Ec717a: Mechanism Design with Costly Communication (M-T JPE 2014)

Dilip Mookherjee

Boston University

Ec 717a, 2020: Lecture 7

Motivation

- Most real-world instances of buyer-seller negotiations, intra-firm organization, or regulation involve dynamic interactive communication (offers/counteroffers, meetings, reports, hearings...)

Motivation

- Most real-world instances of buyer-seller negotiations, intra-firm organization, or regulation involve dynamic interactive communication (offers/counteroffers, meetings, reports, hearings...)
- Communication takes place between agents, as well as between agents and principal, especially within firms

Motivation

- Most real-world instances of buyer-seller negotiations, intra-firm organization, or regulation involve dynamic interactive communication (offers/counteroffers, meetings, reports, hearings...)
- Communication takes place between agents, as well as between agents and principal, especially within firms
- And unlike revelation mechanisms, many important production/allocation decisions are decentralized to agents (e.g., allow workers to sort out shop-floor problems by themselves)

Motivation

- Most real-world instances of buyer-seller negotiations, intra-firm organization, or regulation involve dynamic interactive communication (offers/counteroffers, meetings, reports, hearings...)
- Communication takes place between agents, as well as between agents and principal, especially within firms
- And unlike revelation mechanisms, many important production/allocation decisions are decentralized to agents (e.g., allow workers to sort out shop-floor problems by themselves)
- Similar issues in regulatory policy (e.g., pollution control): command-and-control (pollution caps) versus centralized-coordination-cum-decentralized-incentives (pollution taxes)

Motivation, contd.

- One reason why we do not see one-shot revelation mechanisms in practice is that what agents know privately, cannot be summarized in a low dimensional report
- Information is far too rich to be communicated to others in real time

Motivation, contd.

- One reason why we do not see one-shot revelation mechanisms in practice is that what agents know privately, cannot be summarized in a low dimensional report
- Information is far too rich to be communicated to others in real time
- An observation that goes back to Hayek (1945), in critiquing socialist resource allocation mechanisms
- Hayek's argument for decentralization of economic decisions to agents: they will be better-informed about their local environment, even after sending reports to a central HQ

Motivation, contd.

- One reason why we do not see one-shot revelation mechanisms in practice is that what agents know privately, cannot be summarized in a low dimensional report
- Information is far too rich to be communicated to others in real time
- An observation that goes back to Hayek (1945), in critiquing socialist resource allocation mechanisms
- Hayek's argument for decentralization of economic decisions to agents: they will be better-informed about their local environment, even after sending reports to a central HQ
- Hence under decentralization, decisions will be based on better, local information

Motivation, contd.

- Problem with Hayek's argument: it implicitly assumes no attendant incentive problems (agents report truthfully, make decisions in P's interest)
- Gives rise to *key tradeoff in delegation of decision-making:* information versus potential 'loss of control'/abuse of power

Motivation, contd.

- Problem with Hayek's argument: it implicitly assumes no attendant incentive problems (agents report truthfully, make decisions in P's interest)
- Gives rise to *key tradeoff in delegation of decision-making:* information versus potential 'loss of control'/abuse of power
- To study this trade-off, we need a model of information gaps (even after communication), owing to communication constraints
- How to model communication constraints?

Our Approach

- Communication capacity in any given round: finite message set $\mathcal{R}_i$ for agent i representing language restrictions
- Message m_i has length $l(m_i)$ which represents time or other resources required to compose/write/express/send m_i
- Longer messages involve higher communication costs (resources, or time delays)
- If we fix a (finite) budget for communication, it imposes a constraint on the total amount of communication that can take place prior to decision-making

Our Approach, contd.

- For any given communication budget, select an optimal mechanism (incorporating incentive constraints)
- Compare different types of mechanisms (centralized versus decentralized), for any given communication budget

Our Approach, contd.

- For any given communication budget, select an optimal mechanism (incorporating incentive constraints)
- Compare different types of mechanisms (centralized versus decentralized), for any given communication budget
- Hope is to get a ranking that does not depend on the specific budget

Our Approach, contd.

