Ec717a: Incomplete Contracts and Complexity 1 (Segal RES1999)

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Introduction to Incomplete Contracts

- Coase (‘What is a Firm?’, 1937) first distinguished between ‘market transactions’ and ‘intra-firm transactions’
- e.g., difference between ‘outsourcing’ and ‘employment’ relationships
- outsourcing/market transactions: exchange of a specific service between a buyer and a seller (independent agents, belonging to separate firms)
- employment relationship: exchange of work for compensation between employer and employee within a single firm

Market versus Intra-firm Transactions

- compared to outsourcing, employment usually represents:
    - a longer term contract covering a larger range of tasks to be carried out
    - hierarchical relationship: employer has authority to specify the task needed to be performed at any date and contingency
    - an 'incomplete' contract, which does not specify upfront all the contingencies, required actions and associated payments in advance

Property Rights and Incomplete Contracts

- Notion of 'property right' over any asset: right to decide how the asset is to be used
- In a world with complete contracts, allocation of property rights is irrelevant (Revelation Principle argument; analogous to Modigliani-Miller Theorem in finance)
- Restate Revelation Principle: wlog attention can be restricted to a complete, comprehensive contract/mechanism which is incentive compatible, if there are no 'transaction costs', i.e., :
  - (a) all contingencies can be foreseen and costlessly written into contracts
  - (b) no costs of communication (messages from A to P) or computation (by P, given messages from A)
  - (c) P can commit to the contract; no renegotiation

Property Rights and Incomplete Contracts, contd.

- Most real-world contracts are incomplete owing to nontrivial transaction costs
- Do not incorporate or specify all relevant contingencies, allow for ex post renegotiation
- Given incomplete contracts, Grossman-Hart (1986) define ownership/property rights: ex post 'residual' rights of control over the asset, which can be allocated in different ways (buyer, seller, joint,..)
- In a world with incomplete contracts, allocation of property rights matters for allocative and productive efficiency

Williamson: Boundaries between Firms and Markets

- Williamson (1975, 85) develops a verbal, intuitive and somewhat vague theory of trade-offs between 'firm' and 'market' relationships, with three key ingredients:
  - 'transaction costs' of writing complete contracts
  - asymmetric information and opportunistic behavior
  - investment in specific assets
- With incomplete contracts and opportunistic behavior, ex post exchanges will be the result of 'renegotiation' involving bargaining/haggling
- Which can lead to 'hold-up', anticipation of which can generate ex ante under-investment in specific assets
- Under-investment and ex ante welfare depends on property right allocation (e.g., vertical integration vs. separate ownership): formalized by Grossman-Hart (1986) and Hart-Moore (1990)

Illustrative Model (Edlin-Reichelstein AER 1996)

- Buyer B, Seller S, single good to be exchanged, bilateral monopoly
- Ex ante contract specifies quantity q, price p
- Ex post Payoffs:
  - for B: $V(q, \theta) - pq$
  - for S: $pq - C(q, \theta, s) - s$ where s is specific investment and θ is state of the world
  - $V_q > 0, V_{qq} < 0,\ C_q > 0, C_{qq} < 0, C_s < 0, C_{qs} < 0$
- $q = 0$ implies $V = C = 0$, zero outside options
- B and S share common beliefs (F cdf over θ)
- Information about s and θ realization: symmetric, but non-verifiable by a third-party; p, q verifiable

Timing

- 1. Ex ante contract signed (zero ex ante outside options)
- 2. S chooses s, observed by B (but no one else)
- 3. θ realized, observed by B and S (but no one else)
- 4. B and S renegotiate the contract via Nash bargaining, with the ex ante contract as the status quo (no outside options)

Efficient Allocation

- $q^*(\theta, s)$ maximizes $[V(q, \theta) - C(q, \theta, s)]$, for all θ, s, so:

$$V_q(q^*(\theta, s), \theta) = C_q(q^*(\theta, s), \theta, s)$$

- s^* maximizes $E_\theta[V(q^*(\theta, s), \theta) - C(q^*(\theta, s), \theta, s)] - s$, so

$$-E_\theta[C_s(q^*(\theta, s^*), \theta, s^*)] = 1$$

- Since θ, s are non-verifiable, they cannot be written into the contract; $q^*(\theta, s^*)$ varies with θ so it is differs from any fixed q with high probability

Grossman-Hart-Moore Theory of Separate Ownership

- *Key Simplifying Assumption:* No ex ante contract between B and S can be written at stage 1
- Hence ex post (stage 4) they bargain, with null contract (no trade) as the status quo option
- Nash bargaining with symmetric information over θ, s results in ex post efficient quantity $q^*(\theta, s)$
- Payment from B to S splits gains from trade (costs of s are sunk and do not matter): $V(q^*(\theta, s), s) - P = P - C(q^*(\theta, s), \theta, s))$, so

$$P(\theta, s) = \frac{1}{2}[V(q^*(\theta, s), \theta) + C(q^*(\theta, s), \theta, s)]$$

Grossman-Hart-Moore Theory, contd.

