high collateral requirements that leave lenders with insufficient incentives to screen projects. Legal restrictions on collateral mitigate this inefficiency. If the credit market were more monopolistic, this inefficiency would also tend to be mitigated as lenders internalize the effects of superior project quality. Our theory, in contrast, implies that the contracting externality across borrowers owing to the GE effect is magnified when the credit market is less competitive.

Our focus on the distributional impact of the law is shared by a number of recent theoretical papers on the political economy of law and finance. A political process determines investor protection, employment protection or nature of bank regulation in a number of papers (Perotti and von Thadden 2006; Pagano and Volpin 2005; Perotti and Volpin 2012; Aney *et al*. 2015). The most closely related paper is by Biais and Mariotti (2009), who consider defaulting firms and their access to credit when corporate bankruptcy law regulating liquidation of firms is changed. All of these papers focus on general equilibrium spillovers from the credit market to the labour market, while our focus is on general equilibrium effects *within* the credit market, and in particular the effects on redistribution across borrowers of heterogeneous wealth. Moreover, we focus on issues specific to personal rather than corporate bankruptcy law.[7]

Finally, other papers studying stable allocations in matching markets with contracts include Dam and Perez-Castrillo (2006), Besley and Ghatak (2005), and Legros and

Newman (2007). Methodologically, our result concerning equivalence of stable allocations and Walrasian allocations of contracting equilibrium models may be of some independent interest, as it provides a convenient and simple method for deriving comparative static effects of parametric changes.

II. Model

Technology, endowments

The economy has a population of $m \geq 2$ principals denoted $j = 1,\ldots,m$, and n agents denoted $i = 1,\ldots,n$. Each principal owns an asset such as a plot of land, equipment (real estate, taxis) or a franchise that requires the effort of an agent to generate income, in combination with working capital funded from either the agent's wealth or loans provided by the principal. Agents are prospective tenants who do not own the asset themselves; principals are asset owners who are unable to provide the labour necessary to generate income from these assets. Agents are wealth-constrained. They are ordered in terms of their *ex ante* wealth: $w_1 \geq w_2 \geq w_3 \geq \ldots$; the wealth distribution is given. Some results in the paper are non-vacuous only if the wealth upper bound w_1 of borrowers is sufficiently large.[8] Principals are not wealth-constrained.

Each agent can work at a scale of $\gamma = 1,2,\ldots$, which represents the number of assets leased and operated. For instance, a tenant farmer may lease multiple plots of land. An entrepreneur may start a project at one of many different scales. There are diminishing returns to project scale, described in more detail below.

A key assumption of the model is the existence of capacity limits on principals. We consider the extreme case where each principal owns a single unit of the relevant asset. Principal j is subject to a fixed (overhead or monitoring) cost f_j: its net profit is its operating profit (defined by net transfers, as explained below) less this fixed cost. The results extend straightforwardly to a wider class of asset distributions across principals. Specifically, if a principal owns $q \geq 2$ assets and is subject to an overhead cost of r per asset, then the same results obtain if there are at least q other principals owning one asset each with an overhead cost of r. The existence of such a 'competitive fringe' of small asset owners will eliminate any possible monopoly power of large asset owners. Our results extend to contexts where the supply side is competitive in this particular sense.

The model also applies to a pure credit context, where entrepreneurs or borrowers do not need to lease assets from principals, and need only to borrow funds from the latter. In such a case the agents own or have free access to all other assets required to execute the project. In the simple version that we exposit below, each principal has the capacity to lend enough to finance a single project at unit scale; the analysis applies with more general distributions of loanable funds among lenders satisfying an analogous 'competitive fringe' property.[9]

An agent leasing γ assets will form a coalition with γ principals. In order to operate each asset, an up front (working capital or investment) cost of I must be incurred, so the total up front financing need is $\gamma \cdot I$, which must be distributed between the agent and the principals in the coalition.[10] If the agent leases γ assets, then we say that the project scale is γ. The agent subsequently selects effort $e \in [0,1]$, whence the project results in a success with probability e, and failure otherwise. If successful (outcome s), the return is $\gamma^{\beta} y_s$; if failure (outcome f), it is $\gamma^{\beta} y_f$, where $y_s > I > y_f$ and $\beta \in (0,1)$ represents the extent of diminishing returns with respect to scale. Effort e entails a non-pecuniary cost of $D(e)$ for the agent, where $D(0) = 0$,

$D'(e) > 0$, $D''(e) > 0$, $D'''(e) > 0$ for all $e > 0$. The assumption $y_s > I > y_f$ ensures that the income from the project will be negative if unsuccessful, and positive if successful. In addition, we assume there exists $e \in (0,1)$ such that $e(y_s - y_f) + y_f > I + D(e)$, that is, at unit scale the project returns an expected net income in excess of the effort cost of the agent. Without such an assumption, the technology does not allow any agent to be viable at any scale, even in a first-best setting.

Contracts and default

An agent with *ex ante* wealth w_i can contribute all or part of it ($d \leq w_i$) towards the up front financing cost $\gamma \cdot I$, borrowing the remainder from the principals in the coalition (denoted C_i). They design the contract defining contributions of each member of the coalition towards the up front cost (d for the agent, I_j for each principal $j \in C_i$) and mandated financial transfers t_{kj} from the agent to each principal $j \in C_i$ after the project is completed, conditional on the outcome ($k = s, f$). After the project is completed, the agent obtains the return ($\gamma^{\beta} y_k$ in state k) from the project, in addition to an exogenously determined income $\sigma(w)$ from other sources.[11] We assume that $\sigma(\cdot)$ is a strictly increasing function. The *ex post* wealth of the agent will be the sum of: (a) $w - d$, the portion of *ex ante* wealth remaining after the up front contribution; (b) the project return $\gamma^{\beta} y_k$ in state k; and (c) outside income $\sigma(w)$. From this wealth, the agent decides what transfers to make to each principal, and consumes the rest. Consumption must be non-negative, hence physical feasibility requires aggregate transfers to not exceed the *ex post* wealth of the agent.

Default occurs following outcome k if the agent fails to make the required transfer t_{kj} to some principal $j \in C_i$. Liability rules then specify a penalty of $p(W)$ to be incurred by the agent, where W denotes the agent's *ex post* wealth net of project returns at the point of default. In the event of default, the project returns accrue to the principals in C_i: principal j is entitled to s_{kj}, where $\sum_{j \in C_i} s_{kj} = \gamma^{\beta} y_k$. Feasibility dictates that $W \geq p(W)$. We also assume that $p(W)$ and $W - p(W)$ are both non-decreasing in W. The former assumption seems natural: increases in the capacity of contract enforcers to impose punishments should not result in lower punishments. The latter assumption is also a natural consequence of agents having the option of destroying their own wealth.

We assume that $p(W)$ either accrues to the government or outside parties, or represents pure social deadweight losses (e.g. court or lawyer fees, costs of imprisonment or other penalties). Part of these could also represent mandated punitive transfers to the principals in the coalition. The exact destination of $p(W)$ will make no difference, as default will not actually occur in equilibrium.

In the event that the project outcome is $k \in \{s, f\}$ and the agent does not default, the agent's net payoff will be

$$w_i + \sigma(w_i) - d - \sum_{j \in C_i} t_{kj} + \gamma^{\beta} y_k - D(e),$$

and that of principal $j \in C_i$ will be

$$t_{kj} - I_j - f_j.$$

And if the agent defaults, then the agent will earn

$$w_i + \sigma(w_i) - d - p(w_i + \sigma(w_i) - d) - D(e),$$

and principal j will earn

$$s_{kj} - I_j - f_j.$$

It follows that default occurs if and only if (assuming that the agent does not default unless strictly advantageous)

$$\sum_{j \in C_i} t_{kj} > \gamma^{\beta} y_k + p(w_i + \sigma(w_i) - d). \tag{1}$$

Once outcome k is realized, the agent and the principals in the coalition can renegotiate the contractual payments t_{kj} provided that they are all better off from that point onwards.

The exact sequence of events is thus:

- each agent is matched with a coalition of principals;
- contracts are written;
- the agent selects effort e;
- outcome k is realized;
- mandated transfers are renegotiated if there is scope for an *ex post* Pareto improvement;
- the agent decides whether or not to default, following which payoffs are realized.

Lemma 1. Given any coalition C_i of principals associated with agent i, without loss of generality, attention can be restricted to contracts in which:

(i) there is no default in equilibrium, i.e. equation (1) does not hold;
(ii) $d = w_i$, i.e. the agent contributes his entire *ex ante* wealth as downpayment;
(iii) principal j acquires a constant share δ_j of the project, in the sense that she contributes $I_j = \delta_j(\gamma \cdot I - w_i)$ up front and receives transfer $t_{kj} = \delta_j \sum_{l \in C_i} t_{kl}$.

