EC721 MIDTERM EXAMINATION SOLUTIONS, FALL 2015

1. (5+10+10=25 marks) An economy has a population of poor entrepreneurs of varying ability $a \in [0,1]$ that seek to borrow in order to finance an investment project. All entrepreneurs have zero wealth of their own, and zero outside option. An entrepreneur of ability a needs to borrow $(1-a)$ to start the project, which is successful with probability e and a failure otherwise. The project generates $Q > 1$ if successful and nothing otherwise. There is only one lender L who they can approach for funds. Thie lender can lend any amount at a constant opportunity cost of ρ. Loans are subject to limited liability. The probability of success e is chosen privately by the borrower after getting a loan, at a personal cost of $\frac{e^2}{2k}$. L can observe any borrowers ability before deciding whether to lend, but not their choice of e. The parameters satisfy $4\rho > kQ^2 > 2\rho$ and $kQ < 1$. All parties are risk-neutral.

(a) Suppose L acts as a monopolist. For entrepreneurs of any given ability type a, will L agree to lend to them and if so at what interest rate?

Effort e of borrower given repayment obligation of $R \geq 0$ in the event of success maximizes $e(Q-R) - \frac{e^2}{2k}$, so $e = k(Q-R) \leq kQ < 1$. Hence expected payoff of lender in lending $1-a$ with a repayment obligation R is $\Pi_L(R,a) = k(Q-R)R - \rho(1-a)$. Any such loan will generate a non-negative payoff to the borrower (since the borrower always has the option of selecting $e=0$) so the participation constraint of the borrower can be ignored. The value of R that maximizes Π_L is $R = \frac{Q}{2}$, whereupon L earns a payoff of $\Pi_L(\frac{Q}{2}, a) = \frac{kQ^2}{4} - \rho(1-a)$, which is non-negative if and only if $a \geq 1 - \frac{kQ^2}{4\rho} \equiv \underline{a} \in (0, \frac{1}{2})$ since by hypothesis $4\rho > kQ^2 > 2\rho$. Hence all entrepreneurs with a below $\underline{a}$ will not get a loan. Those with a above $\underline{a}$ will receive a loan of $(1-a)$ at an interest rate of $r_M(a) \equiv \frac{Q}{2(1-a)} - 1$.

(b) Suppose the government imposes a cap of $\frac{Q}{4} - 1$ on the interest rate that lenders can charge. What will the impact of this be on loan outcomes, borrower welfare and social (utilitarian) welfare for each ability type?

Note that the interest rate is increasing in a, since the repayment amount is independent of a while the loan size is decreasing in a. The lowest interest rate is the one charged to type $\underline{a}$. This is $r_M(\underline{a}) = \frac{Q}{kQ^2/2\rho} - 1$ which is larger than $\frac{Q}{4} - 1$ given the parameter restrictions. Hence the interest rate cap is binding for all agents who get a loan. Note that for the agent of type $\underline{a}$ the monopoly lender just broke even with a repayment obligation of $\frac{Q}{2}$, so will lose money in the presence of a cap. So type $\underline{a}$ will not get a loan with the cap.

To check which types above $\underline{a}$ will still receive a loan, we calculate the maximum profit the lender can make in the presence of the cap with type a. Since the monopolist's profit function is quadratic in R and the cap is binding for all $a > \underline{a}$, it will be optimal for the monopolist to set $R(a) = (1-a)\frac{Q}{4}$. He will then earn a payoff of $\Pi_L(R(a), a) = (1-a)\rho[\frac{(3+a)kQ^2}{16\rho} - 1] < (1-a)\rho[\frac{kQ^2}{4\rho} - 1] < 0$. Hence the lender will lend to nobody in the presence of the cap: the market will shut down. Both lender and borrower's payoffs (for types above $\underline{a}$) will be lower: they will lose the payoffs they were attaining in the monopoly allocation, and there will be a fall in utilitarian welfare (equal to the sum of these payoffs).

(c) Describe how your answer to (a) and (b) would change if the credit market involved Bertrand competition between L and another lender in the market with the same opportunity cost of lending.