- For any given communication budget, select an optimal mechanism (incorporating incentive constraints)
- Compare different types of mechanisms (centralized versus decentralized), for any given communication budget
- Hope is to get a ranking that does not depend on the specific budget
- *Main Technical Problem:* we cannot restrict attention to static (one-shot) communication mechanisms when communication is restricted; so have to allow for arbitrary dynamic mechanisms

The Model

- Principal (P), two agents 1 and 2
- Agent i produces $q_i \geq 0$ at cost $\theta_i q_i$
- θ_i is real-valued: (cannot be communicated entirely in finite time, or in finite number of bits)
- θ_i has cdf F_i, positive density f_i over $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ satisfying monotone hazard rate; θ_1, θ_2 are independent
- Zero outside options, risk-neutral

Production, Transfers and Payoffs

- P's gross payoff: $V(q_1, q_2)$, non-separable (so requires coordination)
- Possible technological (jointness) restrictions: $(q_1, q_2) \in Q \subset R_+ \times R_+$
- P transfers t_i to agent i
- Agent i payoff: $t_i - \theta_i.q_i$
- P's payoff: $V(q_1, q_2) - \lambda_1(t_1 + t_2) - \lambda_2(\theta_1 q_1 + \theta_2 q_2)$

Applications

- *Profit maximizing Principal:* $\lambda_1 = 1, \lambda_2 = 0$
- *Welfare Maximizing Regulator:* $\lambda_1 = \lambda > 0$, $\lambda_2 = 1$

Applications

- *Profit maximizing Principal:* $\lambda_1 = 1, \lambda_2 = 0$
- *Welfare Maximizing Regulator:* $\lambda_1 = \lambda > 0$, $\lambda_2 = 1$
- *Allocating private goods:* $\theta_i < 0$, $-t_i$ is amount paid by i, $Q = \{(q_1, q_2) \in R_+ \times R_+ \mid q_1 + q_2 \leq q\}$
- *Public good decisions:* jointness restriction $q_1 = q_2 = q$, $V(q, q) = -C(q)$ where C is cost of public good quantity q, $-t_i$ is tax paid by i and $-\theta_i$ is value placed on the good by i

Timing

- At $t = -1$, P offers mechanism.
- At $t = 0$ each agent observes θ_i and decides whether or not to participate. If both agree to participate, game continues.
- *Communication phase:* rounds of communication $t = 1, \ldots, T$
- *Production/allocation decision:* made at T either by P (if mechanism is centralized) and by agents (if it is decentralized)
- Transfers made *ex post* based on messages reported and productions (in case of decentralized mechanism)

Potential Value of Multiple Rounds of Communication: Example

- Abstract from incentive problems
- Two agents $i = 1, 2$ jointly produce common output $q \in \{0, 1, 2\}$ at personal cost $\theta_i.q$
- Gross benefit to Principal: $V(0) = 0$, $V(1) = 38$, $V(2) = 50$
- Prob $(\theta_1 = 0)$=Prob $(\theta_1 = 10)$=$\frac{1}{2}$
- Prob $(\theta_2 = 0)$=Prob $(\theta_2 = 100)$=$\frac{1}{4}$, Prob$(\theta_2 = 30)$=$\frac{1}{2}$

Example, contd.

- Communication constraint: each agent can only send a binary message only once
- With one round of communication, can confine attention to *threshold* reporting strategies, i.e.. whether $\theta_2 > c$ or not
- Contrast one round of simultaneous binary reports, with sequential reports

