Ec717a: Introduction

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Background

- In Ec703 you have seen some examples of useful applications of mechanism design theory to auctions, public goods and bargaining (e.g., possibility of attaining efficient allocations, revenue maximizing auctions)
- Some common assumptions of those models:
  - agent i are privately informed about their *one dimensional* valuation v_i of a good
  - values $v_1, \ldots, v_n$ are independently distributed (*private values*)
  - prior beliefs of P(rincipal) and other agents over v_i are *common knowledge*

Background, contd.

- Equilibrium concept: noncooperative (Bayesian) equilibrium (ignore possibility of collusion among agents)
- Implementation notion: *'partial' implementation* (there is a Bayesian equilibrium resulting in a desired allocation): ignores the possibility that other equilibria may also exist
- Ignore issues of *complexity or communication costs*, which may necessitate 'simpler' mechanisms
- *Commitment:* implicitly assume that P can commit to following through with implementation of the mechanism

Extensions: Various Directions

- Subsequently there has been much effort devoted to extending the theory in various directions:
    - multidimensional valuations, multiple goods
    - ‘full’ implementation (all equilibria must result in desired allocations)
    - correlated valuations (interdependent values)
    - possible non-robustness to various details
    - incorporate costs of complexity, communication
    - prospect of collusion
- Other concerns (addressed later in this course): incorporate weaker commitment power for P (relevant especially in dynamic settings)

Structure of This Course

- Part 1 (two weeks): Interdependent Valuations, Robust Design
- Part 2 (two weeks): Complexity, Communication Costs
- Part 3 (two weeks): Delegation, Collusion
- Part 4 (two weeks): Dynamic Relational Contracts

Interdependent Values and Robust Design: Introduction

- Part 1: I shall focus on extensions to interdependent values, and robustness issues (esp. with respect to the common prior assumption)
- Ignore substantial literature on:
  - multidimensional mechanism design (difficult, technical)
  - full implementation (easy, but involves making mechanism more complex by augmenting message spaces)

Recap: Designing Efficient Auctions in Private Value Settings

- Suppose P is auctioning off an indivisible good (eg spectrum license) to n risk neutral bidders with independent private values $v_1, \ldots, v_n$
- i's payoff $v_i d_i - p_i$, where $d_i \in \{0, 1\}$ denotes whether the good is allocated to i, and p_i is net amount paid by i
- Common prior beliefs: v_i distributed with C^1 positive density f_i on $[\underline{v}_i, \bar{v}_i]$ (ignore ties)

Recap: Designing Efficient Auctions in Private Value Settings, contd.

- P's objective: efficiency, i.e., award the good to the bidder with the highest valuation: $d_i^*(v_1, \ldots, v_n) = 1$ if $v_i > \max_{j \neq i}\{v_j\}$, and 0 otherwise
- P has not have own reserve value for the good; does not care about revenues raised;
- zero outside option payoff for all bidders (need to ensure voluntary participation)

Designing Efficient Auctions in Private Value Settings: The Problem

- **Problem of (Partial) Bayesian Implementation (given beliefs $\{f_i\}$):** Does there exist a sealed-bid auction $(d_i(b_1, \ldots, b_n), p_i(b_1, \ldots, b_n))$ which yields a Bayesian equilibrium given beliefs $\{f_i\}$ which results in an efficient outcome in all states $(v_1, \ldots, v_n)$?

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- If yes:
  - how sensitive is this equilibrium to the beliefs ?
  - how sensitive is the auction design to these beliefs?
- The first question is about **existence** of an implementing mechanism, the subsequent ones are concerns about its **robustness** to changes in beliefs

Robustness Criteria

- **Definitions:**
  - The Bayesian equilibrium $\{b_i(v_i)\}_i$ is **robust** with respect to prior beliefs if it is an equilibrium for *every* possible set of beliefs $\{\tilde{f}_i\}_i$.

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  - The Bayesian implementation is **robust** if there is a robust Bayesian equilibrium which results in an efficient outcome in all states.
- **Observation** *With private values, a Bayesian equilibrium is robust if and only if it is a dominant strategy equilibrium.* (proof of only if part is straightforward: consider degenerate beliefs concentrated on any state, repeat for all states)
- Hence the problem of (partial) robust Bayesian implementation reduces to problem of (partial) dominant strategy implementation: does there exist an auction $\{d_i(.), p_i(.)\}_i$ which has a dominant strategy equilibrium that results in efficient outcomes in all states?

