Ec717 PROBLEM SET 2

1. You have been appointed arbitrator for negotiations for sale of an indivisible input produced by a selling firm S to a buying firm B with *ex ante* uncertain costs and valuations. They respectively have payoffs $t - q\theta_s$ and $q\theta_b - t$ if $q \in \{0, 1\}$ denotes whether or not a sale takes place, and t is the expected monetary transfer from B to S. It is common knowledge that θ_s and θ_b are drawn independently from an interval $[\underline{\theta}, \bar{\theta}]$ accordingly to cdfs F_s, F_b with positive density functions f_s, f_b. Each firm will privately observe its own cost or valuation parameter; neither you or the other firm know the realization of this parameter.

(a) Suppose you selected a sealed-bid double auction (where each party reports their respective valuations, trade occurs if and only if the buyer reports a higher valuation, at a price which is the average of the two reports). Derive first-order conditions for truthful reporting to constitute a Bayesian equilibrium of this game.

(b) Is truthful reporting a Bayesian equilibrium?

(c) Suppose both distributions are uniform on $[0, 1]$. Derive a Bayesian equilibrium in which reports are linear and increasing in true valuations. When is the outcome of this equilibrium inefficient *ex post*?

(d) Is it possible to design some other trading rule which is *ex post* efficient, Bayesian incentive compatible and *ex ante* individually rational (i.e., both parties will be willing to participate if they have to commit to participation **before** observing their respective costs/valuations)? If not, provide a proof. If yes, construct a mechanism with the required properties.

2. You want to sell an indivisible object which you personally do not value. There are two potential bidders, with independent private values. Bidder 1's value is drawn uniformly over $[0, 1]$, while bidder 2's value is drawn uniformly over $[\alpha, 1 - \alpha]$, where $\alpha \in (\frac{1}{3}, \frac{1}{2})$. Derive the optimal expected revenue maximizing sealed-bid auction in which bidders have incentives to report their true valuations. Describe the way that the bidding rules in the optimal auction 'favor' one bidder over another, and provide some intuition for this result.