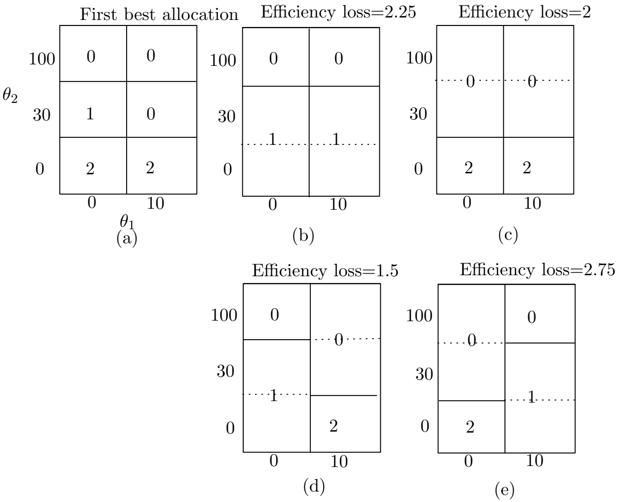

Table: Example 1

Complications with Incentives in Dynamic Communication Protocols

- Multiple communication rounds implies agents get to learn other agents' information along the way
- This can affect their incentives to report truthfully
- So there could be a trade-off between information benefits and incentive costs of dynamic communication: P may want to prevent information spillovers across agents for strategic reasons
- To address this, we need to incorporate necessary and sufficient conditions for incentive compatibility in dynamic communication mechanisms (the key technical problem)

Communication Technology

- Communication capacity in any given round: finite message set $\mathcal{R}_i$ for i representing language restrictions
- Message m_i has length $l(m_i)$ which represents time or other resources required to compose/write/express/send m_i
- **Finite Language Assumption:** For any $k < \infty$, there exists an integer $n < \infty$ such that

$$^{\#}\{m_i \in \mathcal{R}^i \mid l(m_i) < k\} < n$$

(i.e., message of finite length can communicate only finite amount of information)

Possible Communication Constraints

- CC1: constraint on total length of messages sent by each agent
$$\Sigma_{t=1}^{T} l(m_{it}) \leq k_i$$
- CC2: constraint on total length of messages aggregating across agents:
$$\Sigma_{i\in\{1,2\}}\Sigma_{t=1}^{T} l(m_{it}) \leq k$$
- CC3: constraint on communication delay, where delay in each round:
$$\Sigma_{t=1}^{T} \max\{l(m_{1t}), l(m_{2t})\} \leq D$$

Communication Protocol

- Assume receiving/reading messages is costless, and so is sending messages to multiple receivers
- Shall show later it is then optimal to send messages to the other agent as well as P in each round
- History of messages until end of round t denoted by h_t
- A *communication protocol* specifies the number of rounds T, and for every round $t \in \{1, \ldots, T\}$ and every agent i, a message set $M_i(h_{t-1}) \subseteq \mathcal{R}^i$ or $M_i(h_{t-1}) = \emptyset$ for every possible history h_{t-1}
- $\mathcal{P}$ denotes the set of feasible protocols p, satisfying CC1/2/3

Communication Plans and Strategies

- Given protocol p, a *communication plan* for agent i specifies for every round t a message $m_{it}(h_{t-1}) \in M_i(h_{t-1})$ for every possible history h_{t-1}
- Set of possible communication plans for i in protocol p is denoted $C_i(p)$, **a finite set**
- A *communication strategy* for agent i is a mapping $c_i(\theta_i) : \Theta_i \to C_i(p)$
- Communication constraints (finiteness of $C_i(p)$) force different types to pool (since Θ_i is a real interval)

Centralization

- In a centralized mechanism, P makes production decisions at T, based on $h \equiv h_T$
- A *centralized mechanism* is a communication protocol $p \in \mathcal{P}$ and an associated contract $(q(h), t(h)) : \mathcal{H} \rightarrow Q \times \Re \times \Re$.

