EC 717, PROBLEM SET NO. 3

1. Consider a monopolist who produces a good at zero cost, and sells to a population of customers each of whom buy at most one unit of the good at most once, and whose valuations are uniformly distributed on $[0,1]$. There are a finite number T of periods. As in the standard model, the monopolist cannot commit to future price offers, and everyone discounts the future at a constant rate $\delta \in (0,1)$.

(a) Consider the last period T where customers still left waiting to purchase the good are those whose valuations are below $v_T \in (0,1)$. Calculate the optimal selling strategy and profits of the monopolist at T as a function of v_T.

(b) Now consider period $T-1$, and suppose customers still left waiting to purchase the good are those whose valuations are below $v_{T-1} \in (0,1)$. Suppose the monopolist uses a strategy of pricing the product $p_{T-1} = \beta_{T-1} v_{T-1}$ and customers purchases the product at any given price p if and only if their valuation exceeds $\alpha_{T-1} p$. Using the solution to (a) above, calculate equilibrium values of α_{T-1} and β_{T-1} as a function of δ alone. Calculate the limit of the equilibrium and its outcomes as δ approaches 1, and interpret the result.

(c) Using induction, show that equilibrium profits of the monopolist at any date t where customers waiting to buy are those with valuations below v_t, is of the form $\frac{\kappa_t}{2}.[v_t]^2$, where κ_t is a function of δ and t. Obtain a difference equation characterizing equilibrium values of α_t, β_t.

2. Consider the following version of the Levin (2003) model of relational contracts between a Principal P and an Agent A. P obtains benefit $\beta.e$ from effort e of A. A's effort cost is $\frac{\theta}{2}e^2$ depends on the realization of θ, which takes two possible values θ_1 with probability p and $\theta_2 > \theta_1$ with probability $1-p$. A observes the realization of θ_i before choosing effort. Both parties have an outside option of 0 and a discount factor of $\delta \in (0,1)$. As in the Levin model, the realization of P's benefit is mutually observed by both parties but cannot be contracted upon as it is not observed by any third party.

(a) Obtain expressions for the first-best effort of the agent in each state θ_i.

(b) Find a condition on parameters that implies that the first-best cannot be implemented.

(c) Assuming the condition in (b) holds, characterize the second-best effort of the agent in each state, and compare it with the corresponding first-best level.