Introduction

- So far we have abstracted from possible problems in contract enforcement
- Implicit assumption: existence of third party enforcers
- In particular, P has no problem committing to the terms of the contract
- In practice, many measures of performance on which agent’s compensation needs to be conditioned, are not verifiable by a third party
- Agent’s performance is evaluated by P, often in a subjective manner
P could then be tempted to provide a poor performance evaluation, in order to reduce payment to A.

Many examples of this provided by Levin: manager bonuses in First Boston bank in 1990s were lower than expected.

Owners claimed this was because of disappointing financial results, but managers were still upset and left the company.

Similar incident in 1994 in Goldman Sachs.
In such contexts, credible contracts have to incorporate P’s incentive constraints to abide by the terms of the contract.

How can P be disciplined?

Relational contract literature models the role of reputation: if P reneges, A will punish P by quitting or shirking in the future.

In other words, need to model long-term relationships between P and A, and has to be self-enforcing.

Especially relevant for contexts with poor legal system (developing countries) and illegal collusive side-contracts (‘honor among thieves’).
The model includes both a formal (fixed wage) and informal component (bonus based on evaluation) of agent’s compensation.

Fixed wage conditioned on whether A is employed (which is verifiable).

Agent’s performance is mutually observed by P and A, but not verifiable.

Model incorporates A’s private information (regarding productivity) and moral hazard (unobserved effort).
Introduction, contd.

- However the model is set up in such a way that private information or moral hazard do not create any distortions:
  - A is risk neutral
  - A observes productivity after agreeing to participate

- So all distortions arise owing to the requirement that the contract be self-enforcing

- The constraint limits the extent of variation in bonuses that P can credibly commit to

- This prevents attainment of first-best; specific distortions are different from the standard ones
Model

- Dates $t = 0, 1, 2, ..$; A delivers $y_t$ to P at $t$ where $y_t$ is stochastic and depends on effort $e_t$

- Conditional on $e_t$, $y_t$ is i.i.d, with cdf $F(\cdot | e_t)$

- A’s personal effort cost $c(e_t; \theta_t)$ where $\theta_t$ is i.i.d. productivity shock with cdf $P$ on $[\underline{\theta}, \bar{\theta}]$

- Contract: fixed wage $w_t$ conditional on employment, bonus $b_t$ which can depend on $\phi_t \equiv \{y_t, y_{t-1}, y_{t-2}, ..\}$

- Payoffs at $t$: $w_t + b_t - c(e_t; \theta_t)$ for A, $y_t - w_t - b_t$ for P; common discount factor $\delta \in (0, 1)$

- Outside option payoffs (per period): $\bar{u}$ for A, $\bar{\pi}$ for P
Stages

At date $t$:

1. P offers contract $w_t, b_t(.)$, can be positive or negative

2. A accepts ($d_t = 1$) or rejects ($d_t = 0$); if rejects they get $\bar{u}, \bar{\pi}$ resp, otherwise continue

3. A observes $\theta_t$, selects $e_t$

4. $y_t$ realized, observed by P and A (not anyone else)

5. P (resp A) decides whether to pay promised bonus (minus bonus) if it is positive (negative)
Equilibrium

- concept is PBE

- where P and A maximize continuation PV of future payoffs at each date, given history until $t$ (current and past contract offers, employment, performance, payments)

- Allows punishments for deviations from equilibrium play (eg if P reneges on promise to pay bonus, A could quit or shirk at later dates)
Equilibrium, contd.

- Strategies could be very complex

- However, model is set up so that (using results in theory of repeated games) attention can be restricted to a class of stationary contracts/strategies (on equi path)

- This result exploits the assumption of transferable utility (risk-neutrality)
**Optimal contracts**

A relational contract (specifying history dependent strategies at each $t$) is **(Pareto) optimal** if it maximizes sum of date 0 payoffs $\pi_0 + u_0$, over all PBEs where

$$\pi_t \equiv (1 - \delta)E \sum_{\tau=t}^{\infty} \delta^{\tau-1} \left[ d_\tau(y_\tau - w_\tau - b_\tau(\phi_\tau)) + (1 - d_\tau)\bar{\pi} \right]$$

$$u_t \equiv (1 - \delta)E \sum_{\tau=t}^{\infty} \delta^{\tau-1} \left[ d_\tau(w_\tau + b_\tau(\phi_\tau) - c_\tau(e_\tau; \theta_\tau)) + (1 - d_\tau)\bar{u} \right]$$

A contract is **stationary** if on the equilibrium path:

$w_t = w, b_t(\phi_t) = b(y_t), e_t = e(\theta_t)$ for all $t$
Optimal contracts, contd.

