1. Consider a class of homogenous farmers seeking to maximize two period utility $c_0 + c_1^2 - e$, where $c_i$ denotes consumption at date $i = 0, 1$, and $e \in [0, 1]$ the (unobservable) effort selected by the farmer, also equal to the probability of a normal crop. Incomes are 0 at date 0, and $w$ at date 1 if the crop is normal, and 0 otherwise, where $1 > w > 0$. Consumption is always nonnegative. The farmer seeks to borrow $B$ at date 0, repay $R$ at date 1 if the crop is normal, and defaults on the loan otherwise. The interest cost of capital for lenders (all of whom are risk neutral) is zero.

(i) Given a loan contract $(B, R)$, derive the optimal effort selected by the farmer.

The borrower selects $e$ to maximize $e(w - R)^{\frac{1}{2}} - e^2$, the solution for which is $e(R) = \frac{1}{2}(w - R)^{\frac{1}{2}}$ if $R \leq w$, and 0 otherwise.

(ii) In $(B, R)$ space, describe the set of loan contracts that break even on average for lenders, and the indifference curves of borrowers, using your answer to (i).

The breakeven constraint is $B = Re(R) = \frac{1}{2}R(w - R)^{\frac{1}{2}}$ if $R \leq w$, and 0 otherwise, assuming that the riskless cost of credit for lenders is zero. Without loss of generality, thus, we can confine attention hereafter to $R \leq w$. Clearly, $e(R)$ is increasing in $R$ from 0 until $\frac{2}{3}w$, and decreasing thereafter until $w$ (see Figure 2).

The borrower’s expected utility is

$$B^{\frac{1}{2}} + e(R)(w - R)^{\frac{1}{2}} - e(R)^2$$

$$= B^{\frac{1}{2}} + \frac{1}{4}(w - R)^{\frac{5}{2}}$$

so the marginal rate of substitution is $\frac{dR}{dB} = 2B^{-\frac{1}{2}}$. So the indifference curves are upward sloping, and concave to the $B$ axis.

(iii) Describe the optimization problem that characterizes the competitive equilibrium loan contract (assume that each borrower can obtain a loan from at most one lender). Depict the solution diagrammatically in $(B, R)$ space.

The second-best contract involves maximizing the borrower’s expected utility $B^{\frac{1}{2}} + \frac{1}{3}(w - R)$ subject to the break-even constraint $B = \frac{1}{2}R(w - R)^{\frac{1}{2}}$ if $R \leq w$, and 0. It is characterized by tangency of the borrower’s indifference curve to the breakeven line. So the contract must involve $R < \frac{2}{3}w$. The precise value of $R$ can be obtained from the tangency condition:

$$\frac{1}{2}B^{\frac{1}{2}} = \frac{1}{2}[(w - R)^{\frac{3}{2}} + \frac{R}{2}(w - R)^{-\frac{1}{2}}],$$

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which upon substituting for $B$ from the breakeven condition, simplifies to

$$(w - R)\frac{1}{2} = \frac{R}{2}(w - R)^{-1}.$$

(iv) Is there credit rationing in the equilibrium? If so, of what kind? Provide a precise definition of credit rationing, and explain your answer either algebraically or diagrammatically.

Yes, there is micro-credit rationing, in the sense that at the (average) interest rate in the equilibrium, the borrower would like to borrow more if given the opportunity to obtain an additional loan at the same interest rate. That there must be credit rationing is evident from Figure 2: since the breakeven locus is concave to the $R-$ axis, the borrower must be offered a marginal interest rate that is rising in the loan size (incorporating the fall in borrower’s effort with a rise in $R$). So the marginal interest rate must exceed the average interest rate. The borrower equates his marginal rate of substitution $\frac{dR}{dB}$ to the marginal interest rate, which is higher than the average interest rate. Hence if offered the option of borrowing at a constant rate equal to the average interest rate, the borrower would prefer to borrow more.

