1. Consider the model of a credit market with ex ante moral hazard and limited liability considered in class, and extend it to include collateral. A borrower seeks to borrow $1 to finance a project which is successful with probability $e$ and attains a return $y > 0$, and attains a zero return otherwise. The borrower selects $e$ after receiving the loan and this is not observable by lenders or any third party. The borrower incurs a personal cost of $\frac{e^2}{2k}$. The borrower can repay a loan with an interest obligation $r$ in the successful state if $r \leq y$. The borrower cannot repay in the unsuccessful state. He posts collateral worth $c$ which is collected by the lender in this state. Lenders compete in Bertrand fashion, and all incur cost $(1 + \rho)$. The parameters satisfy $k \frac{y^2}{2} > 1 + \rho, k(y + C) < 1$ where $C$ is an upper bound to $c$. Calculate the competitive interest rate as a function of $c$. Under what condition will the interest rate be a declining function of $c$? Interpret this condition verbally.

2. Consider the following special case of the Coate-Ravallion model: the two agents have log utility, and each faces uncertainty over the realization of his income in the following fashion: income is either $y$ or $y - d$ with equal probability. The income risk of the two agents are uncorrelated. Each has an interest rate $r$. Characterize the optimal incentive compatible (symmetric) insurance scheme. Provide a condition on parameter values under which: (i) first-best insurance can be achieved; (ii) no insurance can be achieved. Is the optimal insurance scheme continuous in $r$?

3. Consider the Ghatak model of joint liability loans with adverse selection, with two types with success probabilities $p_h, p_l$ respectively with $1 > p_h > p_l > 0$, outside option $u$, loan size of 1 and cost of lending $1 + \rho$ for lenders. Suppose that the assumptions made in class hold. Show that if $p_h > 1 - p_l$ there exists a joint liability loan contract $(r_h, c_h) >> 0$ which if offered in conjunction with an individual liability loan contract at interest rate $\frac{1 + \rho}{p_l}$ will achieve the first-best allocation.