Final Examination

Economics 703

Spring 2018

Instructions: Answer **TWO** of the following questions. The questions are equally weighted. None of the questions are intentionally misleading, so if you are not sure what the question is looking for, please ask me.

1. Consider the following two bidder auction with an externality. If agent 1 gets the good with probability y_1 and receives transfer t_1 (or pays price $-t_1$), her payoff is $y_1\theta_1 + t_1$ where θ_1 is her type. If agent 2 gets the good with probability y_2 and receives transfer t_2 (or pays price $-t_2$), her payoff is $y_2\theta_2 - \frac{1}{2}y_1\theta_1 + t_2$. In other words, agent 2 is hurt by agent 1's purchasing the good, hurt more the larger is 1's type. Assume the types are independent and that $\theta_i \sim U[0,1]$ for $i = 1, 2$. Assume the seller uses a first–price auction.

(a) Show that there is an equilibrium where each agent i bids $\alpha_i\theta_i$ and find α_1 and α_2.

(b) Is the equilibrium outcome ex post efficient? (Big hint: Compute the total payoffs to the two agents and the seller as a function of y_1. What is the optimal y_1 and how does this compare to the equilibrium?)

2. Consider two agents, 1 and 2, in a public goods setting. There is a public good which can either be provided or not. The cost of providing the good is 1. The value to i of having the public good is θ_i where the common prior is that the θ_i's are iid $U[0,1]$. More specifically, if the outcome is that the public good is provided and i receives a transfer of t_i (pays $-t_i$), then i's payoff is $\theta_i + t_i$. If the public good is not provided and i receives a transfer of t_i (pays $-t_i$), then i's payoff is t_i.

To specify a mechanism, let $y(\theta_1, \theta_2)$ be the probability the good is provided as a function of the types and let $t_i(\theta_1, \theta_2)$ be the transfer to i as a function of the types.

Suppose the mechanism achieves the efficient public good provision in the sense that

$$y(\theta_1, \theta_2) = \begin{cases} 1, & \text{if } \theta_1 + \theta_2 > 1; \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the expected transfer to agent i as a function of θ_i — that is, $\bar{t}_i(\theta_i) = \mathrm{E}_{\theta_j} t_i(\theta_1, \theta_2)$.

(b) Assume this mechanism is individually rational in the sense that $\mathcal{U}_i(\theta_i) \geq 0$ for all θ_i where $\mathcal{U}_i(\theta_i)$ is θ_i's expected utility in the mechanism. What is the largest amount type $\theta_i = 1$ could expect to pay? What is the probability that $\theta_i = 1$ expects the good to be provided?

(c) Use your answers to (b) to show that any such mechanism requires outside funding. That is, this is impossible if we require $-t_1(\theta_1, \theta_2) - t_2(\theta_1, \theta_2) \geq 1$ whenever $y(\theta_1, \theta_2) = 1$.

3. A principal hires an agent to work for him. The agent has two possible effort levels, e_L and e_H. Effort e_L costs her nothing, while effort e_H costs her c in utility. Assume $c \in (0, 10)$. The agent maximizes the wage she receives minus her effort cost.

The principal receives profits over two periods as a function of the agent's effort. If the agent chooses e_L, the principal receives profits of 15 in each of the two periods. If the agent chooses e_H, the principal receives profits of 15 in each period with probability 1/2 and profits of 25 in each period with probability 1/2. That is, the stream of profits is either $(15, 15)$ or $(25, 25)$, each with probability 1/2.

The principal must pay the agent at the end of the first period, after seeing the first period profits but before seeing the second.

If the agent does not work for the principal, she earns utility $\bar{u}$. The principal does not observe the agent's effort.

(a) What is the optimal contract if the principal wants to induce effort e_L? If he wants to induce e_H? Given this, what is the best contract for the principal?

(b) Now assume the agent has a third possible effort level, $\hat{e}$. If she chooses effort $\hat{e}$, the principal receives profits of 25 in the first period and 0 in the second. The effort cost for this action is $\hat{c} \in (0, c)$. Again, the principal does not observe the agent's effort and must pay the agent after seeing the first period profits but before seeing the second. Now what is the optimal contract for the principal to offer?