Proposition

*If there exists an optimal contract, there exists a stationary optimal contract*

- Uses results in theory of repeated games, based on dynamic programming (Abreu 1988): lack of one-step deviations at every stage followed by maximal punishments (worst continuation PBE)
- The maximal punishment here is *termination*, where both parties earn outside option payoffs thereafter
- Owing to linearity of utility in transfers, poor performance of A can be punished immediately, instead of spreading it out into the future
- Hence from tomorrow, A can start with a clean slate
Stationary Contracts

Stationary contract \((w, b(y), e(\theta))\) payoffs:

\[
\pi \equiv E_{\theta,y}[y - w - b(y) | e = e(\theta)] \\
u \equiv E_{\theta,y}[w + b(y) - c(e(\theta); \theta) | e = e(\theta)]
\]

Joint surplus: \(s \equiv E_{\theta,y}[y - c(e(\theta); \theta) | e = e(\theta)]\)
Self-Enforcing Constraints

- If \( b(y) > 0 \), \( P \) should not want to avoid paying this bonus:

  \[
  \sup_y b(y) \leq \frac{\delta}{1 - \delta} (\pi - \bar{\pi})
  \]

- If \( b(y) < 0 \), \( A \) should not want to avoid paying \(-b\):

  \[
  - \inf_y b(y) \leq \frac{\delta}{1 - \delta} (u - \bar{u})
  \]

- Adding these, we obtain a necessary condition (where \( \bar{s} \equiv \bar{u} + \bar{\pi} \))

  \[
  \frac{\delta}{1 - \delta} (s - \bar{s}) \geq \sup_y b(y) - \inf_y b(y)
  \]

  which limits maximal variation in bonus
Since there exists a side payment $w$ which enters linearly in payoffs with opposite signs, we can find $w$ to ensure IR constraints for both; hence this condition is necessary and sufficient for feasibility:

$$\frac{\delta}{1-\delta}(s - \bar{s}) \geq \sup_y b(y) - \inf_y b(y)$$  \hspace{1cm} (i)

A’s incentive constraint ($\forall \theta$):

$$e(\theta) \in \arg\max_e E_y[w + b(y) e] - c(e; \theta)$$  \hspace{1cm} (ii)
Special Case: Hidden Information

- Suppose $e$ is observable by $P$, in addition to $y$
- Stationary contracts are still optimal, while bonus $b$ is based on $\phi \equiv (y, e)$ rather than $y$ alone
- W.l.o.g. bonus depends on $e$ alone (conditional on effort, $y$ is pure noise and therefore uninformative)
- But $P$ cannot observe $\theta_t$: pure hidden information model
- Recall that $A$ observes $\theta_t$ after agreeing to participate, and is risk-neutral; hence in the standard setting with externally enforced contracts the first-best can be achieved (sell the firm to $A$)
- What distortion does the self-enforcing constraint impose, if any?
Optimal Stationary Contract with pure Hidden Info

The problem is to choose \((w, b(e), e(\theta))\) to maximize

\[
s \equiv \int_{\overline{\theta}}^{\theta} [E(y|e(\theta)) - c(e(\theta); \theta)]dP(\theta)
\]

subject to:

\[
\frac{\delta}{1 - \delta}(s - \overline{s}) \geq \sup_y b(y) - \inf_y b(y) \quad (i)
\]

\[
e(\theta) \in \arg \max_e E_y[w + b(y)|e] - c(e; \theta) \quad (ii)
\]
Optimal Stationary Contract with pure Hidden Info

- As in the standard setting, we can use a two-step method (recall Ec703): for given $e(\theta)$, manipulate constraints to substitute out the bonus function.

