Ec717a: Relational Contracts (Levin, AER 2003)

Dilip Mookherjee

Boston University

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Introduction

- So far we have abstracted from possible problems in contract enforcement
- Implicit assumption: existence of third party enforcers
- In particular, P has no problem committing to the terms of the contract
- In practice, many measures of performance on which agent's compensation needs to be conditioned, are not verifiable by a third party
- Agent's performance is evaluated by P, often in a subjective manner

Introduction, contd.

- P could then be tempted to provide a poor performance evaluation, in order to reduce payment to A
- Many examples of this provided by Levin: manager bonuses in First Boston bank in 1990s were lower than expected
- Owners claimed this was because of disappointing financial results, but managers were still upset and left the company
- Similar incident in 1994 in Goldman Sachs

Introduction, contd.

- In such contexts, credible contracts have to incorporate P's incentive constraints to abide by the terms of the contract
- How can P be disciplined?
- Relational contract literature models the role of *reputation*: if P reneges, A will punish P by quitting or shirking in the future
- In other words, need to model long-term relationships between P and A, and has to be **self-enforcing**
- Especially relevant for contexts with poor legal system (developing countries) and illegal collusive side-contracts ('honor among thieves')

Introduction, contd.

- The model includes both a formal (fixed wage) and informal component (bonus based on evaluation) of agent's compensation
- Fixed wage conditioned on whether A is employed (which is verifiable)
- Agent's performance is mutually observed by P and A, but not verifiable
- Model incorporates A's private information (regarding productivity) and moral hazard (unobserved effort)

Introduction, contd.

- However the model is set up in such a way that private information or moral hazard do not create any distortions:
  - A is risk neutral
  - A observes productivity **after** agreeing to participate
- So all distortions arise owing to the requirement that the contract be self-enforcing
- The constraint limits the extent of variation in bonuses that P can credibly commit to
- This prevents attainment of first-best; specific distortions are different from the standard ones

Model

- Dates $t = 0, 1, 2, ..$; A delivers y_t to P at t where y_t is stochastic and depends on effort e_t
- Conditional on e_t, y_t is i.i.d, with cdf $F(.|e_t)$
- A's personal effort cost $c(e_t; \theta_t)$ where θ_t is i.i.d. productivity shock with cdf P on $[\underline{\theta}, \bar{\theta}]$
- Contract: fixed wage w_t conditional on employment, bonus b_t which can depend on $\phi_t \equiv \{y_t, y_{t-1}, y_{t-2}, ..\}$
- Payoffs at t: $w_t + b_t - c(e_t; \theta_t)$ for A, $y_t - w_t - b_t$ for P; common discount factor $\delta \in (0, 1)$
- Outside option payoffs (per period): $\bar{u}$ for A, $\bar{\pi}$ for P

Stages

At date t:

- 1. P offers contract $w_t, b_t(.)$, can be positive or negative
- 2. A accepts ($d_t = 1$) or rejects ($d_t = 0$); if rejects they get $\bar{u}, \bar{\pi}$ resp, otherwise continue
- 3. A observes θ_t, selects e_t
- 4. y_t realized, observed by P and A (not anyone else)
- 5. P (resp A) decides whether to pay promised bonus (minus bonus) if it is positive (negative)

Equilibrium

- concept is PBE
- where P and A maximize continuation PV of future payoffs at each date, given history until t (current and past contract offers, employment, performance, payments)
- Allows punishments for deviations from equilibrium play (eg if P reneges on promise to pay bonus, A could quit or shirk at later dates)

Equilibrium, contd.

- Strategies could be very complex
- However, model is set up so that (using results in theory of repeated games) attention can be restricted to a class of stationary contracts/strategies (on equi path)
- This result exploits the assumption of transferable utility (risk-neutrality)

Optimal contracts

- A relational contract (specifying history dependent strategies at each t) is **(Pareto) optimal** if it maximizes sum of date 0 payoffs $\pi_0 + u_0$, over all PBEs where

$$\pi_t \equiv (1-\delta)E\sum_{\tau=t}^{\infty}\delta^{\tau-1}\left[d_\tau(y_\tau - w_\tau - b_\tau(\phi_\tau)) + (1-d_\tau)\bar{\pi}\right]$$

$$u_t \equiv (1-\delta)E\sum_{\tau=t}^{\infty}\delta^{\tau-1}\left[d_\tau(w_\tau + b_\tau(\phi_\tau) - c_\tau(e_\tau;\theta_\tau)) + (1-d_\tau)\bar{u}\right]$$

- A contract is **stationary** if on the equilibrium path: $w_t = w, b_t(\phi_t) = b(y_t), e_t = e(\theta_t)$ for all t

Optimal contracts, contd.

