1. The Principal Car Sales Company has hired two sales agents \( r \) (Regular Joe) and \( e \) (Eager Beaver). Sales revenue achieved by agent \( i \) \((i = r, e)\) equals \( x_i \equiv a_i + \epsilon_i \), where \( a_i \) is effort, and \( \epsilon_r, \epsilon_e \) are normal, i.i.d., zero mean variables with variance \( \sigma^2 \). Both agents are risk-neutral and effort averse, but agent \( e \) has a lower disutility of effort. So agent \( r \)'s payoff is \( w_r - a_r^2 \), while agent \( e \)'s is \( w_e - \beta a_e^2 \), where \( \beta \in (0, 1) \) and \( w_i \) denotes the wage paid to \( i \). Both agents have the same outside option utility of \( \bar{U} \). The car company’s payoff is \( x_r + x_e - w_r - w_e \).

(a) If the car company can monitor effort and contract with the agents based on their observed efforts, derive the (first-best) contracts and effort levels.

(b) Now suppose that the car company cannot monitor effort nor the actual revenues achieved by each agent. It can only observe which agent achieved the highest sales. Show that there is a symmetric rank order tournament (with a base salary of \( L \) and a ‘best sales agent’ prize bonus of \( B \)) which can ‘implement’ the first-best efforts (in the sense that these satisfy the local first and second order conditions for a Nash equilibrium, in which each agent attains at least his outside option in expectation).

(c) Show that the first-best level of expected profit cannot be attained by the car company with a symmetric tournament (using the notion of ‘implementation’ described above).

(d) Can you suggest a modification of the tournament incentive scheme (i.e., based only on ordering of the sales of the two agents) which will implement the first-best?

2. A risk-neutral principal \( P \) designs a dynamic credit-cum-insurance program for a representative agent \( A \) who is subject to i.i.d. endowment shocks, with the endowment \( \omega_t \) of \( A \) drawn at each date \( t = 1, 2, \ldots \) from a distribution with finite support (with \( N \) possible realizations \( \omega_1, \ldots, \omega_N \), with corresponding probabilities \( f_1, \ldots, f_N \)). At each date \( t \), \( A \) observes the realization of \( \omega_t \) privately and makes a report \( \tilde{\omega}_t \) of it to \( P \). If \( h_t \) denotes the history of reports made by \( A \) until date \( t \), \( P \) mandates a transfer \( b_t(h_t) \) (which can be
positive or negative) to A at \( t \) based on this history. P can precommit to this policy. A’s payoff is \( E[\sum_{t=1}^{\infty} \delta^t u(\omega_t + b_t(h_t))] \), while P’s payoff is \(-E[\sum_{t=1}^{\infty} \delta^t b_t(h_t)]\), where \( \delta \in (0, 1) \) is a common discount factor. A Pareto optimal (PO) contract maximizes the payoff of P, subject to allowing A to reach a minimum pre-specified payoff, and incentive compatibility constraint for A (wherein it is optimal for A to report the true state at every date, following every history).

Show that in every PO contract, the same inverse Euler condition as in the Rogerson repeated moral hazard model holds: for every date \( t \), history \( h_{t-1} \) and every possible \( \omega_t = \omega_i \) realization:

\[
\frac{1}{u'(x_{it}|h_{t-1})} = \sum_{j=1}^{N} f_j \frac{1}{u'(x_{jt+1}|h_t = h_{t-1} \cup \{i\})}
\]

where \( x_{it}|h_{t-1} \) denotes A’s consumption at date \( t \), following a truthful report of \( \omega_t = \omega_i \) at \( t \) and history \( h_{t-1} \) of reports until \( t - 1 \).

3. A Principal P wishes to procure an indivisible object that can be produced either by firm 1 or firm 2. Firm \( i \)'s cost of production is drawn from a uniform distribution over the support \([0, i]\), \( i = 1, 2\); each firm privately observes its own cost realization. P wishes to contract with one of the two firms (called the prime contractor), and delegate to that firm the decision whether it will produce the object itself, or subcontract it to the other firm. Specifically, P will offer a take-it-or-leave-it contract to the prime contractor, which the latter will have to respond with a yes-no decision to P before making a take-it-or-leave-it subcontract offer to the other firm (but after it has observed its own cost realization). P will not observe the subcontract, nor who ultimately produced the object, nor the money transferred between the two firms. All parties are risk neutral. A firm’s payoff is defined by the difference between the net transfer it receives and its cost of production, while P wishes to minimize the expected cost of procuring the object.

(a) Derive the outcome (in terms of production assignments, transfers and expected cost incurred by P) of appointing agent \( i \) the prime contractor. Based on this, which agent should be appointed the prime contractor?
(b) Would P do better to personally contract with the two agents (i.e., design a procurement auction), instead of appointing one of them to act as a prime contractor? If so, explain how much P would gain, and how the production assignments would differ.