Ec721 SOLUTIONS TO PROBLEM SET 3

1. *Suppose the policy space is a one dimensional real variable p, there are a finite (odd) number of voters each with single peaked preferences, with a median ideal point p^*_M. Two parties $i = A, B$ compete in an election, and select policy platforms p_A, p_B before the election, to which they are subsequently committed, in the event that are elected. In contrast to the Downsian model, suppose that each party i has ideological policy preferences represented by a von Neumann utility function $W_i(p)$ which is also single peaked, with ideal point p^*_i. So if ϕ_A denotes the probability that party A wins, party i's objective is to maximize $\phi_A W_i(p_A) + (1 - \phi_A) W_i(p_B)$.*

(a) (6 pts.) Suppose that p^*_M lies in between p^*_A and p^*_B. What is the set of Nash equilibrium policy platforms? Provide complete proofs.

The unique Nash equilibrium is (p^*_m, p^*_m). That this is a Nash equilibrium is obvious, because if one candidate selects p^*_m then deviating from p^*_m by the other candidate guarantees that the latter loses, so does not affect the resulting policy. So the main thing to prove is that there is no other Nash equilibrium.

Without loss of generality assume that $p^*_A < p^*_m < p^*_B$.

Consider first any other convergent platform pair (p, p). If $p < p^*_A$ then A can deviate to p^*_A, win the election and thereby benefit from the deviation. If $p = p^*_A$ then B can profitably deviate to p^*_m. If $p \in (p^*_A, p^*_m)$ then also B can profitably deviate to p^*_m. A symmetric argument takes care of any $p > p^*_m$.

So consider divergent platforms (p_A, p_B) where $p_A \neq p_B$. There are then various cases to consider. I exclude cases where some strict inequalities can be replaced by weak inequalities, where the argument is similar.

(1) $p_B > p^*_m > p_A$. If one candidate wins for sure, say B, then the other candidate (A here) can profitably deviate to p^*_m. If they win with equal probability, then one candidate can move slightly closer to p^*_m and win the election for sure. This will be a profitable deviation because the slight move away from its own ideal point will be dominated by the switch of the eventual policy from that chosen by the other party with probability half.

(2) $p_A > p^*_m > p_B$. If B wins for sure then B can do better to move to p^*_m, and continue to win the election. If they win with equal probability then also B will do better to move to p^*_m and win for sure.

(3) p_A and p_B are both on the same side of p^*_m. Lets suppose they are both less than p^*_m. In this case the candidate with the policy closer to p^*_m will win. If it is B, then B will do better to deviate to p^*_m. If it is A then also B will do better to deviate to p^*_m.

(b) (4 pts.) Now suppose that $p^*_B > p^*_A > p^*_M$. What can you say now about the policy that will be chosen by the winning party (in Nash equilibrium)?

Consider any $p \in [p^*_m, p^*_A]$. Then (p, p) is a Nash equilibrium. Any deviation that changes the election result causes the outcome to move further away from the candidate's ideal point, so is not worthwhile.

There cannot be any other convergent pair (p, p) that constitutes an equilibrium. If $p > p^*_A$ then A can deviate profitably to p^*_A. If $p < p^*_m$ then A can deviate profitably to p^*_m.

For the same reason the winning policy has to be in the range $[p^*_m, p^*_A]$, for if it is not A can deviate in the way described in the preceding paragraph.

2. Suppose there are two parties A, B and m voter groups, where the voter group j comprises α_j fraction of the voter population, $j = 1, \ldots, m$. Party $k = A, B$ selects a policy p^k from the set P of possible policy platforms. All voters in group j have the same utility function $U_j(p)$ over policies p, but differ in terms of their relative loyalties to the two parties. Voter i in group j thus votes for party A if $U_j(p^A) > U_j(p^B) + x_{ij} + y$, where loyalty x_{ij} within group j is distributed uniformly over $[-\frac{1}{2X_j}, \frac{1}{2X_j}]$. y represents a nation-wide loyalty swing variable, distributed uniformly over $[-\frac{1}{2Y}, \frac{1}{2Y}]$. Parties do not know the realization of y at the time they select their platforms. The party winning a majority of the votes wins the election. Assume that Y and X_j are sufficiently large that all equilibria will be interior (i.e., involve positive vote shares for both parties).

(a) (5 pts.) Obtain expressions for the probability that party A wins, as a function of p^A and p^B.

Voter i in group j votes for A (conditional on y) if $U_j(p^A) - U_j(p^B) > x_{ij} + y$. Hence the proportion of group j voters who will vote for A, conditional on y, will equal (upon applying the law of large numbers): $\Pr[x_{ij} < U_j(p^A) - U_j(p^B) - y] = \frac{1}{2} + X_j[U_j(p^A) - U_j(p^B) - y]$. Aggregating over groups, the overall vote share for A conditional on y will be $\frac{1}{2} - y\sum_j \alpha_j X_j + \sum_j \alpha_j X_j[U_j(p^A) - U_j(p^B)$. So the probability that A wins conditional on y is $\Pr[\frac{1}{\sum_j \alpha_j X_j}\sum_j \alpha_j X_j[U_j(p^A) - U_j(p^B) > y]$. The probability that A wins is thus $E_y\{\Pr[\frac{1}{\sum_j \alpha_j X_j}\sum_j \alpha_j X_j[U_j(p^A) - U_j(p^B) > y]\}$.

(b) (5 pts.) Derive the equilibrium policy platforms of the two parties. Contrast the equilibrium policy with the welfare optimal policy, and provide an interpretation for why the two may diverge.

It follows from the expression of the probability of A winning, that each party will have a dominant strategy of selecting a policy p to maximize $\sum_j \alpha_j X_j U_j(p)$. In contrast the welfare optimal policy maximizes $\sum_j \alpha_j U_j(p)$. The two diverge if X_j is not the same across all groups. In that case groups differ in the responsiveness of their voting behavior to the utility differences resulting from the policies of the two parties. So groups that are more responsive get more welfare weight from competing candidates.