Ec721 PROBLEM SET 2

1. Suppose the policy space is a one dimensional real variable $p$, there are a finite (odd) number of voters each with single peaked preferences, with a median ideal point $p_M$. Two parties $i = A, B$ compete in an election, and select policy platforms $p_A, p_B$ before the election, to which they are subsequently committed, in the event that are elected. In contrast to the Downsian model, suppose that each party $i$ has ideological policy preferences represented by a von Neumann utility function $W_i(p)$ which is also single peaked, with ideal point $p_i^*$. So if $\phi_A$ denotes the probability that party $A$ wins, party $i$'s objective is to maximize $\phi_A W_i(p_A) + (1 - \phi_A) W_i(p_B)$.

(a) Suppose that $p_M^*$ lies in between $p_A^*$ and $p_B^*$. What is the set of Nash equilibrium policy platforms? Provide complete proofs.

(b) Now suppose that $p_B^* > p_A^* > p_M^*$. What can you say now about the policy that will be chosen by the winning party (in Nash equilibrium)?

2. There are two parties $L$ and $R$ competing in an election. Voters are divided into $n$ groups, with group $i$ containing $N_i$ members all of whom have the same exogenous pre-tax income $y_i$. Party $k = L, R$ selects a tax-transfer policy $t_{ik}$ satisfying the budget constraint $\sum N_i t_{ik} = 0$. If party $k$ wins, the post-tax income or consumption of group $i$ members is $c_{ik} = y_i + t_{ik}$. All voters share the same utility function over consumption $u(c) = \frac{Ac^{\epsilon - 1}}{1-\epsilon}$ where $\epsilon > 0, \neq 1$. A voter with loyalty $l_R$ in favor of party $R$ will vote for this party if $u(c_{iL}) - u(c_{iR}) < l_R$; within group $i$ $l_R$ is distributed according to the cdf $\phi_i$ (which is nonuniform). There are no lobbies or campaign finance to influence the election.

(a) Show that (interior) Nash equilibrium of this model involves policy convergence.

(b) Show that the extent to which the equilibrium policy favors different groups depends on the relative values of the parameter $\phi'_i(0)$, where $\phi'_i$ denotes the density of $\phi_i$. Interpret this result.

3. Suppose there are two parties $A, B$ and $m$ voter groups, where the voter group $j$ comprises $\alpha_j$ fraction of the voter population, $j = 1, \ldots, m$. Party $k = A, B$ selects a policy $p^k$ from the set $P$ of possible policy platforms. All voters in group $j$ have the same utility function $U_j(p)$ over policies $p$, but differ in terms of their relative loyalties to the two parties. Voter $i$ in group $j$ thus votes for party $A$ if $U_j(p^A) > U_j(p^B) + x_{ij} + y$, where loyalty $x_{ij}$ within group $j$ is distributed uniformly over $[-\frac{1}{2\epsilon}, \frac{1}{2\epsilon}]$. $y$ represents a nation-wide loyalty swing variable, distributed uniformly over $[-\frac{1}{2\epsilon}, \frac{1}{2\epsilon}]$. Parties do not know the realization of $y$ at the time they select their platforms. The party winning a majority of the votes wins the election. Assume that $Y$ and $X_j$ are sufficiently large that all equilibria will be interior (i.e., involve positive vote shares for both parties).

(a) Obtain expressions for the probability that party $A$ wins, as a function of $p^A$ and $p^B$.

(b) Derive the equilibrium policy platforms of the two parties. Contrast the equilibrium policy with the welfare optimal policy, and provide an interpretation for why the two may diverge.

4. Show that if all voters are perfectly informed in the Grossman-Helpman model of electoral competition with a single lobby and two parties, the equilibrium policy will be welfare optimal.
Contrast this result with the model in problem 3 above, and explain why the two models generate distinct conclusions.