There will be no macro credit rationing, because there is an infinitely elastic supply of loanable funds. This is implicit in the statement that the cost of capital is given.

(v) What is the equilibrium interest rate (defined as $\frac{R}{B} - 1$)? How does it relate to the cost of capital for lenders?

The equilibrium interest rate equals $\frac{R}{B} - 1$, which is clearly positive, hence greater than the cost of capital which is 0. The difference arises from the fact that lenders must cover the default risk.

(vi) Now suppose that after the private credit market has operated and the farmers have obtained their respective equilibrium loan $(B^*, R^*)$ the government offers each farmer a supplementary loan at an interest rate lower than the private market. Describe the implications for (a) borrower effort level; (b) borrower welfare; (c) profitability of private lenders; (d) profitability of government loans (assuming that the government incurs the same cost of capital as private lenders). Explain your reasoning algebraically or diagrammatically.

The answer depends on the size of the supplementary loan offered by the government. If it is not too large then it is evident that borrowers welfare will increase as it takes them to a higher indifference curve. They were credit rationed in the market equilibrium, so would have preferred to borrow more at the same interest rate. If the government gives
them additional loans at a lower interest rate, that should make then even better off — provided that the quantum of the government loan is not too large.

If it does make them better off, they will take the additional loan. (Otherwise they do not, and nothing changes). Then the size of their total debt repayment obligation (on private plus government loans) increases, which will cause them to apply less effort. This in turn will cause private lenders to lose money on average (because they were breaking even previously, and now the default risk has risen). The government must also lose money on average, since they lend at a lower interest rate than the private lenders but are subject to the same default risk.

3. A borrower seeks to borrow $B$ at date 0, in exchange for a repayment obligation of $R$ at date 1. Her income at date 0 is $y_0$, and at date 1 is $y_1 > y_0$; these are deterministic and publicly known. The loan is accompanied by a collateral of size $W$, which is forfeited in the event of a loan default at date 1, where this is defined to be any repayment less than the stipulated obligation $R$. The borrower’s utility is given by $u(c_0) + u(c_1)$, where $u$ is an increasing, strictly concave function. The borrower is free to decide how much to repay at date 1. In this two period setting, assume there is an infinite number of potential lenders all of whom are risk neutral, and have access to funds at the constant risk-free interest rate of $r$.

(i) Under what conditions will the borrower default on a loan?

If he doesn’t default, his consumption at date 1 is $y_1 - R$, whereas if he does default it is $y_1 - W$. Hence default occurs if $R > W$. Note that the net payment from the borrower to the lender is $\min\{R, W\}$.

(ii) Describe the nature of competitive equilibrium in this credit market.

Competitive equilibrium must involve selection of a contract $(B, R)$ to maximize the borrower’s utility $u(y_0 + B) + u(y_1 - \min\{R, W\})$, subject to the breakeven condition for lenders: $(1 + r)B = \min\{R, W\}$. This is equivalent to maximizing $u(y_0 + B) + u(y_1 - R)$, subject to $(1 + r)B = R$ and $R \leq W$. The exact solution depends on the borrower’s utility function.

(iii) Provide the precise condition under which the equilibrium is characterized by credit rationing.

This requires that the solution to the unconstrained problem: maximize $u(y_0 + B) + u(y_1 - R)$ subject to $(1 + r)B = R$ involves a solution where $R > W$. The first-order condition which is necessary and sufficient to define the optimal $R^*$ is:

$$\frac{1}{1 + r} U''(Y_0 + \frac{R^*}{1 + r}) = U''(Y_1 - R^*).$$
It is evident that $R^* > 0$ if and only if

$$\frac{1}{1 + r} U'(Y_0) > U'(Y_1).$$

(1)

If (1) is not satisfied then the borrower’s demand for credit is always zero, and there is no credit rationing. If (1) is satisfied, then there is credit rationing if and only if $W < R^*$, i.e., if the borrower’s wealth is small enough relative to demand for credit at constant interest cost $r$. 
