Boston University
PhD Topics course in Contract Theory
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# Selections from Recent EC717 Exams

*Do not write your name on your blue books, only your ID number. You do not need to re-derive expressions or formulae that were derived in class, but be explicit regarding which of these you are invoking. Explain in detail the steps of your reasoning.*

1.

(a) Show that every Bayesian incentive compatible mechanism implementing first-best decisions in the standard model of $n$ agents with quasi-linear utility and independently distributed one dimensional type $\theta_i$ (i.e., $U_i = v_i(d;\theta_i) + t_i$ where $d$, $\theta_i$ and $t_i$ are real-valued and $v_i$ is twice continuously differentiable) must have transfers belong to the *expected* Groves-Clark family, i.e., satisfying:

$$E_{\theta_{-i}}[t_i(\theta_i;\theta_{-i})] = E_{\theta_{-i}}[\sum_{j\neq i} U_j(d_F(\theta_i,\theta_{-i}),\theta_j)] + K_i$$

where $d_F(.)$ denotes the first-best decision rule, $\theta_i$ denotes the type reported by $i$, $\theta_{-i}$ denotes the vector or types reported by agents other than $i$, $E_{\theta_{-i}}$ denotes expectation over the prior distribution of $\theta_{-i}$, and $K_i$ is an arbitrary constant.

(b) Consider a monopolist selling a divisible good which is produced at a constant per unit cost of $c$ to a population of customers with a payoff of $\theta q - t$, where $q \in [0,1]$ denotes the quantity sold and $t$ the payment made, with $\theta$ known privately by each consumer, distributed in the population over some support $[\underline{\theta},\bar{\theta}]$ (where $c \in (\underline{\theta},\bar{\theta})$) according to a distribution function $F$ and associated density $f$ (with an inverse hazard rate $\frac{1-F(\theta)}{f(\theta)}$ which is nonincreasing). Each consumer has an outside option payoff of 0. The monopolist can choose any nonlinear pricing mechanism. Show that it is optimal for the monopolist to set a constant per unit price $p^*$ of the good, and let each consumer decide whether and how much to buy. Calculate the price $p^*$ in terms of the parameters of the model.

2. A production process requires $n$ workers to work together. The revenue generated equals $R(a_1, \ldots, a_n)$ where $a_i \geq 0$ is an input provided by worker $i$, which is contractible. Worker $i$'s utility is $t_i - \theta_i g(a_i)$ where $t_i$ is a transfer payment to $i$, and $\theta_i$ is distributed iid on support $[\underline{\theta}, \bar{\theta}]$ according to a distribution function $F$ which is common knowledge and satisfies the monotone hazard rate property. The effort disutility function $g$ is strictly increasing and strictly convex with $g'(0) = 0$, while $R$ is strictly increasing, strictly concave and satisfies Inada conditions. Each agent privately observes the realization of $\theta_i$, and has an outside option utility of 0.

(a) Suppose the firm is owned by a third party agent called a capitalist $C$, who contracts with the workers, and is a risk-neutral residual claimant (i.e., has a payoff $R - \sum_i t_i$). Characterize the optimal payments and associated production assignments $a_1, \ldots, a_n$ in any given state $(\theta_1, \ldots, \theta_n)$.

(b) Suppose the firm is organized as a co-op, which is jointly owned by the workers. The co-op management contracts with its members, and has access to a competitive insurance market. It seeks to maximize the sum of expected value of payoffs of its members (i.e., $\sum_i [t_i - \theta_i g(a_i)]$) subject to the breakeven constraint which states that the expected value of the firm's revenues covers expected payments to its members, in addition to the same incentive and participation constraints as in the capitalist firm. Characterize the optimal payments made to any given member and associated production assignments in any given state.

(c) Compare the production assignments in (a) and (b) above. Which form of organization is more efficient (measured by expected total surplus)? Provide some intuition for the result.