Mechanism Design: Multiple Agents, Introduction

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Multiple Agent Problems

- Now consider mechanism design problems with multiple agents and one Principal
- Applications include public good provision, efficient bilateral trade, and auction design
- Start with a canonical general model that includes all these as special cases, then consider each context separately
- Agents type: continuous

The Canonical Model

- n agents, one P who selects a decision $x \in X$, and transfers $t_1, \ldots, t_n$ to the agents
- Agent i's (quasi-linear) utility $V_i(x; \theta_i) + t_i$, where $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$ is known privately by i
- Bayesian (private values) formulation: common knowledge that θ_i is drawn (independently) according to cdf F_i on $[\underline{\theta}_i, \bar{\theta}_i]$
- P can be a profit-maximizing monopolist, with profit $\Pi \equiv V_0(x) - \sum_i t_i$
- Or P is a planner with utilitarian welfare objective $W \equiv \sum_i V_i(x; \theta_i)$, and subject to a budget balance (BB) constraint $V_0(x) \geq \sum_i t_i$

Application 1: Public Goods/Policy and Free-Rider Problem

- $x \geq 0$ (or $x \in \{0, 1\}$) is a decision regarding a public good (indivisible public good, or a policy alternative $x = 1$ to the status quo $x = 0$)
- Cost of the public good is $C(x)$, financed by taxes paid $T_1, \ldots, T_n$ by the n citizens
- Citizen i utility $U_i(x; \theta_i) - T_i$
- Define $t_i \equiv -[T_i - \frac{1}{n}C(x)]$, and $V_i(x; \theta_i) \equiv U_i(x; \theta_i) - \frac{1}{n}C(x)$
- So i's utility is $U_i(x; \theta_i) - \frac{1}{n}C(x) + t_i = V_i(x; \theta_i) + t_i$

Application 1: Public Goods/Policy and Free-Rider Problem, contd.

- P's welfare objective $\sum_i V_i(x;\theta_i) \equiv \sum_i U_i(x;\theta_i) - C(x)$; budget constraint: $\sum_i t_i \leq 0$
- Notation: $\theta \equiv (\theta_1, \ldots, \theta_n)$, the state of the world
- **Free-rider problem:** first-best efficient policy $x^F(\theta)$ and $t_i = 0$ (equal cost-sharing) requires P to know the realization of $\theta \equiv (\theta_1, \ldots, \theta_n)$
- P will have to elicit this information from citizens through a 'political economy' process — voting, referendums, Congressional procedures etc
- Citizens (or their elected representatives) will typically have incentives to lie about their true type
- Is it possible to design a mechanism of transfers that will induce citizens to reveal their true valuations?

Application 2: Cooperative (Team) Production and Free-Rider Problem

- A cooperative firm has n worker-members, with member i supplying effort/input $x_i \geq 0$ at personal cost $C_i(x_i, \theta_i)$ which is privately known to i
- Production/revenue function $V_0(x)$ where $x \equiv (x_1, \ldots, x_n)$ is the effort vector
- Member i is distributed dividend or reward t_i, budget constraint $\sum_i t_i \leq V_0(x)$

Application 2: Cooperative (Team) Production and Free-Rider Problem, contd.

- (First-best) efficient allocation $x_F(\theta)$ maximizes $\sum_i [t_i - C_i(x_i, \theta_i)]$ subject to the budget constraint
- **Free-rider problem:** first-best efficient policy $x^F(\theta)$ and equal division of revenue will induce members to pretend to have higher private effort cost than the actual cost
- Is it possible to design an incentive system which deviates from equal sharing of revenues, to implement the efficient allocation?
- If not, what is the second-best incentive system?

Application 3: Bilateral Trade

- There are two agents: a seller S of an indivisible asset (e.g., house, painting) and a single potential buyer B
- Seller's valuation of the asset is θ_S, buyer's valuation is θ_B
- They are privately informed about their own valuation
- Payoffs: $U_B = (\theta_B - p)x, U_S = (p - \theta_S)x$, where x is probability of sale, and p is the price paid in the event of a sale
- Efficiency requires $x = 1(0)$ if $\theta_B > (<)\theta_S$
- Participation Constraint (PC): sale is voluntary, each side is free to walk away and realize payoff zero associated with no sale

Application 3: Bilateral Trade, contd.

