Ec717a PROBLEM SET NO. 2

1. You have been appointed arbitrator for negotiations for sale of an indivisible input produced by a selling firm S to a buying firm B with *ex ante* uncertain costs and valuations. They respectively have payoffs $t - q\theta_s$ and $q\theta_b - t$ if $q \in \{0, 1\}$ denotes whether or not a sale takes place, and t is the expected monetary transfer from B to S. Parameters θ_s and θ_b are drawn independently from intervals $[\underline{\theta}_b, \bar{\theta}_b], [\underline{\theta}_b, \bar{\theta}_b]$ respectively accordingly to cdf's F_s, F_b with positive densities f_s, f_b which are common knowledge. Each firm will privately observe its own cost or valuation parameter; neither you or the other firm will know the realization of this parameter. Design a mechanism which is *ex post* efficient, Bayesian incentive compatible and *ex ante* individually rational (i.e., both parties will be willing to participate if they have to commit to participation **before** observing their respective costs/valuations).

2. You want to sell an indivisible object which you personally do not value. There are two potential bidders, with independent private values. Bidder 1's value is drawn uniformly over $[0, 1]$, while bidder 2's value is drawn uniformly over $[\alpha, 1-\alpha]$, where $\alpha \in (\frac{1}{3}, \frac{1}{2})$. Derive the optimal expected revenue maximizing sealed-bid auction in which bidders have incentives to report their true valuations. Describe the way that the bidding rules in the optimal auction 'favor' one bidder over another, and try to explain why.

3. The Principal Car Sales Company has hired two sales agents r(Regular Joe) and e (Eager Beaver). Sales revenue achieved by agent i ($i = r, e$) equals $x_i \equiv a_i + \epsilon_i$, where a_i is effort, and ϵ_r, ϵ_e are normal, i.i.d., zero mean variables with variance $\frac{\sigma^2}{2}$. Both agents are risk-neutral and effort averse, but agent e has a lower disutility of effort. So agent r's payoff is $w_r - \frac{a_r^2}{2}$, while agent e's is $w_e - \beta\frac{a_e^2}{2}$, where $\beta \in (0, 1)$ and w_i denotes the wage paid to i. Both agents have the same outside option utility of $\bar{U}$. The car company's payoff is $x_r + x_e - w_r - w_e$.

(a) If the car company can monitor effort and contract with the agents based on their observed efforts, derive the (first-best) contracts and effort levels.

(b) Now suppose that the car company cannot monitor effort nor the actual revenues achieved by each agent. It can only observe which agent achieved the highest sales. Show that there is a symmetric rank order tournament (with a base salary of L and a 'best sales agent' prize bonus of B) which can 'implement' the first-best efforts (in the sense that these satisfy the local first and second order conditions for a Nash equilibrium, in which each agent attains at least his outside option in expectation).

(c) Show that the first-best level of expected profit cannot be attained by the car company with a symmetric tournament (using the notion of 'implementation' described above).

(d) Can you suggest a modification of the tournament incentive scheme (i.e., based only on ordering of the sales of the two agents) which will implement the first-best?