Proposition

If there exists an optimal contract, there exists a stationary optimal contract

- Uses results in theory of repeated games, based on dynamic programming (Abreu 1988): lack of one-step deviations at every stage followed by maximal punishments (worst continuation PBE)
- The maximal punishment here is **termination**, where both parties earn outside option payoffs thereafter
- Owing to linearity of utility in transfers, poor performance of A can be punished immediately, instead of spreading it out into the future
- Hence from tomorrow, A can start with a clean slate

Stationary Contracts

- Stationary contract $(w, b(y), e(\theta))$ payoffs:

$$\begin{aligned}\pi &\equiv E_{\theta,y}\left[y - w - b(y)|e = e(\theta)\right]\\ u &\equiv E_{\theta,y}\left[w + b(y) - c(e(\theta);\theta)|e = e(\theta)\right]\end{aligned}$$

- Joint surplus: $s \equiv E_{\theta,y}\left[y - c(e(\theta);\theta)|e = e(\theta)\right]$

Self-Enforcing Constraints

- If $b(y) > 0$, P should not want to avoid paying this bonus:

$$\sup_y b(y) \leq \frac{\delta}{1-\delta}(\pi - \bar{\pi})$$

- If $b(y) < 0$, A should not want to avoid paying $-b$:

$$-\inf_y b(y) \leq \frac{\delta}{1-\delta}(u - \bar{u})$$

- Adding these, we obtain a necessary condition (where $\bar{s} \equiv \bar{u} + \bar{\pi}$:)

$$\frac{\delta}{1-\delta}(s - \bar{s}) \geq \sup_y b(y) - \inf_y b(y)$$

which limits maximal variation in bonus

Constraints, contd.

- Since there exists a side payment w which enters linearly in payoffs with opposite signs, we can find w to ensure IR constraints for both; hence this condition is necessary **and** sufficient for feasibility

$$\frac{\delta}{1-\delta}(s-\bar{s}) \geq \sup_y b(y) - \inf_y b(y) \qquad (i)$$

- A's incentive constraint $(\forall\theta)$:

$$e(\theta) \in \arg\max_e E_y[w + b(y)|e] - c(e;\theta) \qquad (ii)$$

Special Case: Hidden Information

- Suppose e is observable by P, in addition to y
- Stationary contracts are still optimal, while bonus b is based on $\phi \equiv (y, e)$ rather than y alone
- W.l.o.g. bonus depends on e alone (conditional on effort, y is pure noise and therefore uninformative)
- But P cannot observe θ_t: pure hidden information model
- Recall that A observes θ_t after agreeing to participate, and is risk-neutral; hence in the standard setting with externally enforced contracts the first-best can be achieved (sell the firm to A)
- What distortion does the self-enforcing constraint impose, if any?

Optimal Stationary Contract with pure Hidden Info

- The problem is to choose $(w, b(e), e(\theta))$ to maximize

$$s \equiv \int_{\underline{\theta}}^{\bar{\theta}} [E(y|e(\theta)) - c(e(\theta);\theta)] dP(\theta)$$

subject to:

$$\frac{\delta}{1-\delta}(s - \bar{s}) \geq \sup_y b(y) - \inf_y b(y) \qquad (i)$$

$$e(\theta) \in \arg\max_e E_y[w + b(y)|e] - c(e;\theta) \qquad (ii)$$

Optimal Stationary Contract with pure Hidden Info

- As in the standard setting, we can use a two-step method (recall Ec703): for given $e(\theta)$, manipulate constraints to substitute out the bonus function
- Levin assumes:
  - $P(\theta)$ is concave (to ensure monotone hazard rate)
  - set of possible efforts is $[0, E]$
  - cost function satisfies: $c(0, \theta) = 0$, c_e, c_{ee}, c_θ, $c_{e\theta} > 0$, $c_{\theta ee}, c_{\theta e\theta} \geq 0$ (which would be satisfied if $c = \psi(\theta)\gamma(e)$ where ψ, γ are strictly increasing and strictly convex, with $\gamma(0) = 0$)

