Mechanism Design: Public Goods

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Public Goods and The Free Rider Problem

- Consider the problem of P making a public decision or selecting how much public good to provide ($d \in \mathcal{D}$)
- Citizen i's utility $V_i(d;\theta_i) + t_i$, $i = 1, \ldots, n$, where $V_i(d;\theta_i) \equiv U_i(d;\theta_i) - \frac{1}{n}C(d)$
- P's objective: maximize $\sum_i V_i(d;\theta_i) + \sum_i t_i$, subject to $\sum_i t_i \leq 0$
- Efficient allocation: $\sum_i t_i \equiv 0$, $d^F(\theta)$ maximizes $\sum_i V_i(d;\theta_i)$ (Samuelson rule $\sum_i U_i'(d^F(\theta);\theta_i) = C'(d^F(\theta))$)
- *Free Rider problem:* P does not know θ_i, and citizens may not be willing to to reveal their valuations truthfully

Free Rider Problem

- For instance suppose d is a continuous variable, and all relevant functions are differentiable
- In the mechanism $d^F(\theta)$ and equal cost sharing ($t_i \equiv 0$), citizen i will report her true type θ_i truthfully only if

$$\frac{\partial U_i(d^F(\theta); \theta_i))}{\partial d} = \frac{1}{n} C'(d^F(\theta))$$

- E.g. if $U_i = \theta_i \log d$ and $C(d) = cd$, then $d^F(\theta) = \frac{\sum_j \theta_j}{c}$ and we would need $\frac{\theta_i}{\sum_j \theta_j} = \frac{1}{n}$ for all θ, i.e., all citizens must value the good equally with probability one.
- So if (with some probability) some citizens have higher marginal utility than others, individual optimality will be inconsistent with the Samuelson rule for Pareto optimality

Free Rider Problem, contd.

- Could try to adjust for this, by assigning higher cost shares to those that value the public good more
- Is there a way of designing the supplementary transfers t_i as a function of reported types, so as to ensure that all citizens will want to reveal their types truthfully, in all states of the world?
- Ensure truthful reporting is a Bayesian Nash equilibrium, or better still a dominant strategy equilibrium

The Vickrey-Clarke-Groves (VCG) Mechanism

Proposition

The following transfers ensure truthful reporting is a dominant strategy equilibrium:

$$t_i(\theta_i, \theta_{-i}) = \sum_{j \neq i} V_j(d^F(\theta_i, \theta_{-i}), \theta_j) + h_i(\theta_{-i}) \qquad (VCGT)$$

where $h_i(.)$ is an arbitrary function of θ_{-i}.

The Vickrey-Clarke-Groves (VCG) Mechanism, contd.

Proof: Given arbitrary reports θ_{-i} of others, consider i's problem with true type θ_i: report $\tilde{\theta}_i$ to maximize

$$V_i(d^F(\tilde{\theta}_i, \theta_{-i}), \theta_i) + \sum_{j \neq i} V_j(d^F(\tilde{\theta}_i, \theta_{-i}), \theta_j)$$

This is the social planner's problem corresponding to true state (θ_i, θ_{-i}), so $\tilde{\theta}_i = \theta_i$ is optimal. □

Intuition: supplementary transfers represent 'externality taxes', which induce perfect internalization of consequences of own reports on payoffs of others

This works, irrespective of whether others report truthfully or not!!

Balancing the Budget

- To be feasible, the transfers must aggregate to a nonpositive number in every state
- The $h_i(\theta_{-i})$ function can be used to achieve this, e.g. so that $(\forall \theta_{-i})$:

$$h_i(\theta_{-i}) \leq -\max_{\theta_i} \sum_{j \neq i} V_j(d^F(\theta_i, \theta_{-i}), \theta_j)$$

- This could however generate some waste of the private good, i.e., $\sum_i t_i$ could be negative for some states — outcome would not be Pareto optimal

Can the VCG Budget be Balanced? Examples

- Indivisible public project/policy problem ($d \in \{0,1\}$, $V_i(d,\theta_i) = \theta_i d, \theta_i \in \Re$):

$$d^F(\theta) = 1(0), \quad \text{if} \quad \sum_i \theta_i > (<) 0$$

- **Vickrey-Clarke mechanism:** Impose a tax on i only when i is *pivotal* (causes a switch in the public decision):

$$\begin{aligned} t_i(\theta_i, \theta_{-i}) &= \sum_{j \neq i} \theta_j, \quad \text{if} \quad \sum_{j \neq i} \theta_j < 0, \theta_i + \sum_{j \neq i} \theta_j > 0 \\ &= -\sum_{j \neq i} \theta_j, \quad \text{if} \quad \sum_{j \neq i} \theta_j > 0, \theta_i + \sum_{j \neq i} \theta_j < 0 \end{aligned}$$

Vickrey-Clarke mechanism, contd.

- The VC mechanism corresponds to $h_i(\theta_{-i}) = \sum_{j \neq i} \theta_j$ if $\sum_{j \neq i} \theta_j < 0$, and 0 otherwise

Vickrey-Clarke mechanism, contd.

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- Transfers are always non-positive by construction, so the mechanism is feasible
- They are non-zero only for pivotal agents
- In a large economy with θ_i drawn from iid distribution with non-zero mean, there will be 'very few' pivotal agents with high probability
- Hence probability of any waste goes to zero as $n \to \infty$

Do Efficient Solutions Exist in Small Economies?

- Groves (1971) proved that if the public good quantity is continuous and nonnegative ($d \geq 0$), utility functions belong to the quadratic family: $V_i(d) = \theta_i d - \frac{d^2}{n}$, and $n \geq 3$, there is a VCG mechanism which achieves perfect budget balance in every state
- However, this is the **only** known case where full Pareto optimality can be achieved!!
- Various impossibility theorems (eg with $n = 2$, d^F differentiable with non-vanishing partial derivative, and $\sum_i V_i(d^F(\theta))$ strictly concave in θ) (Laffont-Maskin Ecta 1979)

The ADAV Mechanism

- d'Aspremont and Gerard-Varet (1979) and Arrow (1980) showed that full Pareto optimality can be achieved if the implementation concept is weakened to requiring that truthful reporting be a Bayesian (rather than dominant strategy) equilibrium
- Recall the requirement of Bayesian Incentive Compatibility (BIC): $(\forall i, \theta_i:)$

$$\tilde{\theta}_i = \theta_i \quad \text{maximizes} \quad E_{\theta_{-i}}[V_i(d^F(\tilde{\theta}_i, \theta_{-i}), \theta_i) + t_i(\tilde{\theta}_i, \theta_{-i})]$$

ADAV Mechanism

Proposition

Suppose $\theta_1, \ldots, \theta_n$ are drawn from mutually independent distributions. Define the 'expected VCG transfer' function

$$H_i(\theta_i) \equiv E_{\theta_{-i}}[\sum_{j \neq i} V_j(d^F(\theta_i, \theta_{-i}), \theta_j)]$$

and consider

$$t_i(\theta_i, \theta_{-i}) = H_i(\theta_i) - \frac{1}{n-1} \sum_{j \neq i} H_j(\theta_j).$$

These transfers are balanced in every state, and ensure truthful reporting is a Bayesian equilibrium.

ADAV Mechanism: Remarks

- Proof is straightforward (but you should check!)
- Caveats:
  - While truthful reporting is a Bayesian equilibrium, it is typically not a dominant strategy equilibrium
  - There may be alternative Bayesian equilibria, so it does require citizens to coordinate on the truthful equilibrium
  - Assumes θ_i to be independent of θ_{-i}, for all i; extension to the case of correlated types sometimes works but is complicated