The proofs of this and many subsequent results are relegated to the Appendix. The underlying idea is simple. Default does not arise in equilibrium, since any contract inducing default generates deadweight losses that can be avoided via an *ex post* Pareto-improving renegotiation. Hence attention can be restricted to contracts that do not provide borrowers with an incentive to default *ex post*. This imposes an incentive compatibility restriction on the agent's default incentive, apart from the conventional incentive constraint associated with *ex ante* effort choice. The no-default constraint is defined by the level of transfers that the bankruptcy law would allow *ex post*; a weaker bankruptcy law would lower such transfers, thus restricting the set of feasible default-free contracts. Increasing downpayments made by the borrower helps to relax these constraints, since these reduce *ex post* wealth of the borrower by more than they increase *ex post* transfers in the event of default. Finally, risk-neutrality of principals implies that they care only about their expected net returns. Hence we do not have to take into account any risk-sharing considerations across principals. Rather, we can view principals as if they pool payments to finance the project and pool the payments received from the agent. Each principal then finances a share of the project *ex ante* and receives the same fraction of the payments *ex post*.

In what follows we restrict attention to contracts depicted in Lemma 1. It helps to denote contracts in terms of wealth v_k of the agent following outcome k:

$$v_k \equiv \gamma^{\beta} y_k + \sigma(w_i) - T_k, \tag{2}$$

where $T_k \equiv \sum_{j \in C_i} t_{kj}$ denotes the aggregate transfer paid by the agent in state k. The agent's net payoff in state k following effort choice e is $v_k - D(e)$, and the expected payoff is $ev_s + (1-e)v_f - D(e)$. Denoting aggregate (expected) operating profit of the principals by

$$\Pi \equiv eT_s + (1-e)T_f - [\gamma \cdot I - w_i],$$

the expected payoff of principal $j \in C_i$ is $\delta_j \Pi - f_j$. The contract can then be represented by the aggregate financial transfers and shares of different principals: $(T_s, T_f, \{\delta_j\}_{j \in C_i})$, besides project scale γ and effort e. Equivalently, we can represent it in terms of $(v_s, v_f, \{\delta_j\}_{j \in C_i}, \gamma, e)$, using equation (2). The no-default condition in state k requires

$$T_k \leq \gamma^{\beta} \cdot y_k + p(\sigma(w_i)),$$

which reduces to

$$\sigma(w_i) \leq v_k + p(\sigma(w_i)). \tag{3}$$

The contract $(v_s, v_f, \{\delta_j\}_{j \in C_i}, \gamma, e)$ with shares $\delta_j \geq 0$, $\sum_{j \in C_i} \delta_j = 1$ is *feasible* if it satisfies the following constraints:

$$v_s - v_f = D'(e) \tag{IC}$$

$$v_k \geq \sigma(w_i) - p(\sigma(w_i)), \quad k = s, f, \tag{LL}$$

$$\Pi \equiv \gamma^{\beta}[ey_s + (1-e)y_f] - [ev_s + (1-e)v_f] + \sigma(w_i) + w_i - \gamma \cdot I \geq \sum_{j \in C_i} f_j, \tag{PCP}$$

$$ev_s + (1-e)v_f - D(e) \geq w_i + \sigma(w_i). \tag{PCA}$$

Here (IC) refers to the effort incentive constraint, (LL) to the no-default constraint, and (PCA) to the participation constraint for the agent. The constraint (PCP) is clearly necessary in order for each principal to break even. It is also sufficient in the sense that one can find shares δ_j, $j \in C_i$, such that j breaks even in expectation (i.e. $\delta_j \cdot \Pi \geq f_j$). Accordingly, we can simplify the definition of a contract to a tuple $(v_s, v_f, \gamma \geq 1, e)$, and call it feasible for coalition C_i of principals if it satisfies (IC), (LL), (PCP) and (PCA). We can then call an agent with wealth w_i *viable* if the set of feasible contracts is non-empty for that agent, for some coalition C_i of principals.

Note the role of bankruptcy law, in its stipulation of default penalties imposed on the agent: these define the limit of the agent's liability, as represented by (LL). A stronger bankruptcy law pertains to a penalty function $\widetilde{p}(\cdot)$ that uniformly dominates another $p(\cdot)$, that is, if $\widetilde{p}(W) \geq p(W)$ for all W. An example of a specific bankruptcy law is one involving an exemption limit E, with a zero marginal tax rate below the limit and 100% above the limit: $p(W;E) = \max\{0, W-E\}$. A lower exemption limit then corresponds to a strengthening of bankruptcy law, and a corresponding weakening of the (LL) constraint. This enlarges the set of feasible contracts for any given agent.

It is easy to check that viability of agents is positively related to their wealth. Intuitively, this occurs because wealthier agents require less external finance, and thus need to repay less to the lenders: this effect outweighs the higher payoff option available to them if they default.[12]

Lemma 2. Suppose that agent i with wealth w_i is viable. Then every agent l with $w_l > w_i$ is also viable.

III. Stable Allocations

An *allocation* is a matching of each agent $i = 1,\ldots,n$ with a coalition C_i of principals such that $C_i \cap C_l = \emptyset$ when $i \neq l$, and a contract for each matched agent i that is feasible for the coalition C_i. An agent i is *unmatched* if $C_i = \emptyset$: such an agent gets payoff $w_i + \sigma(w_i)$. A principal j is *unmatched* if j does not belong to C_i for any agent i. Such a principal earns a payoff of 0. Payoffs for matched agents and principals are defined by the contracts into which they enter.

A common solution concept for matching models is the set of stable allocations (Gale and Shapley 1962; Roth and Sotomayor 1990). This is defined as follows.

An allocation is said to be *stable* if there does not exist any agent i and a coalition $\widehat{C}_i$ of principals that can enter into a feasible contract with i that generates a higher payoff for i and every principal in $\widehat{C}_i$.

A characterization of stable allocations will turn out to greatly simplify our analysis, besides being a result of some independent methodological interest as it could be fruitfully applied in the analysis of contracts in many other matching contexts. We will show below that the set of stable allocations coincides with a particular notion of Walrasian allocations. To introduce this concept, we will be thinking of the market for contracts for leasing one unit of the asset, with the 'price' π of such contracts represented by the rate of operating profit per asset leased. Principals and agents take this profit rate as given: each principal decides whether to enter the market and offer her asset for lease. Each agent decides on how many assets to lease, and designs a contract to maximize her own utility, subject to the constraint of generating a profit of at least π for each asset that she leases.

The Walrasian demand for contracts by an agent i corresponds to the solution of the following problem, given the profit rate π: select contract (v_s, v_f, e, γ) to maximize expected payoff $ev_s + (1-e)v_f - D(e)$ subject to (IC), (LL), (PCA) and the following 'budget constraint':

$$\gamma^{\beta}[ey_s + (1-e)y_f] - [ev_s + (1-e)v_f] + w_i + \sigma(w_i) \geq \gamma \cdot (I + \pi). \qquad (BC)$$

If the feasible contract set is empty, set $\gamma = 0$. We will call any solution of this problem an *A-optimal contract* for agent i, given profit rate π per asset leased.

A *Walrasian allocation* is defined to be an allocation and an operating profit rate π per asset such that we have the following.

(a) For any agent i, the contract $(v_s^i, v_f^i, e^i, \gamma^i)$ assigned to agent i is A-optimal for i relative to π.
(b) The 'supply' of assets (or number of active principals) is determined as follows: any principal with $f_j > \pi$ is inactive; any principal with $f_j < \pi$ is active. Every active principal receives the same expected operating profit π.
(c) The total demand for assets $\sum_i \gamma^i$ equals the supply. Agent i is assigned a coalition C_i consisting of γ^i principals, arbitrarily selected from the set of active principals.

We now provide the main result linking stable and Walrasian allocations.

Proposition 1. An allocation is stable if and only if it is Walrasian.

The intuitive reasoning underlying this result is as follows. Owing to competition among lenders, they must all attain the same rate of operating profit per unit leased. Any principal that can cover its fixed costs with the common rate of operating profit will be willing to enter, others will not be willing to enter. Hence supply decisions are as if lenders take the rate of operating profit as given and decide on their profit-maximizing responses. Moreover, every borrower must select an optimal contract, subject to the 'budget' constraint of paying the going expected rate of return to all its lenders (apart from effort and no-default incentive constraints). Otherwise it is possible to find a Pareto-improving coalition: a contract can be designed to provide the borrower with higher expected utility, and its lenders a higher rate of profit. In this sense, the demand for projects is also Walrasian. Finally, matching implies that supply and demand are balanced.

This result allows us to focus on Walrasian allocations for the rest of our analysis. Since the supply side of the market is simple, we need to understand how A-optimal demands for project scale for borrowers of differing wealths are affected. We turn to this next.

A-optimal contracts

The A-optimal problem for an agent with wealth w can be represented more simply as follows. Using (IC) to substitute for v_s in terms of v_f and e, the problem is to choose (v_f, e, γ) to maximize

$$v_f + eD'(e) - D(e)$$

subject to

$$\gamma^{\beta} R(e) - eD'(e) - \gamma \cdot (I + \pi) + \sigma(w) + w \geq v_f \geq \sigma(w) - p(\sigma(w)),$$

where $R(e) \equiv ey_s + (1-e)y_f$, the first constraint is the budget constraint, and the second constraint is (LL). The feasible set in this problem is non-empty if there exists (e,γ) such that

$$\gamma^{\beta}R(e) - eD'(e) - \gamma \cdot (I + \pi) \geq -w - p(\sigma(w)) \quad (F)$$

If the feasible set is empty, we can set $\gamma = e = 0$. Otherwise, for any (e,γ) satisfying (F), it is optimal to set

$$v_f = \gamma^{\beta}R(e) - eD'(e) - \gamma \cdot (I + \pi) + w + \sigma(w),$$

so we can restate the A-optimal problem as selection of (e,γ) to maximize

$$[\gamma^{\beta}R(e) - D(e) - \gamma \cdot (I + \pi)] + w + \sigma(w) \quad \text{(AO)}$$

subject to constraint (F).