Incentive Constraints

- A's incentive constraint (ii) reduces to: $e(\theta)$ maximizes $b(e) - c(e;\theta)$ over the set $\{e = 0, \quad \text{or} \quad e(\theta) \quad \text{for some} \quad \theta \in [\underline{\theta}, \bar{\theta}]\}$
- Using standard Envelope Theorem argument provides necessary condition (if $W(e) \equiv w + b(e)$, $U(\theta) \equiv W(e(\theta)) - c(e(\theta);\theta)$):

$$U(\theta) \equiv W(e(\theta)) - c(e(\theta);\theta) = \int_{\theta}^{\bar{\theta}} c_{\theta}(e(\tau);\tau)d\tau + U(\bar{\theta})$$

- standard Revealed Preference argument provides another necessary condition: $e(.)$ is non-increasing

Incentive Constraints, contd.

- Also observe that since A could always deviate to zero effort which involves zero cost: $U(\bar{\theta}) \geq W(0) \equiv b(0)$ is also necessary
- Maximal variation in bonus is

$$b(e(\underline{\theta})) - b(0)(= W(\underline{\theta}) - W(0)) \geq c(e(\underline{\theta}); \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} c_\theta(e(\tau); \tau) d\tau \tag{IC'}$$

- Conversely, any non-increasing effort schedule satisfying (IC') can be implemented by some wage function $W(e)$ (e.g., set $W(e(\bar{\theta})) = c(e(\bar{\theta}); \bar{\theta}) + W(0)$)

Optimal Contract w Pure Hidden Info: Characterization

- Therefore constraints (i) and (ii) can be replaced by the single constraint:

$$\frac{\delta}{1-\delta}(s-\bar{s})(\geq b(e(\underline{\theta}))-b(0)) \geq c(e(\underline{\theta});\underline{\theta})+\int_{\underline{\theta}}^{\bar{\theta}} c_{\theta}(e(\tau);\tau)d\tau \quad (C)$$

- And the problem reduces to maximizing $s \equiv E_{\theta,y}\left[y-c(e(\theta);\theta)|e=e(\theta)\right]$ subject to (C)
- If δ is close enough to 1, the first-best can be implemented; so assume here onwards this is not possible (so (C) is binding)

Properties of the Optimal Contract

Proposition

In any second-best contract:

(a) effort is strictly smaller than the first-best for **all** θ*,*

(b) constant over $[\underline{\theta}, \hat{\theta})$ *for some* $\hat{\theta} > \underline{\theta}$*, and strictly decreasing thereafter (if* $\hat{\theta} < \bar{\theta}$*). (***Pooling***)*

(a) says effort is lowered for all types, in contrast to standard model where it remains first-best for $\underline{\theta}$; Intuition :

- the effort (local) incentive constraint implies $b'(e(\theta)) = c_e(e(\theta), \theta)$
- $c_{ee} > 0$ implies that raising e above any $e(\theta)$ requires the slope of $b(.)$ at $e(\theta)$ to increase, thus raising $[b(e(\underline{\theta})) - b(0)]$ and causing (C) to be violated
- this applies at every θ

Case of Pure Moral Hazard

- The other polar case is where θ does not vary, and y equals e plus some random noise
- Levin assumes e is continuous, $c_e, c_{ee} > 0$, and $F(y|e)$ satisfies MLRP and CDFC (ensures validity of first order condition approach to A's incentive constraint)
- Shows that optimal contract involves just two levels of bonus $\bar{b} > \underline{b}$ where $b(y) = \bar{b}$ iff y is above some threshold y^*
- Get such a 'bang-bang' solution partly because agent is risk neutral

Extension: Subjective Performance Measures

- What if y is privately observed by P, and not by A?
- As in repeated games with private monitoring, its much harder to sustain incentives (if $b(y) > b(y')$ for $y \neq y'$, P would be tempted to misreport A's performance as y' when y occurs)
- Then may no longer be able to confine attention to stationary contracts (need to keep track of histories to check if P is deviating)

Subjective Performance Measures, contd.

- One feasible way of providing incentives: A and P are **both** punished if P reports low performance
- For example: if y falls below some threshold A is paid w and relationship is terminated, otherwise paid $w + b$ and relationship continues (and P is indifferent between reporting low and high performance)
- Such contracts are optimal within a special class of 'full review' contracts (see paper for details)