Mookherjee EC 721

SOLUTIONS TO PROBLEM SET 1

1. (5+10+10=25 marks) An economy has a population of poor entrepreneurs of varying ability $a \in [0,1]$ that seek to borrow in order to finance an investment project. All entrepreneurs have zero wealth of their own, and zero outside option. An entrepreneur of ability a needs to borrow $(1-a)$ to start the project, which is successful with probability e and a failure otherwise. The project generates $Q > 1$ if successful and nothing otherwise. There is only one lender L who they can approach for funds. Thie lender can lend any amount at a constant opportunity cost of ρ. Loans are subject to limited liability. The probability of success e is chosen privately by the borrower after getting a loan, at a personal cost of $\frac{e^2}{2k}$. L can observe any borrowers ability before deciding whether to lend, but not their choice of e. The parameters satisfy $4\rho > kQ^2 > 2\rho$ and $kQ < 1$. All parties are risk-neutral.

(a) Suppose L acts as a monopolist. For entrepreneurs of any given ability type a, will L agree to lend to them and if so at what interest rate?

Effort e of borrower given repayment obligation of $R \geq 0$ in the event of success maximizes $e(Q-R) - \frac{e^2}{2k}$, so $e = k(Q-R) \leq kQ < 1$. Hence expected payoff of lender in lending $1-a$ with a repayment obligation R is $\Pi_L(R,a) = k(Q-R)R - \rho(1-a)$. Any such loan will generate a non-negative payoff to the borrower (since the borrower always has the option of selecting $e = 0$) so the participation constraint of the borrower can be ignored. The value of R that maximizes Π_L is $R = \frac{Q}{2}$, whereupon L earns a payoff of $\Pi_L(\frac{Q}{2}, a) = \frac{kQ^2}{4} - \rho(1-a)$, which is non-negative if and only if $a \geq 1 - \frac{kQ^2}{4\rho} \equiv \underline{a} \in (0, \frac{1}{2})$ since by hypothesis $4\rho > kQ^2 > 2\rho$. Hence all entrepreneurs with a below $\underline{a}$ will not get a loan. Those with a above $\underline{a}$ will receive a loan of $(1-a)$ at an interest rate of $r_M(a) \equiv \frac{Q}{2(1-a)} - 1$.

(b) Suppose the government imposes a cap of $\frac{Q}{4} - 1$ on the interest rate that lenders can charge. What will the impact of this be on loan outcomes, borrower welfare and social (utilitarian) welfare for each ability type?

Note that the interest rate is increasing in a, since the repayment amount is independent of a while the loan size is decreasing in a. The lowest interest rate is the one charged to type $\underline{a}$. This is $r_M(\underline{a}) = \frac{Q}{kQ^2/2\rho} - 1$ which is larger than $\frac{Q}{4} - 1$ given the parameter restrictions. Hence the interest rate cap is binding for all agents who get a loan. Note that for the agent of type $\underline{a}$ the monopoly lender just broke even with a repayment obligation of $\frac{Q}{2}$, so will

lose money in the presence of a cap. So type $\underline{a}$ will not get a loan with the cap.

To check which types above $\underline{a}$ will still receive a loan, we calculate the maximum profit the lender can make in the presence of the cap with type a. Since the monopolist's profit function is quadratic in R and the cap is binding for all $a > \underline{a}$, it will be optimal for the monopolist to set $R(a) = (1-a)\frac{Q}{4}$. He will then earn a payoff of $\Pi_L(R(a), a) = (1-a)\rho[\frac{(3+a)kQ^2}{16\rho} - 1] < (1-a)\rho[\frac{kQ^2}{4\rho} - 1] < 0$. Hence the lender will lend to nobody in the presence of the cap: the market will shut down. Both lender and borrower's payoffs (for types above $\underline{a}$) will be lower: they will lose the payoffs they were attaining in the monopoly allocation, and there will be a fall in utilitarian welfare (equal to the sum of these payoffs).

(c) Describe how your answer to (a) and (b) would change if the credit market involved Bertrand competition between L and another lender in the market with the same opportunity cost of lending.

