# Mechanism Design: Bargaining

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# The Bargaining Problem

- Two agents: S, seller and B, a prospective buyer, of an indivisible good
- They know their own valuations of the good: $\theta_s \in [\underline{\theta}_s, \bar{\theta}_s], \theta_b \in [\underline{\theta}_b, \bar{\theta}_b]$
- Common knowledge that $\theta_b, \theta_s$ are drawn independently according to cdf's $F_s, F_b$
- $x$: probability of sale, $p$ price in the event of a sale
- Payoffs $U_S \equiv (p - \theta_s)x, U_B \equiv (\theta_b - p)x$
- Trade must be voluntary: each agent has the option not to participate (attain 0 payoff from $x = 0$)

# Negotiations and Haggling

- Most actual bargaining situations involve a dynamic negotiation game
- E.g. the seller offers to sell at an asking price, the buyer responds by saying yes, or refuses and makes a counteroffer, to which the seller responds...
- Suppose game ends at each round with a fixed probability $q$
- Can study the outcome of a perfect Bayesian equilibrium of this game
- Each agent will tend to keep negotiating for a 'better' price, so the game may end without any sale occurring, despite the existence of gains from trade ($\theta_b > \theta_s$)

# Chatterjee-Samuelson Bargaining Game (F-T Chapter 6, Example 6.4)

- Chatterjee-Samuelson (1983) studied a 'double auction' game with one round of simultaneous offers, where both valuations are uniform on $[0, 1]$
- Buyer submits a bid $\tilde{\theta}_b$, seller asks for $\tilde{\theta}_s$; trade occurs iff the bid exceeds the asking price, at a price equal to their average ($p = \frac{\tilde{\theta}_b + \tilde{\theta}_s}{2}$)
- A Bayesian equilibrium where bids and asks are linear in the true valuations: $\tilde{\theta}_b = \frac{1}{12} + \frac{2}{3}\theta_b; \tilde{\theta}_s = \frac{1}{4} + \frac{2}{3}\theta_s$
- Trade occurs iff $\theta_b - \theta_s \geq \frac{1}{4}$
- If $\frac{1}{4} > \theta_b - \theta_s > 0$, there is no sale despite the existence of gains from trade

# Scope for Designing the Bargaining Game

- Maybe there is scope for reducing the inefficiency, by adding more rounds, or going to a sequential procedure...?
- Could a negotiation game be designed which always generates efficient outcomes in all possible states?
- Difficult to use a trial and error process to answer this question, there are infinite number of possible negotiation games
- Can cut through this problem, using the Revelation Principle!
- RP states that if there exists an efficient negotiation protocol, there must also exist a static revelation mechanism which results in efficient trade and satisfies the Partiicipation Constraint (PC)

# Bargaining Revelation Mechanisms

- In a revelation mechanism, buyer and seller simultaneously report $\tilde{\theta}_s, \tilde{\theta}_b$, which determines $x(\tilde{\theta}_s, \tilde{\theta}_b), t_s(\tilde{\theta}_s, \tilde{\theta}_b), t_b(\tilde{\theta}_s, \tilde{\theta}_b)$, where $t_s, t_b$ denote expected transfers to (from) the seller (buyer)
- (if trade probability is $x^* \equiv x(\tilde{\theta}_s, \tilde{\theta}_b)$, price in event of trade is $p^* \equiv p(\tilde{\theta}_b, \tilde{\theta}_s)$ and there is no broker commission or entry fee, then $t_s(\tilde{\theta}_s, \tilde{\theta}_b) = p^* x^* = -t_b(\tilde{\theta}_s, \tilde{\theta}_b)$)
- (Interim) Payoffs:

$$\begin{aligned} U_s(\tilde{\theta}_s; \theta_s) &\equiv E_{\theta_b}[t_s(\tilde{\theta}_s, \theta_b) - \theta_s x(\tilde{\theta}_s, \theta_b)] \\ U_b(\tilde{\theta}_b; \theta_b) &\equiv E_{\theta_s}[\theta_b x(\theta_s, \tilde{\theta}_b) - t_b(\theta_s, \tilde{\theta}_b)] \end{aligned}$$

# Bargaining Revelation Mechanisms, contd.

- BB: $t_s(\theta_s, \theta_b) + t_b(\theta_s, \theta_b) = 0$ for all $\theta_b, \theta_s$
- PE: Sale occurs (does not occur) ($x = 1(0)$) if $\theta_b > (<)\theta_s$
- PC: $U_b(\theta_b; \theta_b) \geq 0$, $U_s(\theta_s; \theta_s) \geq 0$ for all $\theta_b, \theta_s$
- BIC: $\tilde{\theta}_b = \theta_b$ maximizes $U_b(\tilde{\theta}_b; \theta_b)$, $\tilde{\theta}_s = \theta_s$ maximizes $U_s(\tilde{\theta}_s; \theta_s)$, for all $\theta_b, \theta_s$
- **The Problem:** Does there exist a mechanism satisfying BB, PE, PC and BIC?

