SOLUTIONS TO PROBLEM SET 1

1. Consider the model of farming households in an economy with a smooth, strictly concave CRS production function $f(h, n)$ of land and labor used in cultivation, and a utility function given by $Y + U(R)$, Y denoting income and R its leisure. U is strictly increasing and strictly concave; both U and f satisfy Inada conditions. Each household has an endowment of one time unit, can apply its own labor on its own farm, and hire in or out its own labor on a competitive labor market. Households differ in their landholdings, and can lease in or out their land on a competitive rental market. Households are subject to a working capital constraint, which depends on how much land they own. In contrast to the model studies in class, hired labor is perfectly trustworthy and does not need to be supervised.

(a) For poor cultivating households who are forced to hire out their labor to augment their working capital, show that they are indifferent between applying their own labor or hiring in workers to work on their own farm. Prove that as these households become richer, they hire out less labor.

The household will seek to maximize $\Pi \equiv P\beta f(\frac{B+wt-wL}{v}, l+L) + U(1-t-l)$, and for B close to zero, $t > 0$ and satisfies the first order condition $P\beta f_h \frac{w}{v} = U'(R)$. Moreover, $\frac{\partial \Pi}{\partial L} = P\beta[f_h \frac{-w}{v} + f_L] = P\beta f_l - U'(R) = \frac{\partial \Pi}{\partial l}$, so the household is indifferent between own-labor and hired labor.

Without loss of generality, set $L = 0$. The first order conditions defining optimal choices of the household are

$$f_h(\frac{B+wt}{v}, l)\frac{w}{v} = \frac{U'(R)}{P\beta}$$

$$f_l(\frac{B+wt}{v}, l) = \frac{U'(R)}{P\beta}$$

These imply that the productive efficiency condition $\frac{f_h}{f_l} = \frac{v}{w}$ is met. This determines the land-labor ratio, and the marginal product of labor, and hence also the amount of leisure. So R is independent of B.

Differentiating the first order conditions above with respect to B, we thus obtain

$$f_{hh}(\frac{1}{v} + \frac{w}{v}t_B) + f_{hl}l_B = 0$$

$$f_{lh}(\frac{1}{v} + \frac{w}{v}t_B) + f_{ll}l_B = 0$$

Since $R_B = 0$, it follows that $t_B + l_B = 0$. Using this, we then solve

$$t_B = \frac{f_{hh}}{v f_{hl} - w f_{hh}}$$

which is negative by virtue of the concavity of the production function.

(b) Now consider households who own enough land that they do not hire out their own labor. Show that these farms are as productive as those of the poorer households.

Given $t = 0$ the farm's problem is to select l, L to maximize $P\beta f(\frac{B-wL}{v}, l + L) + u(1 - l)$, which is a strictly concave function. Hence the solution is characterized by the first order conditions $\frac{\partial \Pi}{\partial L} = P\beta[f_h \frac{-w}{v} + f_l] = 0$, $\frac{\partial \Pi}{\partial l} = P\beta f_l - u'(R) = 0$, which represent two equations in the two unknowns. These yield the result that $\frac{f_h}{f_l} = \frac{v}{w}$, i.e., the land-labor ratio is chosen to be productively efficient, exactly as in the case of the poorer households, implying they have equal productivity.

2. Consider the following variation in the nature of the credit market in the Galor-Zeira model: a borrower can 'take the money and run', i.e., not repay a loan at all, but incurs a fixed cost of d from this which does not vary with loan size. The credit market is competitive, and the cost of lending is constant (r, say). Each individual lives for two periods, and leaves a bequest to a single heir. In the first period, an individual either works as an unskilled worker or decides to acquire human capital at a fixed schooling cost of h. In the second period, the individual works as a skilled worker if human capital was acquired previously, and as an unskilled worker otherwise. Labor supply is inelastic, and there is no consumption in the first period. Second period wealth is divided in fixed proportions between consumption and bequest (as a result of Cobb-Douglas 'warm glow' bequest motives). The wage rate for unskilled and skilled labor are also fixed: w_n and w_s respectively, where $w_s > w_n$.

(a) Describe the way that the credit market functions.

A loan of z at interest rate i will be repaid if $z(1 + i) \leq d$, i.e., if the loan size does not exceed $\frac{d}{1+i}$. Hence the credit ceiling will be $\frac{d}{1+i}$ at the interest rate i. Competition in lending will imply that the interest rate i equals the cost of capital r. Hence loans upto a ceiling of $\frac{d}{1+r}$ will be available at interest rate r.

(b) Provide a condition for human capital investments to be undertaken at all. In what follows, assume this condition holds.

For wealthy individuals with an inheritance W exceeding h, going to school leads to a lifetime wealth of $w_s - (h - W)(1 + r)$, whereas not going to school yields $w_n + (w_n + W)(1 + r)$. Hence they will go to school if $w_s - w_n \geq (w_n + h)(1 + r)$. Those

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with inheritance W less than $h - \frac{d}{1+r}$ will be unable to afford to go to school. For those with W between $h - \frac{d}{1+r}$ and h, the calculations are exactly the same as for those with inheritance above h, as the borrowing rate of interest exceeds the lending rate of interest. So the same condition $w_s - w_n \geq (w_n + h)(1 + r)$ ensures that they will also go to school.

(c) Given the distribution of inherited wealth for any given generation, characterize the nature of human capital investment decisions in the population.

Already described above: those with $W \geq h - \frac{d}{1+r}$ will go to school, and the rest will not.

(d) Describe the resulting dynamics of the distribution of inherited wealth. Provide a condition for the emergence of wealth inequality in the long run.

If $W_t < h - \frac{d}{1+r}$ then

$$W_{t+1} = \alpha[W_t(1 + r) + w_n(2 + r)]$$

while if $W_t \geq h - \frac{d}{1+r}$ then

$$W_{t+1} = \alpha[W_t(1 + r) + w_s - h(1 + r)]$$

Note that the intercept is strictly positive, so $\alpha(1 + r) < 1$ is necessary for existence of a steady state. There will be multiple steady states if the return to human capital is strictly positive, and satisfies the following inequality:

$$\alpha[w_s - h(1 + r)] > [1 - \alpha(1 + r)][h - \frac{d}{1 + r}] > \alpha w_n(2 + r).$$

This inequality is derived from the condition that the left limit of W_{t+1} at the threshold lies below the 45 degree line, while the right limit lies above this line. Under this condition there is an upward jump in the wealth evolution equation at the threshold $h - \frac{d}{1+r}$, in such a way as to create one steady state below this threshold, and one above. Clearly the dynamics are such as to ensure that any family that starts below the threshold will converge to the lower steady state, while the one that starts above will converge to the upper steady state. See the Figure.