Dominant Strategy Implementation of Efficient Outcomes with Private Values

- Vickrey (second-price) auction: awards the good to the highest bidder $d_i = 1$ iff $b_i > \max_{j \neq i}\{b_j\}$, who is required to pay the second highest bid ($p_i = \max_{j \neq i}\{b_j\}$) and others pay nothing
- Each bidder has a dominant strategy: bid truthfully ($b_i = v_i$), which results in an efficient allocation

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- Each bidder has a dominant strategy: bid truthfully ($b_i = v_i$), which results in an efficient allocation
- However, the argument uses the private values assumption (each bidder **knows** own valuation, does not vary with other's valuation)

Generalizing to Interdependent Values (Dasgupta-Maskin QJE 2000)

- Suppose $v_i = v + \epsilon_i$ where v is an unknown common value component distributed according to some density f on $[\underline{v}, \bar{v}]$, and ϵ_i's are independent private value components
- Bidder i's information: signal $s_i = v + \delta_i$ of the common value, with independent noise δ_i
- Bidder i's valuation depends on own signal s_i as well as of others s_{-i}, but observes only s_i
- Reformulate state of the world: $(s_1, \ldots, s_n)$, where i is privately informed about s_i, and has valuation $v_i(s_i, s_{-i})$ where $\frac{\partial v_i}{\partial s_i} > 0$
- In this context, also true that $\frac{\partial v_i}{\partial s_j} > 0$, but DM do not impose this (I shall, to simplify)

Efficient Auctions with Interdependent Values, contd.

- For now, assume common prior beliefs over $(s_1, \ldots, s_n)$, and these are **not** independent (i's beliefs over s_{-i} will depend on s_i)
- Bayesian equilibrium bidding strategies in a sealed bid auction given these beliefs: $b = b_i(s_i)$ maximizes

$$E_{s_{-i}|s_i}[v_i(s_i, s_{-i})d_i(b, b_{-i}(s_{-i})) - p_i(b, b_{-i}(s_{-i}))]$$

- Can no longer define dominant strategy in the usual 'belief-free' manner: because i does not 'know' own true value of the good, and would learn from the bids of the others
- i cannot disregard the information of other bidders; beliefs over this information is needed in order to formulate i's objective

Ex Post Equilibrium

- We can extend the definition of dominant strategy equilibrium, however, by requiring optimality of bidding strategies in every state of the world (rather than irrespective of bids of others)

- $\{b_i(s_i)\}_i$ is an **Ex Post equilibrium (EPE)** if $b = b_i(s_i)$ maximizes

$$v_i(s_i, s_{-i})d_i(b, b_{-i}(s_{-i})) - p_i(b, b_{-i}(s_{-i}))$$

  for every possible state (s_i, s_{-i})

- **Observation** *With interdependent values, a Bayesian equilibrium is robust if and only if it is an EPE.*

Robust Implementation with Interdependent Values

- Dasgupta-Maskin (2000) show answer is yes, provided the following (Monotonicity (M)) assumption holds:

$$\frac{\partial v_i}{\partial s_i} > \frac{\partial v_j}{\partial s_i}(i \neq j) \quad \text{whenever} v_i = v_j = \max_k \{v_k\}$$

- This condition is also necessary (if we have $<$ instead, an efficient EPE equilibrium does not exist)
- A **generalized Vickrey** auction, in which bidders submit bid (conditional valuation) functions $b_i(v_{-i})$ where v_{-i} is the vector of bids of others

Generalized Vickrey Auction

- Illustrate in the case of two bidders: bidder $i = 1, 2$ submits $b_i(v_j)$, which must satisfy $|\frac{\partial b_i}{\partial v_j}| < 1$
- P calculates fixed point $(v_1^0, v_2^0) = (b_1(v_2^0), b_2(v_1^0))$ if one exists, otherwise does not allocate the good
- M ensures fixed point, if it exists, is unique
- The good is awarded to the bidder with a higher valuation: e.g., 1 wins if $v_1^0 > v_2^0$, and pays v_1^* which is a fixed point of $b_2(.)$, i.e., $v_1^* = b_2(v_1^*)$.
- Generalizes second price auction in the following sense: if 1 were constrained to a constant bid, v_1^* is the minimum bid at which 1 would win the good

Truthful Bidding is an EPE: Proof

- Suppose 2 bids truthfully: $b_2(v_1(s_1, s_2))) = v_2(s_1, s_2)$ for all (s_1, s_2)
- Take any state (s_1, s_2), suppose 1 knows the state: show that it is optimal for 1 to also submit a truthful bid $b_1(v_2(s_1, s_2)) = v_1(s_1, s_2))$
- **Observation 1:** If both bid truthfully, good will be allocated efficiently
- **Observation 2:** Conditional on winning, bidder 1's payoff is $v_1(s_1, s_2) - v_1^*$, independent of what he bid (depends only on v_1^* which depends on 2's strategy)
- Hence it suffices to show that truthful bidding will result in 1 winning if and only if $v_1(s_1, s_2) - v_1^*$ is positive

Truthful Bidding is an EPE: Proof (contd)

- $v_1(s_1, s_2) - v_1^*$ is positive iff $v_1(s_1, s_2) > v_1^*$, iff

$$b_2(v_1(s_1, s_2)) - b_2(v_1^*) < v_1(s_1, s_2) - v_1^*$$

(given restriction on slope of bid functions)

- Since $b_2(v_1^*) = v_1^*$, and 2 bids truthfully this is equivalent to

$$b_2(v_1(s_1, s_2)) = v_2(s_1, s_2) < v_1(s_1, s_2)$$

which is the outcome of truthful bidding by 1 when 1 wins