Denote the solution to the A-optimal problem (AO) by $\gamma(\pi,w)$, $e(\pi,w)$, with the convention that $\gamma(\pi,w) = e = 0$ if the maximized value of (AO) falls below the autarchic payoff of $w + \sigma(w)$. And denote the corresponding problem of maximizing (AO) without any constraints the *first-best problem*, with solution $\gamma^*(\pi)$, $e^*(\pi)$. Note that the discreteness of project scale implies that the optimal contract may be non-unique in either first-best or second-best situations; hence $(\gamma(\pi,w), e(\pi,w))$ and $(\gamma^*(\pi),e^*(\pi))$ are correspondences.

Note also that the first-best generates positive surplus to the agent (i.e. above the autarchic payoff of $w + \sigma(w)$) if $\pi = 0$, since there exists e (with $\gamma = 1$) such that $R(e)-D(e) > I$. On the other hand for π sufficiently large, a positive surplus cannot be generated. This will impose an upper bound $\overline{\pi}$ to the profit rate π consistent with a positive demand for projects from the agent.

Lemma 3.

(a) For any given profit rate $\pi \geq 0$, there exists $\overline{w}(\pi)$, a non-decreasing function of π, such that every A-optimal contract for an agent with wealth w above $\overline{w}(\pi)$ is first-best: $(\gamma(\pi,w),e(\pi,w)) = (\gamma^*(\pi), e^*(\pi))$. Conversely, $w < \overline{w}(\pi)$ implies that the first-best cannot be attained.

(b) The first-best contract $(\gamma^*(\pi), e^*(\pi))$ is non-increasing in π, in the sense that $\pi_2 > \pi_1$ implies that $\gamma_2 \leq \gamma_1$ and $e_2 \leq e_1$ for any first-best choice $(\gamma_m, e_m) \in (\gamma^*(\pi_m), e^*(\pi_m))$, $m = 1,2$.

(c) If $w < \overline{w}(\pi)$, then every A-optimal contract involves lower effort than the first-best $(e(\pi,w) < e^*(\pi))$, and $\gamma(\pi,w) \leq \gamma^*(\pi)$.

This lemma states that for agents with wealth above some threshold, A-optimal contracts will be first-best: the first-best contract is an unconstrained maximizer of (AO), it is independent of the wealth of the agent, and it satisfies constraint (F) if this wealth is sufficiently high. As the required profit rate to be paid increases, it reduces the desired project scale, and in turn this reduces the borrower's *ex ante* effort. For those borrowers not wealthy enough to be able to implement the first-best contract, the scale of the project has to be reduced in order to meet constraint (F). While these results appear reasonable enough, proving them is somewhat complicated because the problem of selecting an A-optimal contract is not a convex optimization problem (owing to the complementarity between project scale and effort).

The next set of results shows that A-optimal demand for project scale is non-decreasing in the agent's wealth, and non-increasing in the going profit rate.

Lemma 4. If $w_1 < w_2 < \overline{w}(\pi)$, then $e(\pi,w_1) \le e(\pi,w_2)$ and $\gamma(\pi,w_1) \le \gamma(\pi,w_2)$ for any selection of A-optimal contracts.

Lemma 5. If $\pi_1 < \pi_2$, then $e(\pi_1,w) \ge e(\pi_2,w)$ and $\gamma(\pi_1,w) \ge \gamma(\pi_2,w)$ for any selection of A-optimal contracts.

Moreover, weakening bankruptcy law tightens the no-default constraint, which in turn reduces A-optimal demand for project scale.

Lemma 6. Consider a weakening of bankruptcy rules from $p_2(\cdot)$ to $p_1(\cdot)$ in the sense that $p_1(W) \le p_2(W)$ for all W. Consider any (π,w), and let γ_l denote the A-optimal project scale under rule p_l, $l = 1,2$. Then if the A-optimal payoff differs under the two rules, $\gamma_1 \le \gamma_2$.

Distributional impact of changing bankruptcy law

These results enable us to derive our central result.

Proposition 2. Consider a weakening of bankruptcy rule from $p_2(\cdot)$ to $p_1(\cdot)$ (in the sense that $p_2(W) \ge p_1(W)$ for all W). Let A_i denote a Walrasian allocation resulting under rule $p_i(\cdot)$, and let π_i be the corresponding profit rate. Suppose that A_2 is not a Walrasian allocation under rule p_1, in the following sense: there does not exist a Walrasian allocation at p_1 with the same total number of assets leased and the same profit rate as in A_2. Then:

(a) the profit rate is lower, with the weaker rule: $\pi_1 \le \pi_2$;
(b) for agents with $w > \overline{w}(\pi_2;p_1)$, project scale γ, effort e and payoff are higher (or remain the same) in A_1, the Walrasian allocation corresponding to the weaker rule p_1;
(c) the total number of assets leased by agents with $w < \overline{w}(\pi_2;p_1)$ is (weakly) lower in A_1.

Proof Let S_2 be the total number of assets leased in the Walrasian allocation under p_2. By hypothesis, when the rule is changed to p_1, there is no Walrasian allocation with S_2 projects leased and the same profit rate π_2. In other words, we cannot find A-optimal project scales $\gamma(\pi_2,w_i,p_1)$ under rule p_1 such that $\Sigma_i\, \gamma(\pi_2,w_i,p_1) = S_2$. But there were A-optimal project scales $\gamma(\pi_2,w_i,p_2)$ under rule p_2 such that $\Sigma_i\, \gamma(\pi_2,w_i,p_2) = S_2$. By Lemma 6 we know that $\gamma(\pi_2,w_i,p_1) \le \gamma(\pi_2,w_i,p_2)$. So it must be the case that $\Sigma_i\, \gamma(\pi_2,w_i,p_1) < S_2$ for any set of A-optimal project choices at p_1.

Suppose that $\pi_1 > \pi_2$. Then any principal that was active under p_2 will continue to be active under p_1. Hence the supply of assets under p_1 cannot be smaller than under p_2: $S_1 \ge S_2$. On the other hand, Lemma 5 ensures that $\gamma(\pi_1,w_i,p_1) \le \gamma(\pi_2,w_i,p_1)$. This implies $S_1 > \Sigma_i\, \gamma(\pi_1,w_i,p_1)$, i.e. there cannot be a Walrasian allocation at π_1 under p_1. Therefore $\pi_1 \le \pi_2$, establishing (a).

Wealthy agents with $w > \overline{w}(\pi_2, p_1)$ will achieve the first-best in both allocations. Since $\pi_1 \le \pi_2$, they are (weakly) better off, and strictly better off if the profit rate falls. By Lemma 3, their A-optimal project scale and effort will increase or remain the same.

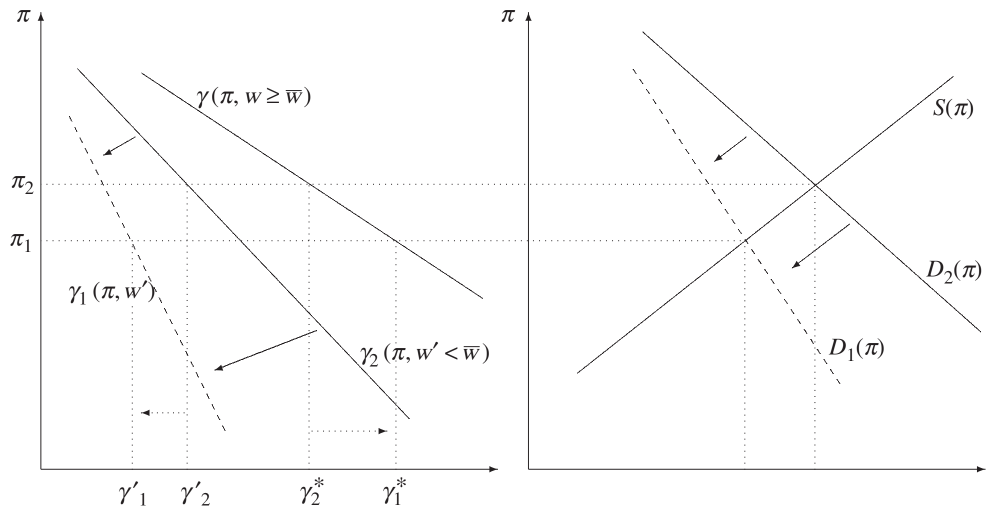

FIGURE 1. Impact of changing liability law on poor and rich agents.