With Bertrand competition, the lenders will break-even, implying that for any type a the repayment obligation will be $R_c(a)$ which is the smaller root of $k(Q-R)R - (1-a)\rho = 0$. This equation has real roots if $a \geq \underline{a}$. Hence the range of types that will get a loan will be the same as under monopoly: $a \geq \underline{a}$. For any such a,

$$R_c(a) = \frac{Q}{2} - \frac{1}{2}[Q^2 - 4\frac{(1-a)\rho}{k}]^{\frac{1}{2}}$$

The corresponding interest rate is $r_c(a)$ where

$$1 + r_c(a) \equiv \frac{Q}{2(1-a)} - \frac{1}{2(1-a)}[Q^2 - 4\frac{(1-a)\rho}{k}]^{\frac{1}{2}}$$

Now $1 + r_c(a)$ tends to $\frac{\rho}{kQ} > \frac{Q}{4}$ as a approaches 1, and it tends to $\frac{2\rho}{kQ}$ as a approaches $\underline{a}$. Hence the interest cap is binding in the presence of Bertrand competition, so it also reduces welfare in this case.

2. Consider the model of farming households in an economy with a smooth, strictly concave CRS production function $f(h, n)$ of land and labor used in cultivation, and a utility function given by $Y + U(R)$, Y denoting income and R its leisure. U is strictly increasing and strictly concave; both U and f satisfy Inada conditions. Each household has an endowment of one time unit, can apply its own labor on its own farm, and hire in or out its own labor on a competitive labor market. Households differ in their landholdings, and can lease in or out their land on a competitive rental market. Households are subject *to a working capital constraint, which depends on how much land they own. In contrast to the model studies in class, hired labor is perfectly trustworthy and does not need to be supervised.*

(a) For poor cultivating households who are forced to hire out their labor to augment their working capital, show that they are indifferent between applying their own labor or hiring in workers to work on their own farm. Prove that as these households become richer, they hire out less labor.

The household will seek to maximize $\Pi \equiv P\beta f(\frac{B+wt-wL}{v}, l+L) + U(1-t-l)$, and for B close to zero, $t > 0$ and satisfies the first order condition $P\beta f_h \frac{w}{v} = U'(R)$. Moreover, $\frac{\partial \Pi}{\partial L} = P\beta[f_h \frac{-w}{v} + f_L] = P\beta f_l - U'(R) = \frac{\partial \Pi}{\partial l}$, so the household is indifferent between own-labor and hired labor.

Without loss of generality, set $L = 0$. The first order conditions defining optimal choices of the household are

$$f_h(\frac{B+wt}{v}, l)\frac{w}{v} = \frac{U'(R)}{P\beta}$$

$$f_l(\frac{B+wt}{v}, l) = \frac{U'(R)}{P\beta}$$

These imply that the productive efficiency condition $\frac{f_h}{f_l} = \frac{v}{w}$ is met. This determines the land-labor ratio, and the marginal product of labor, and hence also the amount of leisure. So R is independent of B.

Differentiating the first order conditions above with respect to B, we thus obtain

$$f_{hh}(\frac{1}{v} + \frac{w}{v}t_B) + f_{hl}l_B = 0$$

$$f_{lh}(\frac{1}{v} + \frac{w}{v}t_B) + f_{ll}l_B = 0$$

Since $R_B = 0$, it follows that $t_B + l_B = 0$. Using this, we then solve

$$t_B = \frac{f_{hh}}{vf_{hl} - wf_{hh}}$$

which is negative by virtue of the concavity of the production function.

(b) Now consider households who own enough land that they do not hire out their own labor. Show that these farms are as productive as those of the poorer households.

Given $t = 0$ the farm's problem is to select l, L to maximize $P\beta f(\frac{B-wL}{v}, l + L) + u(1-l)$, which is a strictly concave function. Hence the solution is characterized by the first order conditions $\frac{\partial \Pi}{\partial L} = P\beta[f_h \frac{-w}{v} + f_l] = 0$, $\frac{\partial \Pi}{\partial l} = P\beta f_l - u'(R) =$

0, **which represent two equations in the two unknowns. These yield the result that** $\frac{f_h}{f_l} = \frac{v}{w}$**, i.e., the land-labor ratio is chosen to be productively efficient, exactly as in the case of the poorer households, implying they have equal productivity.**