1. You have been appointed arbitrator for negotiations for sale of an indivisible input produced by a selling firm $S$ to a buying firm $B$ with \textit{ex ante} uncertain costs and valuations. They respectively have payoffs $t - q\theta_s$ and $q\theta_b - t$ if $q \in \{0, 1\}$ denotes whether or not a sale takes place, and $t$ is the expected monetary transfer from $B$ to $S$. Parameters $\theta_s$ and $\theta_b$ are drawn independently from intervals $[\theta_b, \bar{\theta}_b], [\theta_b, \bar{\theta}_b]$ respectively accordingly to cdf’s $F_s, F_b$ with positive densities $f_s, f_b$ which are common knowledge. Each firm will privately observe its own cost or valuation parameter; neither you or the other firm will know the realization of this parameter. Is it possible to design a mechanism which is \textit{ex post} efficient, Bayesian incentive compatible and \textit{ex ante} individually rational (i.e., both parties will be willing to participate if they have to commit to participation \textbf{before} observing their respective costs/valuations)? If not, provide a proof. If yes, construct a mechanism with the required properties.

2. You want to sell an indivisible object which you personally do not value. There are two potential bidders, with independent private values. Bidder 1’s value is drawn uniformly over $[0, 1]$, while bidder 2’s value is drawn uniformly over $[\alpha, 1 - \alpha]$, where $\alpha \in (\frac{1}{3}, \frac{1}{2})$. Derive the optimal expected revenue maximizing sealed-bid auction in which bidders have incentives to report their true valuations. Describe the way that the bidding rules in the optimal auction ‘favor’ one bidder over another, and try to explain why.

3. A Principal $P$ wishes to procure an indivisible object that can be produced either by firm 1 or firm 2. Firm $i$’s cost of production is drawn from a uniform distribution over the support $[0, i], i = 1, 2$; each firm privately observes its own cost realization. $P$ wishes to contract with one of the two firms (called the prime contractor), and delegate to that firm the decision whether it will produce the object itself, or subcontract it to the other firm. Specifically, $P$ will offer a take-it-or-leave-it contract to the prime contractor, which the latter will have to respond with a yes-no decision to $P$ before making a take-it-or-leave-it subcontract offer to the other firm (but after it has observed its own cost realization). $P$ will not observe the subcontract, nor who ultimately produced the object, nor the money
transferred between the two firms. All parties are risk neutral. A firm’s payoff is defined by
the difference between the net transfer it receives and its cost of production, while P wishes
to minimize the expected cost of procuring the object.

(a) Derive the outcome (in terms of production assignments, transfers and expected cost
incurred by P) of appointing agent $i$ the prime contractor. Based on this, which agent
should be appointed the prime contractor?

(b) Would P do better to personally contract with the two agents (i.e., design a procure-
ment auction), instead of appointing one of them to act as a prime contractor? If so,
explain how much P would gain, and how the production assignments would differ.