The total supply of assets cannot increase because $\pi_1 \leq \pi_2$. Therefore the total number of assets leased by agents with wealth below $\overline{w}(\pi_2, p_1)$ cannot increase. □

The intuitive reasoning is as follows. A weaker bankruptcy rule lowers the demand for projects from poorer agents that cannot achieve the first-best, and leaves the demand of wealthier agents unchanged. This reduces the total demand for assets, creating an excess supply, which lowers the profit rate. This in turn *increases* the demand from wealthy agents, and raises their payoffs. The reduction in the profit rate restricts supply of assets. Hence there is a reallocation of assets from poorer to wealthier agents. The poorer the agent, the more important is the (LL) constraint, so they tend to be the most adversely impacted by the weakening of the bankruptcy rule. Some of them may be excluded from the market altogether.

Figure 1 highlights the driving forces behind the redistribution of assets leased.[13] Individual demand for assets of an agent with wealth w' under law $p_2(\cdot)$ is given as $\gamma_2(\pi, w' < \overline{w})$ and depicted in the left-hand graph of Figure 1. Weakening liability law shifts demand downwards to $\gamma_1(\pi, w')$. In contrast, demand of rich enough agents is unaffected and given as $\gamma(\pi, w \geq \overline{w})$ under either law. As a result, aggregate demand D_2 shifts downwards to D_1 (right-hand graph of Figure 1). A reduction in aggregate demand leads to a reduction of the profit rate from π_2 to π_1. Now, individual equilibrium demand for assets is affected differently for rich and poor agents. For rich agents, equilibrium demand *increases* from γ_2^* to γ_1^*, which is due to the reduced profit rate. However, for poor agents demand *decreases* from γ'_2 to γ_1 because the partial equilibrium impact of weakening liability law overrules the general equilibrium effect of lower profit rates.

These results are in line with empirical evidence found by Gropp *et al.* (1997).[14] Consistent with Proposition 2(c), Gropp *et al.* (1997, Table III, p. 238) report that the amount of debt is decreasing in the exemption limit for poor borrowers in the lowest two quartiles of the wealth distribution. The increase in debt for the top two quartiles of wealth is consistent with Proposition 2. These differences are significant and economically meaningful. Gropp *et al.* (1997, Table V, p. 242) compare the predicted value of debt for an observationally equivalent household living in two hypothetical states with different exemption limits.[15] If the household is 'poor', with assets worth \$47,000, belonging to the second quartile of the wealth distribution, then the estimated debt holding is \$28,105 in a state with low exemption limit (\$6000), which *decreases* to \$10,551 if the same household lives in a state with a high exemption limit (\$50,000). These differences change their sign if the household is rich, with \$150,000 worth of assets, belonging to the highest wealth quartile. The rich household has a predicted debt of \$36,136 in the low exemption limit state, which *increases* to \$72,076 in the high exemption limit state.

Gropp *et al.* (1997) find that *ex ante* (nominal) interest rates increase substantially for poor borrowers. For example, going from a zero exemption limit state to an unlimited exemption limit state increases interest rates for a borrower of the lowest asset quartile by 5%, while it slightly decreases the interest rate for a borrower from the highest asset quartile (albeit in a statistically insignificant manner). The latter finding is in line with our theoretical analysis. In order to solve the model, we used standard contract theory techniques and limited attention to state-contingent, renegotiation proof contracts. A more realistic setup (which could be the direct mechanism that is actually being played to obtain the outcome of our indirect mechanism) would consider a loan with an interest rate corresponding to the payments made in the good state of the world. In case of failure (the bad state of the world), renegotiation with the bank then leads to a downward adjustment of payments and hence state-contingent contracts. If we now take the payment in the good state of the world as our measure of the interest rate, then it becomes clear that absent any GE effects, the interest rate must increase due to an increase in exemption limits. With GE effects, the implications on the interest rate are not so clear as there are two opposing effects. The GE effect decreases the interest rate, while the direct commitment effect increases the interest rate.

Proposition 2 does not describe the impact of weakening bankruptcy law on any given wealth-constrained borrower; it states a reduction in the scale of projects aggregating across all wealth-constrained borrowers. A more detailed result is possible if we impose the additional restriction that the bankruptcy law is weakened more for poorer borrowers.[16] Hence a relaxation which appears to be 'progressive' *ex post* ends up having a 'regressive' impact *ex ante*.

Proposition 3. Consider a weakening of the bankruptcy rule from $p_2(\cdot)$ to $p_1(\cdot)$ with the property that $p_2(W) \geq p_1(W)$ for all W, and $p_2(W) - p_1(W)$ is non-increasing in W. Then there exists $\widetilde{w} \leq \overline{w}(\pi_2; p_2)$ such that the payoff, effort and project scale of all borrowers with wealth above (respectively, below) $\widetilde{w}$ (weakly) increases (respectively, decreases).

Proof. By Proposition 2, we have $\pi_1 \leq \pi_2$. Since equilibrium project scale is non-decreasing in w, the fact that $p_2(W) - p_1(W)$ is non-increasing implies that

$$\gamma(w; p_2)[\pi_2 - \pi_1] - [p_2(\sigma(w)) - p_1(\sigma(w))] \tag{4}$$

is non-decreasing in w. Define $\widetilde{w}$ to be the smallest w such that (4) is nonnegative. Then weakening the bankruptcy law to p_1 causes constraint (F) to be relaxed at the A-optimal contract under p_2 for any $w > \widetilde{w}$, and strengthened otherwise. Using arguments analogous to those in previous results, it follows that project scale, effort and payoff of agents will rise (weakly) above $\widetilde{w}$ and fall otherwise. From Proposition 2 we know that those above $\overline{w}(\pi_2;p_2)$ are better off, so it must be the case that $\widetilde{w} \leq \overline{w}(\pi_2;p_2)$. □

IV. Fixed Project Size

We now consider the special case where the returns to scale diminish fast enough that project scale is at most 1 for any borrower. This is a special case of our model where β is sufficiently small.[17]

Fixing project scale restricts the scope of asset reallocations across borrowers: it is no longer possible for wealthy borrowers to borrow more when bankruptcy laws are eased. Nevertheless, this case is of practical interest in many situations: for example, tenant households rarely want to rent more than one apartment, and taxi drivers can rarely drive more than one taxi. In such contexts, we can obtain some additional results concerning the optimal shape of bankruptcy law, and more detailed distributional and incentive effects of changing the law.

We first show that in the setting where project size is fixed, a normative justification can be provided for the widespread practice of using asset exemption limits as the form of bankruptcy law: they Pareto-dominate any other law.

Proposition 4. For any allocation that is Walrasian under an initial law $p(\cdot)$ with operating profit rate π per asset, there exists a profit-preserving exemption limit E^* in the following sense. Under exemption limit law $\widetilde{p}(W) = \max\{W - E^*, 0\}$, there is a Walrasian allocation with the same operating profit rate π per asset as under the Walrasian allocation of the initial law $p(\cdot)$.

Consider a change in the bankruptcy law from the initial $p(\cdot)$ to the corresponding profit-preserving exemption limit law $\widetilde{p}(W) = \max\{W - E^*, 0\}$. Then the following is true.

1. Suppose that β is small enough that the first-best project size is at most 1. Then any initial bankruptcy law is (weakly) Pareto-dominated by its profit-preserving exemption limit law: every borrower is weakly better off.
2. There exists $\widetilde{w}$ such that the payoff, effort and project scale of all borrowers with wealth above (respectively, below) $\widetilde{w}$ (weakly) increases (respectively, decreases).

Proof. Existence of a profit-preserving exemption limit can be shown with the following arguments. Consider first an exemption limit $E = 0$ that is the most stringent feasible bankruptcy law (denoted p_2). By Lemma 6 we know that $\gamma(\pi,w_i,p_2(\cdot)) \leq \gamma(\pi,w_i,p_2)$ because for exemption limit $E = 0$ we have $p_2(W) = \max\{W{-}0,0\} = W \geq p_2(\cdot)$, where the last inequality is due to feasibility. So it must be the case that $\sum_i \gamma(\pi,w_i,p_2) \geq S$ for any set of A-optimal project choices with $E = 0$. Next, let p_2 denote a large enough exemption with $\overline{E} = \sigma(w_1)$; this is the weakest possible bankruptcy law in our setting since Lemma 1 implies $d = w_i$ and hence $p_2(W) = \max\{W{-}\sigma(w_1),0\} = 0 \leq p(\cdot)$ for all agents. By Lemma 6 we know that $\gamma(\pi,w_i,p(\cdot)) \leq \gamma(\pi,w_i,p_2)$. Finally, for every agent i, the Walrasian demand is stepwise decreasing in E. As a result we can find some intermediate value $E^* \in [0,\overline{E}]$ with a market-clearing Walrasian allocation corresponding to profit rate π.

Part 1: Let $\underline{n}$ be the poorest matched agent in the Walrasian allocation under law $p(\cdot)$ with operating profit rate π per asset and contracts $(v_s^i, v_f^i, e^i, \gamma^i)$ for all i. From Lemma 4 it follows that all richer agents are also matched: $\gamma^i = 1$ for all $i \leq \underline{n}$. Choose $E^* = \sigma(w_{\underline{n}}) - p(\sigma(w_{\underline{n}}))$.

