PROBLEM SET NO. 1

1. A monopolist wishes to sell a good produced at constant unit cost c to a large population of consumers with heterogenous preferences: a consumer of type θ has a payoff $\theta \log q - t$ for consuming q units of the good, and paying t dollars for it. θ is distributed uniformly on $[0, 1]$. The monopolist cannot identify the type of any given consumer.

(a) If $q(\theta)$ denotes the quantity sold to type θ, find a condition on this function $q(.)$ that ensures that it is IC (incentive compatible, i.e., there exists some pricing rule $t(q)$ for which $q(\theta)$ is the optimal purchase of type θ)).

(b) For any such IC $q(.)$, what is the associated set of payments (i.e., $t(\theta)$) that customers (of type θ) make to the monopolist?

(c) Obtain an expression for total profit of the monopolist as a function only of the selling strategy $q(.)$.

(d) Calculate the optimal selling strategy $q^*(\theta)$, and the pricing function $t(q)$ which implements it (in the sense of (a) above). (*Note:* Be careful about corner solutions.)

2. A risk-neutral principal P hires an agent A, who chooses an effort $a \geq 0$, which results in gross profit $x = a + \epsilon$ for P, where ϵ is uniformly distributed on $[0, 1]$. A's payoff equals $\frac{w^{1-\rho}}{1-\rho} - \frac{a^2}{2}$, where w denotes a non-negative wage paid by P, and $\rho > 0, \neq 1$ is a parameter of risk-aversion. A has an outside option payoff of $\bar{U}$ which is non-negative if $\rho < 1$ and negative if $\rho > 1$.

(a) If a is contractible, characterize the first-best wage and effort levels.

(b) If a is not contractible, but the profit x is contractible, and $\rho \in (0, 1)$, find a condition on the parameters of the problem (specifically, on ρ) which ensure that the first-best profit can be achieved by P. If $\bar{U} = 0$, when is this condition satisfied?

(c) If $\rho > 1$ what can you say about implementability of the first-best profit when a is not contractible? How would you interpret these results?