Mookherjee EC 721

Ec721 PROBLEM SET 3 SOLUTIONS

1. Suppose the policy space is a one dimensional real variable p, there are a finite (odd) number of voters each with single peaked preferences, with a median ideal point p_M^*. Two parties $i = A, B$ compete in an election, and select policy platforms p_A, p_B before the election, to which they are subsequently committed, in the event that are elected. In contrast to the Downsian model, suppose that each party i has (ideological) policy preferences represented by a von Neumann utility function $W_i(p)$ which is also single peaked, with ideal point p_i^*. So if ϕ_A denotes the probability that party A wins, party i's objective is to maximize $\phi_A W_i(p_A) + (1 - \phi_A) W_i(p_B)$.

(a) Suppose that p_M^* lies in between p_A^* and p_B^*. What is the set of Nash equilibrium policy platforms? Provide complete proofs.

The unique Nash equilibrium is (p_m^*, p_m^*). That this is a Nash equilibrium is obvious, because if one candidate selects p_m^* then deviating from p_m^* by the other candidate guarantees that the latter loses, so does not affect the resulting policy. So the main thing to prove is that there is no other Nash equilibrium.

Without loss of generality assume that $p_A^* < p_m^* < p_B^*$.

Consider first any other convergent platform pair (p, p). If $p < p_A^*$ then A can deviate to p_A^*, win the election and thereby benefit from the deviation. If $p = p_A^*$ then B can profitably deviate to p_m^*. If $p \in (p_A^*, p_m^*)$ then also B can profitably deviate to p_m^*. A symmetric argument takes care of any $p > p_m^*$.

So consider divergent platforms (p_A, p_B) where $p_A \neq p_B$. There are then various cases to consider. I exclude cases where some strict inequalities can be replaced by weak inequalities, where the argument is similar.

(1) $p_B > p_m^* > p_A$. If one candidate wins for sure, say B, then the other candidate (A here) can profitably deviate to p_m^*. If they win with equal probability, then one candidate can move slightly closer to p_m^* and win the election for sure. This will be a profitable deviation because the slight move away from its own ideal point will be dominated by the switch of the eventual policy from that chosen by the other party with probability half.

(2) $p_A > p_m^* > p_B$. If B wins for sure then B can do better to move to p_m^*, and continue to win the election. If they win with equal probability then also B will do better to move to p_m^* and win for sure.

(3) p_A and p_B are both on the same side of p_m^*. Lets suppose they are both less than p_m^*. In this case the candidate with the policy closer to p_m^* will win. If it is B, then B will do better to deviate to p_m^*. If it is A then also B will do better to deviate to p_m^*.

(b) Now suppose that $p_B^* > p_A^* > p_M^*$. What can you say now about the policy that will be chosen by the winning party (in Nash equilibrium)?

Consider any $p \in [p_m^*, p_A^*]$. Then (p, p) is a Nash equilibrium. Any deviation that changes the election result causes the outcome to move further away from the candidate's ideal point, so is not worthwhile.

There cannot be any other convergent pair (p, p) that constitutes an equilibrium. If $p > p_A^*$ then A can deviate profitably to p_A^*. If $p < p_m^*$ then A can deviate profitably to p_m^*.

For the same reason the winning policy has to be in the range $[p_m^*, p_A^*]$, for if it is not A can deviate in the way described in the preceding paragraph.

2. Consider the Besley-Coate (1997) model of citizen candidates, applied to the following economy. There are four policies p_0, p_1, p_2, p_3 and three types of citizens $i = 1, 2, 3$. Citizens care only about policies chosen by elected candidates and not the latter's identity. Preferences for every citizen are strict between every pair of policies. Policy p_0 is ranked last by every citizen, this results when no candidate runs for election. Type 1 citizens prefer p_1 to p_2 to p_3. Type 2 citizens strictly prefer p_2 to either p_1 or p_3. Type 3 citizens strictly prefer p_3 to p_2 to p_1. The proportion of type 2 citizens is less than a third of the population, while there are an equal proportion of citizens of types 1 and 3.

Any citizen can run for office, at an entry cost of δ which is positive but close to zero. In the following, consider equilibria for all sufficiently small values of δ.

(a) Does a single candidate equilibrium exist? If so, characterize the set of such equilibria.

Note first that policy p_2 is a Condorcet winner, i.e., would win in any pairwise contest with any of the other policies if voters vote sincerely. Hence there is a one candidate equilibrium where a candidate of type 2 enters and wins unopposed. Candidates of types 1 or 3 would not want to enter, since they would definitely lose in the resulting two-candidate election in which voters would vote sincerely. And candidate of type 2 wants to run because otherwise there will be no candidates and the policy would be p_0.

(b) Does a two candidate equilibrium exist? If so, characterize the set of such equilibria.

No it does not. Because if it did, the two candidates that enter must split the vote evenly and win with probability half. In a two candidate race, voting will be sincere. If 1 and 3 enter, all voters of type 2 will vote for 1 if they prefer p_1 to p_3, and for 3 otherwise. In the former (resp. latter) case, candidate 1 (resp. 3) will win for sure since voters of types 1 and 3 will vote for their own candidate. If 2 and 3 enter, 2 will definitely win since all voters of types 1 and 2 will vote for 2. If 1 and 2 enter, 2 will again definitely win as all voters of types 2 and 3 will vote for 2. So we get a contradiction.

(c) Does a three candidate equilibrium exist? If so, characterize the set of such equilibria.

No it does not. If all three types enter, all three policies p_1, p_2, p_3 are on the ballot.

If there is an equilibrium where all three obtain the same number of votes, every voter is pivotal and must be voting for her favorite policy, and 2 must get fewer votes than the other two candidates, so we get a contradiction.

Suppose there is an equilibrium where 1 and 3 win with equal probability, while 2 loses for sure. Voters of types 1 and 3 must be voting for their own candidate since they are pivotal in this comparison. Voters of type 2 must all either vote for the candidate they prefer among 1 and 3, which implies that this candidate must win for sure, a contradiction.

Suppose there is an equilibrium where 1 and 2 win with equal probability. Then voters of type 1 and 2 must vote for their own candidate. Voters of type 3 must vote for candidate 2. Then

2 must win the election, a contradiction. A similar argument applies to eliminate the possibility that 2 and 3 win with equal probability.

Finally suppose there is an equilibrium where one candidate wins for sure. Then at least one of the two losing candidates must prefer to withdraw from the race. For example, if 2 wins for sure, this will continue to be the case if either candidate 1 or 2 withdraw, so the latter will want to do so. If 3 wins for sure, then 1 will want to withdraw since in that case there will be a contest between candidates 2 and 3, and 2 will win, which will make 1 better off. A similar argument applies in the case that 1 wins for sure: then 3 will want to withdraw.