With Bertrand competition, the lenders will break-even, implying that for any type a the repayment obligation will be $R_c(a)$ which is the smaller root of $k(Q-R)R - (1-a)\rho = 0$. This equation has real roots if $a \geq \underline{a}$. Hence the range of types that will get a loan will be the same as under monopoly: $a \geq \underline{a}$. For any such a,

$$R_c(a) = \frac{Q}{2} - \frac{1}{2}[Q^2 - 4\frac{(1-a)\rho}{k}]^{\frac{1}{2}}$$

The corresponding interest rate is $r_c(a)$ where

$$1 + r_c(a) \equiv \frac{Q}{2(1-a)} - \frac{1}{2(1-a)}[Q^2 - 4\frac{(1-a)\rho}{k}]^{\frac{1}{2}}$$

Now $1 + r_c(a)$ tends to $\frac{\rho}{kQ} > \frac{Q}{4}$ as a approaches 1, and it tends to $\frac{2\rho}{kQ}$ as a approaches $\underline{a}$. Hence the interest cap is binding in the presence of Bertrand competition, so it also reduces welfare in this case.

2. (4+5+8+8=25 pts.) An economy has a given distribution of pre-tax incomes y which is lognormal, where m and d denote the mean and standard deviation of log income (so mean income $\bar{y} = e^{m+\frac{d^2}{2}}$ while median income equals e^m). Government policy consists of a linear tax rate t between 0 and 1, which funds a public good. A voter with pre-tax income y obtains a utility equal to $y(1-t) + t(1-q)\bar{y} - \frac{a}{2}t^2\bar{y}$, where $a > 1-q > 0$, with a denoting a parameter of deadweight loss of tax collection, and q denotes a parameter of efficiency of public good provision.

(a) What is the ideal tax rate from the point of view of a voter with income y?

This is $\tau(y) = \max[\frac{1-q}{a} - \frac{y}{a\bar{y}}, 0]$.

(b) What does the Downsian model predict the equilibrium tax rate to be (as a function of parameter values)?

The median income is e^m while $\bar{y} = e^{m+\frac{d^2}{2}}$. Hence the ideal tax rate for the median voter is $\max[\frac{1-q-e^{-\frac{d^2}{2}}}{a}, 0]$.

(c) Does there exist an equilibrium of the citizen candidate model (with positive but negligible entry costs) in which a single candidate runs unopposed and wins? If so, describe the set of such equilibria and how they differ from the Downsian prediction.

Yes. A single candidate with the median income runs and selects his own ideal point, no one else does. This is an equilibrium since every other voter with a different income expects to lose to the median voter. Given that no one else is running, it is optimal for this candidate to run (he gets to select his own favorite tax policy, whereas the government would shut down if he does not run, and entry costs are negligible). And it is not optimal for any other voter with the same median income to run, since one of them is running anyway (and

entry costs are positive). This equilibrium generates the same prediction for policy as the Downsian model.

Any other single candidate equilibrium must involve a candidate 'close enough' to the median voter that runs for election, if entry costs are small. Otherwise, the median voter would have an incentive to enter and change the policy: this will happen whenever the benefit to the median voter from the change in policy outweighs the entry cost. Hence all single candidate equilibria will be close to the prediction of the Downsian model.

(d) Describe how your answer to (c) is modified in the case of equilibria where two candidates run for election.

There is an equilibrium of the citizen candidate model where two candidates with different incomes y_1, y_2 run, and split the vote equally (the candidates select their own ideal tax rates; if $y_1 < y_2$ everyone with incomes below y_1 vote for y_1, those above y_2 vote above y_2, and those between y_1 and y_2 choose for the candidate whose policy they prefer. y_1 and y_2 are selected on either side of the median income so that the proportions voting for each candidate is exactly one half. Such pairs can be checked to exist: for any $\epsilon > 0$ and small, if we select $y_1 = e^m - \epsilon$, there exists $y_2 > e^m$ such that this is true (since if we select $y_2 = e^m$ then candidate 2 will win more than half the vote, while if we select y_2 sufficiently larger than e^m then candidate 1 will win more than half the vote. By continuity there must exist $y_2 > e^m$ such that candidate 1 will win exactly half the vote against y_2.) Given these two candidates enter, no other candidate will enter under the hypothesis that no one will vote for them (which is sequentially rational, as almost everyone has strict preferences between y_1 and y_2 and everyone is pivotal in deciding which of these two wins. So each wants to vote for their favorite candidate given that everyone else is doing so, and no one votes for the entrant.)

This equilibrium produces a different prediction from the Downsian model.