SOLUTIONS TO EC721 EXAMINATION, FALL 2016

1. (3*15=45 pts.) For each of the following propositions, describe a model (e.g., the context and key assumptions) where it can be shown to hold, and provide an intuitive explanation for why it holds.

(a) *If productivity differences between entrepreneurs are persistent, capital market imperfections will not lead to any misallocation in the long run which lowers economy-wide TFP.*

In the model of Moll (AER 2014), agents vary in (exogenous) productivity and (endogenous) wealth. Optimal allocation requres only the highest productivity agent to produce, borrowing from everyone else. The credit market imperfection restricts borrowing to a constant fraction of wealth, thereby generating misallocation whenever the most productive agent does not own almost all the wealth in the economy. If productivity is persistent, the same individual is the most productive, will become an entrepreneur and earn the highest return on own-wealth. With savings a constant fraction of wealth, the most able agent's wealth will grow the fastest. In the long run this agent will own almost all the wealth and the misallocation will disappear.

(b) *It is more difficult for poorer communities to sustain self-enforcing informal insurance arrangements against idiosyncratic risks.*

In the model of Coate-Ravallion (JDE1993), there is an infinite horizon economy with two identical risk averse individuals that have risky endowments. Consider the special case where at any date their risks are perfectly negatively correlated, so agent A earns y_H when agent B earns $y_L(< y_H)$ (with probability a half). With the remaining probability their roles are reversed. Each has utility u over consumption. First-best risk sharing requires the agent earning y_H transfer $\frac{y_H - y_L}{2}$ to the other agent. This is incentive compatible (with utility function u and discount factor δ) with a grim trigger strategy if

$$u(y_H) - u(y_H - \frac{y_H - y_L}{2}) \leq \frac{\delta}{1-\delta}[u(\frac{y_H + y_L}{2} - \frac{u(y_H) + u(y_L)}{2}] \quad (1)$$

Now suppose we consider a richer economy where incomes are $y_H + \Delta$ and $y_L + \Delta$ instead of y_H and y_L with $\Delta > 0$. Concavity of u implies the left-hand-side, the temptation to deviate, will be lower in the richer economy, as the required transfer is the same as in the poor economy. What about the right-hand-side, the value of future insurance? It is now

$$\frac{\delta}{1-\delta}[u(\frac{y_H + y_L}{2} + \Delta) - \frac{u(y_H + \Delta) + u(y_L + \Delta)}{2}]$$

If u has constant absolute risk aversion, this is increasing in Δ. Hence the value of future insurance will be higher in the richer economy, implying that first best risk-sharing can be sustained for a wider range of discount factors in the richer economy.

(c) *Allowing access to rainfall insurance to landless workers will make their employers worse off.*

In Mobarak and Rosenzweig (2014), demand for labor and hence wage rates co-move with rainfall shocks. Hence workers are worse off in droughts. The income effect causes their labor supply to go up in droughts, lowering wages even further. Rainfall insurance would pay out in droughts, collecting premiums in normal rainfall periods. This will lower labor supply in droughts, raising it in normal periods, which would reduce wage volatility and cause the benefits of insurance to also spread to uninsured workers. Employers will be worse off in droughts and better off in normal periods as a result of the insurance. This generates two effects on employers: (i) Since profits are convex in wages, this will increase expected profits (wages rise when they employ less workers, and fall when they employ more). (ii) But employers earn less profits in droughts than in normal periods, and the wage change will increase the spread further. If employers are risk-averse, this will make them worse off. If they are sufficiently risk-averse, the second effect will dominate.