Decentralization

- In a decentralized mechanism, agent i decides q_i at T (based on information (θ_i, h_T), richer than information h available to P at T)
- A *decentralized mechanism* is:
  - a communication protocol p
  - a feasible output space $Q = \Re_+ \times \Re_+$
  - contract for agent i: transfer rule $t_i(q_i, h) : \Re_+ \times \mathcal{H} \rightarrow \Re$
- Associated quantity allocation $q_i(\theta_i, h) : \Theta_i \times \mathcal{H} \rightarrow \Re_+$ maximizes $[t_i(q_i, h) - \theta_i q_i]$ with respect to choice of q_i

Truly Decentralized Mechanisms

- Can view any centralized mechanism as a decentralized mechanism in which q_i is measurable with respect to h, and transfers t_i ‘force’ agent i to abide by the assigned target q_i
- A *truly decentralized* mechanism is one in which q_i is not measurable with respect to h, so agents have true discretion *ex post*
- The interesting question concerns comparison between centralized and truly decentralized mechanisms

Communication-Feasible Production Allocations

- Seek to extend standard methods based on Revenue Equivalence Theorem
- A *production allocation* is a mapping $q(\theta) \equiv (q_1(\theta), q_2(\theta)) : \Theta_1 \times \Theta_2 \to Q$
- A production allocation $q(\theta)$ is said to be *communication-feasible* if: (a) the mechanism involves a communication protocol p satisfying the specified constraints on communication, and (b) there exist communication strategies $c(\theta) = (c_i(\theta_i), c_j(\theta_j)) \in C(p)$ and output decisions of agents $q_i(\theta_i, h) : \Theta_i \times \mathcal{H} \to \Re_+$, such that $q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta))))$ for all $\theta \in \Theta$

Incentive-Feasibility

- A communication-feasible production allocation $\tilde{q}(\theta)$ is said to be *incentive-feasible* in a mechanism if there exists a Perfect Bayesian Equilibrium (PBE) of the game induced by the mechanism which implements the production allocation
- *Key Technical Problem:* characterization of incentive feasible production allocations

Characterizing Incentive Feasible Allocations

Lemma Given any strategy configuration $(c_1(\theta_i), c_2(\theta_2))$ and any history h_t until the end of round t in a communication protocol, the set of types (θ_1, θ_2) that could have generated the history h_t can be expressed as the Cartesian product of subsets $\Theta_1(h_t), \Theta_2(h_t)$ such that

$$\{(\theta_1, \theta_2) \mid h_t(c(\theta_1, \theta_2)) = h_t\} = \Theta_i(h_t) \times \Theta_j(h_t).$$

- Necessary condition for incentive-feasibility of a production allocation $q(\theta)$ which is communication-feasible in a protocol p and supported by communication strategies $c(\theta)$:
- For any $t = 1, \ldots, T$, any $h_t \in H_t$ and any $i = 1, 2$:

$$E[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)] \text{ is non-increasing in } \theta_i \text{ on } \Theta_i(h_t) \tag{1}$$

- *Notation:* H_t denotes the set of possible histories until round t generated with positive probability in the protocol when $c(\theta)$ is played, and $\Theta_i(h_t)$ denotes the set of types of i who arrive at h_t with positive probability under the communication strategies $c(\theta)$.

Sufficient Conditions in the Literature

- If we strengthen solution concept to *ex post incentive compatibility (EPIC)*, the following condition is necessary and sufficient (Van Zandt (2007), Fadel and Segal (2009)):

$$q_i(\theta_i, \theta_j) \text{ is globally non-increasing in } \theta_i \text{ for every } \theta_j \in \Theta_j$$

- Get this condition “for free” in specific single-round settings (Melumad, Mookherjee and Reichelstein (1992,1997), Blumrosen and Feldman (2006), Blumrosen, Nisan and Segal (2007), Kos (2011b))
- This property is not satisfied in Example 1
- Another sufficiency condition in Fadel and Segal (2009) for a centralized mechanism:

$$E[q_i(\theta_i, \theta_j) | \theta_j \in \Theta_j(h_t)] \text{ is globally non-increasing in } \theta_i$$

The Necessary Condition is Sufficient

Proposition

Condition (1) is sufficient for incentive-feasibility of a production allocation q(θ) *which is communication-feasible in a protocol p and supported by communication strategies c*(θ)*, provided the protocol is parsimonious with respect to c*(θ)*.*

- Any protocol can be pruned to make it parsimonious relative to a given set of strategies, which preserves feasibility
- Hence (1) is both necessary and sufficient for feasibility

Restating the Design Problem

- Since $\lambda_1 \geq 0$ it is optimal to set transfers that incentivize any given output allocation rule $q(\theta)$ satisfying (1) such that the expected payoff of the highest cost type $\bar{\theta}_i$ equals zero for each i
- The expected transfers to the agents then equal (*a la* Revenue Equivalence Theorem):

$$\Sigma_{i=1}^{2} E[v_i(\theta_i) q_i(\theta_i, \theta_j)]$$

where $v_i(\theta_i) \equiv \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$

Restating the Design Problem, contd.