For every agent $i \leq \underline{n}$, the set of contracts obeying constraint (F) increases due to a change from $p(\cdot)$ to E^* if π is constant. In contrast, for every agent $i > \underline{n}$, the set of contracts obeying (F) shrinks. This is true due to the corresponding change in $\sigma - p(\sigma)$ as depicted in Figure 2, where $\sigma^* = \sigma_{\underline{n}}$ for the first part of the proposition.[18]

Construct a new Walrasian allocation as follows, with the same profit rate π. Borrowers with $i > \underline{n}$ do not demand any project. Those with $i \leq \underline{n}$ demand one project, and are assigned an A-optimal contract corresponding to profit rate π and exemption limit law E^*. The same principals are active. This is a Walrasian allocation as long as the assigned project scales are A-optimal for every agent.

For an agent with $i > \underline{n}$, 0 is an A-optimal project scale, because (F) has become tighter for them. For $i \leq \underline{n}$, (F) has become more relaxed. See Figure 2. Therefore by Lemma 6, their A-optimal project scale cannot decrease (as the profit rate is the same). The A-optimal project scale for them was 1 previously, so it must continue to be 1. Therefore the constructed allocation is Walrasian.

Note finally that agents with $i \geq \underline{n}$ are as well off as before. Those with $i \leq \underline{n}$ with wealth high enough to attain the first-best will also be left unaffected. Others will be better off, as (F) was binding to start with, and has been relaxed.

Part 2: Constraint (F) of the A-optimal contract in the Walrasian allocation (and consequently the Walrasian allocation) is unaffected for an agent with *ex ante* wealth w if $\sigma(w) - p(\cdot) = \max\{\sigma(w) - E^*, 0\}$, and we consider the richest unaffected agent $\widetilde{w}$. Note that it must be true that $\max\{\sigma(\widetilde{w}) - E^*, 0\} = \sigma(\widetilde{w}) - E^*$. This follows from the fact that $\sigma(w) - p(\cdot) - \max\{\sigma(w) - E^*, 0\}$ is inverse U-shaped and (weakly) increasing in w until $\sigma(w) = E^*$. For wealth level with $\sigma(w) > E^*$, $\sigma(w) - p(\cdot) - \max\{\sigma(w) - E^*, 0\}$ is decreasing in w. Due to the inverse U-shaped nature of $\sigma(w) - p(\cdot) - \max\{\sigma(w) - E^*, 0\}$, it must be that the largest unaffected agent has wealth $\widetilde{w} > E^*$. Otherwise, all agents with $w < w^*$ would be unaffected and Walrasian A-optimal demand for agents with wealth $w \geq \widetilde{w}$

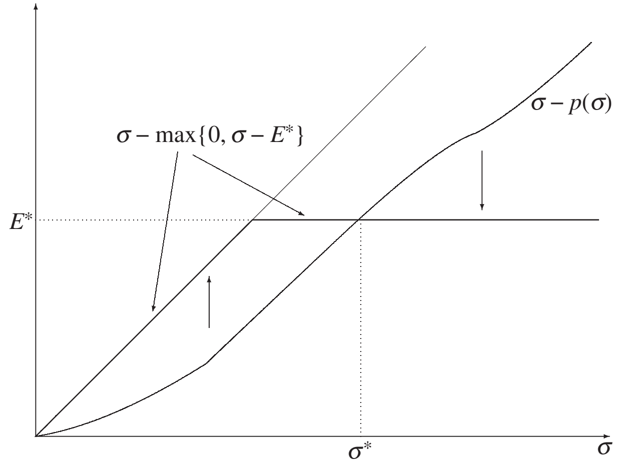

FIGURE 2. Replacing $p(\cdot)$ with exemption limit law E^*.

would increase. This is a contradiction to market clearing. From this it follows that constraint (F) is less binding for all agents with wealth above $\widetilde{w}$ and more binding or unaffected for agents with wealth $w < \widetilde{w}$. Using arguments that are almost identical to those in Proposition 3 implies that utility, effort and project scale must be weakly increasing. A graphical illustration of these arguments is given in Figure 2, where $\sigma(\widetilde{w}) = \sigma^*$ in the second part of the proposition. □

Figure 2 conveys the underlying idea: fixing the exemption limit to equal the liability limit of the marginal agent active in the market ensures that liability is raised for excluded agents, and lowered for intramarginal active agents. Then the demand pattern is unaffected: excluded agents continue to demand no project, while intramarginal agents demand a single project. Hence there is an equilibrium with the same profit rate and the same allocation of projects; active agents can now commit credibly to higher repayments in time of distress and obtain credit on easier terms as a result. The logic does not extend if project scales could exceed unity: intramarginal agents may then demand more projects, creating excess demand and raising the profit rate. This may cause some marginal agents to get excluded from the market, so a Pareto improvement no longer results. However, as shown in part 2 of Proposition 4 in the case of variable project scale, replacing any bankruptcy law $p(\cdot)$ with a profit-preserving exemption limit is beneficial to rich borrowers.

The case of fixed project scale also permits a more detailed description of the distributional impact of weakening bankruptcy law, if we further assume that all principals are identical, that is, have the same overhead cost f. Let the bankruptcy law be represented by exemption limit E. An agent is viable at the exemption limit E if there exists a contract for that agent, feasible with this bankruptcy law, that generates an operating profit of at least f. Let $n(E)$ denote the number of viable agents at exemption limit E; this is a non-increasing function by virtue of Lemma 6.

Walrasian allocations can be computed as follows. Without loss of generality, equilibrium π must be at least f.[19] Compute the A-optimal demand for each viable agent when $\pi = f$, and the exemption limit is E. If the resulting aggregate A-optimal demand exceeds m, the number of principals, then the Walrasian allocation must involve $\pi > f$. In that case, all principals will be active, and some viable agents will be excluded from the market. In this case, the principals are on the short side of the market.

Conversely, if aggregate A-optimal demand (when evaluated at $\pi = f$ and exemption limit E) does not exceed m, then there will be a Walrasian allocation at $\pi = f$ in which all viable agents are matched but some principals are not. This is the case where the agents are on the short side of the market.

Now suppose that the exemption limit is raised from E to E'. There are three cases to consider.

(A) *Principals are on the short side of the market at both E and E′*. Then there is no effect on the total volume of leasing or credit; the allocation of credit across agents remains unaltered, but the profit rate falls or remains unchanged (since the A-optimal demand for every agent falls or remains unchanged as the exemption limit rises, at any given profit rate). Then *every active agent is better off, while every active principal is worse off*. The result is a redistribution from active principals to active agents. Moreover, it can be shown that wealthier active agents benefit more, while the effort level declines (weakly) for all active agents.[20] The intuitive reason is that the beneficial GE effect applies equally to all active agents, while the adverse PE effect of a higher exemption limit is less significant for wealthier agents. Nevertheless, the former outweighs the latter for all active agents, not just the wealthiest ones.

(B) *Agents are on the short side of the market both before and after the change*. Then the equilibrium profit rate is unchanged (at f); there is no GE effect. All agents are (weakly) worse off, owing to the strengthening of the no-default constraint. Principals are unaffected, so the result is a Pareto-deterioration of welfare.

(C) *Principals are on the short side at E but on the long side at E'*. Then the profit rate drops from $\pi(E) > f$ to $\pi(E') = f$; the number of assets leased falls, and the poorest agents active at E get excluded from the market a E'. On the other hand, the wealthiest agents are better off owing to the drop in the profit rate. In this case the weakening of the bankruptcy law makes lenders and poor borrowers worse off, while wealthy borrowers are better off. It can be shown that the effort level declines or remains constant for all borrowers that continue to be active. In this case, aggregate welfare also declines.

V. Effects of Means-tested Exemption Limits

In this section, we discuss a simple variation of the model that helps to predict the impact of the current change in US bankruptcy law undertaken in the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005. We focus on one particular aspect of BAPCPA: the abolition of free choice between Chapter 7 and Chapter 13.

Prior to the change in the US bankruptcy law, defaulting borrowers were free to choose between the Chapter 7 code and the Chapter 13 code in most instances. The Chapter 7 code is a much weaker bankruptcy law that allows defaulting borrowers to keep a large fraction of wealth and all of their future labour income. So prior to 2005, most debtors filed under Chapter 7 (approximately 70% of all households, according to White (1987)).

Following BAPCPA, households are allowed to file under Chapter 7 only if they pass a means test, effectively requiring that their *ex post* income during the six months prior to filing does not exceed the median (household size adjusted) income of the state in which the debtor is living (White 2007). If the income exceeds this threshold, then the household is not allowed to file for bankruptcy under Chapter 7 unless it passes a second test, the repayment test, which checks whether average consumption prior to filing exceeds a certain threshold. If it does, then the household must file under Chapter 13. If average consumption prior to filing does not exceed the threshold, the household may file under Chapter 7 if a certain requirement relating disposable income to secured debt is met.

In what follows we will use the following simple interpretation of US bankruptcy law before and after BAPCPA to understand the impact of the new law, especially the means test used in BAPCPA. First, we only consider the impact of the means test under BAPCPA, assuming that the procedures under Chapter 7 and Chapter 13 codes are unchanged.[21] Second, we exaggerate the attractiveness of Chapter 7 vs. Chapter 13 from the standpoint of defaulting borrowers: what we call Chapter 7 will always be more attractive to defaulting borrowers *ex post*.[22] Alternatively, we restrict attention to the majority of borrowers for whom Chapter 7 is more attractive.