- S then selects s^e to maximize

$$\begin{aligned}\Pi_S &\equiv E_\theta[P(\theta,s) - C(q^*(\theta,s),\theta,s)] - s \\ &= \frac{1}{2}E_\theta[V(q^*(\theta,s),\theta) - C(q^*(\theta,s),\theta,s)] - s\end{aligned}$$

- Underinvestment: $s^e < s^*$
- Hence separate ownership will result in too little investment in specific assets
- If S acquires B's firm, this inefficiency will be eliminated

Edlin-Reichelstein criticism

- Allow for a fixed ex ante contract $\bar{q}, \bar{P}$
- Renegotiation surplus ex post given (θ, s):

$$RS(s, \bar{q}, \theta) \equiv V(q^*(\theta, s), \theta) - C(q^*(\theta, s), \theta, s) - [V(\bar{q}, \theta) - C(\bar{q}, \theta, s)]$$

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- Resulting ex ante objective of S:

$$\begin{aligned} \Pi_S(\bar{q}) &\equiv E[\bar{P} - C(\bar{q}, \theta, s) - s + \frac{1}{2} RS(s, \bar{q}, \theta)] \\ &= \frac{1}{2} E[V(q^*(\theta, s), \theta) - C(q^*(\theta, s), \theta, s) - C(\bar{q}, \theta, s)] - s \\ &\quad + [\bar{P} - \frac{1}{2} V(\bar{q}, \theta)] \end{aligned}$$

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- FOC:

$$-\frac{1}{2} E[C_s(q^*(\theta, s^e), \theta, s^e) + C_s(\bar{q}, \theta, s^e)] = 1$$

Edlin-Reichelstein criticism, contd.

- Set $\bar{q}$ such that

$$E[C_s(\bar{q}, \theta, s^*)] = E[C_s(q^*(\theta, s^*), \theta, s^*)$$

- Then first-best investment will result: $s^e = s^*$
- Quantity $\bar{q}$ in the ex ante contract 'protects' S's interest, and prevents ex post hold-up by B
- Can set $\bar{P}$ to ensure ex ante participation incentives for both parties

Edlin-Reichelstein criticism, contd.

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- Quantity $\bar{q}$ in the ex ante contract 'protects' S's interest, and prevents ex post hold-up by B
- Can set $\bar{P}$ to ensure ex ante participation incentives for both parties
- So a simple ex ante uncontingent contract can ensure efficient outcomes; no need for S to acquire B's firm
- Result extends to a two-sided investment context, under a separability assumption

Segal 1999 Model

- Segal constructs a variation on this model, where quantity of trade cannot be varied (S provides 0 or 1 unit of the good, a widget)
- Instead, there are n different types of widgets
- Trade options: $w \in \{0, 1, \ldots, n\}$ where 0 denotes no widget is supplied, and i denotes one unit of widget of type i is supplied
- Buyers values: $\mathbf{v} \equiv (v_0 = 0, v_1, \ldots v_n)$, Seller's costs: $\mathbf{c} \equiv (c_0 = 0, c_1, \ldots, c_n)$
- Two-sided investment in specific assets: I, s for B and S respectively, affects values $v_i(I, \theta)$ and costs $c_i(s, \theta)$ observed mutually ex post

Segal model, contd.

- Ex post surplus

$$S(\mathbf{v}, \mathbf{c}) = \max_{w \in \{0,1,..,n\}} [v_w - c_w]$$

- Efficient investments:

$$(I^*, s^*) \in \arg\max_{I,s} E_\theta[S(\mathbf{v}(I, \theta), \mathbf{c}(s, \theta))] - I - s$$

- Ex post Payoffs (incorporating renegotiation) from ex ante contract stipulating trade of w at price P:

$$\begin{aligned} U_B &= -P + \frac{v_w + c_w}{2} + \frac{1}{2} S(\mathbf{v}, \mathbf{c}) \\ U_S &= P - \frac{v_w + c_w}{2} + \frac{1}{2} S(\mathbf{v}, \mathbf{c}) \end{aligned}$$