- Resulting expected payoff of P:

$$E[V(q_i(\theta_i,\theta_j),q_j(\theta_i,\theta_j)) - w_i(\theta_i)q_i(\theta_i,\theta_j) - w_j(\theta_j)q_j(\theta_i,\theta_j)] \tag{2}$$

where $w_i(\theta_i) \equiv (\lambda_1+\lambda_2)\theta_i + \lambda_1 \frac{F_i(\theta_i)}{f_i(\theta_i)}$

Restating the Mechanism Design Problem

- Select a protocol $p \in \mathcal{P}$, communication strategies $c(\theta)$ in p and output allocation $q(\theta)$ to maximize

$$E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i)q_i(\theta_i, \theta_j) - w_j(\theta_j)q_j(\theta_i, \theta_j)]$$

- subject to:
  - (i) there exists a set of output decision strategies $q_i(\theta_i, h), i = 1, 2$ such that $q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta))))$ for all $\theta \in \Theta$,
  - (ii) the output allocation satisfies condition (1)
- Monotone hazard rate condition implies condition (ii) is redundant: IC constraints have no bite

The Main Result

Proposition

The mechanism design problem can be reduced to the following. Given the set $\mathcal{P}$ of feasible communication protocols defined by the communication constraints, select a protocol $p \in \mathcal{P}$, communication strategies $c(\theta)$ in p and output allocation $q(\theta)$ to maximize (2), subject to the constraint of communication feasibility alone, i.e., there exists a set of output decision strategies $q_i(\theta_i, h), i = 1, 2$ such that

$$q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta)))), \forall \theta \in \Theta. \qquad (3)$$

Implications

- Separation between incentive problem and communication constraints
- Can ignore incentive problem after incorporating costs of incentive rents into P's objective function
- Given any set of communication strategies in a given protocol, in state (θ_i, θ_j) agent i learns that θ_j lies in the set $\Theta_j(h(c_i(\theta_i), c_j(\theta_j)))$, which generates an information partition for agent i over agent j's type
- *Principle of Informational Efficiency:* protocols that generate finer partitions are better; hence select communication protocol and assignment of decision-making authority to maximize informational efficiency

Implications for Value of Decentralization

- Absent communication constraints, decentralized mechanisms cannot outperform centralized ones
- With communication constraints, there is a non-trivial tradeoff between more informed decision-making and attendant incentive problems
- Principle of Informational Efficiency: implies incentive problems have no additional bite, so it is better to let agents make output decisions

Truly Decentralized Mechanisms are Superior

Proposition

Suppose that (i) outputs of the two agents can be chosen independently ($Q = \Re_+ \times \Re_+$); and (ii) $V(q_1, q_2)$ is twice continuously differentiable, strictly concave and satisfies the Inada condition $\frac{\partial V}{\partial q_i} \to \infty$ as $q_i \to 0$. Then given any feasible centralized mechanism there exists a corresponding truly decentralized mechanism which generates a strictly higher payoff to the Principal.

Implications for Choice of Protocol, contd.

- Suppose agents send information in the form of 0-1 bits, and each bit takes one unit of time

Proposition

(i) Suppose either Communication Constraint 1 or 2 applies. Then an optimal protocol has the feature that only one agent sends messages in any given communication round.
(ii) Suppose Communication Constraint 3 applies, limiting the total delay to time taken to send D bits. Then the optimal protocol involves D rounds of communication with both agents simultaneously sending one bit of information in each round.