

EC 717a, PROBLEM SET NO. 1

1. A monopolist wishes to sell a good produced at constant unit cost $c \in (0, 1)$ to a large population of consumers with heterogeneous preferences: a consumer of type θ has a payoff $\theta \log(q + 1) - t$ for consuming $q \geq 0$ units of the good and paying t dollars for it. The monopolist cannot identify the type of any given consumer. Each customer has an outside option of 0.

(a) Suppose θ can take three possible values 3, 2, 1, with probabilities $\beta_3, \beta_2, \beta_1 > 0$ satisfying $\beta_3 + \beta_2 + \beta_1 = 1$. Provide conditions on parameters ensuring that

(i) the solution is interior and fully separating, i.e., $q_3 > q_2 > q_1 > 0$.

(ii) the solution is interior ($q_3, q_2, q_1 > 0$) and not fully separating. In the latter case show that the solution must entail $q_3 > q_2 = q_1$.

(b) Now consider the case where θ is distributed uniformly on the interval $[0, 1]$.

(i) If $q(\theta)$ denotes the quantity sold to type θ , find a condition on this function $q(\cdot)$ that ensures that it is IC (incentive compatible, i.e., there exists some pricing rule $t(q)$ for which $q(\theta)$ is the optimal purchase of type θ).

(ii) For any such IC $q(\cdot)$, what is the associated set of payments (i.e., $t(\theta)$) that customers (of type θ) make to the monopolist?

(iii) Obtain an expression for total profit of the monopolist as a function only of the selling strategy $q(\cdot)$ and payoff of consumer of type 0.

(iv) Calculate the optimal selling strategy $q^*(\theta)$, and find the corresponding schedule of payments $t^*(\theta)$.

(v) Find the payment rule $t(q)$ that implements this outcome, i.e., where a consumer of type θ selects $q^*(\theta)$ to maximize $\theta \log[1 + q] - t(q)$ and $t^*(\theta) = t(q^*(\theta))$. Does the optimal nonlinear pricing rule involve unit price discounts or premia for high q purchases?