- Sale at what price? Should be set somewhere between θ_B and θ_S to satisfy PC
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- Setting $p = \alpha\theta_B + (1-\alpha)\theta_S$ where $\alpha \in (0,1)$ implies the sale price depends partly on the valuation of both parties
- Incentive to manipulate the price by overstatement of valuation by the seller, and understatement by the buyer ('haggling'), which might jeopardize the sale
- *The Problem:* Is it possible to design a trading mechanism which achieves efficient trades, provides each party with incentives to report their values truthfully, and participate in the mechanism?
- **New feature (compared with public goods problem):** have to incorporate participation constraint as well

Application 4: Competition/Auction Design

- P is a seller of an indivisible object, with $n > 1$ potential buyers
- Bidder i payoff is $\theta_i x_i - t_i$, where x_i is the probability of selling to i, and t_i is payment of i to P (allows all-pay auctions, or auction fees)
- Voluntary participation; non-participation ($x_i = t_i = 0$) payoff: 0
- Constraints: $\sum_i x_i \leq 1$, and each bidder is willing to participate in the auction (attain non-negative expected payoff)
- P's personal valuation of the object θ_0, payoff is either profit $\sum_i [t_i - \theta_0 x_i]$, or efficiency $\sum_i [\theta_i - \theta_0] x_i$

Bayesian Mechanism Design Problem

- Stages:
  - Mechanism is a game designed by P (who commits to it)
  - Each agent observes her own θ_i realization, and decides whether to participate (if relevant, i.e., in bargaining/auction problem)
  - Game played by agents that decide to participate
  - Solution concept: Bayesian Nash equilibrium (or refinement)

Revelation Principle, once again

- **Revelation Principle:** P can confine attention to revelation mechanisms $x(\theta), t_i(\theta), i = 1, \ldots, n$ where $\theta \equiv (\theta_1, \ldots, \theta_n)$ are the types reported by the agents, which satisfy Bayesian Incentive Compatibility (BIC):

- **BIC:** Truthful reporting is a Bayesian Nash equilibrium (for all i and all $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$):

$$\tilde{\theta}_i = \theta_i \quad \text{Max} \quad W_i(\tilde{\theta}_i; \theta_i) \equiv E_{\theta_{-i}}[V_i(x(\tilde{\theta}_i; \theta_{-i}), \theta_i) + t_i(\tilde{\theta}_i; \theta_{-i})] \qquad (BIC)$$

  where θ_{-i} denotes $(\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n)$

- plus PC (if relevant)

- Same construction and logic as in the single agent problem (Check!)

Revelation Principle with Multiple Agents: Some Qualifications

- However, difference between single and multiple agent case is that latter notion of 'implementation' is more fragile in the following senses:
- Possibility of multiple equilibria: there may exist alternative non-truthful equilibria that are not payoff-equivalent
- If alternative equilibrium generates higher payoffs to all agents in all states, they may coordinate on that one instead of the truthful one (*tacit collusion*)

Revelation Principle with Multiple Agents: Qualifications, contd.

- *Explicit collusion:* agents may enter into a side-contract where they coordinate their reports to P accompanied by hidden side-payments (bribes)

- Even if problems of collusion do not arise, the equilibrium is based on common knowledge assumptions and may not be robust to small perturbations of the prior

- E.g., suppose agent 1 is not absolutely sure what agent 2's beliefs are, may doubt whether latter will report truthfully, motivating 1 to also deviate from the truth

Implementation in Dominant Strategies: An Alternative

- A more robust notion of implementation is the requirement that truth-telling be a dominant strategy for every agent
- **Dominant Strategy Incentive Compatibility:** for all i and θ_i, θ_{-i}:

$$\tilde{\theta}_i = \theta_i \quad \text{Max} \quad X_i(\tilde{\theta}_i; \theta_{-i}) \equiv [V_i(x(\tilde{\theta}_i; \theta_{-i}), \theta_i) + t_i(\tilde{\theta}_i; \theta_{-i})] \;\; (DSIC)$$

- No longer require a common knowledge prior; agents do not have to worry about what other agents will report
- More demanding notion of implementation
- (May not, however, eliminate scope for collusion, since truthful reporting may not be a strongly dominant strategy)