EC 717a, PROBLEM SET NO. 3

1. Consider the problem of a durable good monopolist who produces at zero cost, and sells to a population of customers each of whom buy at most one unit of the good at most once, and whose valuations are uniformly distributed on $[0, 1]$. There are a finite number T of periods. As in the standard model, the monopolist cannot commit to future price offers, and everyone discounts the future at a constant rate $\delta \in (0, 1)$.

(a) Consider the last period T where customers still left waiting to purchase the good are those whose valuations are below $v_T \in (0, 1)$. Calculate the optimal selling strategy and profits of the monopolist at T as a function of v_T.

(b) Now consider period $T - 1$, and suppose customers still left waiting to purchase the good are those whose valuations are below $v_{T-1} \in (0, 1)$. Suppose the monopolist uses a strategy of pricing the product $p_{T-1} = \beta_{T-1} v_{T-1}$ and customers purchases the product at any given price p if and only if their valuation exceeds $\alpha_{T-1} p$. Using the solution to (a) above, calculate equilibrium values of α_{T-1} and β_{T-1} as a function of δ alone. Calculate the limit of the equilibrium and its outcomes as δ approaches 1, and interpret the result.

(c) *Optional:* Using induction, show that equilibrium profits of the monopolist at any date t where customers waiting to buy are those with valuations below v_t, is of the form $\frac{\kappa_t}{2} . [v_t]^2$, where κ_t is a function of δ and t. Obtain a difference equation characterizing equilibrium values of α_t, β_t.

2. A risk-neutral principal P designs a dynamic credit-cum-insurance program for a representative agent A who is subject to i.i.d. endowment shocks, with the endowment ω_t of A drawn at each date $t = 1, 2, \ldots$ from a distribution with finite support (with N possible realizations $\omega_1, \ldots, \omega_N$, with corresponding probabilities $f_1, \ldots, f_N$ which are given). At each date t, A observes the realization of ω_t privately and makes a report $\tilde{\omega}_t$ of it to P. If h_t denotes the history of reports made by A until date t, P mandates a transfer $b_t(h_t)$ (which can be positive or negative) to A at t based on this history. P can precommit to this policy. A's payoff is $E[\sum_{t=1}^{\infty} \delta^t u(\omega_t + b_t(h_t))]$, while P's payoff is $-E[\sum_{t=1}^{\infty} \delta^t b_t(h_t)]$, where $\delta \in (0, 1)$

is a common discount factor. A Pareto optimal (PO) contract maximizes the payoff of P, subject to allowing A to reach a minimum pre-specified payoff, and incentive compatibility constraint for A (wherein it is optimal for A to report the true state at every date, following every history).

Show that in every PO contract, the same inverse Euler condition as in the Rogerson repeated moral hazard model holds: for every date t, history h_{t-1} and every possible $\omega_t = \omega_i$ realization:

$$\frac{1}{u'(x_{it}|h_{t-1})} = \sum_{j=1}^{N} f_j \frac{1}{u'(x_{j,t+1}|h_t = h_{t-1} \cup \{i\})}$$

where $x_{it}|h_{t-1}$ denotes A's consumption at date t, following a truthful report of $\omega_t = \omega_i$ at t and history h_{t-1} of reports until $t-1$.