1. Consider a class of homogenous farmers seeking to maximize two period utility $c_0^{\frac{1}{2}} + c_1^{\frac{1}{2}} - \epsilon^2$, where $c_i$ denotes consumption at date $i = 0, 1$, and $\epsilon \in [0, 1]$ the (unobservable) effort selected by the farmer, also equal to the probability of a normal crop. Incomes are 0 at date 0, and $w$ at date 1 if the crop is normal, and 0 otherwise, where $1 > w > 0$. Consumption is always nonnegative.

The farmer seeks to borrow $B$ at date 0, repay $R$ at date 1 if the crop is normal, and defaults on the loan otherwise. The interest cost of capital for lenders (all of whom are risk neutral) is zero.

(i) Given a loan contract $(B, R)$, derive the optimal effort selected by the farmer.

(ii) In $(B, R)$ space, describe the set of loan contracts that break even on average for lenders, and the indifference curves of borrowers, using your answer to (i).

(iii) Describe the optimization problem that characterizes the competitive equilibrium loan contract (assume that each borrower can obtain a loan from at most one lender). Depict the solution diagrammatically in $(B, R)$ space.

(iv) Is there credit rationing in the equilibrium? If so, of what kind? Provide a precise definition of credit rationing, and explain your answer either algebraically or diagrammatically.

(v) What is the equilibrium interest rate (defined as $\frac{R}{B} - 1$)? How does it relate to the cost of capital for lenders?

(vi) Now suppose that after the private credit market has operated and the farmers have obtained their respective equilibrium loan $(B^*, R^*)$ the government offers each farmer a supplementary loan at an interest rate lower than the private market. Describe the implications for (a) borrower effort level; (b) borrower welfare; (c) profitability of private lenders; (d) profitability of government loans (assuming that the government incurs the same cost of capital as private lenders). Explain your reasoning algebraically or diagrammatically.

2. A borrower seeks to borrow $B$ at date 0, in exchange for a repayment obligation of $R$ at date 1. Her income at date 0 is $y_0$, and at date 1 is $y_1 > y_0$; these are deterministic and publicly known.

The loan is accompanied by a collateral of size $W$, which is forfeited in the event of a loan default at date 1, where this is defined to be any repayment less than the stipulated obligation $R$. The borrower’s utility is given by $u(c_0) + u(c_1)$, where $u$ is an increasing, strictly concave function. The borrower is free to decide how much to repay at date 1. In this two period setting, assume there is an infinite number of potential lenders all of whom are risk neutral, and have access to funds at the constant risk-free interest rate of $r$.

(i) Under what conditions will the borrower default on a loan?

(ii) Define a competitive equilibrium of this credit market.

(iii) Provide a precise definition of credit rationing, and describe the circumstances under which the equilibrium is characterized by credit rationing.