2. (5+5+5+15+15+10=55 pts.) *A plot of land is cultivated by a farmer F with zero liquid wealth, utility function $c - D(e)$, where $c \geq 0$ denotes consumption, $e \in [0,1]$ denotes effort. The effort disutility function D is twice differentiable, strictly increasing, strictly concave with $D'(0) = 0, D'(1) = \infty$. If the farmer cultivates fraction $x \in [0,1]$ of the land, he needs to spend Ix on material inputs at $t = 0$. At $t = 1$, the plot then yields output worth sx with probability e, and worth fx with probability $1 - e$, where $s > I > f > 0$. To* *finance the material costs, F takes a loan of size Ix from a lender L. At $t = 1$, F repays R_i in state $i = s, f$. L incurs an interest cost of zero. Both F and L consume only at $t = 1$. Both are risk neutral.*

L and F will negotiate a loan contract in conjunction with the scale x of cultivation. The contract cannot be conditioned on F's effort because L cannot observe it. If they fail to agree to a contract, F will consume 0, select $x = e = 0$, while L will earn nothing.

(a) Show that a contract can be summarized by four variables: F's effort e, the scale of cultivation x, and consumptions c_s, c_f in states s, f respectively. Express expected payoffs of F and L as a function of these four variables. Using this representation of contracts, derive the set of feasible contracts.

Define $c_i \equiv ix - R_i$, whence $R_i = ix - c_i$, and lender's expected profit is

$$\Pi_L = e(sx - c_s) + (1-e)(fx - c_f) - Ix = xR(e) - [ec_s + (1-e)c_f] \tag{2}$$

where $R(e) \equiv es + (1-e)f - I$. The farmer's expected payoff is

$$\Pi_F = ec_s + (1-e)c_f - D(e) \tag{3}$$

Feasibility constraints are participation constraints: $\Pi_L \geq 0, \Pi_F \geq 0$, non-negativity constraints $c_s, c_f \geq 0$ and $1 \geq x \geq 0$, and finally the farmer's incentive constraint $c_s - c_f = D'(e)$ since effort is unobservable.

(b) What is the feasible set in the first-best situation where F's effort e is observable by L? Derive conditions under which there exists a feasible contract which generates positive surplus to both parties (relative to their outside options) in the first-best situation. From this point onwards, assume that these conditions hold.

If e is observable the incentive constraint no longer applies. The participation and non-negativity constraints apply. Existence of a feasible contract with positive surplus requires $xR(e) \geq ec_s + (1-e)c_f \geq D(e)$ (from the two participation constraints), with at least one inequality strict, for some x, e and $c_s, c_f \geq 0$. A necessary and sufficient condition for this is existence of x, e both in $[0, 1]$ such that $xR(e) > D(e)$. Note that $R(0) < 0$ since $f < I$.

Hence e and hence $R(e)$ must be strictly positive in any feasible contract. In that case the necessary and sufficient condition for existence of a feasible contract where both parties earn a positive surplus simplifies to existence of $e \in (0, 1]$ such that $R(e) > D(e)$.

(c) Show that in the first-best situation there is a unique Pareto optimal scale of cultivation and effort, and characterize these as fully as possible as a function of the parameters.

In the first-best a Pareto optimal contract should not allow Π_F to be raised while leaving Π_L unaffected and preserving feasibility. Leaving Π_L unaffected means imposing the constraint $[ec_s + (1-e)c_f] = xR(e) - \Pi_L$, whence $\Pi_F = xR(e) - D(e) + \Pi_L$. So x, e must be chosen to maximize $xR(e) - D(e)$ subject to $x, e \in [0, 1]$. As explained above any feasible contract satisfies $R(e) > 0$. So its always optimal to set $x^* = 1$, and e^* to maximize $R(e) - D(e)$ over $[0, 1]$. Since $R(e) - D(e)$ is strictly concave, e^* is characterized by the condition that $R'(e^*) = s - f = D'(e^*)$.

(d) Now return to the second-best situation where L cannot monitor F's effort. Consider the L-monopoly contract which maximizes L's expected payoff over the set of second-best contracts. Characterize the corresponding optimal scale of cultivation and effort, and compare these with the corresponding first-best solutions.

The second-best L-monopoly contract maximizes $xR(e) - [ec_s + (1-e)c_f]$ subject to the feasibility constraints given in (a) above.

