PROBLEM SET NO. 1

due October 2, 2008

1. A monopolist wishes to sell a good produced at constant unit cost $c$ to a large population of consumers with heterogeneous preferences: a consumer of type $\theta$ has a payoff $\theta \log q - t$ for consuming $q$ units of the good, and paying $t$ dollars for it. $\theta$ is distributed uniformly on $[0,1]$. The monopolist cannot identify the type of any given consumer.

(a) If $q(\theta)$ denotes the quantity sold to type $\theta$, find a condition on this function $q(.)$ that ensures that it is IC (incentive compatible, i.e., there exists some pricing rule $t(q)$ for which $q(\theta)$ is the optimal purchase of type $\theta$).

(b) For any such IC $q(.)$, what is the associated set of payments (i.e., $t(\theta)$) that customers (of type $\theta$) make to the monopolist?

(c) Obtain an expression for total profit of the monopolist as a function only of the selling strategy $q(.)$.

(d) Calculate the optimal selling strategy $q^*(\theta)$, and the pricing function $t(q)$ which implements it (in the sense of (a) above).

2. A risk-neutral principal P hires an agent A, who chooses an effort $a \geq 0$, which results in gross profit $x = a + \epsilon$ for P, where $\epsilon$ is uniformly distributed on $[0,1]$. A’s payoff equals $w^{1-\rho} - a^\rho$, where $w$ denotes a non-negative wage paid by P, and $\rho > 0, \neq 1$ is a parameter of risk-aversion. A has an outside option payoff of $\bar{U}$ which is non-negative if $\rho < 1$ and negative if $\rho > 1$.

(a) If $a$ is contractible, characterize the first-best wage and effort levels.

(b) If $a$ is not contractible, but the profit $x$ is contractible, and $\rho \in (0,1)$, find a condition on the parameters of the problem which ensure that the first-best profit can be achieved by P. If $\bar{U} = 0$, when is this condition satisfied?
(c) If $\rho > 1$ what can you say about implementability of the first-best profit when $a$ is not contractible?

(d) How would you interpret the results in (b) and (c)?

3. You are appointed arbitrator for negotiations for sale of an indivisible input produced by a selling firm $S$ to a buying firm $B$ with \textit{ex ante} uncertain costs and valuations: they respectively have payoffs $t - q\theta_s$ and $q\theta_b - t$ if $q \in \{0, 1\}$ denotes whether or not a sale takes place, and $t$ is the expected monetary transfer from $B$ to $S$. Parameters $\theta_s$ and $\theta_b$ are drawn independently from intervals $[\bar{\theta}_b, \bar{\theta}_b], [\underline{\theta}_b, \bar{\theta}_b]$ respectively according to cdf’s $F_s, F_b$ with positive positive densities $f_s, f_b$ which are common knowledge. Each party will observe its own cost or valuation parameter privately. Design a mechanism which is ex post efficient, Bayesian incentive compatible and \textit{ex ante} individually rational (i.e., both parties will be willing to participate if they have to commit to participation \textbf{before} observing their respective costs/valuations).

[If you need a hint, send me an email]