Equilibrium Investments given ex ante contract

- Suppose there is a fixed, uncontingent ex ante contract (w, P), as in Edlin-Reichelstein
- Ex ante payoff functions:

$$\begin{aligned} EU_B(w, P|I, s) &= -P + E[\frac{v_w + c_w}{2} + \frac{1}{2}S(\mathbf{v}, \mathbf{c})|I, s] - I \\ EU_S(w, P|I, s) &= P - E[\frac{v_w + c_w}{2} - \frac{1}{2}S(\mathbf{v}, \mathbf{c})|I, s] - s \end{aligned}$$

- Nash equilibrium investments $(I^e, s^e)(w, P)$

Preliminary Results

- Proposition 1 deals with consequences of no ex ante contract: there is underinvestment if investment is one-sided, also if it is two-sided and ex post efficient widget choice $w^*(\theta)$ (maximizing $v_w(I, \theta) - c_w(s, \theta)$) does not depend on investments (I, s)

Preliminary Results

- Proposition 1 deals with consequences of no ex ante contract: there is underinvestment if investment is one-sided, also if it is two-sided and ex post efficient widget choice $w^*(\theta)$ (maximizing $v_w(I,\theta) - c_w(s,\theta)$) does not depend on investments (I, s)
- Proposition 2: if a fully contingent contract specifying ex post widget choice $w^*(\theta)$ and price $P(\theta)$ can be written and enforced, efficient investments can be induced
- To rule this out, assume that investments, ex post valuations, and realization of θ are non-verifiable so such contracts are infeasible

Message Games

- Segal allows not only for choice of an *ex ante* contract as in Edlin-Reichelstein, but also for buyer and seller to make reports of $\theta, \mathbf{v}, \mathbf{s}$ to a court (third party enforcer of the contract) once they observe the state
- There can be a complicated contingent contract (status quo contract $w(m_S, m_B|\theta), P(m_S, m_B|\theta)$ where m_S, m_B denote reports)
- Followed by renegotiation, which generates actual trading outcome (efficient, IR bargaining outcome) given the status quo contract
- Contract/mechanism design problem with incentive compatibility constraints for truthful reporting to the court (Nash equilibrium reports)

Segal Example

- ex post, in any state θ: there is only one *reasonable* widget with (v, c), all others are either *gold-plated* with (v_g, c_g) or *bad* with (v_b, c_b), where:

$$\begin{aligned} (v - c) &> \max\{0, v_g - c_g, v_b - c_b\} \\ v_g + c_g &> v + c > v_b + c_b \end{aligned}$$

- Proposing g widget enhances bargaining position for B, b for S, but are ex post inefficient so will not end up being actually chosen following renegotiation
- Only (v, c) of reasonable widget can depend on investments
- Different widgets are reasonable in different states; ex post type of each widget observable (only) by B and S

Segal Example, contd.

- Ex ante, all widgets have equal probability $\frac{1}{n}$ of being the reasonable widget; same likelihood $(\frac{1}{2(n-1)})$ of being gold-plated or bad
- If $n = 2$, it is possible to achieve the first-best with a sequential mechanism where buyer proposes the widget to be traded at a fixed price; seller can accept this or reject it, incur a penalty and propose a different widget
- Complexity of the environment represented by n, number of types of widgets
- Characterize attainable outcomes as $n \to \infty$

Segal Example, contd.

- **Theorem 1:** as complexity $n \to \infty$, achievable payoffs/investment converge to payoffs/investment generated by no ex ante or status quo contract/mechanism

Segal Example, contd.

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- *Intuitive Reason:* As complexity rises, number of incentive compatibility constraints to be incorporated in the contract design problem explodes, and it is no longer possible for the court to screen/infer the actual ex post state and base payments on reports

How Robust is this Result?

- However, the result is delicate, because it does not extend if:
  - the number of goldplated widgets $g(n)$ is ex ante known (i.e., independent of θ) — allow seller to select $g(n)$ widgets to veto, then let buyer select
  - or if the proportion of reasonable widgets is asymptotically certain and bounded away from zero — allow seller to veto half the widgets, then buyer decides

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  - or if the proportion of reasonable widgets is asymptotically certain and bounded away from zero — allow seller to veto half the widgets, then buyer decides
- Segal argues this requires contract to be very complex, in the sense that seller has to describe a very large number of widgets to the court, in order to exercise the veto
- Theorem 2 then focuses on contracts with a finite upper bound to the number of possible reports sent by either party, and shows Theorem 1 continues to apply

Summary

- Providing a foundation for incomplete contracts is not easy
- Depends on what is allowed or incorporated in contract design problem (nature of 'transaction costs'):
  - enforcement problems (e.g., non-verifiability of agents' information by third-party enforcer)
  - renegotiation (essential, as shown by Rogerson (1992))
  - complexity costs (e.g., of writing contingencies, or sending lengthy messages to the court)
- Need **all** of these problems to be present simultaneously