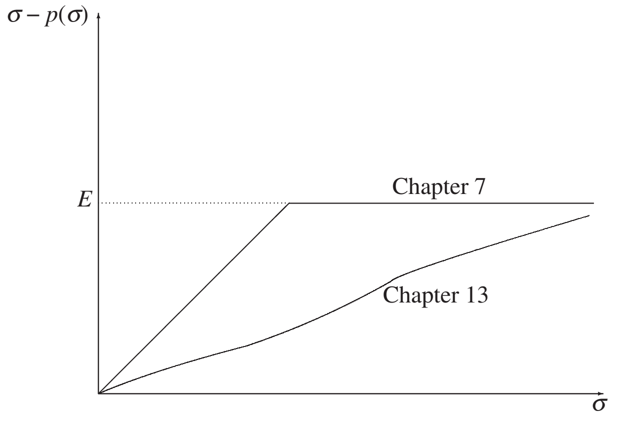

FIGURE 3. Chapter 7 vs. Chapter 13 prior to BAPCPA.

The law before the change is depicted in Figure 3. *Ex post*, Chapter 7 is more attractive to defaulting borrowers than Chapter 13 since the corresponding $\sigma - p(\sigma)$ curve under Chapter 7 is always above the one corresponding to Chapter 13.

In contrast, the law after BAPCPA is depicted in Figure 4. Note that post-BAPCPA law violates our earlier assumption that $W - p(W)$ must be non-decreasing since $W - p(W)$ makes a jump downward at σ^T. Here, σ^T corresponds to the threshold value used in the means test. In what follows we will assume that agents cannot reduce their *ex post* wealth to seek protection under Chapter 7. We expect our arguments to be valid even if we allow for such opportunistic behaviour as long as reducing *ex post* income is costly.

Due to the discrete number of projects, Walrasian allocations need not be unique. In what follows, if there exist multiple Walrasian allocations, we assume that the Walrasian allocation with the highest profit rate is realized.[23]

Proposition 5. Consider a change of bankruptcy law as described above, and restrict attention to the Walrasian allocation with the maximal profit rate π (across all Walrasian allocations at any given bankruptcy law). Then we have the following.

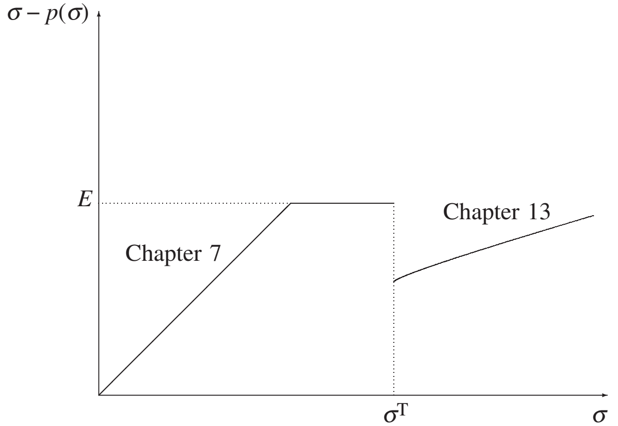

FIGURE 4. Chapter 7 vs. Chapter 13 after BAPCPA.

(a) All agents with wealth $\sigma < \sigma^T$ are weakly worse off due to the change. Agents with wealth $\sigma \geq \sigma^T$ may be better or worse off, and all principals benefit from the change.
(b) The total number of assets leased by agents with $\sigma < \sigma^T$ will decrease (or remain the same).

The argument follows from an inspection of Figures 3 and 4. Prior to BAPCPA, there was a uniform bankruptcy law for all agents, represented by Chapter 7. Due to BAPCPA, agents with $\sigma \geq \sigma^T$ have a potentially beneficial partial equilibrium effect since their $\sigma - p(\sigma)$ curve is shifted downwards. These rich agents are effectively able to post a greater fraction of their *ex post* wealth as collateral. This leads to a (weak) increase in their A-optimal demand. For agents with $\sigma < \sigma^T$, the A-optimal demand patterns are unaltered. Hence the aggregate A-optimal demand curve shifts outwards, raising the equilibrium profit rate. This renders principals (weakly) better off. Whether or not agents with $\sigma \geq \sigma^T$ benefit depends on the interaction of partial and general equilibrium effect, which is not clear in general. However, for agents whose *ex post* wealth is below the threshold σ^T, the effect is clear. There is no beneficial partial equilibrium effect and the (weak) increase in equilibrium profits will render these poor agents worse off. From Lemma 5, the allocation of credit to these agents must fall.

VI. Conclusion

To summarize our principal result, weakening bankruptcy law leads to a redistribution of credit and/or assets from poor to rich borrowers. This explains the findings of cross-sectional analysis employing differences across US state bankruptcy provisions (Gropp *et al*. 1997; Cerqueiro and Penas 2014), as well as across Indian states (von Lilienfeld-Toal *et al*. 2012). Hence the effects emphasized in this paper appear to be quantitatively significant.

Our model neglected dynamic effects of altering bankruptcy rules or collateralizability of assets on savings incentives of agents, and on the ownership distribution of assets in future periods. Enlarging the range of collateralizable assets may allow increased access to credit in the short run, but subsequently renders borrowers more vulnerable to downturns in the economy. Investigation of such dynamic effects remains an important task for future research.

APPENDIX

Proof of Lemma 1 Suppose that there is a contract that satisfies equation (1), and the agent defaults following outcome k. Then $j \in C_i$ receives s_{kj} instead of t_{kj}, the transfer in the event that the agent does not default. Now the earlier contract can be replaced with one where $\widetilde{t}_{kj} = s_{kj}$. This is feasible because

$$\sum_{j \in C_i} \widetilde{t}_{kj} = \sum_{j \in C_i} s_{kj} = \gamma^\beta y_k \leq \gamma^\beta y_k + w_i - d + \sigma(w_i);$$

this inequality holds since $w_i - d \geq 0$, $\sigma(w_i) \geq 0$. The agent will not default under this new contract as

$$\sum_{j \in C_i} \widetilde{t}_{kj} = \sum_{j \in C_i} s_{kj} = \gamma^\beta y_k \leq \gamma^\beta y_k + p(w_i - d + \sigma(w_i)).$$

And each principal $j \in C_i$ earns the same payoff as before when default occurs, while the agent avoids the penalty associated with default. So the new contract *ex post* Pareto-dominates the previous one, which is thus vulnerable to renegotiation. This establishes (i).

For (ii), note that if $d < w_i$, then the downpayment d can be raised by some $\varepsilon>0$, with a corresponding reduction of $\sum_{j\in C_i} t_{kj}$ and of $\sum_{j\in C_i} I_j$ by ε. In the new contract, the agent will also not want to default, as the mandated transfers fall by ε, while default penalty $p(w_i - d + \sigma(w_i))$ falls by at most ε (because the slope of $p(\cdot)$ is between 0 and 1). Then the agent's payoff, as well as that of every $j \in C_i$, in state k is unaltered. This new contract is then payoff-equivalent to the previous one.

To show (iii), fix $d = w_i$, then re-allocate transfers and contributions across $j \in C_i$ in such a way as to leave their expected payoffs unchanged. The aggregate financial transfers vis-à-vis the agent remain unaltered, and so do the agent's incentives and payoffs. Specifically, let π_j denote the expected operating profit of j, which must cover the overhead cost f_j (otherwise j would do better to leave the coalition):

$$\pi_j \equiv et_{sj} + (1-e)t_{fj} - I_j \geq f_j.$$

Let $\Pi \equiv \sum_{j\in C_i} \pi_j \geq \sum_{j\in C_i} f_j$ denote aggregate operating profit. If $\Pi > 0$, define $\delta_j \equiv \pi_j/\Pi$ and select $\widetilde{I}_j = \delta_j \sum_{l\in C_i} I_l$, $\widetilde{t}_{kj} = \delta_j \sum_{l\in C_i} t_{kl}$. Then the payoff of j equals $\widetilde{\pi}_j = \delta_j\Pi = \pi_j$. If $\Pi = 0$, we have $I_j = et_{sj} + (1-e)t_{fj}$ for all $j \in C_i$: select arbitrary $\delta_j \geq 0$ with $\sum_{j\in C_i} \delta_j = 1$, and repeat the above construction. This completes the proof of Lemma 1.

Proof of Lemma 2 Since i is viable, there exists a feasible contract (v_s, v_f, e, γ) for some coalition C_i including i. Let $\delta \equiv (w_l + \sigma(w_l)) - (w_i + \sigma(w_i)) > 0$. For agent l we can select a coalition C_l consisting of l and the same principals j that belong to C_i. Then select the contract $\widehat{v}_s = v_s + \delta$, $\widehat{v}_f = v_f + \delta$, combined with the same e and γ. By construction, this contract satisfies (IC), (PCP) and (PCA). It also satisfies (LL): the increase in the left-hand side of (LL) is δ, while the increase in the right-hand side of (LL) is

$$[\sigma(w_l) - \sigma(w_i)] - [p(\sigma(w_l)) - p(\sigma(w_i))] \leq \sigma(w_l) - \sigma(w_i) < \delta.$$

Proof of Proposition 1 We proceed through a sequence of steps.