- Levin assumes:
  - $P(\theta)$ is concave (to ensure monotone hazard rate)
  - set of possible efforts is $[0, E]$
  - cost function satisfies: $c(0, \theta) = 0, c_e, c_{ee}, c_\theta, c_{e\theta} > 0, c_{\theta ee}, c_{\theta e\theta} \geq 0$
    (which would be satisfied if $c = \psi(\theta)\gamma(e)$ where $\psi, \gamma$ are strictly increasing and strictly convex, with $\gamma(0) = 0$)
Incentive Constraints

- A’s incentive constraint (ii) reduces to: $e(\theta)$ maximizes $b(e) - c(e; \theta)$ over the set \{ $e = 0$, or $e(\theta)$ for some $\theta \in [\underline{\theta}, \bar{\theta}]$ \}

- Using standard Envelope Theorem argument provides necessary condition (if $W(e) \equiv w + b(e)$, $U(\theta) \equiv W(e(\theta)) - c(e(\theta); \theta)$):

  $$U(\theta) \equiv W(e(\theta)) - c(e(\theta); \theta) = \int_{\theta}^{\bar{\theta}} c_\theta(e(\tau); \tau) d\tau + U(\bar{\theta})$$

- standard Revealed Preference argument provides another necessary condition: $e(.)$ is non-increasing
Also observe that since A could always deviate to zero effort which involves zero cost: $U(\bar{\theta}) \geq W(0) \equiv b(0)$ is also necessary.

Maximal variation in bonus is

$$b(e(\bar{\theta})) - b(0)(= W(\theta) - W(0)) \geq c(e(\theta); \bar{\theta}) + \int_{\theta}^{\bar{\theta}} c_{\theta}(e(\tau); \tau) d\tau$$

(\text{IC}')

Conversely, any non-increasing effort schedule satisfying (IC') can be implemented by some wage function $W(e)$ (e.g., set $W(e(\bar{\theta})) = c(e(\bar{\theta}); \bar{\theta}) + W(0)$)
Therefore constraints (i) and (ii) can be replaced by the single constraint:

\[
\frac{\delta}{1-\delta} (s - \bar{s}) \geq b(e(\theta)) - b(0) \geq c(e(\theta); \theta) + \int_{\theta}^{\bar{\theta}} c_\theta(e(\tau); \tau) d\tau \quad (C)
\]

And the problem reduces to maximizing

\[ s \equiv E_{\theta,y} \left[ y - c(e(\theta); \theta) \mid e = e(\theta) \right] \text{ subject to (C)} \]

If \( \delta \) is close enough to 1, the first-best can be implemented; so assume here onwards this is not possible (so (C) is binding)
Properties of the Optimal Contract

Proposition

In any second-best contract:

(a) effort is strictly smaller than the first-best for all \( \theta \),

(b) constant over \([\theta, \hat{\theta}]\) for some \( \hat{\theta} > \theta \), and strictly decreasing thereafter (if \( \hat{\theta} < \bar{\theta} \)). (Pooling)

(a) says effort is lowered for all types, in contrast to standard model where it remains first-best for \( \underline{\theta} \); Intuition:

- the effort (local) incentive constraint implies \( b'(e(\theta)) = c_e(e(\theta), \theta) \)
- \( c_{ee} > 0 \) implies that raising \( e \) above any \( e(\theta) \) requires the slope of \( b(\cdot) \) at \( e(\theta) \) to increase, thus raising \([b(e(\theta)) - b(0)]\) and causing (C) to be violated
- this applies at every \( \theta \)
Case of Pure Moral Hazard

- The other polar case is where $\theta$ does not vary, and $y$ equals $e$ plus some random noise.

- Levin assumes $e$ is continuous, $c_e, c_{ee} > 0$, and $F(y|e)$ satisfies MLRP and CDFC (ensures validity of first order condition approach to A’s incentive constraint).

- Shows that optimal contract involves just two levels of bonus $\bar{b} > b$ where $b(y) = \bar{b}$ iff $y$ is above some threshold $y^*$.

- Get such a ‘bang-bang’ solution partly because agent is risk neutral.
What if $y$ is privately observed by $P$, and not by $A$?

As in repeated games with private monitoring, it's much harder to sustain incentives (if $b(y) > b(y')$ for $y \neq y'$, $P$ would be tempted to misreport $A$’s performance as $y'$ when $y$ occurs)

Then may no longer be able to confine attention to stationary contracts (need to keep track of histories to check if $P$ is deviating)
Subjective Performance Measures, contd.

- One feasible way of providing incentives: A and P are **both** punished if P reports low performance

- For example: if \( y \) falls below some threshold A is paid \( w \) and relationship is terminated, otherwise paid \( w + b \) and relationship continues (and P is indifferent between reporting low and high performance)

- Such contracts are optimal within a special class of ‘full review’ contracts (see paper for details)