Observe first that any feasible contract in the second-best setting must also be feasible in the first-best setting, so must involve $e > 0$. It must satisfy the incentive constraint $c_s - c_f = D'(e)$. Using this the agent's PC reduces to $c_f + eD'(e) - D(e) \geq 0$, while L's expected profit equals $xR(e) - eD'(e) - c_f$. Since $D(e)$ is strictly convex, $eD'(e) > D(e)$ for any $e > 0$. So the agent's PC cannot bind, given the nonnegativity constraint $c_f \geq 0$. (**Intuition:** To provide effort incentives, L needs to provide a bonus for success, by creating a spread between c_s and c_f. He wants to lower the farmers expected consumption as much as possible, while maintaining effort incentives. c_f cannot be lowered below 0. So c_s has to be positive to provide incentives. Given this, the farmer must attain strictly positive utility, as he always has the option to selecting zero effort which will give him his outside option payoff, and he can do better than that by selecting positive effort.)

The problem therefore reduces to maximizing $xR(e) - eD'(e) - c_f$ subject to $c_f \geq 0$ and $1 \geq x \geq 0$. It is optimal to set $c_f = 0$ and $x = 1$. Then select e to maximize $xR(e) - eD'(e)$. This differs from the first-best problem, in that the cost of effort equals $eD'(e)$ instead of $D(e)$ (because of the need to provide effort incentives, which generates rents for the farmer which are effectively paid out of L's pocket). The optimal effort in L-monopoly will satisfy $R'(e_L) = s - f = e_L D''(e_L) + D'(e_L)$. Since $D'' > 0$, it follows that $e_L < e^*$. Hence L-monopoly results in too little effort, but the scale of cultivation is unaffected.

(e) Next consider the F-monopoly contract which maximizes F's expected payoff over the set of second-best contracts. Characterize the corresponding optimal scale of cultivation and effort. Compare these to both the second-best L-monopoly and the first-best solutions. Provide an intuitive explanation for these results.

Now the problem is maximize $ec_s + (1-e)c_f - D(e)$ subject to $\Pi_L \geq 0, c_s - c_f = D'(e)$ and $c_s, c_f \geq 0, 1 \geq x \geq 0$.

$c_s - c_f = D'(e)$ implies the objective function can be rewritten as $c_f + eD'(e) - D(e)$. The PC is $xR(e) \geq c_f + eD'(e)$. Hence x, e, c_f must maximize $c_f + eD'(e) - D(e)$ subject to $xR(e) \geq c_f + eD'(e)$ and $c_f \geq 0, 1 \geq x \geq 0$.

L's participation constraint must bind, otherwise c_f can be raised. So $c_f = xR(e) - eD'(e)$, and F's payoff equals the first-best surplus $xR(e) - D(e)$. The problem then reduces to selecting x, e to maximize first-best surplus $xR(e) - D(e)$ subject to $xR(e) - eD'(e) \geq 0, 1 \geq x \geq 0$.

Clearly optimal $x = 1$ again, and e must maximize $R(e) - D(e)$ subject to $R(e) - eD'(e) \geq 0$. If first-best effort e^* satisfies this constraint, it is the solution to the F-monopoly problem. Otherwise it is $\underline{e}$, the highest e satisfying the constraint, i.e., F-monopoly effort e_F equals $\min\{\underline{e}, e^*\}$. In any case $e_F > e_L$ as long as L makes strictly positive profit $R(e_L) - e_L D'(e_L) > 0$, as starting from e_L it is feasible to raise e slightly and move it closer to the first-best effort.

(f) What implications does this model have for productivity and welfare effects of changes in credit market concentration?

Both productivity and welfare rise when concentration falls. Owing to moral hazard and limited liability the farmer earns some ‘incentive rents’ whenever positive effort is to be elicited, and these rents are rising in the level of the effort. These rents are treated as a ‘cost’ by the lender, who thereby finds it optimal to induce less effort than the welfare maximizing level. Shifting bargaining power to the farmer allows this externality to be internalized.