Step 1: *A Walrasian allocation is stable*. If not, then there will exist an agent i that deviates with a coalition $\widehat{C}_i$ and a contract $(\widehat{v}_s, \widehat{v}_f, \widehat{e}, \widehat{\gamma})$ that generates an expected utility larger than that in the A-optimal contract relative to π, and an expected operating profit for every $j \in \widehat{C}_i$ that exceeds π. This contradicts the definition of an A-optimal contract.

Step 2: *In any stable allocation, all active principals attain the same (expected) operating profit*. If this is false, then there exist two active principals j,m with $\pi_j > \pi_m$, $\pi_j \geq f_j$, $\pi_m \geq f_m$. Let $j \in C_i$. Then i can form a coalition with $\widehat{C}_i \equiv C_i \setminus \{j\} \cup \{m\}$. Let n_i denote the number of principals in C_i, and select any $\varepsilon \in (0, (\pi_j - \pi_m)/(n_i + 1))$. We can then select a contract for the deviating coalition that increases v_s, v_f by ε, and also increases π_h by ε for every $h \in \widehat{C}_i$. This contract is feasible and makes everyone in the deviating coalition better off.

Step 3: *In any stable allocation*, $\pi \geq f_j$ *for every active principal*. If not, then such a principal would be better off unmatched.

Step 4: *In any stable allocation*, $\pi \leq f_j$ *for any unmatched principal*. Otherwise, an unmatched principal j could make positive net profit by being matched with an active agent at the going profit rate of π. An argument analogous to Step 2 can now be used to show that the allocation is not stable.

Step 5: *In any stable allocation, every agent gets an A-optimal contract relative to* π, *the common rate of operating profit earned by active principals*. Otherwise, there exists a feasible contract for i that generates a higher expected payoff to i, while paying an operating profit rate of at least π on each asset leased. We now argue there exists a contract in the neighbourhood of this deviating contract that awards a profit rate greater than π for every asset leased, while still enabling the agent to attain a higher expected payoff compared with that in the stable allocation.

The original contract satisfied (PCA), so the deviating contract satisfied (PCA) with slack. We can thus ignore this aspect of feasibility in what follows.

If (LL) is slack in both states in the deviating contract, then we can reduce v_s,v_f by some common but small ε, which preserves feasibility. In this case the argument is straightforward.

So consider the case where (LL) binds in some state. This must be state f, because $v_s > v_f$ is needed to induce $e > 0$, which in turn is necessary for feasibility ((PCP) requires the project to break even in expectation, and this is not possible if e=0 given that $y_f < I$). So we must have $v_f = \sigma(w_i) - p(\sigma(w_i))$. Now holding v_f fixed at this level, consider varying v_s above v_f, with e adjusted according to (IC), that is, with $e = e(v_s)$ that solves $v_s - \{\sigma(w_i) - p(\sigma(w_i))\} = D'(e)$. Let the corresponding aggregate profit for asset owners be denoted

$$\Pi(v_s) \equiv e(v_s)[\gamma^{\beta} y_s - v_s] + (1 - e(v_s))[\gamma^{\beta} y_f - \{\sigma(w_i) - p(\sigma(w_i))\}],$$

where γ is the project scale in the deviating contract.

Define the P-optimal contract to be one where v_s is selected to maximize $\Pi(v_s)$, subject to $v_s \geq \sigma(w_i) - p(\sigma(w_i))$. This problem can be re-stated as follows (replacing e as the control variable): select $e \in [0,1]$ to maximize $e \cdot [\gamma^{\beta}(y_s - y_f) - D'(e)]$. Noting that $eD'(e)$ is strictly convex, the objective function is strictly concave, and thus has a unique solution. It is also evident that $e > 0$ in the P-optimal contract.

We claim that in the original contract (in the stable allocation), the agent must have attained a utility at least as large as in the P-optimal contract. Otherwise, the agent obtained a smaller payoff, and the contract in the stable allocation was not P-optimal (as the P-optimal contract is unique, as shown above). So aggregate operating profit of the principals in C_i must be less than the profit in the P-optimal contract. Then the agent and principals in C_i could deviate to the P-optimal contract, which would make all of them strictly better off.

Since the agent received a higher payoff in the deviating contract compared with the contract in the stable allocation, it follows that the agent's payoff in the deviating contract is strictly higher than in the P-optimal contract. In both the deviating contract and in the P-optimal contract, we have $v_f = \sigma(w_i) - p(\sigma(w_i))$, so they must differ in v_s, with the deviating contract associated with a higher v_s. It follows that as v_s is lowered from that in the deviating contract to the level in the P-optimal contract, the agent's payoff is (continuously) lowered while aggregate profit of the principals in C_i is (continuously) raised (owing to the strict concavity of aggregate profit with respect to e). Therefore we can find a contract with v_s slightly below that in the deviating contract, which will allow a strictly higher aggregate profit, and a slightly lower payoff for the agent. This allows all members of C_i as well as i to be better off compared to the stable allocation—a contradiction. This completes the proof of Step 5.

The proof of Proposition 1 now follows from combining Steps 1–5 to infer that a stable allocation must be Walrasian.

Proof of Lemma 3 Define

$$S^*(\pi) \equiv \gamma^{*\beta} R(e^*) - e^* D'(e^*) - \gamma^*(I + \pi),$$

where we drop the dependence of the first-best contract on π to avoid notational clutter. Then if $S^*(\pi) \geq -p(\sigma(0))$, the first-best satisfies (F) for all $w \geq 0$, so we can set $\overline{w}(\pi) = 0$ in that case. Otherwise $S^*(\pi) < -p(\sigma(0))$ and there exists $\overline{w}(\pi)$>0 such that $S^*(\pi) = -w - p(\sigma(w))$, since $-w - p(\sigma(w))$ is decreasing in w and goes to $-\infty$ as w becomes arbitrarily large. Then the first-best is attainable with profit rate π if and only if $w \geq \overline{w}(\pi)$, which establishes (a).

For (b), suppose that there exist $\pi_1 < \pi_2$, wealth w and choices of first-best contracts such that $\gamma_2 = \gamma^*(\pi_2,w) > \gamma_1 = \gamma^*(\pi_1,w)$. Then

$$\gamma_1^\beta R(e_1) - D(e_1) - \gamma_1(I+\pi_1) \geq \gamma_2^\beta R(e_2) - D(e_2) - \gamma_2(I+\pi_1)$$

and

$$\gamma_2^\beta R(e_2) - D(e_2) - \gamma_2(I+\pi_2) \geq \gamma_1^\beta R(e_1) - D(e_1) - \gamma_1(I+\pi_2),$$

where e_m denotes $e^*(\pi_m,w)$, $m = 1,2$. Adding these two inequalities, we obtain

$$(\gamma_2 - \gamma_1)(I+\pi_1) \geq [\gamma_2^\beta R(e_2) - D(e_2)] - [\gamma_1^\beta R(e_1) - D(e_1)] \geq (\gamma_2 - \gamma_1)(I+\pi_2),$$

which contradicts the hypothesis that $\gamma_2 > \gamma_1$, as $\pi_1 < \pi_2$. Hence $\gamma^*(\pi,w)$ is non-increasing in π. This in turn implies that $e^*(\pi,w)$ is non-increasing because it maximizes $\gamma^{*\beta}R(e) - D(e)$.

In order to establish (c), note the following

Observation 1: In any A-optimal contract, $\gamma(\pi,w)$ maximizes $\gamma^\beta R(e(\pi,w)) - \gamma(I+\pi)$. Otherwise, we can select a different γ to raise the value of $\gamma^\beta R(e(\pi,w)) - \gamma(I+\pi)$: this both raises the value of the objective function (AO) and helps to relax the constraint (F).

Observation 2: In any first-best contract, $\gamma^*(\pi)$ maximizes $\gamma^\beta R(e^*(\pi)) - \gamma(I+\pi)$. Otherwise, the value of the first-best objective function can be raised by switching to a different γ, while leaving $e^*(\pi)$ unchanged.

We now claim that if $w < \overline{w}(\pi)$, then $e(\pi,w) < e^*(\pi)$ for any selection of second-best and first-best contracts. If this is false, then we can find contracts with $e(\pi,w) \geq e^*(\pi)$. Using Observations 1 and 2, it follows that corresponding second-best and first-best project scales satisfy $\gamma(\pi,w) \geq \gamma^*(\pi)$.

The hypothesis $w < \overline{w}(\pi)$ implies that the first-best cannot be achieved by w at profit rate π. This means that $(\gamma^*(\pi), e^*(\pi))$ violates (F), while by its very nature $(\gamma(\pi,w), e(\pi,w))$ satisfies (F). This implies that

$$\begin{aligned}&[\gamma(\pi,w)]^\beta R(e(\pi,w)) - e(\pi,w)D'(e(\pi,w)) - \gamma(\pi,w)(I+\pi)\\ &> [\gamma^*(\pi)]^\beta R(e^*(\pi)) - e^*(\pi)D'(e^*(\pi)) - \gamma^*(\pi)(I+\pi),\end{aligned}$$

which in turn implies that

$$\begin{aligned}&\{[\gamma(\pi,w)]^\beta R(e(\pi,w)) - \gamma(\pi,w)(I+\pi)\} - \{[\gamma^*(\pi)]^\beta R(e^*(\pi)) - \gamma^*(\pi)(I+\pi)\}\\ &> e(\pi,w)D'(e(\pi,w)) - e^*(\pi)D'(e^*(\pi))\\ &> D(e(\pi,w)) - D(e^*(\pi)),\end{aligned}$$

the last inequality following from the fact that

$$\frac{\partial eD'(e)}{\partial e} = eD''(e) + D'(e) > D'(e).$$

Then it must be the case that the second-best choices yield a higher utility than the first-best choice, which contradicts the definition of the first-best.