# Connection with the Public Good Problem

- We can reformulate it as a 'public decision' problem: $d \equiv x; V_S = -x\theta_S + t_S, V_B = x\theta_B + t_B$
- The ADAV Theorem states that there does exist a set of balanced budget transfers that implement the PO allocation (where truthful reporting of valuations by both agents constitutes a Bayesian equilibrium)
- But what about the Participation Constraint?
- There is no PC in the public goods problem – payment of taxes is not voluntary for most people!

# Cases where Efficient Bargaining Mechanisms Exist

- Suppose there are gains from trade with probability one ($\bar{\theta}_s < \underline{\theta}_b$): set $x \equiv 1$ and $p = \frac{\bar{\theta}_s + \underline{\theta}_b}{2}$, $t_s = p - \bar{\theta}_s$, $t_b = -t_s$
- Suppose there are gains from trade with probability zero ($\bar{\theta}_b < \underline{\theta}_s$): set $x \equiv 0 \equiv t_s \equiv t_b$

# Myerson-Satterthwaite Theorem

**Theorem**

*Suppose there are gains from trade with positive probability less than one ($\bar{\theta}_s > \underline{\theta}_b, \bar{\theta}_b > \underline{\theta}_s$), and $F_s, F_b$ have positive densities $f_s, f_b$ at every interior state $(\theta_s, \theta_b)$. Then there does not exist any bargaining mechanism satisfying BB, BIC, PE and PC.*

# Proof of M-S Theorem

In an efficient mechanism, $x(\theta_b, \theta_s) = 1$ iff $\theta_b > \theta_s$ (ignoring measure zero states where $\theta_b = \theta_s$), hence:

$$U_b(\tilde{\theta}_b; \theta_b) = \theta_b F_s(\tilde{\theta}_b) - T_b(\tilde{\theta}_b), U_s(\tilde{\theta}_s; \theta_s) = T_s(\tilde{\theta}_s) - \theta_s[1 - F_b(\tilde{\theta}_s)]$$

(where $T_s(\theta_s) \equiv E_{\theta_b} t_s(\theta_s, \theta_b)$; $T_b(\theta_b) \equiv E_{\theta_s} t_b(\theta_s, \theta_b)$)

BIC for buyer requires (using Mirrlees-Myerson characterization of IC constraint in single agent problems from L2):

$$U_b(\theta_b; \theta_b) \equiv \theta_b F_s(\theta_b) - T_b(\theta_b) = \underline{\Pi}_b + \int_{\underline{\theta}_b}^{\theta_b} F_s(\tilde{\theta}_b) d\tilde{\theta}_b$$

(where $\underline{\Pi}_b \equiv U_b(\underline{\theta}_b; \underline{\theta}_b)$)

# M-S Proof, contd.

BIC implies:

$$T_b(\theta_b) = \theta_b F_s(\theta_b) - \int_{\underline{\theta}_b}^{\theta_b} F_s(\tilde{\theta}_b) d\tilde{\theta}_b - \underline{\Pi}_b$$

$$T_s(\theta_s) = \theta_s[1 - F_b(\theta_s)] + \int_{\theta_s}^{\bar{\theta}_s} [1 - F_b(\tilde{\theta}_s)] d\tilde{\theta}_s + \bar{\Pi}_s$$

(where $\bar{\Pi}_b$ denotes exp payoff of seller of type $\bar{\theta}_s$)

BB requires $E_{\theta_b} T_b(\theta_b) = E_{\theta_s} T_s(\theta_s)$, or

$$\begin{aligned} E_{\theta_b}[\theta_b F_s(\theta_b) \quad &- \quad \int_{\underline{\theta}_b}^{\theta_b} F_s(\tilde{\theta}_b) d\tilde{\theta}_b] - \underline{\Pi}_b \\ &= \quad E_{\theta_s}[\theta_s[1 - F_b(\theta_s)] + \int_{\theta_s}^{\bar{\theta}_s} [1 - F_b(\tilde{\theta}_s)] d\tilde{\theta}_s] + \bar{\Pi}_s \end{aligned}$$

# M-S Proof, contd.

$$\begin{aligned} E_{\theta_b}[\theta_b F_s(\theta_b) &- \int_{\underline{\theta}_b}^{\theta_b} F_s(\tilde{\theta}_b) d\tilde{\theta}_b] \\ &- E_{\theta_s}[\theta_s[1 - F_b(\theta_s)] + \int_{\theta_s}^{\bar{\theta}_s} [1 - F_b(\tilde{\theta}_s)] d\tilde{\theta}_s] \\ &= \underline{\Pi}_b + \bar{\Pi}_s \geq 0 \end{aligned}$$

(since PC requires $\underline{\Pi}_b, \bar{\Pi}_s \geq 0$)

On the other hand, Integrating LHS by parts, it equals (Check!)

$$-\int_{\underline{\theta}_b}^{\bar{\theta}_s} F_s(\theta)[1 - F_b(\theta)] d\theta$$

which is negative since $\bar{\theta}_s > \underline{\theta}_b$.