Therefore every second-best effort must always be less than any first-best effort. The rest of (c) now follows from Observations 1 and 2.

Proof of Lemma 4 Suppose that there exist A-optimal efforts $e_m \equiv e(\pi, w_m)$, m=1,2, such that $e_1 > e_2$. Then by Observation 1, corresponding A-optimal scales satisfy $\gamma_1 \geq \gamma_2$.

Since $w_m < \overline{w}(\pi)$, the first-best is not achievable at (π, w_m), $m = 1,2$, and constraint (F) must be binding at (γ_m, e_m) for w_m. Since $w_2 > w_1$, the contract (γ_1, e_1) must satisfy (F) with slack at (π, w_2). To see this, note that

$$\begin{aligned}[\gamma_2]^\beta R(e_2) - e_2 D'(e_2) - \gamma_2(I + \pi) &= -w_2 - p(\sigma(w_2)) \\ &< -w_1 - p(\sigma(w_1)) \\ &= [\gamma_1]^\beta R(e_1) - e_1 D'(e_1) - \gamma_1(I + \pi).\end{aligned}$$

This implies that

$$\{[\gamma_1]^\beta R(e_1) - \gamma_1(I + \pi)\} - \{[\gamma_2]^\beta R(e_2) - \gamma_2(I + \pi)\} \geq e_1 D'(e_1) - e_2 D'(e_2) > D(e_1) - D(e_2),$$

the last inequality following from the hypothesis that $e_1 > e_2$. This implies that the contract (γ_1, e_1) generates a higher payoff than (γ_2, e_2), contradicting the A-optimality of the latter, since the former is feasible at the wealth w_2. Hence $e_1 \leq e_2$. The remaining part of the result follows from Observation 1.

Proof of Lemma 5 The argument is virtually identical to that for Lemma 4, in the case that the first-best is not achievable in either situation. And if the first-best is achievable in either or both situations, then we can use Lemma 3.

Proof of Lemma 6 Suppose that the first-best payoff is achievable at (π, w) under rule p_1; then it is achievable under p_2, and the A-optimal payoffs coincide. So suppose that the first-best is not achievable at (π, w) under rule p_1. If the first-best is achieved at (π, w) under rule p_2, then the result follows from Lemma 3. So we need to consider the case where the first-best is not achieved under either rule p_1 or rule p_2.

Let e_l denote an A-optimal effort under rule p_l. An argument analogous to that used for preceding lemmas shows that $e_1 \leq e_2$, which in turn implies that $\gamma_1 \leq \gamma_2$.

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NOTES

1. For more details of cross-country comparisons, see Djankov *et al.* (2007, 2008).

2. This pertains to the annual number of non-business bankruptcy filings in the USA, according to the American Bankruptcy Institute: see http://abi-org.s3.amazonaws.com/Newsroom/ Bankruptcy_Statistics/ Quarterlynonbusinessfilings1994-Present.pdf (accessed 23 September 2015).
3. The different ingredients of the model are necessary in order to derive different parts of our main results. Varying project scale is relevant for understanding the redistribution of credit across borrowers. The moral hazard part is crucial for showing that exemption limits are Pareto-optimal. Only heterogeneity on the side of the principals is not strictly necessary and could be replaced by the simplifying assumption of fixed supply —this generates the strongest form of GE effects.
4. In the case of a fixed project scale, weakening bankruptcy law causes agent efforts to fall or remain unchanged, implying that aggregate welfare in the economy cannot improve. In the more general case, we cannot sign the effect on aggregate welfare.
5. The recent empirical literature on the importance of exemption limits is exemplified by Mahoney (2015). That paper looks at the impact of Chapter 7 bankruptcy filings on demand for health insurance, showing that soft bankruptcy law will serve as a substitute for private health insurance. Dobbie and Song (2015), on the other hand, look at the *ex post* impact of bankruptcy protection while we focus on *ex ante* effects.
6. Other papers looking at Indian reforms intended to increase the range of collateralizable assets and enforcement of debt contracts are Visaria (2009) and Vig (2013).
7. Nevertheless, our predictions of the distributional effects of softening bankruptcy law with respect to rents are similar in some respects, though operating via different channels. In Biais and Mariotti (2009), for instance, softer laws benefit wealthier investors as they cause entry of less wealthy investors to fall, which causes the wage rate to fall. At the same time, their model has no implications with respect to redistribution of credit. In our context, wealthy entrepreneurs benefit from softer laws owing to reduced competition for funds arising from less wealthy entrepreneurs, and this is also mirrored in better access to credit. Therefore both theories predict that wealthy borrowers will prefer soft bankruptcy laws, unlike lenders and less-wealthy borrowers.
8. For instance, our main result (Proposition 2) concerning redistributional effects of changing bankruptcy law requires the existence of borrowers wealthy enough that they attain first-best project scales.
9. One complication in the pure credit context arises if $\sigma(w)\equiv 0$ for all w. In that case, borrowers wealthy enough to achieve the first-best will be able to entirely self-finance their projects, and will thus not need to borrow. Then Proposition 2 concerning redistributive effects of altering bankruptcy law may become vacuous. However, if *ex post* incomes are positive, then this would no longer be true: there can be wealthy borrowers who achieve the first-best and yet would want to borrow (against their future incomes).
10. We ignore the possibility of outside financing on the grounds of the benefits of ‘interlinked’ contracts (see Braverman and Stiglitz 1982): any financial contract offered by outsiders can be replicated by insiders as the latter are not wealth-constrained, with the benefit that common-agency externalities can be avoided.
11. Much of our analysis can be extended to a setting where *ex post* income is stochastic but positively correlated with initial wealth. This income could be due to some illiquid wealth invested in other productive activities. Alternatively, it could constitute labour income earned on a spot labour market.
12. This relies on the assumption that the slope of $p(\cdot)$ is less than 1.
13. Note that Figure 1 depicts the continuous limit of our results and abstracts from minor issues that arise due to the discreteness of project scale.
14. Our explanation differs from that of Gropp *et al.* (1997), which is based on the insurance role of higher exemption limits. This requires the implicit assumption that debt contracts are incomplete. Our theory applies to complete contracts and risk-neutral borrowers. Note also that their categorization of ‘demand’ and ‘supply’ factors differs from ours, in our respective theoretical explanations. They classify among supply factors the effect of a higher exemption limit that raises default risk and lowers the returns to lenders. We classify it as a demand factor, as it is incorporated in the calculation of A-optimal contracts (i.e. it is internalized by borrower–lender coalitions when they negotiate loan contracts). This difference is purely semantic, of course.
15. Gropp *et al.* (1997) are doing the exercise for a family with a 45-year-old male head, $75,000 yearly income, college degree and varying financial wealth.
16. However, this condition is not met when bankruptcy laws take the form of exemption limits: when the exemption limit is raised, the bankruptcy law is weakened more for wealthier borrowers.
17. When $\beta = 0$, this is obviously true: there are no returns at all to increasing project scale beyond one unit. It is also true in a positive neighbourhood of 0. Such a neighbourhood can be found by imposing the requirement that the first-best project scale at $\pi = 0$ equals 1. Since A-optimal project scales are non-increasing in the profit rate, and bounded above by the first-best scale, the A-optimal scale for every agent at any non-negative profit rate will be 0 or 1.
18. Note that $\sigma - p(\sigma)$ is increasing and below the 45-degree line for the class of liability laws that we consider here.
19. If $\pi < f$, then no principal or agent will be active; an equivalent allocation with no activity is obtained with $\pi = f$.
20. This implies that aggregate welfare, the sum of net payoffs across all principals and agents, decreases (weakly). This result is shown in an earlier version of this paper, available on request.

21. Actually, there are two additional important changes in bankruptcy law due to BAPCPA. First, the administrative procedure for filing under both Chapter 7 and Chapter 13 has become more complex, rendering either bankruptcy procedure less attractive to defaulting borrowers. Further, Chapter 13 has been made less attractive since certain kinds of debt can no longer be discharged, mainly debt obtained by fraud.
22. This is true for at least 70% of defaulting borrowers prior to BAPCPA, as White (1987) reports that 70% of defaulting borrowers opted to default under Chapter 7. The remaining 30% defaulting borrowers either prefer Chapter 13 over Chapter 7 (this is in particular the case if their house is under the risk of foreclosure) or are not allowed to file under Chapter 7. Under repeated default, borrowers are no longer allowed to choose between Chapter 13 and Chapter 7 prior to BAPCPA.
23. We conjecture that the results hold for other rules, for example all allocations with minimal profits or allocations with profits that are a weighted average of minimal and maximal profits.

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